Frascati, 27 November '04

Models of Neutrino Masses & Mixings

G. Altarelli
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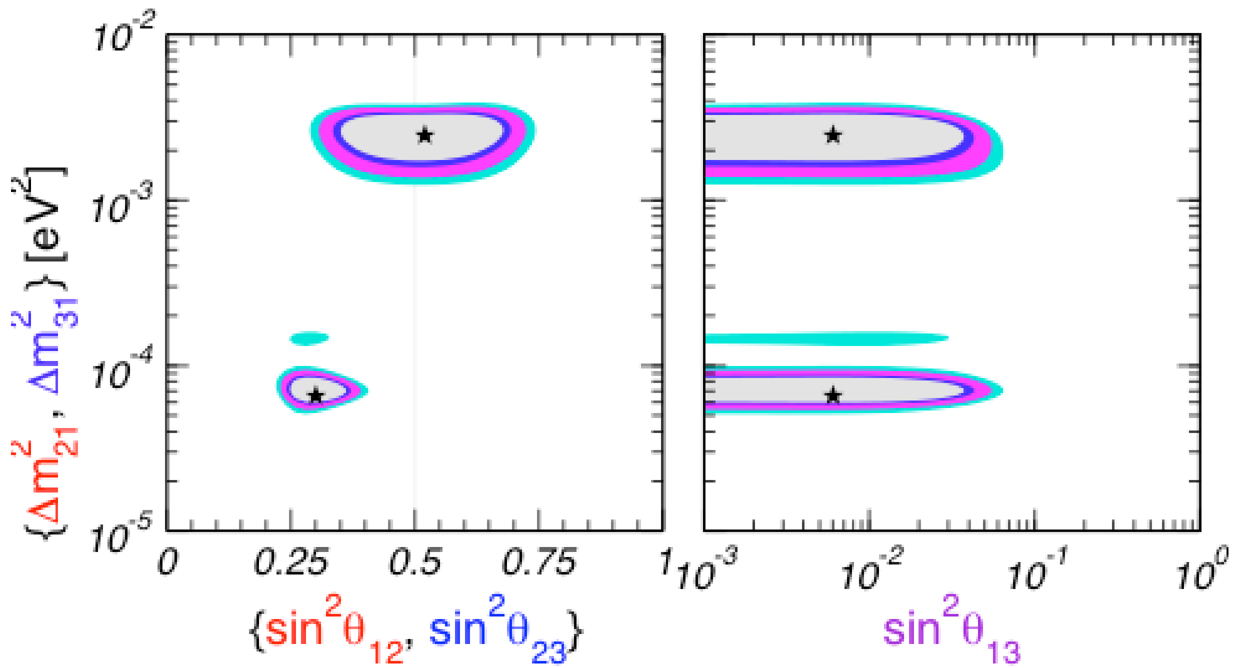
Some recent work by our group
G.A., F. Feruglio, I. Masina, hep-ph/0210342
(Addendum: v2 in Nov. '03), hep-ph/0402155.
Reviews:
G.A., F. Feruglio, hep-ph/0206077/0306265
F. Feruglio, hep-ph/0410131

Neutrino oscillation parameters

Maltoni et al

parameter	best fit	2σ	3σ	5σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	$\lesssim 0.035$	$\lesssim 0.054$	$\lesssim 0.11$



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ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2$

- Direct limits

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin (sub-eV)

$m_{"\nu e"} < 2.2$ eV
$m_{"\nu\mu"} < 170$ KeV
$m_{"\nu\tau"} < 18.2$ MeV

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.5 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\, /94\mathrm{eV}$ $(h^2 \sim 1/2)$

$\Sigma_i m_i < 0.7\text{-}1.8\text{-}?$ eV (dep. on priors)

WMAP, 2dFGRS...

$\Rightarrow$ Any ν mass $< 0.23\text{-}0.6\text{-}?$ eV

Why ν's so much lighter than quarks and leptons?

Lahav

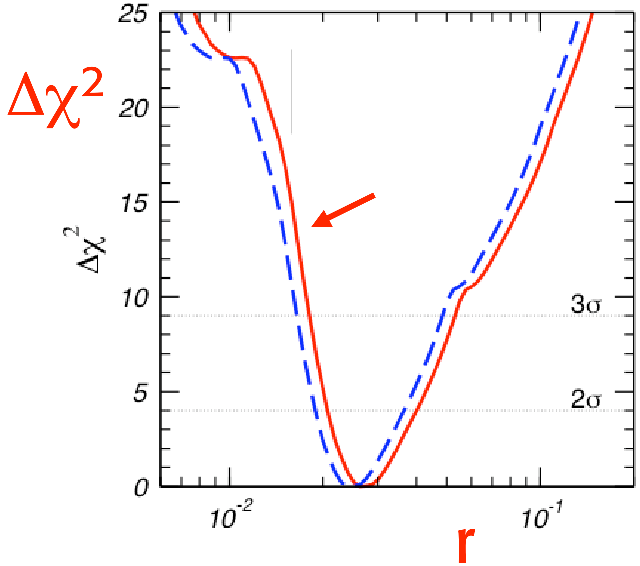
Neutrino mass from Cosmology

Data	Authors	$M_\nu = \Sigma\, m_i$ 95%cl
2dFGRS	Elgaroy et al. 02	< 1.8 eV
WMAP+2dF+…	Spergel et al. 03	< 0.7 eV
WMAP+2dF	Hannestad 03	< 1.0 eV
SDSS+WMAP	Tegmark et al. 04	< 1.7 eV
WMAP+2dF+ SDSS	Crotty et al. 04	< 1.0 eV

By itself CMB (WMAP, ACBAR) do not fix M_ν
Only in combination with galaxy power spectrum (2dFGRS, SDSS) become sensitive.

After KamLAND, SNO and WMAP not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/35$



Precisely at 3σ: $0.018 < r < 0.053$

or

$m_{heaviest} < 1 - 0.6$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $$\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$$

Comparable to: $$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Suggests the same "hierarchy" parameters for q, l, ν

$\longrightarrow$ e.g. θ_{13} not too small!

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- Still large space for non maximal 23 mixing

3-σ interval $0.31 < \sin^2\theta_{23} < 0.72$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small probably accessible to exp.

$\sin\theta_{13} \sim 1/2 \sin\theta_{12}$ not excluded!

Very small θ_{13} theoretically hard

Normal models: θ_{23} large but not maximal, θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)
Exceptional models: θ_{23} maximal or θ_{13} very small
or also: all mixing from the charged lepton sector....

$U = U_e^+ U_\nu$

The current experimental situation is still unclear

- LSND: true or false?
- what is the absolute scale of ν masses?
- ...

Different classes of models are still possible:

If LSND true

sterile ν(s)??

CPT violat'n??

- "3-1"

$\nu_{sterile}$

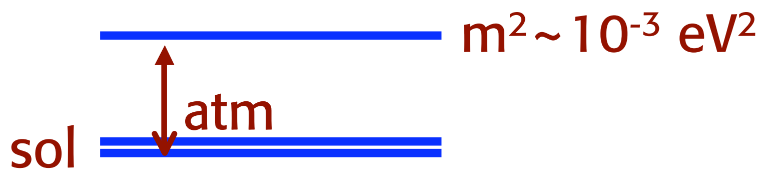
If LSND false → 3 light ν's are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$)

- Inverse hierarchy

- Normal hierarchy



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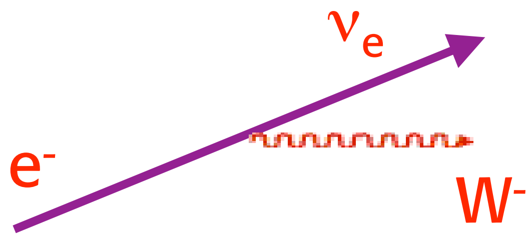
3-ν Models

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}| < \sim 0.2$

$$\sim \begin{bmatrix}c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta}\\ \ldots & \ldots & c_{13}s_{23}\\ \ldots & \ldots & c_{13}c_{23}\end{bmatrix}$$

atm.: ~ max

$$U \cong \begin{bmatrix} c & -s & 0\\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}}\\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

(some signs are conventional)

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$$m_\nu \sim U^* \begin{bmatrix} e^{i\phi_1} m_1 & 0 & 0 \\ 0 & e^{i\phi_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^+$$

In general 9 parameters:
3 masses, 3 angles,
3 phases

$L^T m_\nu L$

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1c^2+m_2s^2 & (m_1-m_2)cs/\sqrt{2} & (m_1-m_2)cs/\sqrt{2} \\ \ldots & (m_1s^2+m_2c^2+m_3)/2 & (m_1s^2+m_2c^2-m_3)/2 \\ \ldots & \ldots & (m_1s^2+m_2c^2+m_3)/2 \end{bmatrix}$$

Note:
- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \text{<->} \nu_\mu) = P(\nu_e \text{<->} \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2\Delta_{sun}$

$P(\nu_\mu \text{<->} \nu_\tau) = \sin^2\Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2\Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \quad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

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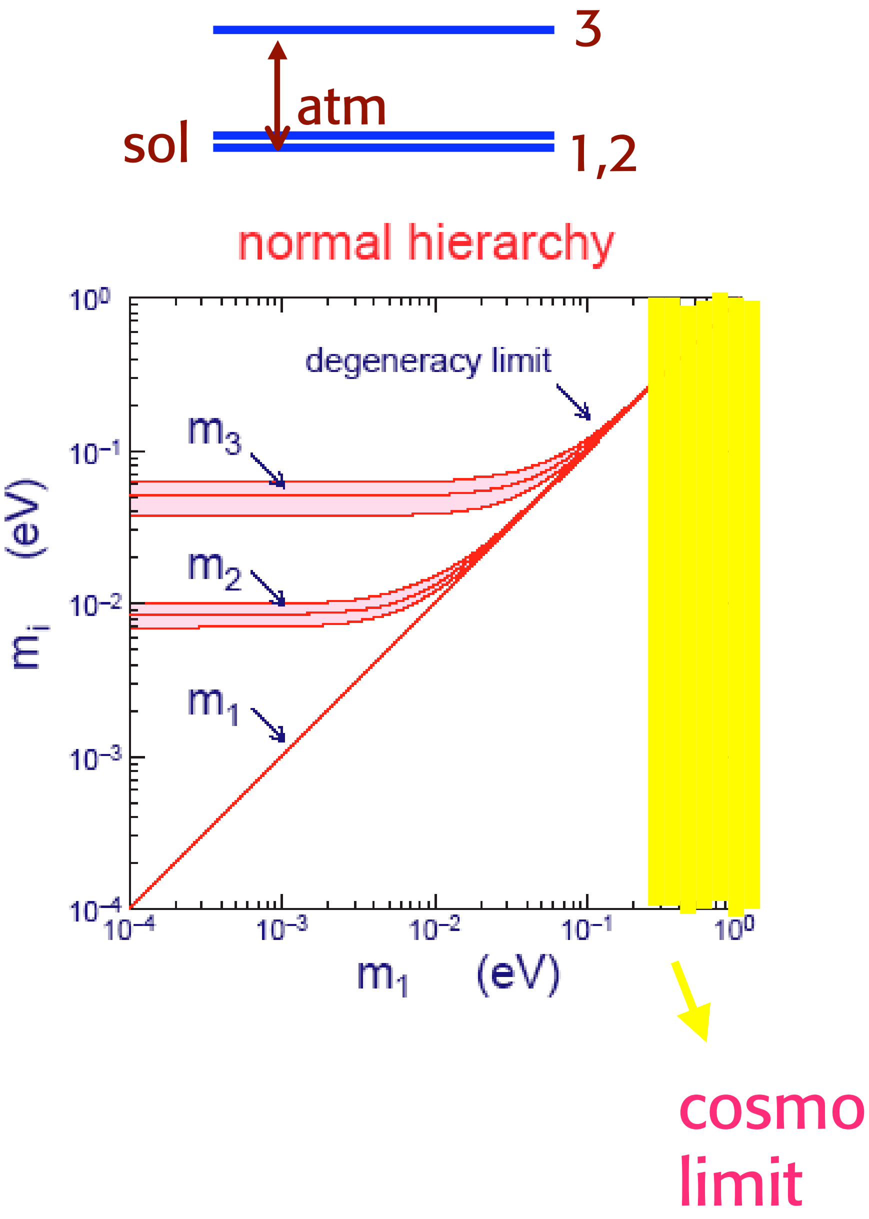
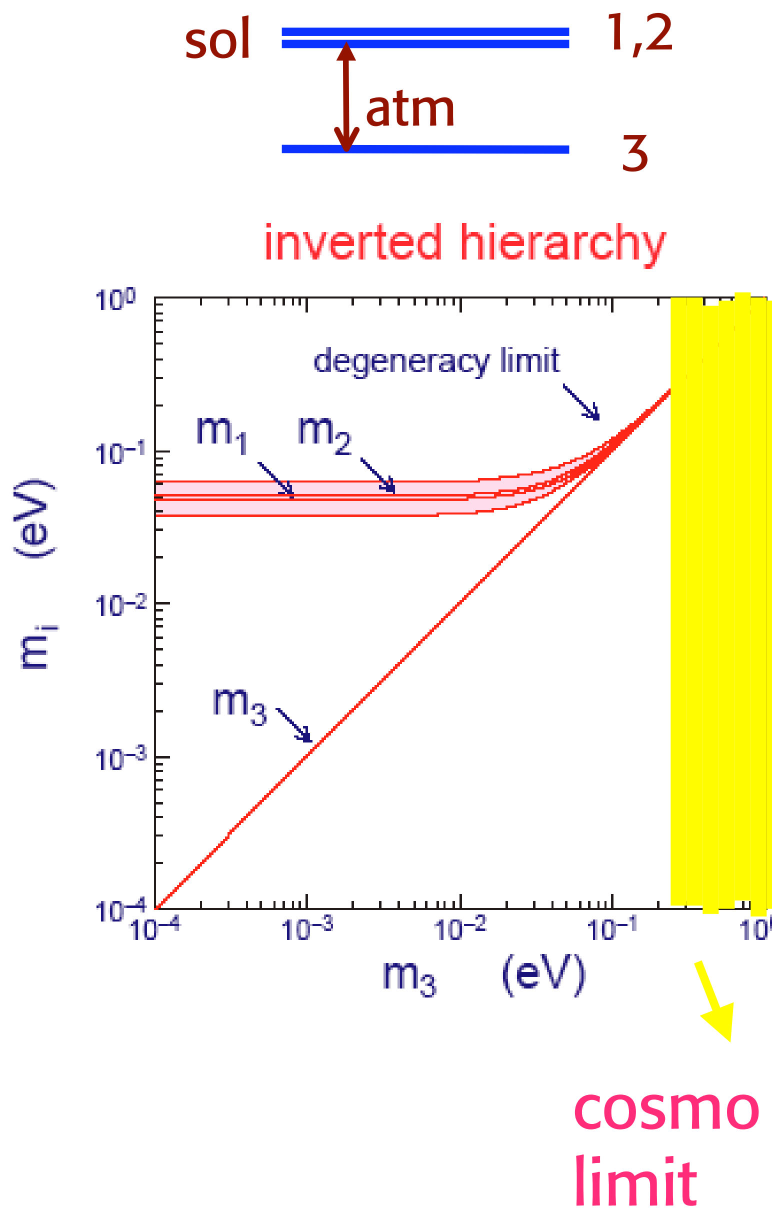
In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0





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Only moderate degeneracy allowed

$0\nu\beta\beta$ can tell degenerate, inverted or normal hierarchy

$$|m_{ee}| = c_{13}^2\,[m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

LA:~0.3-1

Degenerate: $\sim |m|\,|c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim |m|\ (0.3 - 1) < 0.23\text{-}1$ eV

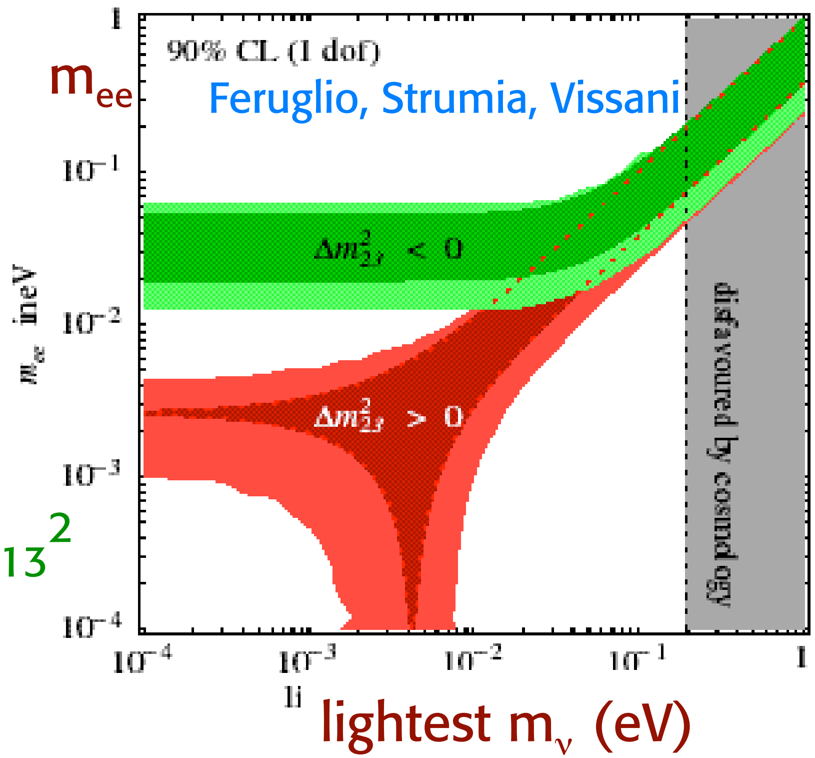
IH: $\sim (\Delta m^2_{atm})^{1/2} |c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim (\Delta m^2_{sol})^{1/2} s_{12}^2 + (\Delta m^2_{atm})^{1/2} e^{i\beta} s_{13}^2$

$|m_{ee}| \sim$ (few) 10^{-3} eV

Full dependence on min m_ν



Present exp. limit: $m_{ee} < 0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

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Degenerate ν's

$m^2 >> \Delta m^2$

- Apriori compatible with hot dark matter ($m \sim 1$-2 eV)
  → was considered by many
- Limits on m_{ee} from $0\nu\beta\beta$ then imply large mixing also for solar oscillations: (Vissani; Georgi,Glashow)

$m_{ee} < 0.3$-0.5 eV (Exp)

$$m_{ee} = c^2_{13}\,(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$$

If $|m_1| \sim |m_2| \sim |m_2| \sim 1$-$2$ eV $\longrightarrow$ $\boxed{m_1 = -m_2 \text{ and } c^2_{12} \sim s^2_{12}}$

LA solution: $\sin^2\theta \sim 0.3$ → $\cos^2\theta - \sin^2\theta \sim 0.4$

a moderate suppression factor!

Trusting WMAP&2dF: $|m| < 0.23$ eV, only a moderate degeneracy is allowed: for LA, $m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

Less constraints from $0\nu\beta\beta$ (both $m_1 = \pm m_2$ allowed)

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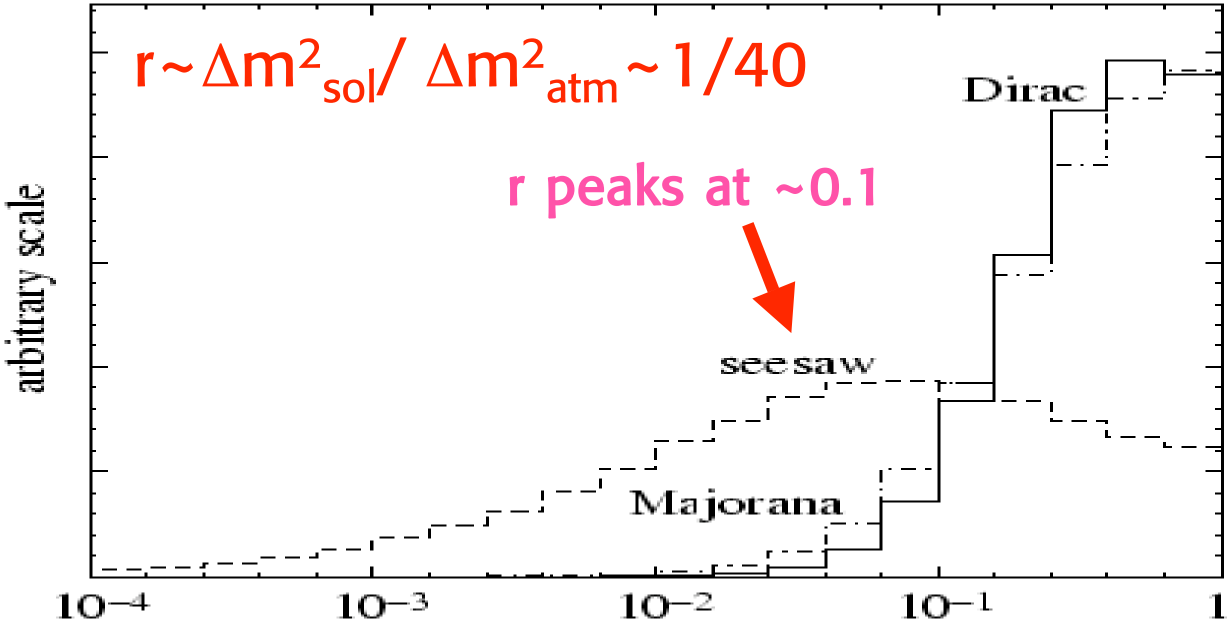
Recall: leptogenesis prefers $|m| < 0.1$ eV

Anarchy (or accidental hierarchy): No structure in the leptonic sector

Hall, Murayama, Weiner

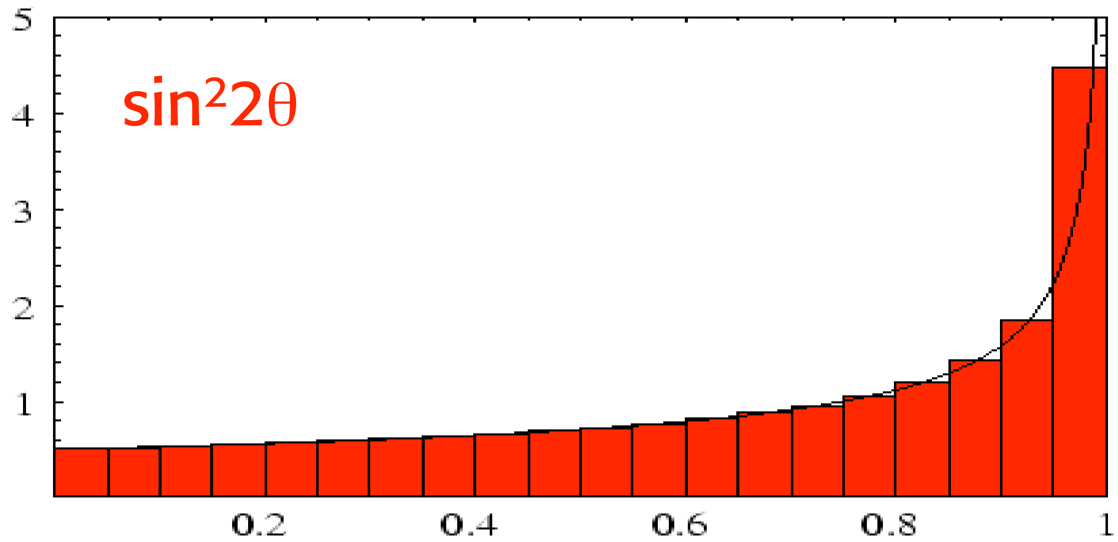
See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy from random m,M

could fit LA



But: all mixing angles should be large

marginal for LA → predicts θ_{13} near bound



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Semianarchy: no structure in 23

Consider a matrix like $m_\nu \sim \begin{bmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{bmatrix}$

Note: $\theta_{13} \sim \lambda$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[λ~1 corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{bmatrix} \lambda^2 & \lambda & 0 \\ \lambda & \eta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Normally two masses are of o(1) and $\theta_{12} \sim \lambda$
But if, accidentally, $\eta \sim \lambda$, then the solar angle is also large.

The advantage over anarchy is that θ_{13} is small, but
the hierarchy $m^2_3 >> m^2_2$ is accidental

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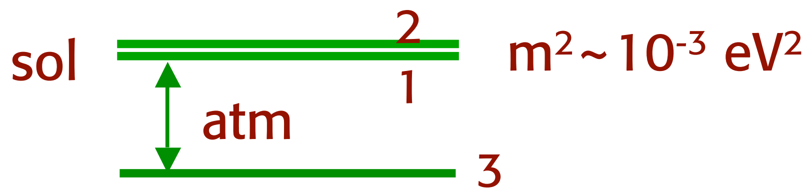
Ramond et al, Buchmuller et al

Inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto



An interesting model:

An exact U(1) L_e-L_μ-L_τ symmetry of predicts:

(a good 1st approximation)

$$m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & c & -s \\ c & 0 & 0 \\ -s & 0 & 0 \end{bmatrix} \quad \text{with} \quad m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- $\theta_{13} = 0$
- $\theta_{12} = \pi/4$
- $\sin^2\theta_{23} = s^2$

θ_{sun} maximal!

θ_{atm} generic

Can arise from see-saw or dim-5 $L^T H H^T L$

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- 1-2 degeneracy stable under rad. corr.'s

1^{st} approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad m_\nu = U m_{\nu diag} U^T = m \begin{bmatrix} 0 & c & -s \\ c & 0 & 0 \\ -s & 0 & 0 \end{bmatrix}$$

- Data? This texture prefers θ_{sol} closer to maximal than θ_{atm}
  i.e $\theta_{sol} - \pi/4$ small for $(\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim 1/40$

In fact: 12-> $\begin{bmatrix} 0 & c \\ c & 0 \end{bmatrix} \rightarrow$ Pseudodirac θ_{12} maximal 23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With perturbations: $\begin{bmatrix} 0 & c & -s \\ c & 0 & 0 \\ -s & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$ (modulo o(1) coeff.s)

one gets $1 - tg^2\,\theta_{12} \sim o(\delta + \eta) \sim (\Delta m^2_{sol}/\Delta m^2_{atm})_{LA}$

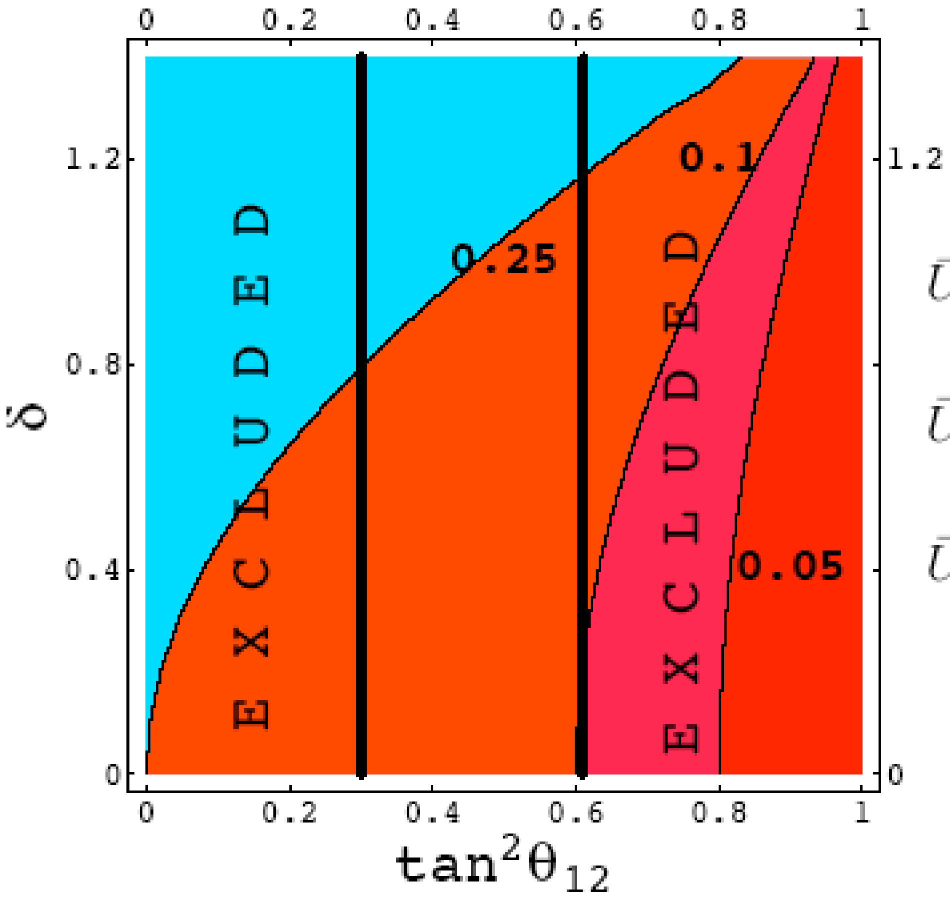
Exp. (3σ): 0.39-0.70 0.024-0.060

- In principle one can use the charged lepton mixing
  to go away from θ_{12} maximal.
  In practice constraints from θ_{13} small ($\delta\theta_{12} \sim \theta_{13}$)

Frampton et al; GA, Feruglio, Masina '04

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina '04



$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

For the corrections to bimixing from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4$

GA, Feruglio, Masina '04

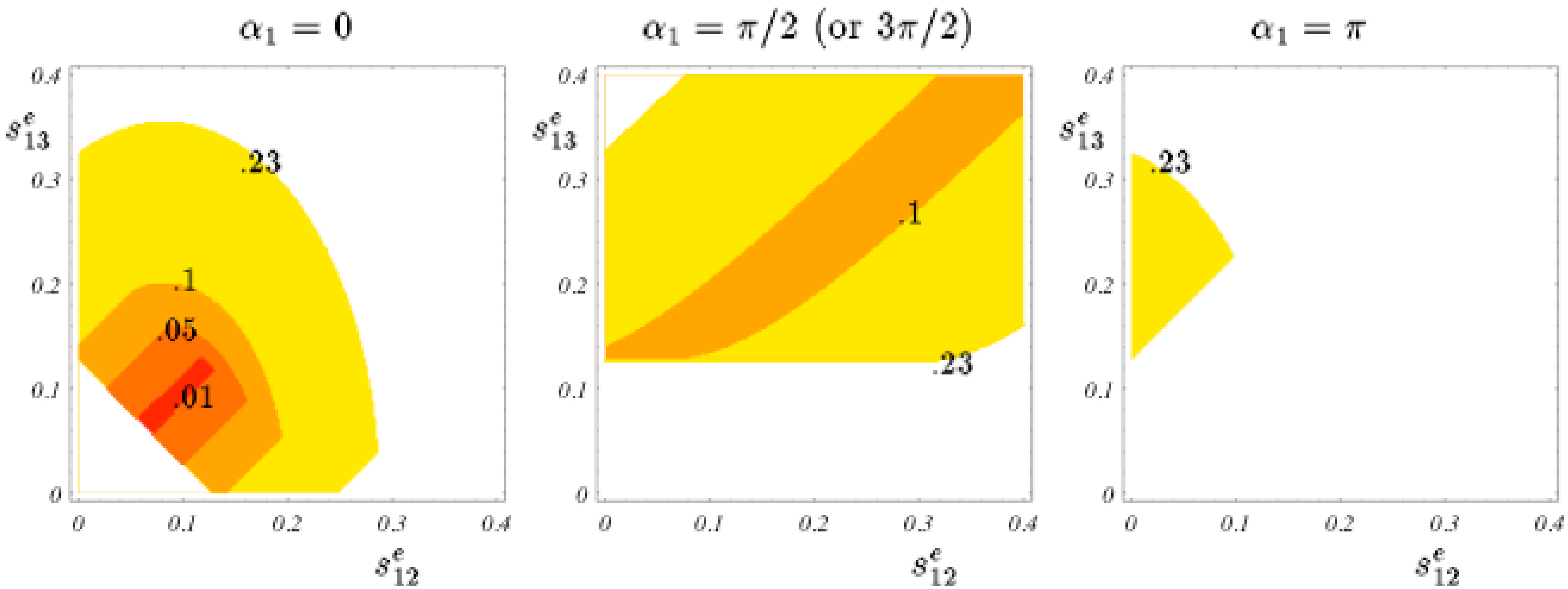


Figure 1: Taking an upper bound on $|U_{e3}|$ respectively equal to $0.23, 0.1, 0.05, 0.01$, we show (from yellow to red) the allowed regions of the plane $[s^e_{12}, s^e_{13}]$. Each plot is obtained by setting α_1 to a particular value, while leaving $\alpha_2 + \delta_e$ free. We keep the present 3 σ window for δ_{sol} [10].

- In general more θ_{12} is close to maximal, more is IH likely

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$\overline{L} m_e R$

$m'_e = U^+ m_e V$

$m'_e m'^+_e = U^+ m_e m_e^+ U$

or $m_e m_e^+$ transforms as $\overline{L}\, L$

L_e-L_μ-L_τ implies:

$$\bar{L}L \sim \begin{bmatrix} 1 & \lambda'^2 & \lambda'^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

λ, λ' from flavons of ± 1 charge

After diagonalisation of charged leptons θ_{23} remains large, while modifications to θ_{13} and θ_{12} are small.

In conclusion IH is viable but prefers θ_{12} close to maximal, and given the exp. value of θ_{12}, needs θ_{13} near its upper bound

[Both anarchy and IH point to θ_{13} near bound]

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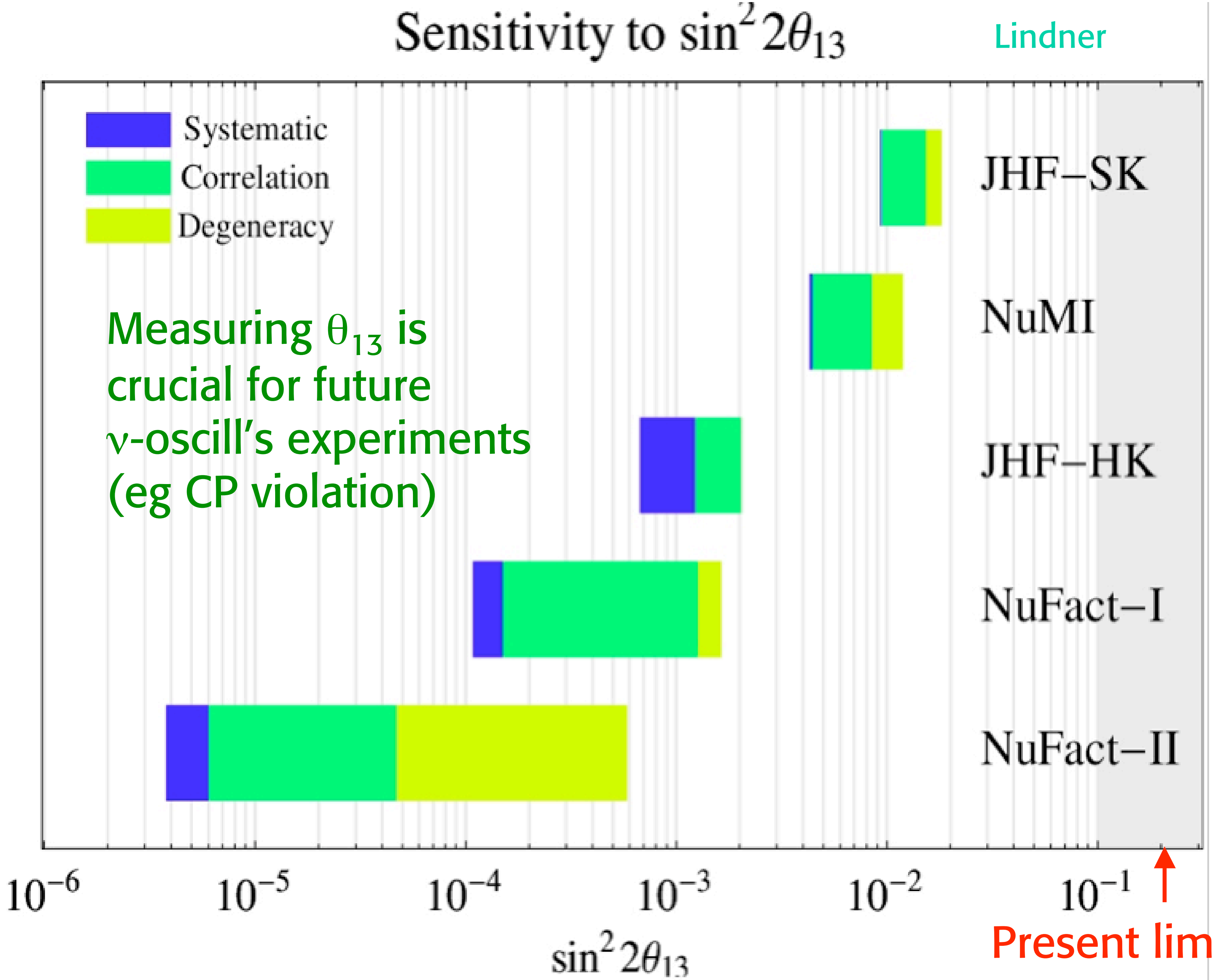
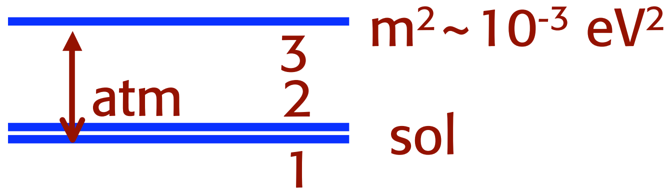
Lindner



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)

Present limit

Normal Hierarchy



- Assume 3 widely split light neutrinos.
- For u, d and l^- Dirac matrices the 3rd generation eigenvalue is dominant.
- May be this is also true for $m_{\nu D}$: diag $m_{\nu D} \sim (0,0,m_{D3})$.

  (but not at all necessary!)
- Assume see-saw is dominant: $m_\nu \sim m^T_D M^{-1} m_D$

  See-saw quadratic in m_D: tends to enhance hierarchy
- Maximally constraining: GUT's relate q, l^-, ν masses!

- A crucial point: in the 2-3 sector we need both large m_3-m_2 splitting and large mixing.

$m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
$m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 8\ 10^{-3}$ eV

- The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i - E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

Det = 0; Eigenvl's: 0, $1+x^2$
Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

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Examples of mechanisms for Det[23]~0

see-saw $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $\qquad m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

Caution: if 0 -> 0(ε), det23=0 could be spoiled by suitable $1/\varepsilon$ terms in M^{-1}

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An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks	$\Longleftrightarrow$	RH for l^- leptons

$$m_d \sim \overset{\bar{5}}{\bar{d}_R}\, \overset{10}{d_L}$$

$$m_e \sim \overset{10}{\bar{e}_R}\, \overset{\bar{5}}{e_L}$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d},}_{R}\ \underbrace{\nu,e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

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- Hierarchical ν's and see-saw dominance

$$L^T m_\nu L \to m_\nu \sim m_D^2/M$$

allow to relate q, l, ν masses and mixings in GUT models.
For dominance of dim-5 operators -> less constraints

$$\lambda^2/M\,(LH)(LH) \to m_\nu \sim \lambda^2 v^2/M$$

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

$SU(5)xU(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

- SO(10) models could be more predictive, as are non abelian flavour symmetries, eg O(3)

Albright, Barr; Babu et al; Buccella et al; Barbieri et al; Raby et al; King, Ross

- The non trivial pattern of fermion masses and mixing demands a flavour structure (symmetry)

- (SUSY) SU(5)XU(1)$_F$ models offer a minimal description of flavour symmetry

- A flexible enough framework used to realize and compare models with anarchy or hierarchy (direct or inverse) in ν sector, with see-saw dominance or not.

- On this basis we found that there is still a significant preference for hierarchy vs anarchy

G.A., F. Feruglio, I. Masina, hep-ph/0210342 (v2 Nov '03)

Previous related work: Haba,Murayama; Hirsch,King; Vissani; Rosenfeld,Rosner; Antonelli et al….

Hierarchy for masses and mixings via horizontal U(1) charges.

Froggatt, Nielsen '79

Principle:

A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1$, L_2, H

U(1) broken by vev of "flavon" field θ with U(1) charge q_θ= -1.
The coupling is allowed: if vev $\theta = w$, and $w/M=\lambda$ we get:

Δ_{charge}

$$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{q1+q2+qH} \qquad m_{12} \to m_{12}\, \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0)etc -> many versions

With suitable charge assignments all relevant patterns can be obtained

1st fam. 2nd 3rd

$$\begin{cases} \Psi_{10}: (5, 3, 0) \\ \Psi_{5}: (2, 0, 0) \\ \Psi_{1}: (1,-1, 0) \end{cases}$$

Equal 2,3 ch. for lopsided

Recall: u ~ 10 10

$d = e^T \sim \bar{5}\ 10$

$\nu_D \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

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All entries are a given power of λ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with ϕ =[0,2π] and ρ= [0.5,2] (default) or [0.8,1.2], or [0.95,1.05] or [0,1] (real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries ~3σ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

$0.018 < r < 0.053$
$|U_{e3}| < 0.23$
$0.30 < \tan^2\theta_{12} < 0.64$
$0.45 < \tan^2\theta_{23} < 2.57$

for each model the λ,λ' values are optimised

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The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

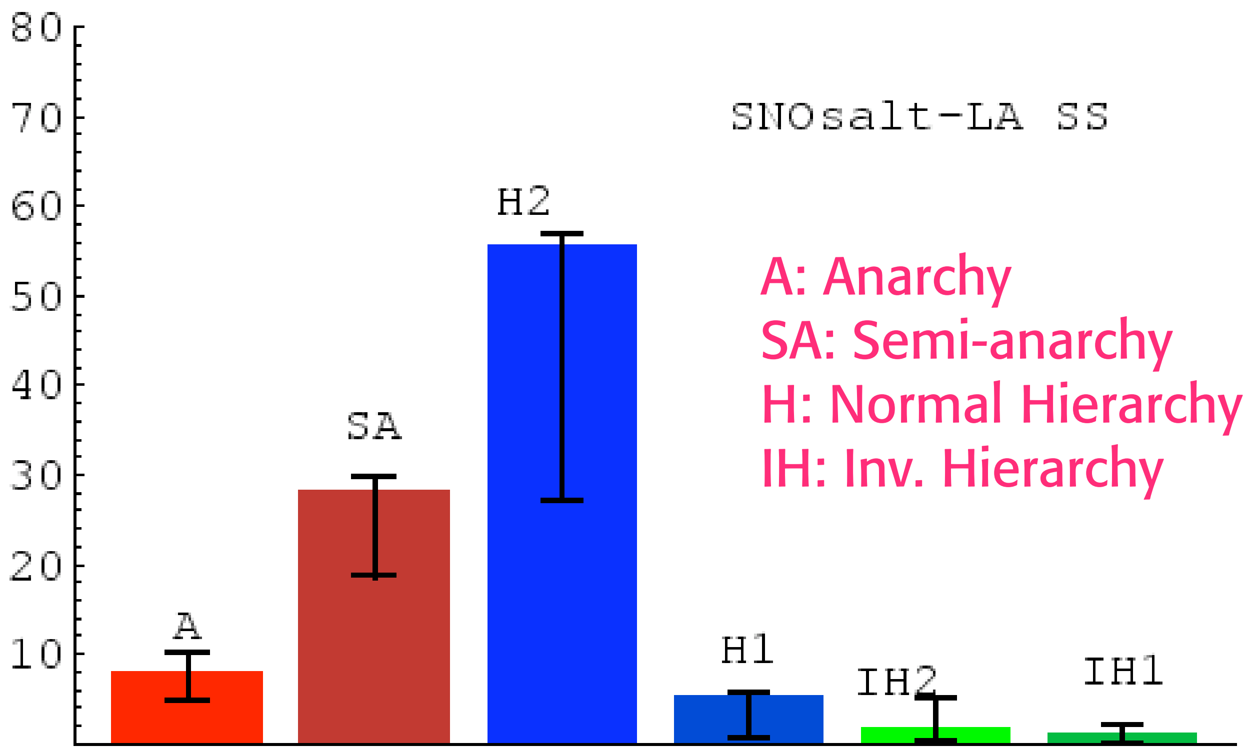
model	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Results with see-saw dominance (updated in Nov. '03):

Scale: Σrates=100

1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero



A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of ρ, real or complex)

H2 is better than SA, better than A, better than IH

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Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$

$q(\bar{5})$: $(2, 0, 0)$

$q(1)$: $(1,-1, 0)$

Note: not all charges positive --> det23 suppression

$q(H) = 0,\ q(\bar{H}) = 0$

$q(\theta) = -1,\ q(\theta') = +1$

In first approx., with $\langle\theta\rangle/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

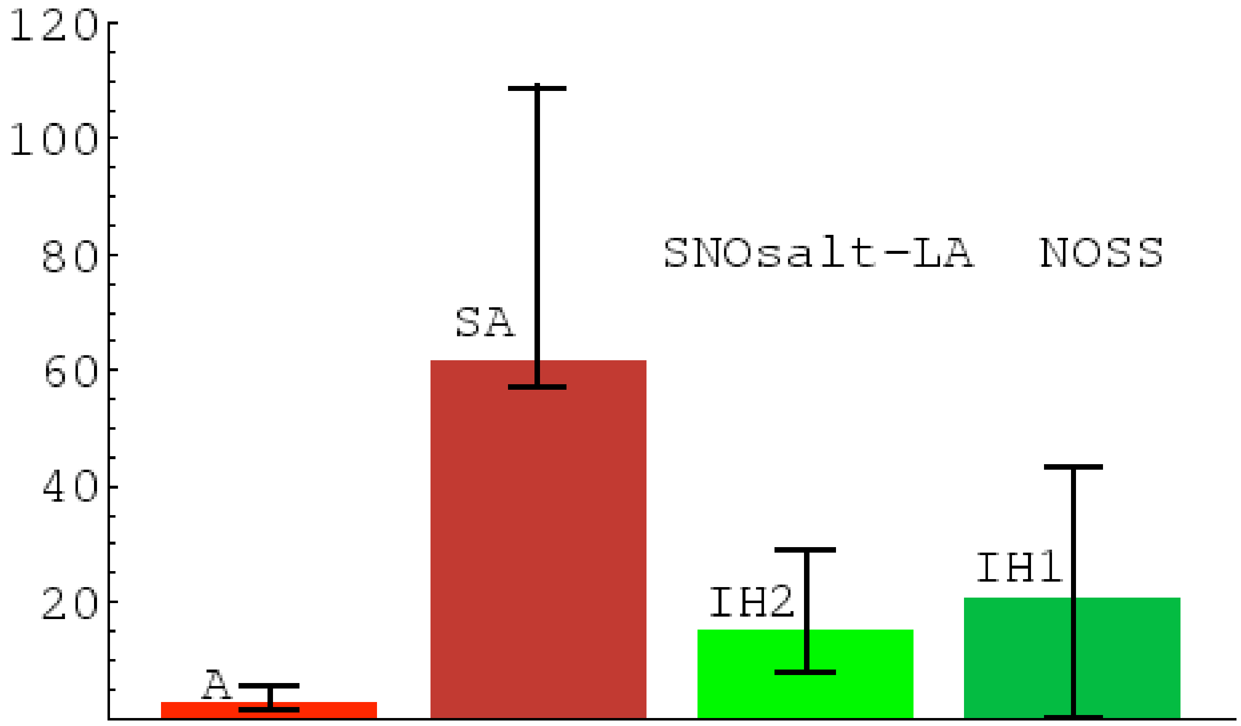
$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted

With no see-saw (m_ν generated directly from $L^T m_\nu L \sim \bar{5}\,\bar{5}$) IH is better than A

[With no-see-saw H coincide with SA]



Note: we always include the effect of diagonalising charged leptons

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What if θ_{23} is really maximal?

Would be challenging!

All existing models invoke peculiar symmetries (non abelian or discrete are crucial) Early models: Barbieri et al, Wetterich....

A set of recent models are based on obtaining, in the basis of (nearly) diagonal charged leptons

Grimus, Lavoura..., Ma,....

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

This predicts θ_{13}=0 and θ_{23} max.

Imposing a 2-3 perm. symmetry on $L^T m_\nu L$ does not work, because $\bar{R}$ L then produces a charged lepton mixing that spoils θ_{23} max.

Rather, discrete broken symmetries are used to make charged leptons and Dirac neutrino masses diagonal, while the perm. symmetry is in the Majorana RR matrix

Can ν mixings arise only from the charged lepton sector?

G.A., Feruglio, Masina '04

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Rightarrow U = U_e^+ U_\nu$$

flavour mass

diag of ch leptons

$m_\nu = U^* \, m_\nu^{diag} U^+$

$m_e = V_e m_e^{diag} U_e^+$

$$\bar{R} m_e L \quad \begin{cases} L_{diag} = U_e L \\ R_{diag} = V_e R \end{cases}$$

Assume that, in the lagrangian basis where all symmetries are specified, we have: $U_\nu \sim 1$. Then: $U \sim U_e^+ \sim$

$$\begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(small effects like s_{13} can be thought to arise from U_ν- 1. Phases dropped for simplicity)

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Given $m_e^{diag} \sim m_\tau diag[0,\eta,1]$ (with $\eta = m_\mu/m_\tau$) we obtain:

$$m_e = V_e m_e^{diag} U \sim V_e m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

For $V_e \sim 1$ this is a generalisation of lopsided (s large) but with $det_{12}=0$

Independent of V_e:

$$m_e^+ m_e \sim U^+ (m_e^{diag})^2 U \sim m_\tau^2 \; \frac{1+\eta^2}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta^2) \\ -cs & c^2 & c(1-2\eta^2) \\ -s(1-2\eta^2) & c(1-2\eta^2) & 1 \end{bmatrix}$$

- all matrix elements of same order (because s is large)
  "democratic" (hierarchy of masses non trivial)
- $s_{13}=0$ (i.e. eigenvector $(c,s,0)^T$) -> first two columns proportional

Note: in minimal SU(5) models $m_e = m_d^T$. This implies $V_e = U_d$

Quark mixings are small: $V_{CKM} = U_u^+ U_d$

Two possibilities:

- Both U_u and U_d nearly diagonal -> $V_e \sim 1$
- $U_u \sim U_d$ nearly equal and non diagonal

This is the way of democratic models:

$U_u \sim U_d \sim U_e$ -> $V_e \sim U_e$

$V_e \sim 1$

$$m_e = m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$V_e \sim U_e$

$$m_e = m_\tau \frac{1+\eta}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta) \\ -cs & c^2 & c(1-2\eta) \\ -s(1-2\eta) & c(1-2\eta) & 1 \end{bmatrix}$$

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The first two columns are proportional

Our general conclusion:

From the charged lepton sector:

a large s_{23} can easily be produced

example: lopsided models

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & s_{23} & c_{23} \end{bmatrix}}_{m_e} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}}_{U_e} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

but different orders for s_{12} and s_{13} is not simple

Still we have formulated a model where all mixings arise naturally from the charged lepton sector.

A set of U(1) charges garantees that m_ν is diagonal

The spectrum of one family is like in the 27 of E6

charged leptons

$$27 = 1 + 10 + 16 = 1 + (5 + \bar{5}\ ) + (1 + \ \bar{5} + 10)$$

E6 SO(10) SU(5)

A see-saw mechanism involving the two sets of $\bar{5}$ leeds to the required zero determinant condition in m_e

The model works but requires a complicated setup of charges and flavons.

Note that it borrows the see-saw tricks from the neutrino model building

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To make $m_\nu \sim 1$ a single U(1) is not enough:

$$m_\nu = \begin{bmatrix} \xi^{2p} & \xi^{p+1} & \xi^p \\ \xi^{p+1} & \xi^2 & \xi \\ \xi^p & \xi & 1 \end{bmatrix} m$$

In fact as $r \sim \xi^4 \sim 1/40$
then $\theta_{23} \sim \xi$ would be large

We need a flavour group

$$F = U(1)_{F_0} \times U(1)_{F_1} \times U(1)_{F_2} \times U(1)_{F_3}$$

F_i act on different light ν's

F_0 fixes quark and lepton hierarchies

	10_1	10_2	10_3	$\bar{5}^l_1$	$\bar{5}^l_2$	$\bar{5}^l_3$	5_H	$\bar{5}_H$	5_1	5_2	$\bar{5}_1$	$\bar{5}_2$
F_0	4	2	0	0	0	0	0	0	0	0	0	0
F_1	2	2	2	1	0	0	0	0	2	0	-2	0
F_2	2	2	2	0	1	0	0	0	2	0	-2	0
F_3	2	2	2	0	0	1	0	0	0	2	0	-2

The model
is natural
but cumbersome!

flavons $\longrightarrow$

	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F_0	-1	0	0	0	0	0	0	0
F_1	0	-2	0	0	-3	0	0	-4
F_0	0	0	-2	0	0	-3	0	-4
F_0	0	0	0	-2	0	0	-3	-4

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We obtain a matrix of the form

$$m_e = \begin{bmatrix} O(\lambda^4) & O(\lambda^4) & O(\lambda^4) \\ x_{21}\lambda^2 & x_{22}\lambda^2 & O(\lambda^2) \\ x_{31} & x_{32} & O(1) \end{bmatrix} m$$

$m_e : m_\mu : m_\tau = \lambda^4 : \lambda^2 : 1$

det

We need $x_{21}x_{32} - x_{22}x_{31} = 0$ to guarantee an eigenvector of $m_e^+ m_e$ $[c,s,0(\lambda^4)]$ with eigenvalue $0(\lambda^8)$: $s/c = -x_{31}/x_{32}$

- The hierarchy in the rows is from the $U(1)_{F0}$
- det=0 is arranged by a see-saw with dominance of a single heavy state in M^{-1} guaranteed by $U(1)_{F1} \times U(1)_{F2} \times U(1)_{F3}$

$\boxed{\text{Note that } \theta_{13} \sim \lambda^4 \text{ in this model}}$

Conclusion

We favour:

Normal models: θ_{23} large but not maximal, θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

- Semi anarchy
- Inverse hierarchy

In particular

- Normal hierarchy with suppressed 23 determinant

Exceptional models: θ_{23} maximal or θ_{13} very small or also: all mixing from the charged lepton sector.... are interesting but not very plausible