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Stanford Linear Accelerator Center
Stanford Synchrotron Radiation Laboratory

Superradiance in a seeded Free Electron Laser

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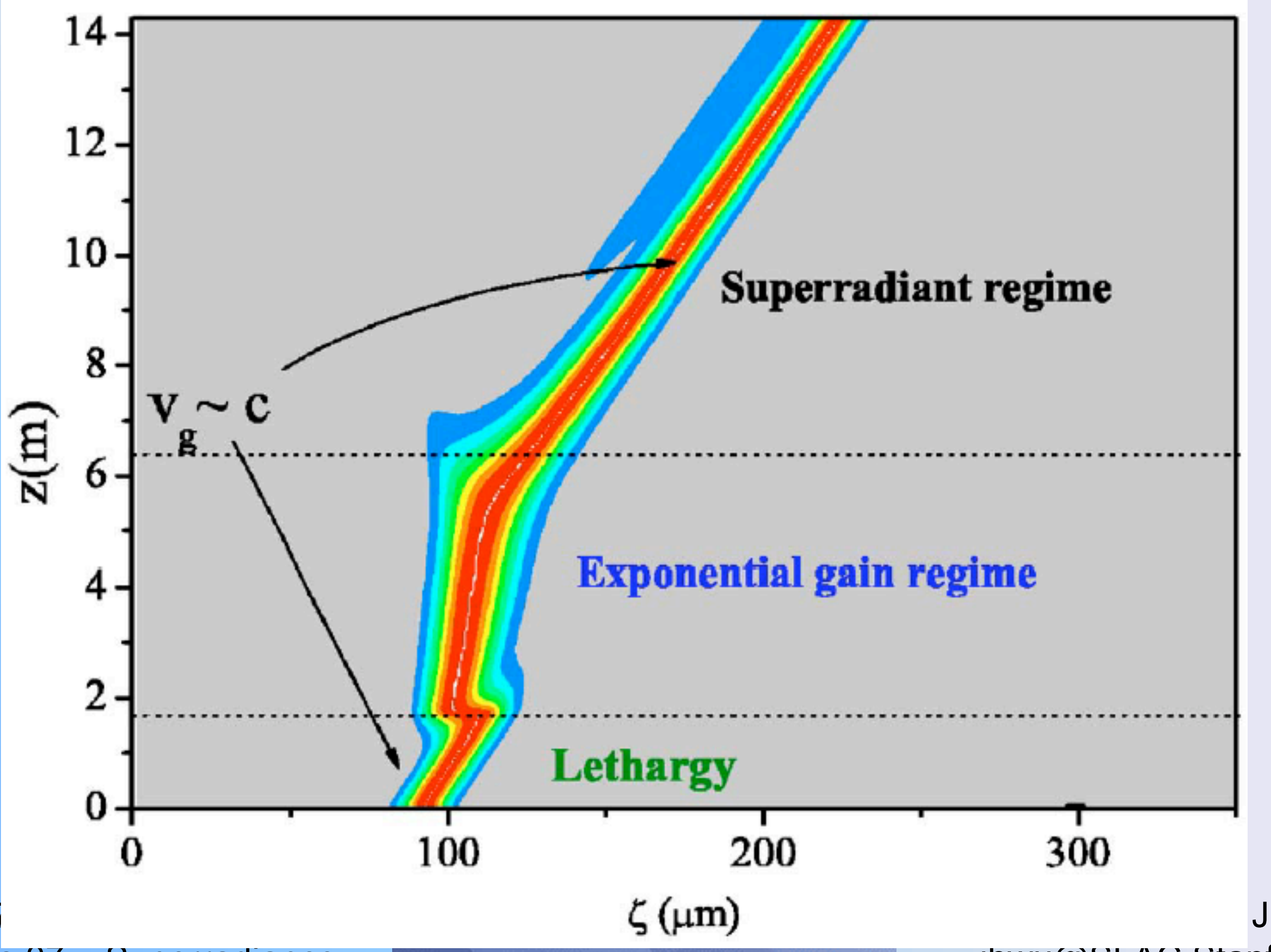
- A unified theory for exponential growth and superradiance → a theory for saturation?
- Vlasov-Maxwell analysis
  - Exponential Growth
  - Superradiance
- Collective Variables analysis
  - Review some pioneering work
  - Comparison of the two difference analyses
- Comparison with the experiment at SDL/NSLS/BNL: qualitative features
- Future plan

- Three regime in a seeded FEL [Giannessi, Musumeci, Spampinati, J. Appl. Phys. (2005)]



Vlasov-Maxwell analysis

$$\frac{\partial \psi}{\partial Z} + p\frac{\partial \psi}{\partial \theta} - \frac{2D_2}{\gamma_0^2}\left(Ae^{i\theta} + A^* e^{-i\theta}\right)\frac{\partial \psi_0}{\partial p} = 0,$$

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial \theta}\right)A(\theta, Z) = \frac{D_1}{\gamma_0}e^{-i\theta}\int dp\,\psi(\theta, p, Z)$$

$\psi(\theta, p, Z)$ is the electron distribution function;

and $E(t,z) \equiv A(\theta, Z)e^{i(\theta - Z)}$ with $Z = k_w z$ and $\theta = (k_s + k_w)z - \omega_s t$

D_1 and D_2 : coupling coefficients

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- Laplace transform to solve the V-M equations to end up with the integral representation for seeded FEL

$$A(\theta, Z) = \int_C \frac{ds}{2\pi i} e^{sZ} \int_{-\infty}^{\theta} d\theta' A(\theta', 0) e^{-s(\theta-\theta') + \frac{i(2\rho)^3(\theta-\theta')}{s^2}}$$

- Let us use variable to absorb ρ.

$$\hat{z} = 2\rho k_w z \text{ and } \hat{s} = \rho\left[\left(k_s + k_w\right)z - \omega_s t\right]$$

To complete the integral, we do the contour integral first

- Green function and then seeded FEL → seed convolved with Green function

$$g(\hat{z},\hat{s}) = 2\int_{\gamma'-i\infty}^{\gamma'+i\infty} \frac{dp}{2\pi i} \exp\left[f(p,\hat{z},\hat{s})\right]$$

- The phase

$$f(p,\hat{z},\hat{s}) = p(\hat{z}-2\hat{s}) + \frac{2i\hat{s}}{p^2}$$

- The Green function is approximated by saddle point approach

$$g(\hat{z},\hat{s}) \approx \frac{2\exp\left[f(p_s,\hat{z},\hat{s})\right]}{\left[2\pi f''(p_s,\hat{z},\hat{s})\right]^{1/2}}$$

Saddle point approximation for the contour integral

$$\frac{df(p)}{dp} = 0 \Rightarrow p^3 - \frac{4i\hat{s}}{\hat{z} - 2\hat{s}} = 0$$

◈ Saddle point condition → FEL resonance condition

$$4\hat{s} = \hat{z} - 2\hat{s}$$

◈ zeroth order solution in detuning parameter

$$P_{s,0} = i^{1/3}.$$

Group velocity from resonance condition

$$4\hat{s} = \hat{z} - 2\hat{s} \Rightarrow \left(k_s + \frac{2}{3}k_w\right)z - \omega_s t = 0$$

$$\Rightarrow v_g = \frac{\omega_s}{k_s + \frac{2}{3}k_w}$$

- Refer to the pioneering work [Bonifacio, DeSalvo Souza,Pierini & Piovella, NIMA 296, 358(1990)]

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Detuning

$$\varsigma = \hat{z} - 6\hat{s}$$

- Solution up to 3rd order in detuning

$$p_s \approx i^{1/3} - \frac{i^{1/3}\varsigma}{2\hat{z}} - \frac{i^{1/3}\varsigma^3}{12\hat{z}^3}$$

- Green function → Exp[f(p_s)] with

$$f(p_s) \approx i^{1/3}\hat{z} - \frac{i^{1/3}\varsigma^2}{4\hat{z}} = i^{1/3}\hat{z} - 9i^{1/3}\frac{(\hat{s} - \hat{z}/6)^2}{\hat{z}}$$

- Exponential growth

- Superradiance: growth mode

$$p^3 - \frac{4i\hat{s}}{\hat{z}-2\hat{s}} = 0 \Rightarrow p_s = \frac{4^{1/3} i^{1/3} \hat{s}^{1/3}}{\left(\hat{z}-2\hat{s}\right)^{1/3}}$$

- Recall: the exponential growth mode generated from the pole

$$\text{zeroth-order} \Rightarrow p_{s,0} = i^{1/3}$$

- Interesting transition

$$\frac{4^{1/3} \hat{s}^{1/3}}{\left(\hat{z}-2\hat{s}\right)^{1/3}} >> 1$$

Superradiance: growth mode

- The phase

$$f(p_s) = \frac{3i^{1/3}\left[\sqrt{\hat{s}}(\hat{z}-2\hat{s})\right]^{2/3}}{2^{1/3}}$$

- The Green function is approximated by saddle point approach, and gives

$$\left|A_{\mathrm{SR}}\right| \propto \exp\left\{\frac{3^{3/2}}{2^{4/3}}\left[\sqrt{\hat{s}}(\hat{z}-2\hat{s})\right]^{2/3}\right\}$$

Group velocity for superradiance

$$\frac{4^{1/3}\hat{s}^{1/3}}{\left(\hat{z}-2\hat{s}\right)^{1/3}} >> 1 \Rightarrow \hat{z}-2\hat{s} << 1$$

$$\text{when } \hat{z}-2\hat{s}=0 \Rightarrow k_s z - \omega_s t = 0$$

$$\Rightarrow v_g = c$$

- Review of pioneering work [Bonifacio, DeSalvo Souza,Pierini & Piovella, NIMA 296, 358(1990)]

$$\begin{cases} \dfrac{\partial A}{\partial \bar{z}} + \dfrac{\partial A}{\partial z_1} & = \quad b + i\delta A \\ \dfrac{\partial b}{\partial \bar{z}} & = \quad -iP \\ \dfrac{\partial P}{\partial \bar{z}} & = \quad -A \end{cases}$$

$$b = \left\langle \exp\left(-i\bar{\theta}\right)\right\rangle \rightarrow \text{bunching}; A \rightarrow \text{field}$$

$$P = \left\langle p\exp\left(-i\bar{\theta}\right)\right\rangle \rightarrow \text{momentum bunching}$$

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Superradiance solution

$$\left|A_{\mathrm{SR}}\right| \approx \frac{b_0}{\sqrt{3\pi}} \frac{z_1}{y} \exp\left[3\frac{\sqrt{3}}{2}\left(\frac{y}{2}\right)^{2/3}\right]$$

$$= \frac{2b_0}{\sqrt{6\pi}} \frac{\sqrt{\hat{s}}}{\hat{z}-2\hat{s}} \exp\left\{\frac{3^{3/2}}{2^{4/3}}\left[\sqrt{\hat{s}}\left(\hat{z}-2\hat{s}\right)\right]^{2/3}\right\}$$

$$\begin{cases} z_1 = 2\hat{s} \\ z_2 = \hat{z}-2\hat{s} \\ \bar{z} = \hat{z} \\ y = \sqrt{2\hat{s}\left(\hat{z}-2\hat{s}\right)} \end{cases}$$

Same expression!

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- Why the resonance condition so special?
  - In general we look for pole which is not a function of $\hat{s}$

$$\text{Saddle point approach} \Rightarrow p^3 - \frac{4i\hat{s}}{\hat{z} - 2\hat{s}} = 0$$

$$\text{Coherent condition}: \ \hat{z} - 2\hat{s} = \eta\hat{s}$$

$$\Rightarrow p_s^3 = \frac{4i}{\eta}$$

$$\Rightarrow f(p_s) = \left(\frac{4i}{\eta}\right)^{1/3}\hat{z} - \left(\frac{4i}{\eta}\right)^{1/3} 2\hat{s} + \frac{2i\hat{s}\eta^{2/3}}{(4i)^{2/3}}$$

$$\Rightarrow -\left(\frac{4i}{\eta}\right)^{1/3} 2\hat{s} + \frac{2i\hat{s}\eta^{2/3}}{(4i)^{2/3}} = 0$$

$$\eta = 4$$

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Coherence condition

$$\text{Coherence condition}: \ \hat{z} - 2\hat{s} = \eta\hat{s}$$

$$\Rightarrow \begin{cases} \eta = 4 \Rightarrow & \text{resonance condition} \\ \eta = 0 \Rightarrow & \text{superradiance} \end{cases}$$

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Superradiance in a seeded FEL

$$A(\theta, Z) = \int_C \frac{ds}{2\pi i} e^{sZ} \int_{-\infty}^{\theta} d\theta' e^{-s(\theta-\theta') + \frac{i(2\rho)^3(\theta-\theta')}{s^2}} A(\theta', 0)$$

$$\approx C \int_0^{\infty} d\xi \frac{\xi^{1/6} A(\theta - \xi, 0)}{(Z - \xi)^{2/3}} e^{3 i^{1/3} 2^{1/3} \rho (Z-\xi)^{2/3} \xi^{1/3}}$$

Gaussian seed

$$A(\theta, 0) = E_0 e^{-\frac{\theta^2}{\omega_s^2}\alpha_0}$$

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Source Development Lab Experiment

σ_t [fs]	90
L_W [m]	10
λ_W [m]	0.039
ρ	10^{-3}
λ_s [nm]	800

$$Z \in \left[0, Z_{up}\right] \text{ with } Z_{up} = 1611$$

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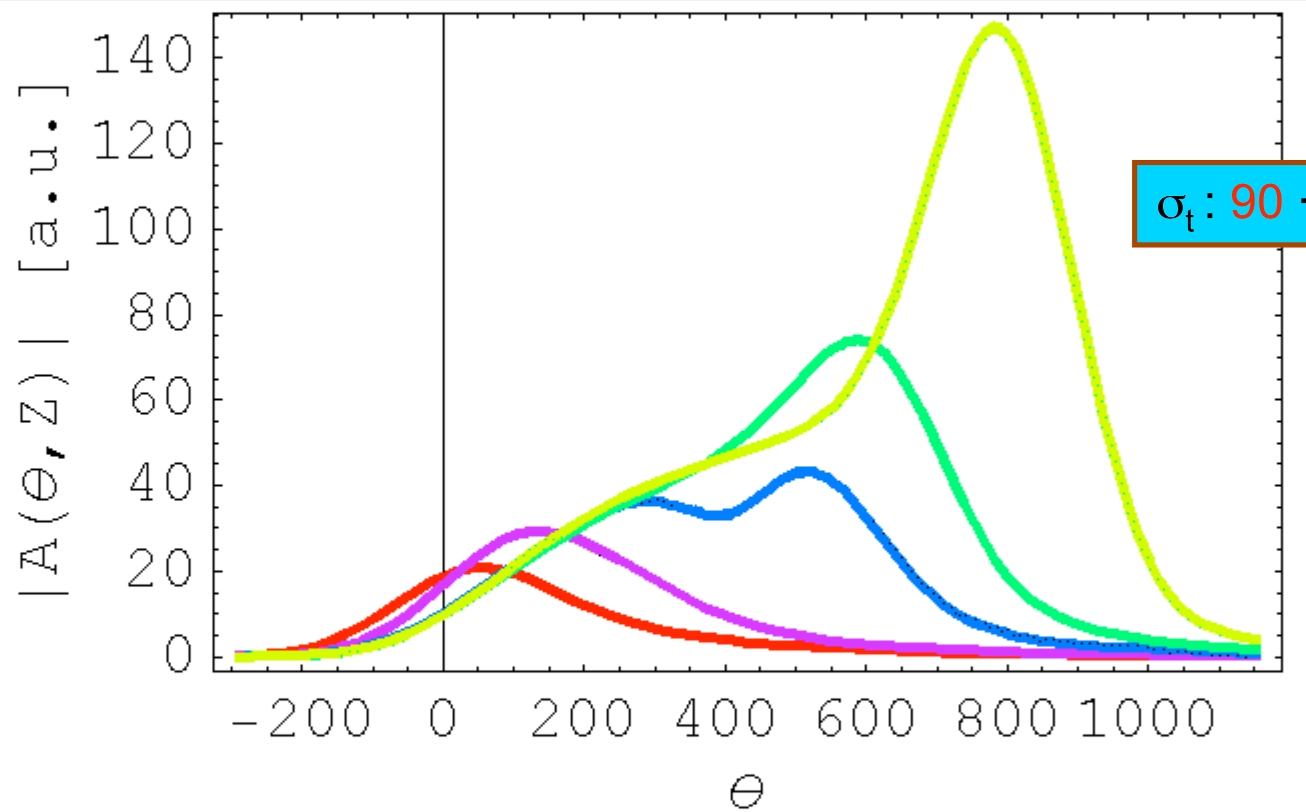
Source Development Lab Experiment



σ_t : 90 → 50 fs

$$|A(\theta, Z)| @ Z = Z_0 [\text{red}] \; Z = Z_0 + 0.1 Z_{up} [\text{purple}]$$

$$Z = Z_0 + 0.3 Z_{up} [\text{blue}] \; Z = Z_0 + 0.4 Z_{up} [\text{green}] \; Z = Z_0 + 0.5 Z_{up} [\text{yellow}]$$

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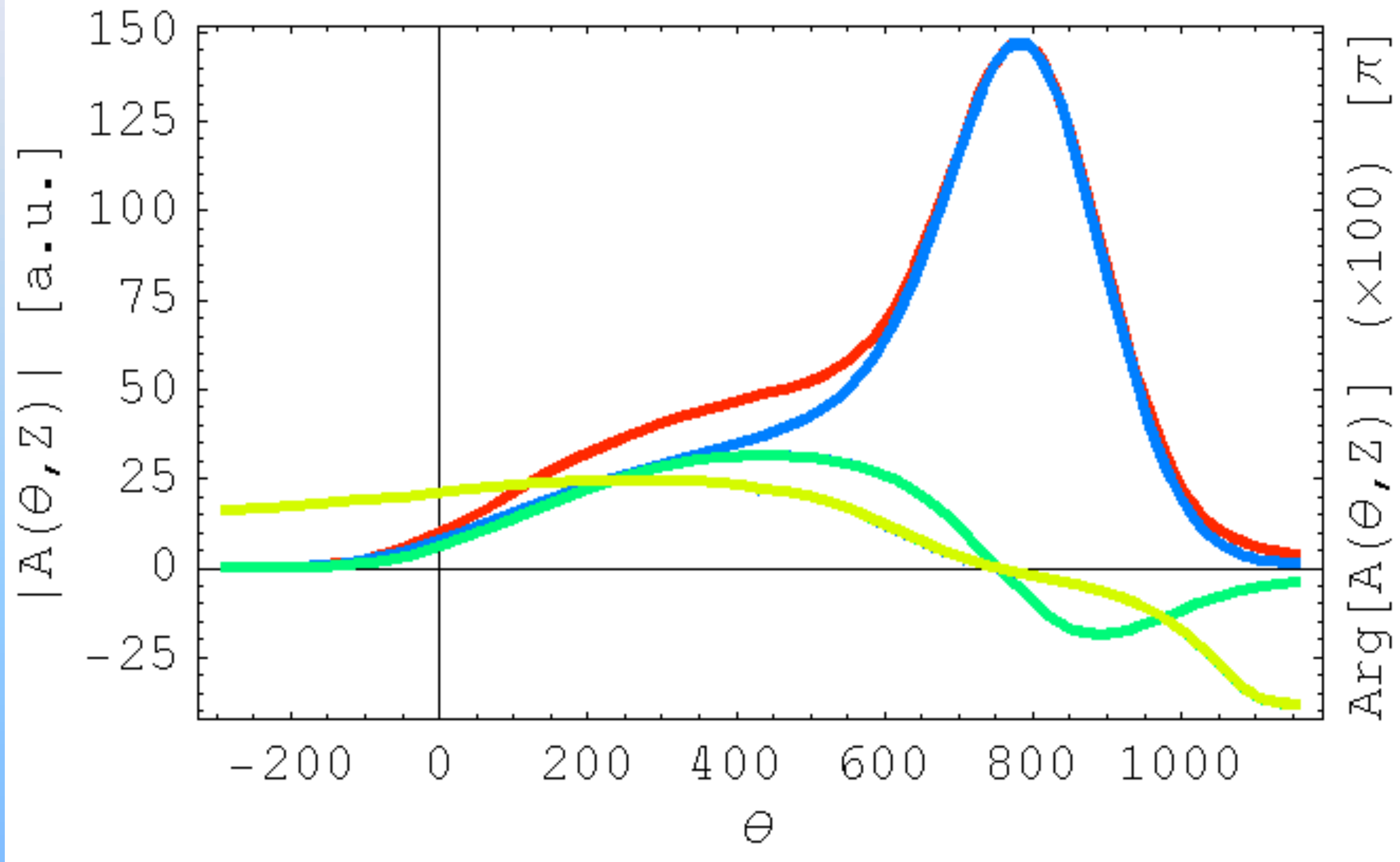
More details

$Z = Z_0 + 0.5\, Z_{up}$



$|A(\theta, Z)|$[red] $\mathrm{Re}|A(\theta, Z)|$[blue] $\mathrm{Im}|A(\theta, Z)|$[green] $\phi(\theta, Z)$[yellow]

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Chirp

$$E(t,z) = |A(\theta, Z)| e^{-i\Phi(t,z)}$$

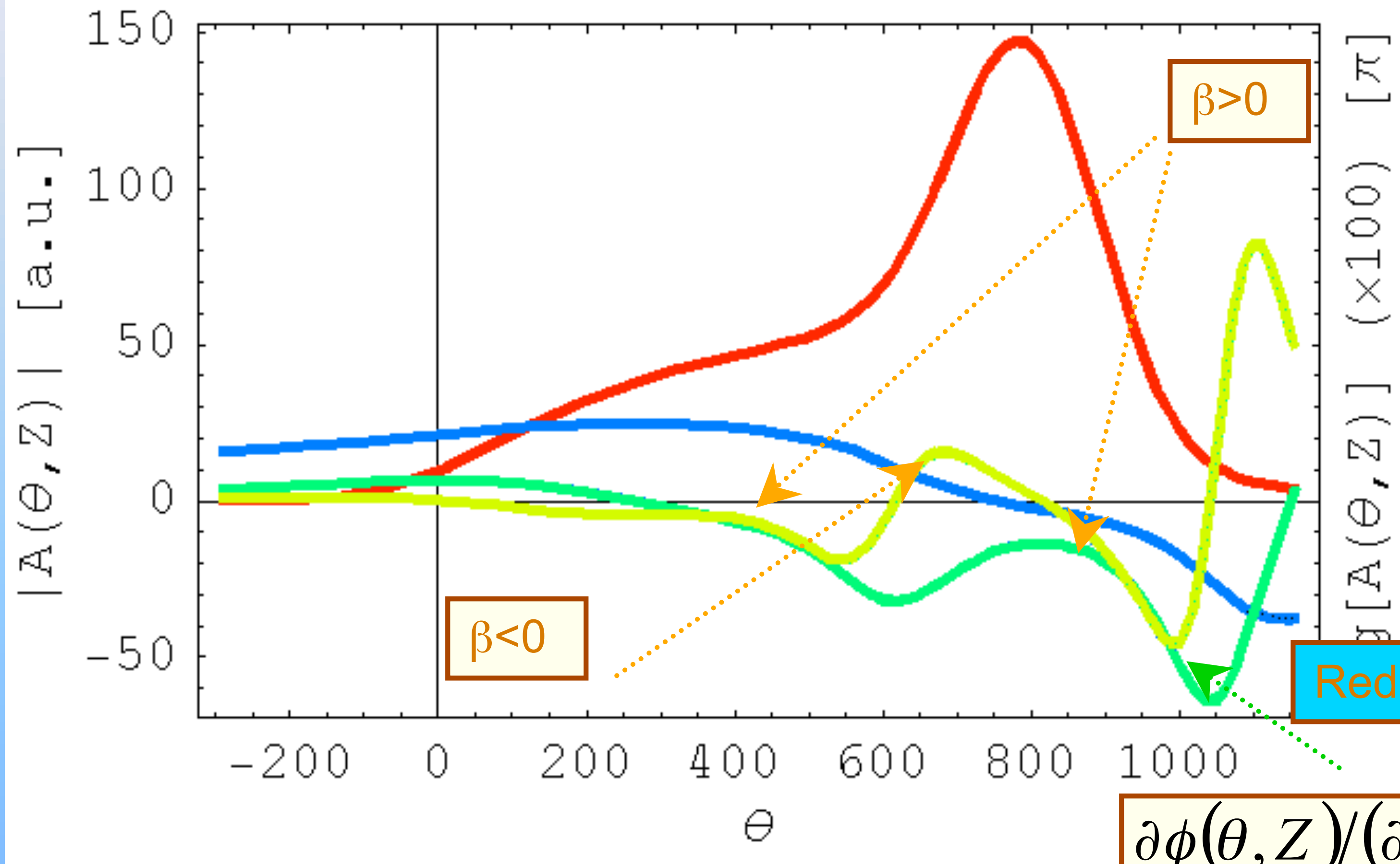
$$\Phi(t,z) = \omega_s t - k_s z - \phi(\theta, Z)$$

$$= \Phi(t_c, z) + \left.\frac{\partial \Phi}{\partial t}\right|_{t=t_c} (t - t_c) + \left.\frac{\partial^2 \Phi}{\partial t^2}\right|_{t=t_c} (t - t_c)^2 + \ldots$$

$$\begin{cases} \dfrac{\partial \Phi}{\partial t} = \omega_s + \omega_s \dfrac{\partial \phi}{\partial \theta} \\ \dfrac{\partial^2 \Phi}{\partial t^2} = -\omega_s^2 \dfrac{\partial^2 \phi}{\partial \theta^2} \end{cases}$$

Chirps

$$Z = Z_0 + 0.5\, Z_{up}$$



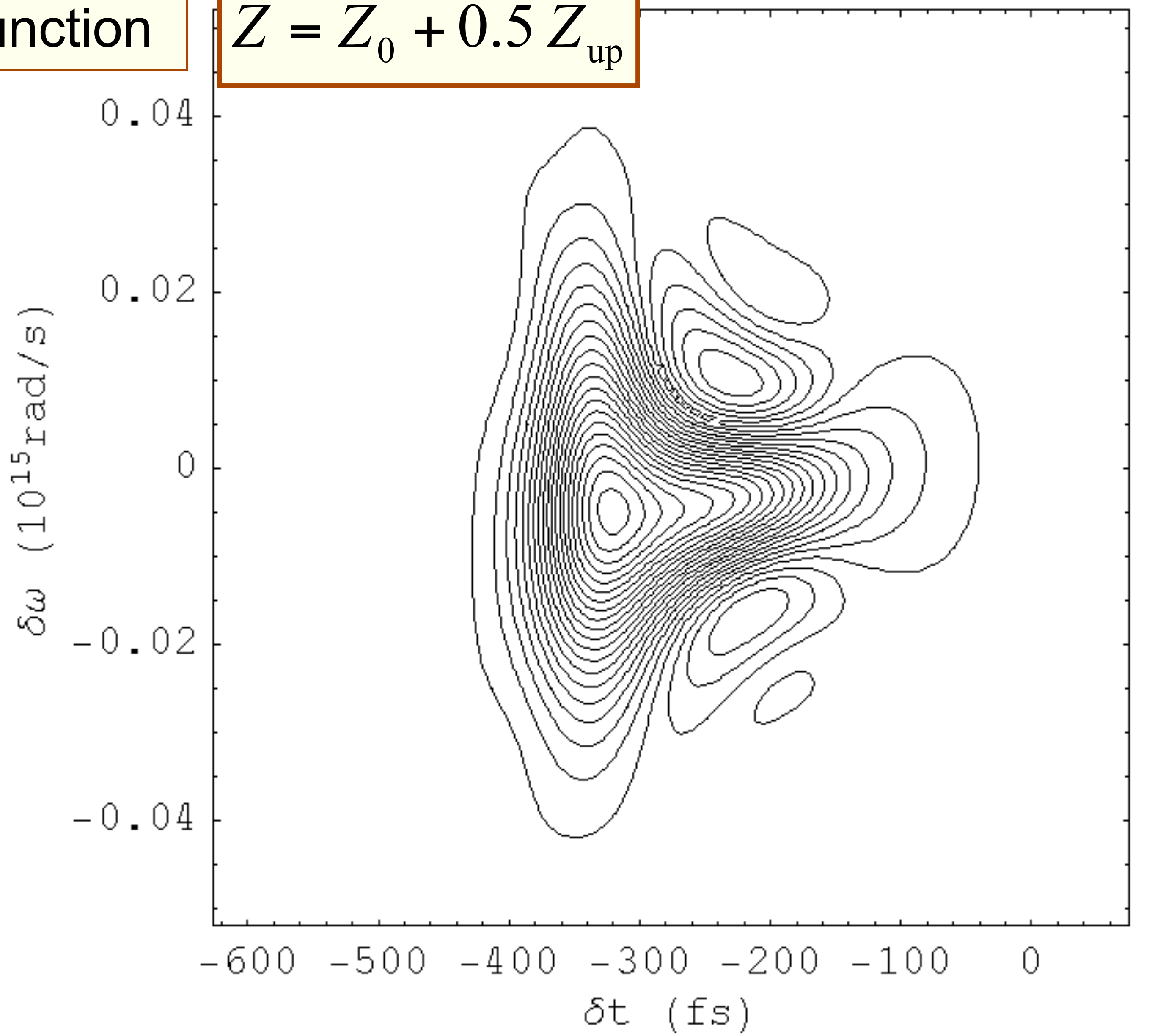
Red shift

$\partial\phi(\theta,Z)/(\partial\theta) < 0$

$|A(\theta,Z)|$[red]; $\phi(\theta,Z)$[blue]; $\partial\phi(\theta,Z)/(\partial\theta)$[green]; $\partial^2\phi(\theta,Z)/(\partial\theta^2)$[yellow]

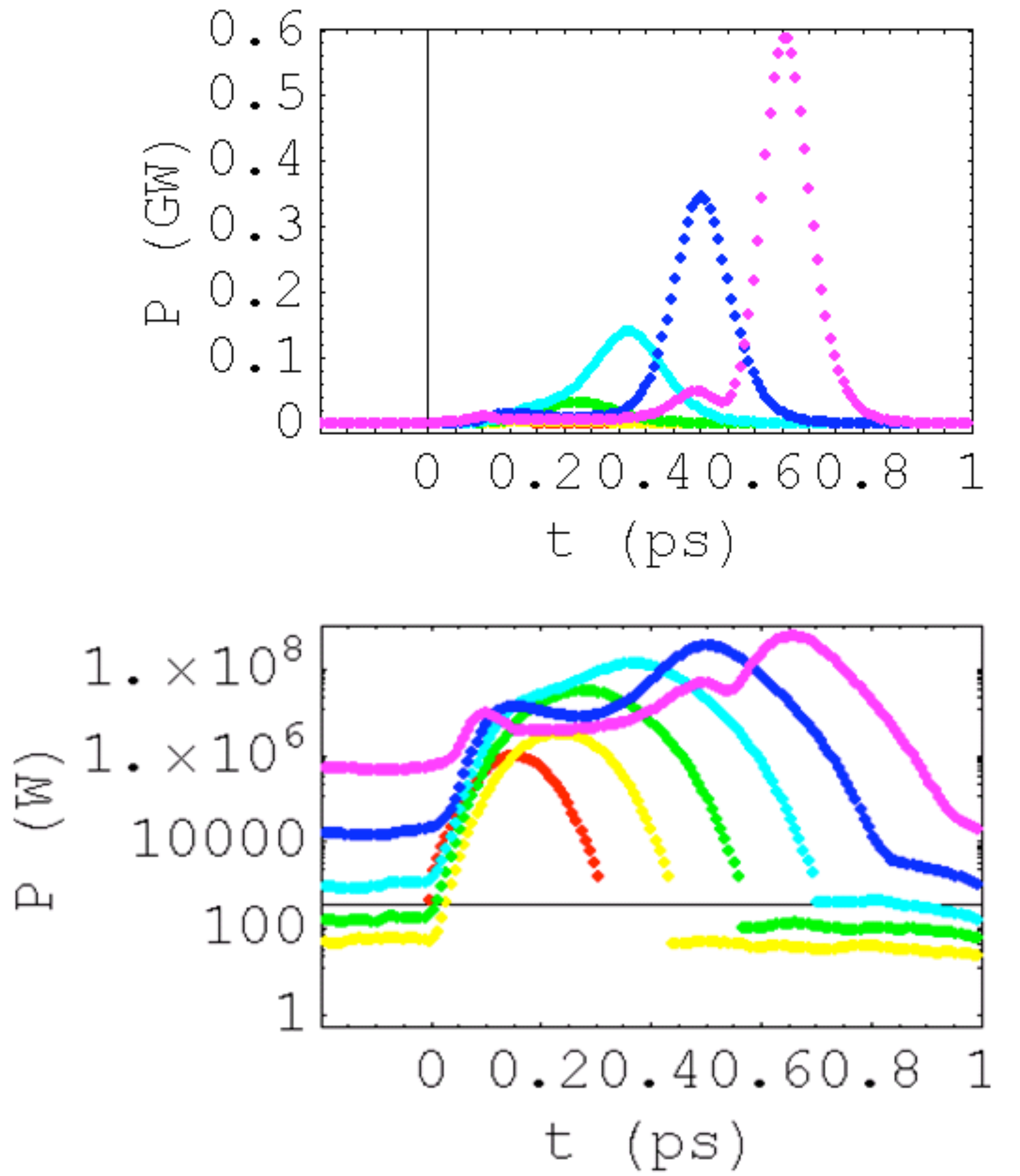
Wigner function

$Z = Z_0 + 0.5\, Z_{up}$



Genesis simulation

z = 1.6 (red),
3.2 (yellow),
4.8 (green),
6.4 (light blue),
8.0 (blue),
10.0 (purple) [m]



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References

[1] T. Watanabe, X.J. Wang, J.B. Murphy, J. Rose, Y. Shen, T, Tsang, L. Giannessi, P. Musumeci, and S. Reiche, *"Experimental Characterization of Superradiance in a Single-Pass High-Gain Laser-Seeded Free-Electron Laser Amplifier"*, Phys. Rev. Lett. **98**, 034802 (2007); **98**, 189903 (2007); **99**, 029502 (2007)

[2] R. Bonifacio, F. Casagrande, D.A. Jaroszynski, B.W. J. McNeil, N. Piovella, and G.R.M. Robb, *"Comment on 'Experimental Characterization of Superradiance in a Single-Pass High-Gain Laser-Seeded Free-Electron Laser Amplifier' "*, Phys. Rev. Lett. **99**, 029501 (2007)

[3] R. Bonifacio, and F. Casagrande, *"The Superradiance Regime of a Free Electron Laser"*, Nucl. Inst. Meth. A **239**, 36 (1985).

[4] R. Bonifacio, C. Maroli and N. Piovella, *"Slippage and Superradiance in the High-gain FEL: Linear Theory"*, Opt. Comm. **68**, 369 (1988).

[5] R. Bonifacio, L. De Salvo Souza, P. Pierini, and N. Piovella, *"The Superradiant Regime of a FEL: Analytical and Numerical Results"*, Nucl. Inst. Meth. A **296**, 358 (1990).

[6] R.H. Dicke, "*Coherence in Spontaneous Radiation Process*", Phys. Rev. **93**, 99 (1954).

[7] L. Giannessi, P. Musumeci, and S. Spampinati "*Nonlinear pulse evolution in seeded free-electron laser amplifiers and in free-electron laser cascades*", J. Appl. Phys. **98**, 043110 (2005).