Chirps in a high-gain free electron laser seeded by high-order harmonic generation and ultrafast source production

J. Wu, P.R. Bolton, and P.J. Emma (LCLS/SLAC)
J.B. Murphy, X.J. Wang, and T. Watanabe (NSLS/BNL)

September 09, 2007

- Chirps in a seeded Free Electron Laser (FEL) in general
  - Frequency chirp along the seed pulse
  - Energy chirp along the electron bunch
  - Intrinsic frequency chirp developed during FEL process
  - Interplay of the chirps
- ABCD formalism
- High-order Harmonic Generation (HHG) seed is attractive
  - Ultrashort, VUV to soft x-ray, attosecond pulse train (APT)
  - smearing of APT and APT restoration
- LCLS-type high brightness electron bunch seeded by HHG
  - Ultrashort, powerful, hard x-ray FEL,

Juhao Wu
jhwu@SLAC.Stanford.EDU

Wigner Function for a Chirped Gaussian Seed Pulse

- The electric field of the chirped seed laser is assumed to be

$$E_s(t,z) = E_0 e^{i(k_s z - \omega_s t)} e^{-(\alpha + i\beta)\omega_s^2 (t - z/v_{g,0})^2}$$

Seed Chirp

with $v_{g,0} = \omega_s / (k_s + 2k_w/3)$

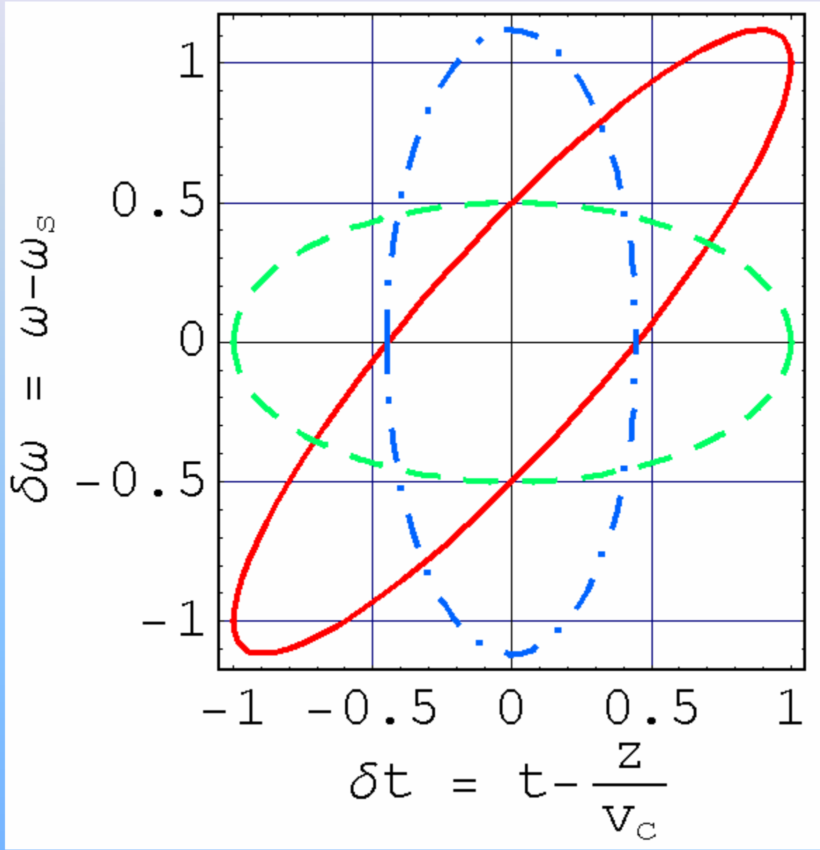
- The Wigner function is defined as

$$W(t,\omega,z) = \int E(t-\tau/2, z) E^*(t+\tau/2, z) e^{-i\omega\tau} d\tau$$

- The Wigner function for the seed laser is a Bi-Gaussian:

Juhao Wu
jhwu@SLAC.Stanford.EDU

Contour plot of the Wigner function for the seed laser



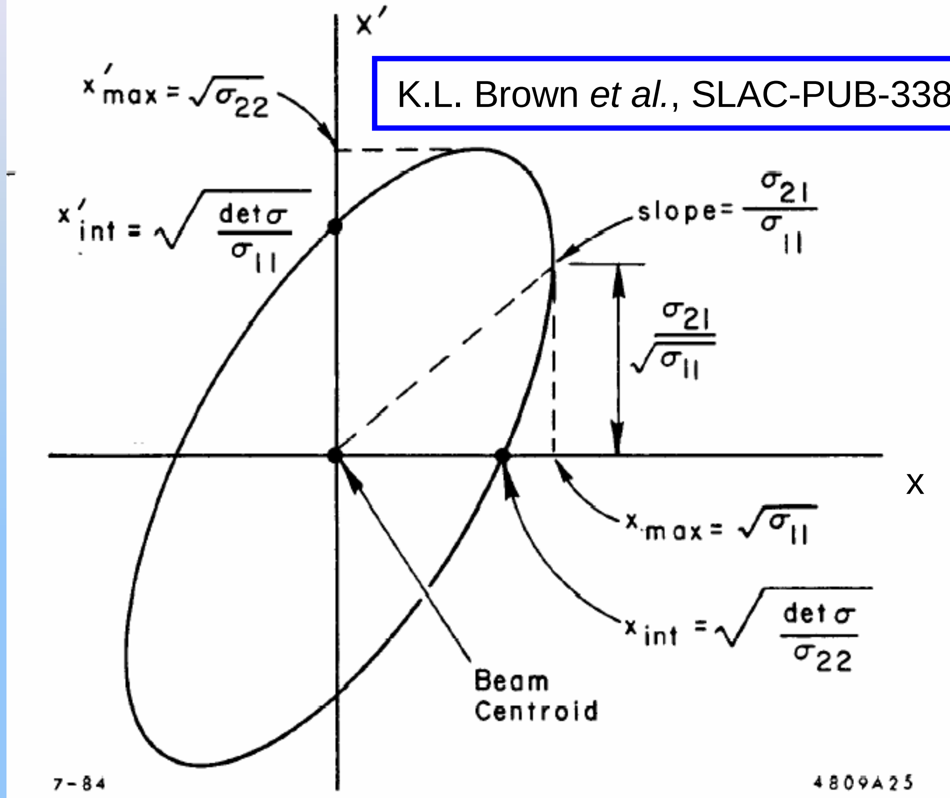


Fig. 10. A beam ellipse based on the σ matrix. The maximum extent of the ellipse and its orientation are shown as a function of the matrix elements.

K.L. Brown *et al.*, SLAC-PUB-3381

Electron bunch: RF cavity and Bunch Compressor

Juhao Wu
jhwu@SLAC.Stanford.EDU

FEL process – group velocity dispersion and gain

- Integral representation of a seeded FEL

$$A(\theta,Z) \cong e^{\rho\left(\sqrt{3}+i\right)Z}\int d\xi A(\theta-\xi,0)e^{i\mu\theta(Z-\xi)}e^{-\rho\left(\sqrt{3}+i\right)\left[9(\xi-Z/3)^2/(4Z)\right]}e^{-i(\mu/2)(Z-\xi)\xi}$$

where $E(t,z) \equiv A(\theta,Z)e^{i(\theta-Z)}$ with $Z=k_w z$ and $\theta=(k_s+k_w)z-\omega_s t$

- Energy chirp in the electron bunch

$$\mu \equiv \frac{2}{\gamma_0\omega_s}\frac{d\gamma}{dt}$$

- The FEL electric field in (t, z) coordinates

$$E_{\text{FEL}}(t,z) = E_{0,\text{FEL}}e^{\rho\left(\sqrt{3}+i\right)k_w z}e^{i(k_s z-\omega_s t)}e^{-\left[\alpha_{s,f}(z)+i\beta_{s,f}(z)\right]\omega_s^2(t-z/v_c)^2}$$

where $\alpha_{s,f}(z)=\left[4\sigma_{t,s,f}^2(z)\omega_s^2\right]^{-1}, \beta_{s,f}^2(z)=\alpha_{s,f}(z)\sigma_{\omega,s,f}^2(z)\big/\omega_s^2-\alpha_{s,f}^2(z)$

Juhao Wu
jhwu@SLAC.Stanford.EDU

Wigner Function for the FEL Pulse is again Bi-Gaussian:

$$W(t,\omega,z) \propto \exp\left[-\frac{\dfrac{\delta t^2}{\sigma_{t,s,f}^2} - 2r\dfrac{\delta t\,\delta\omega}{\sigma_{t,s,f}\sigma_{\omega,s,f}} + \dfrac{\delta\omega^2}{\sigma_{\omega,s,f}^2}}{2\left(1-r^2\right)}\right]$$

$\delta t = t - \langle t\rangle$ and $\delta\omega = \omega - \langle\omega\rangle$

$$r = 2\beta_{s,f}\frac{\sigma_{t,s,f}\,\omega_s^2}{\sigma_{\omega,s,f}} = \frac{\beta_{s,f}}{2\alpha_{s,f}}\frac{1}{\sigma_{t,s,f}\sigma_{\omega,s,f}} = \frac{\left\langle\left(t-\langle t\rangle\right)\left(\omega-\langle\omega\rangle\right)\right\rangle}{\sigma_{t,s,f}\sigma_{\omega,s,f}}$$

Juhao Wu
jhwu@SLAC.Stanford.EDU

Second moments:

$$
\begin{cases}
\sigma^2_{t,s,f}(z) = \dfrac{\vec{U}}{\omega_s^2 \vec{V}} \\[2ex]
\sigma^2_{\omega,s,f}(z) = \dfrac{\omega_s^2\left(\vec{V}^2 + \vec{W}^2\right)}{4\vec{U}\vec{V}} \\[2ex]
\langle (t-\langle t\rangle)(\omega-\langle \omega\rangle)\rangle \equiv \dfrac{\beta_{s,f}(z)}{2\alpha_{s,f}(z)} = \dfrac{\vec{W}}{2\vec{V}}
\end{cases}
$$

◆The longitudinal emittance of the Gaussian seed is defined as

$$
\varepsilon_{\text{Light}} \equiv \sqrt{\langle (t-\langle t\rangle)^2\rangle \langle (\omega-\langle \omega\rangle)^2\rangle - \langle (t-\langle t\rangle)(\omega-\langle \omega\rangle)\rangle^2} = 1/2
$$

◆The longitudinal Compute $\varepsilon(z) = 1/2 \Rightarrow$ Coherence is Preserved!

Notations:

$$\begin{cases} \vec{U}' & \equiv & 3+P^2\left[Q+\left(6+4R^2\right)\alpha_s+\sqrt{3}\left(2\beta_s-\mu\right)\right] \\ \vec{V} & \equiv & 3\left[Q+4\left(1+R^2\right)\alpha_s\right] \\ \vec{W}' & \equiv & 4\left(\sqrt{3}R^2\alpha_s+3\beta_s\right)+P^2\mu\left[2Q+4\left(3+2R^2\right)\alpha_s-\sqrt{3}\mu\right] \end{cases}$$

$$\begin{cases} P & \equiv & \frac{\omega_s}{\sigma_{\omega,\mathrm{GF}}} \\ Q & \equiv & \frac{\mu(\mu-4\beta_s)\omega_s^2}{\sigma_{\omega,\mathrm{GF}}^2} \\ R & \equiv & \frac{\sigma_{\omega,s}}{\sigma_{\omega,\mathrm{GF}}} \end{cases}, \quad \text{with} \quad \mu \equiv \frac{2}{\gamma_0\omega_s}\frac{d\gamma}{dt}, \quad \text{and} \quad \sigma_{\omega,\mathrm{GF}}(z) \equiv \sqrt{\frac{3\sqrt{3}\rho\,\omega_s^2}{k_w z}}$$

Notice that, even for $\beta_s = \mu = 0$, $W \neq 0$, hence FEL intrinsic chirp

Centrovelocity:

$$v_c^{-1}(z) \equiv \langle t\rangle / z = v_{g,0}^{-1} + \sqrt{3}\mu\omega_s k_w\left(2\alpha_s - 2\sqrt{3}\beta_s + \sqrt{3}\mu\right) / \left(2\vec{V}\sigma_{\omega,\mathrm{GF}}^2\right)$$

Juhao Wu
jhwu@SLAC.Stanford.EDU

Ellipse evolution

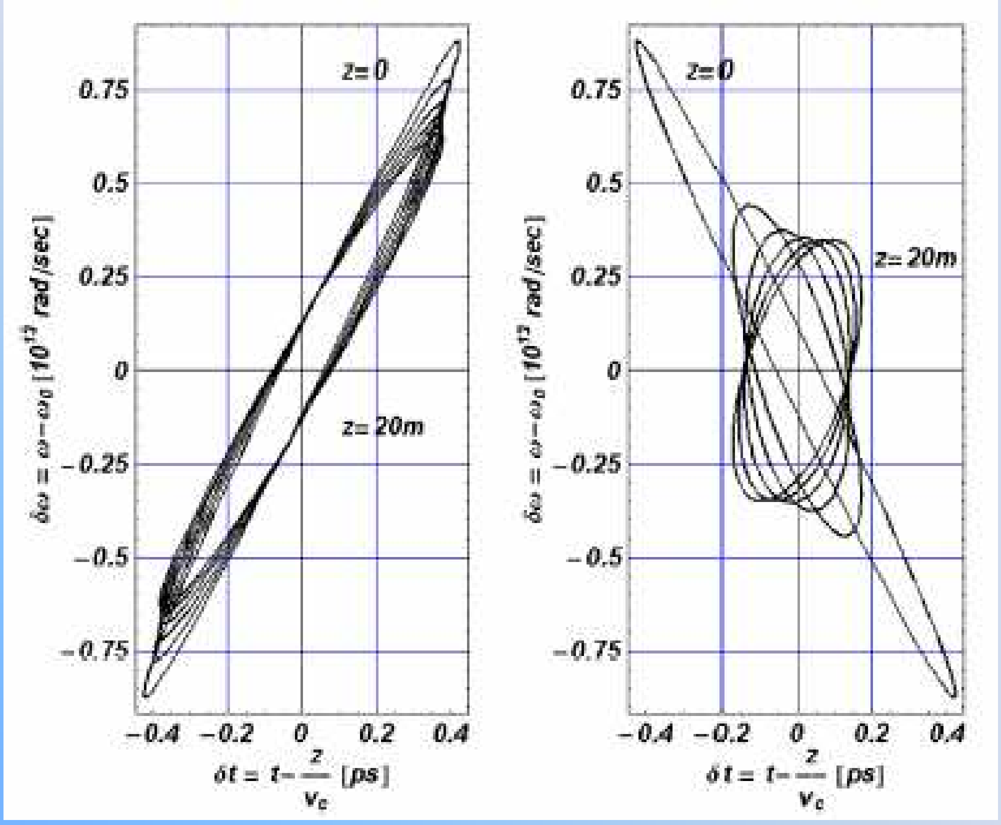


Fig. 7. (Color online) Contour plot of the Wigner function $W(t - z/v_c, \omega - \omega_0)$ of the FEL light for $z = [0, 20]$ m. The centrovelocity v_c is given in Eq. (32). The electron beam has a positive energy chirp. The left (right) plot is for an initial positive (negative) seed laser chirp of equal magnitude.

Juhao Wu
jhwu@SLAC.Stanford.EDU

Ellipse evolution

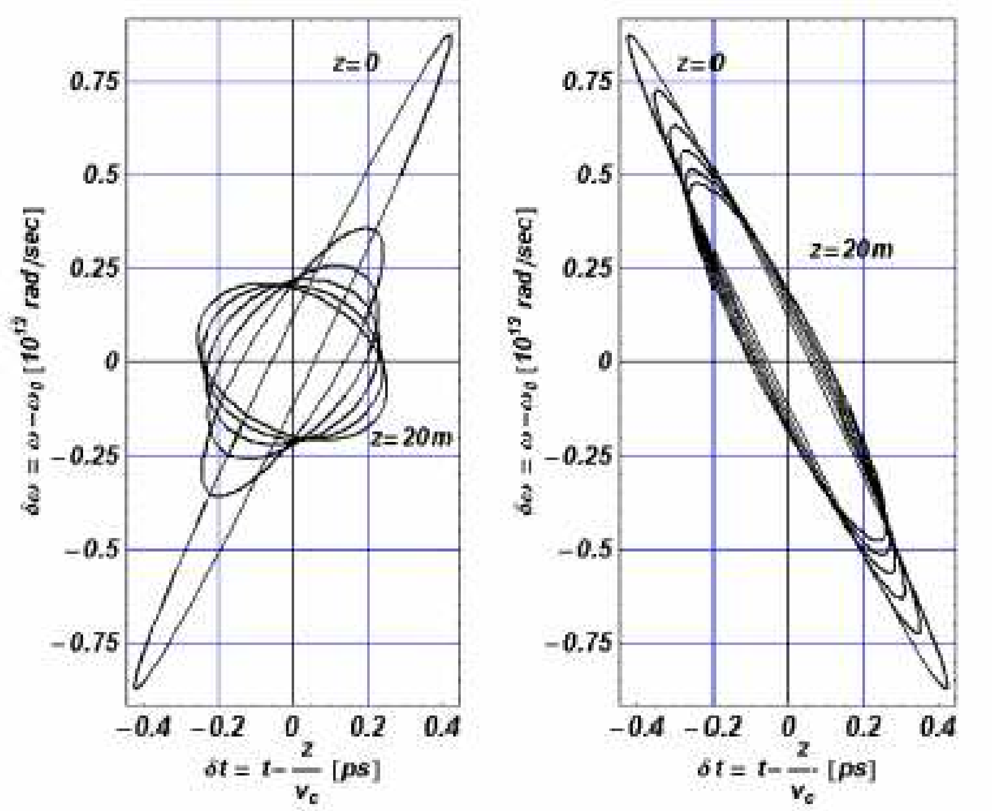


Fig. 9. (Color online) Contour plot of the Wigner function $W(t - z/v_c, \omega - \omega_0)$ of the FEL light for $z \in [0, 20]$ m. The centrovelocity v_c is given in Eq. (32). The electron beam has a negative energy chirp. The left (right) plot is for an initial positive (negative) seed laser chirp of equal magnitude.

Juhao Wu
jhwu@SLAC.Stanford.EDU

Chirped pulse compression

- Preundulator – introduce the chirp via horizontal shearing – stretch temporally
- Postundulator – chirped pulse compression via horizontal shearing – compress temporally
- Yet, the FEL is a group velocity dispersive medium with gain
- For μ = 0

$$\sigma_{t,f}\big|_{\mu=0} = \frac{1}{2\sigma_{\omega}\big|_{\mu=0}} = \sigma_{t,i}\sqrt{1+\frac{1}{4C^2}}$$

$$C = \sigma_{t,i}\sigma_{\omega,\mathrm{GF}}$$

Positively defined, inevitably stretch the pulse temporal duration

Juhao Wu
jhwu@SLAC.Stanford.EDU

However, for μ ≠ 0

$$\sigma_{t,f} \approx \sigma_{t,i}\left[1 + \frac{1}{4C^2} - \frac{1+4C^2}{36C^2}\eta - \frac{4}{27}C^2\epsilon\eta - \frac{1+2C^2}{54}\eta^2 + \frac{2(3+4C^2)}{243}C^2\epsilon\eta^2 + \frac{16}{729}C^6\epsilon^2\eta^2\right]^{1/2}.$$

$$\epsilon \equiv \frac{3\sqrt{3}\beta_0}{\sigma^2_{\omega,\mathrm{GF}}}, \quad \eta \equiv \frac{3\sqrt{3}\mu\omega_0^2}{\sigma^2_{\omega,\mathrm{GF}}}$$

Juhao Wu
jhwu@SLAC.Stanford.EDU

Example, interplay of the chirps and optimization for compression

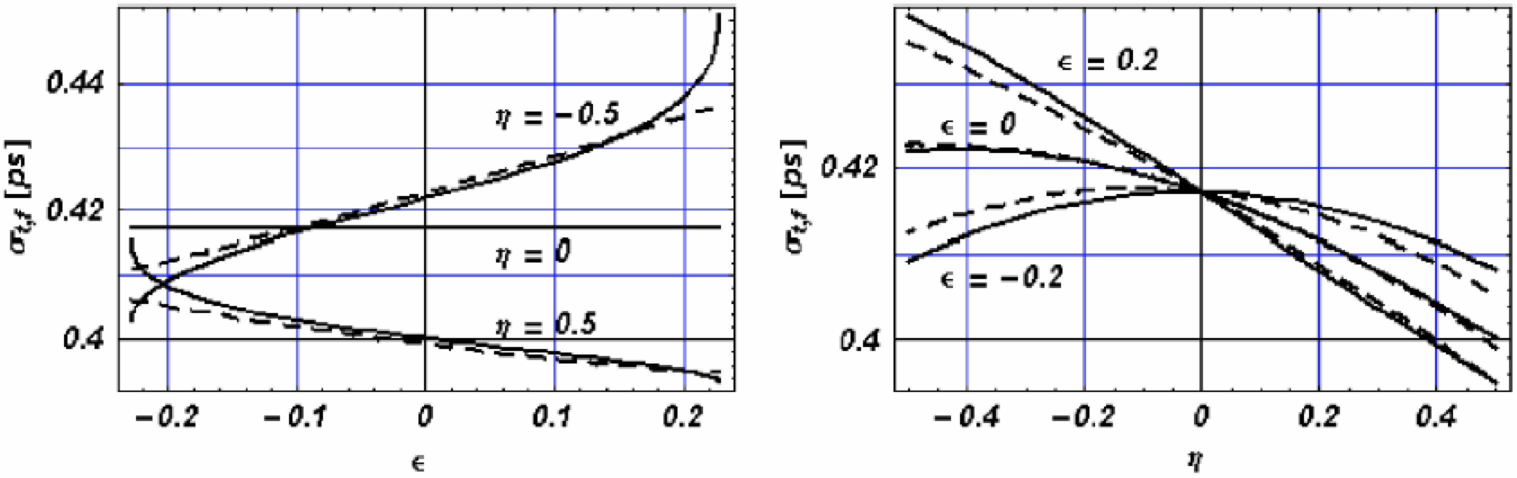


Fig. 11. (Color online) Plot of the FEL pulse duration $\sigma_{t,f}$ after postundulator pulse compression versus the dimensionless frequency chirp ϵ in the seed laser (left plot) and the dimensionless energy chirp η (right plot) in the electron beam introduced in Eq. (43).

Juhao Wu
jhwu@SLAC.Stanford.EDU

ABCD formalism – transfer the complex Gaussian parameter

$$\frac{1}{p(z)} \equiv -2\beta_{s,f}(z)\omega_s + i2\alpha_{s,f}(z)\omega_s, \quad \text{as} \quad p(z) = \frac{Ap(0)+B}{Cp(0)+D}$$

◆ Symplectic *ABCD* matrix

$$M_{ABCD} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2ik_W z}{9(i+\sqrt{3})\rho\,\omega_s} \\ C & D \end{pmatrix}$$

$$C = \frac{(i\mathscr{V} - \mathscr{W})\,\omega_s}{2\mathscr{U}} - \frac{\left(i\,\mathscr{V}|_{\mu=0} - \mathscr{W}|_{\mu=0}\right)\omega_s}{2\,\mathscr{U}|_{\mu=0}}, \quad \text{and} \quad D = 1 + BC$$

Only for $\mu \neq 0$, $C \neq 0$

Juhao Wu
jhwu@SLAC.Stanford.EDU

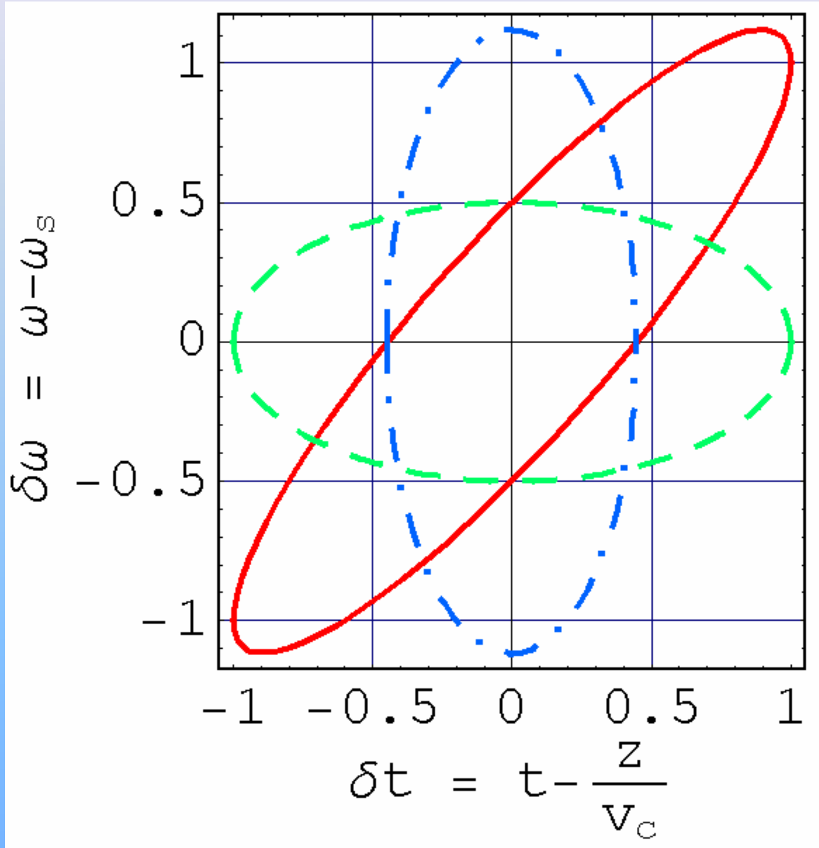
ABCD canonical transformation

$$\begin{pmatrix} \tau \\ \frac{d\tau}{d\zeta} \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{1\to 2} \begin{pmatrix} \tau \\ \frac{d\tau}{d\zeta} \end{pmatrix}_1, \text{where} \begin{cases} \tau &= t - z\left.\frac{dk}{d\omega}\right|_{\omega=\omega_s} = t - \frac{z}{v_{g,0}} \\ \zeta &= \omega_s z \left.\frac{d^2k}{d\omega^2}\right|_{\omega=\omega_s} \end{cases}$$

- It is now clear that B represents a horizontal shearing, and C for a vertical shearing in the t–ω ellipse

Juhao Wu
jhwu@SLAC.Stanford.EDU

Contour plot of the Wigner function for the seed laser



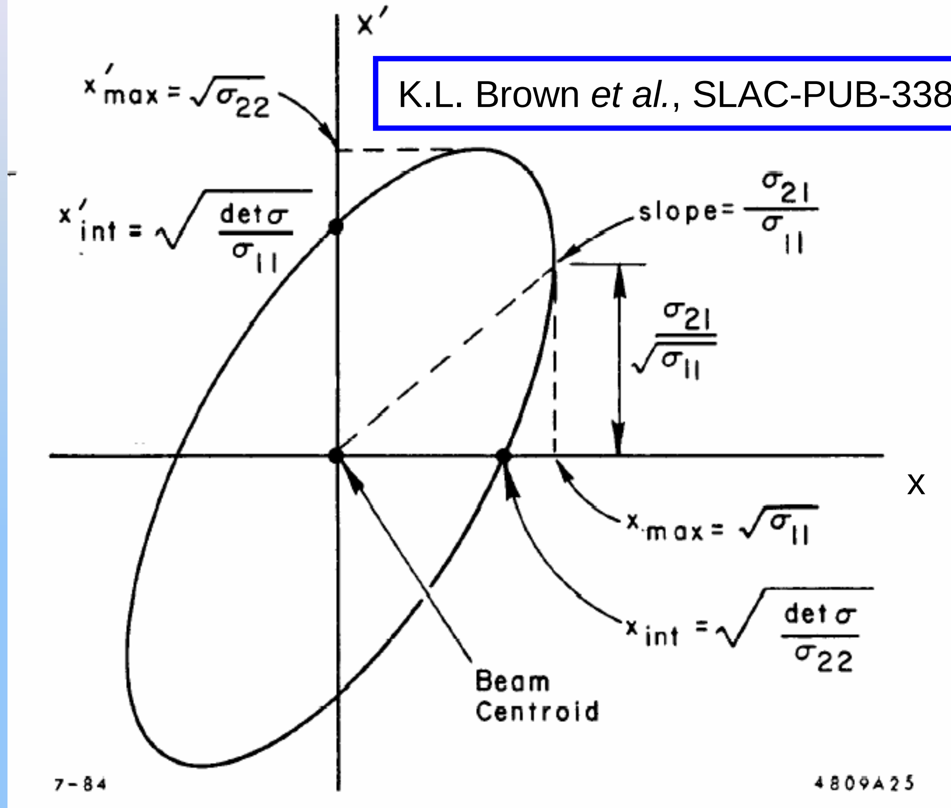


Fig. 10. A beam ellipse based on the σ matrix. The maximum extent of the ellipse and its orientation are shown as a function of the matrix elements.

Electron bunch: RF cavity and Bunch Compressor

HHG Seed

$$E_s(t,z) = E_{s,0} e^{i(k_s z - \omega_s t)} e^{-i\beta_s \omega_s^2 t^2} \sum_{n=-N}^{N} e^{-\frac{t_n^2}{4\sigma_{t,0}^2}} e^{-\alpha_s \omega_s^2 [(t-t_n)-z/c]^2}$$

- Attosecond pulse train
  - $t_n = n\,\tau / 2$
  - $\beta_s \approx -\frac{A_s I_{ir}}{2(s\omega_{ir}\sigma_{t,ir})^2} + \frac{b_{ir}}{2s\omega_{ir}^2}$
- Multiple harmonic order, yet FEL is a narrow band filter

Juhao Wu
jhwu@SLAC.Stanford.EDU

- Attosecond pulselet smearing: example: λ_{ir} = 800 nm, $t_n = n\, \tau_{ir} / 2$, $\sigma_{t,0}$ = 10 fs, $\sigma_{t,s} = \tau_{ir}/10$ = 267 attosec, and s = 27

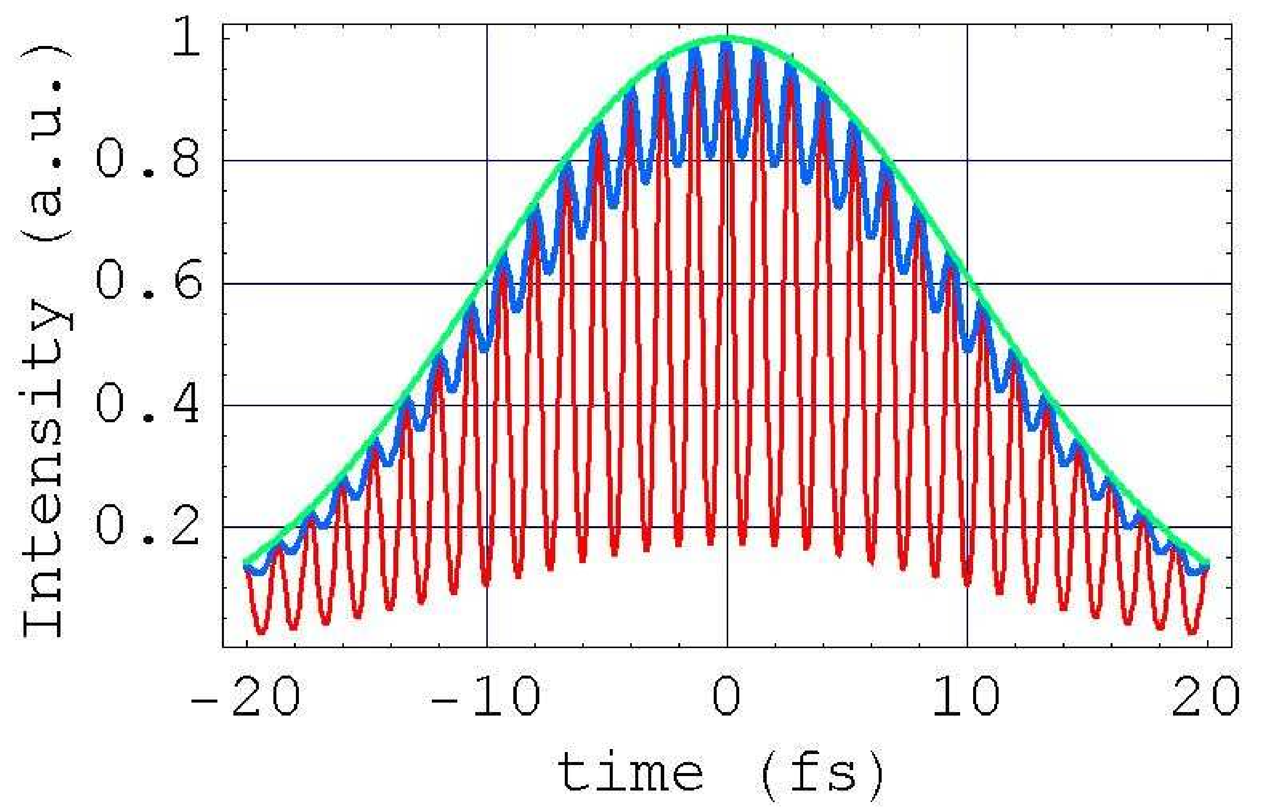


Illustration of smearing effect. Red @ z = 0; blue @ z = 18 cm into exponential growth regime; green @ z = 4.2 m approach saturation.

- Attosecond restoration via energy chirp? example: λ_{ir} = 800 nm, $t_n = n\,\tau_{ir} / 2$, $\sigma_{t,0}$ = 10 fs, $\sigma_{t,s} = \tau_{ir}/10$ = 267 attosec, and s = 27

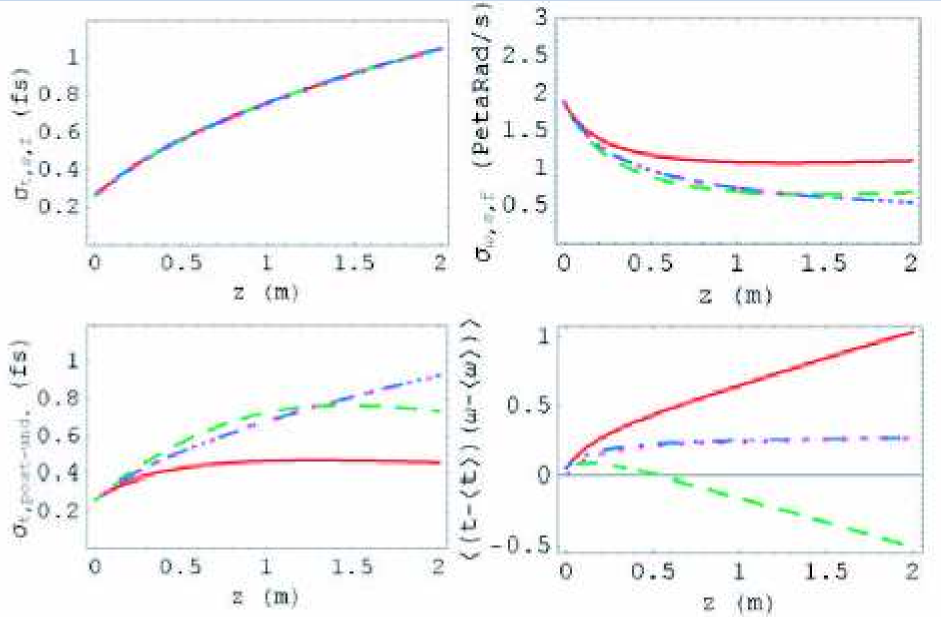


Fig. 1. The FEL pulse rms duration (upper left), the rms bandwidth (upper right), the rms duration after post-undulator compression (lower left), and the time-frequency correlation (lower right) as a function of the location into the undulator. The solid (red) curve is for $\mu = 2\beta_s$, the dashed (green) for $\mu = -2\beta_s$, and the dash-dotted (blue) for $\mu = 0$. For all these three cases, $\beta_s \approx 8.7 \times 10^{-5}$. The dotted (purple) curve is for $\mu = \beta_s = 0$.

Juhao Wu
jhwu@SLAC.Stanford.EDU

APT HHG seeded FEL

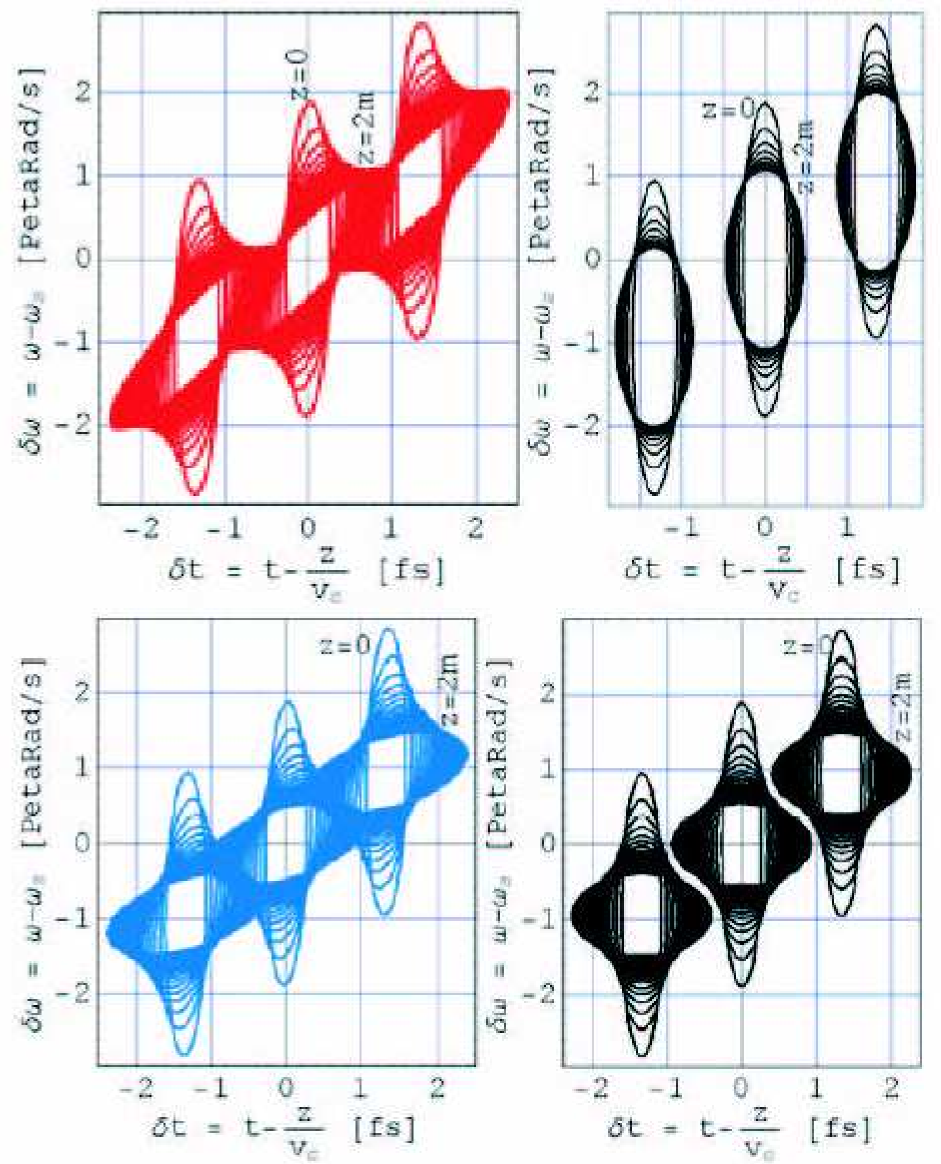
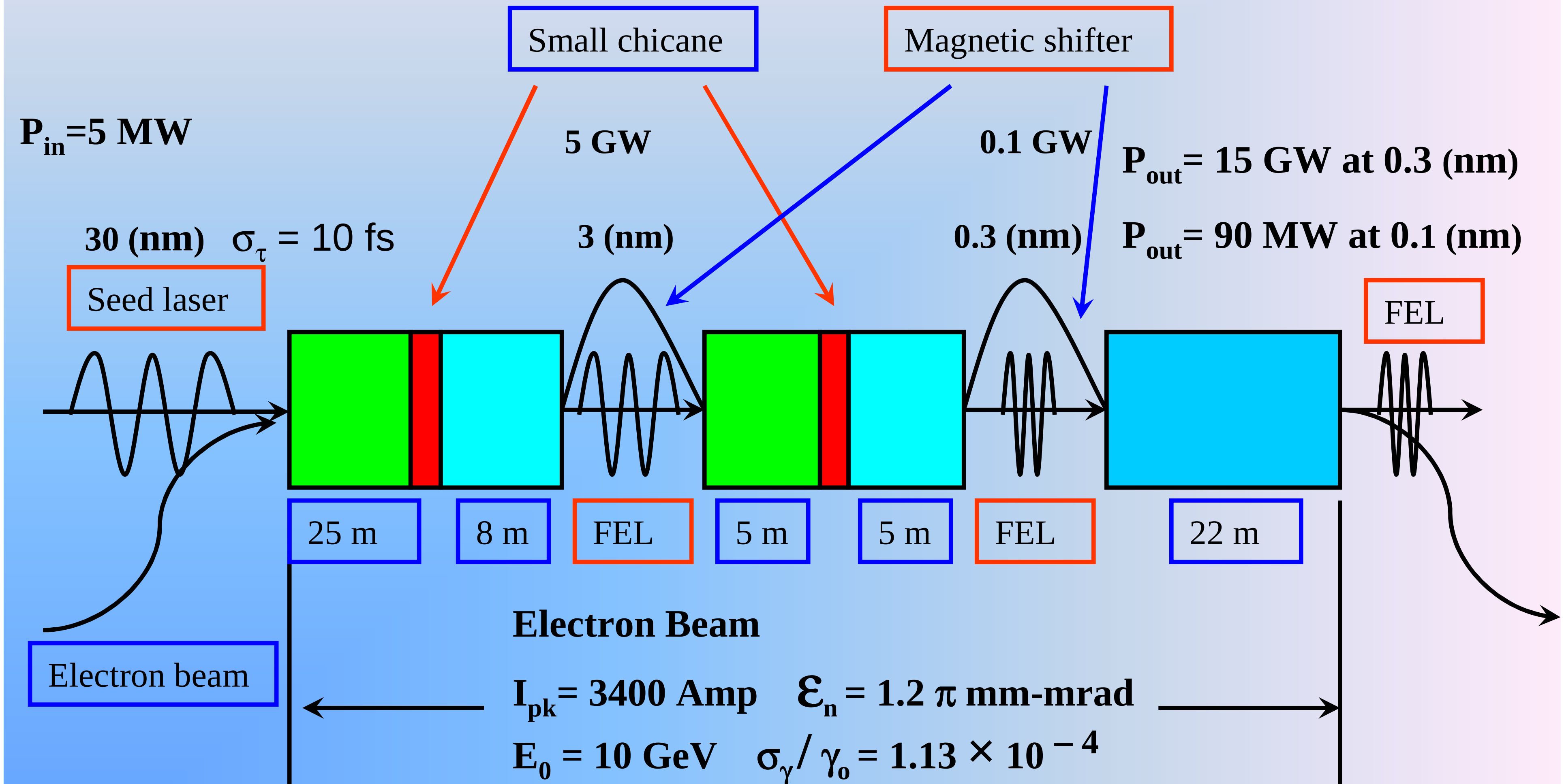


Fig. 2. The evolution of the Wigner function ellipse as a function of the location into the undulator is shown in the left subplots. For clarity, only three pulselets are shown. In the right subplots, each ellipse stands for experiencing a postundulator compression. In the upper row, the energy chirp in the electron bunch is $\mu = 2\beta_s$. In the lower row, $\mu = 0$. In all the plots, $\beta_s \approx 8.7 \times 10^{-5}$.

Juhao Wu
jhwu@SLAC.Stanford.EDU

High-order Harmonic Generation (HHG) Cascaded HGHG FEL for LCLS



Discussions

- FEL Coasting Beam Green Function can be characterized by an *ABCD* Canonical Transformation
- FEL Process = Group Velocity Dispersion and Gain Modifies the Seed pulse duration, spectral bandwidth, and chirp
- Longitudinal Coherence of the Seed is Preserved in the High Gain Exponential Regime
- Energy Chirp in the electron bunch is necessary to achieve FEL pulse temporal net compression
  - Attosecond pulse train can be preserved with proper energy chirp in the electron bunch with postundulator process
- With LCLS-type electron bunch, cascaded HGHG scheme with HHG seed for LCLS

Juhao Wu
jhwu@SLAC.Stanford.EDU

Acknowledgements

Work supported by USDOE contract DE-AC02-76SF00515 (JW, PRB, EJP) and DE-AC02-98CH10886 & Office of Naval Research (JBM, XJW, TW)

References

[1] J.B. Murphy, J. Wu, X.J. Wang, and T. Watanabe, *"Longitudinal Coherence Preservation and Chirp Evolution in a High Gain Laser Seeded Free Electron Laser Amplifier"*, Phys. Rev. Lett. (submitted).

[2] J. Wu, P.R. Bolton, J.B. Murphy, and X. Zhong, *"Free electron laser seeded by ir laser driven high-order harmonic generation"*, Appl. Phys. Lett. **90**, 021109 (2007).

[3] J. Wu, J.B. Murphy, P.J. Emma, X.J. Wang, T. Watanabe, and X. Zhong, *"Interplay of the chirps and chirped pulse compression in a high-gain seeded free-electron laser"*, J. Opt. Soc. Am. B **24**, 484 (2007).

[4] J. Wu, P.R. Bolton, J.B. Murphy, and K. Wang, *"ABCD Formalism and Attosecond Few-cycle Pulse via Chirp Manipulation of a Seeded Free Electron Laser"*, Optics Express **15**, No. 19, p. ???? in press (2007).

[5] J. Wu, P.R. Bolton, "*Coherent X-Ray Production by Cascading Stages of High Gain Harmonic Generation Free Electron Lasers Seeded by IR Laser Driven High-Order Harmonic Generation* ", Proceedings FEL'06 p. 186 (2006).

[6] J. Wu, L.H. Yu, "*Coherent hard X-ray production by cascading stages of High Gain Harmonic Generation*", Nucl. Inst. and Meth. A 475, 104 (2001).

References

[1] J. Arthur et al., *Linac Coherent Light Source Design Study Report*, SLAC Report No. SLAC-R-521 (1998); R. Brinkman et al., *TESLA XFEL: First Stage of the X- Ray Laser Laboratory*, TESLA Report No. TESLA FEL2002-09 (2002).

[2] J.-M. Wang and L.-H. Yu, NIM A 250, 484 (1986).

[3] K.J. Kim, NIM A 250, 396 (1986).

[4] K.J. Kim, Phys. Rev. Lett. 57, 1871 (1986).

[5] K.J. Kim, LBNL Report No. 40672 (1997).

[6] S. Krinsky & Z. Huang, Phys. Rev. ST Accel. Beams 6, 050702 (2003).

[7] E.L. Saldin, E.A. Schneidmiller, & M.V. Yurkov, *FEL2005*, 258 (2005).

[8] E. Wigner, Phys. Rev. 40, 749 (1932).

[9] M.J. Bastiaans, Optik 82, 173 (1989).

[10] R. Bonifacio, et al, NIM A 296, 358 (1990).

[11] G.T. Moore and N. Piovella, IEEE Jour. Quantum Elec.27, 2522 (1991).

[12] K.B. Wolf, *Integral Transforms in Science and Engineering*, Plenum (1979).

[13] R. Ortega-Martinez, et al, Rev. Mex. de Fisica 48, 565 (2002).

[14] S.P. Dijaili, A. Dienes, & J.S. Smith, IEEE Jour. Quantum Elec. 26, 1158 (1990).

[15] B.H. Kolner, IEEE Jour. Quantum Elec. 30, 1951 (1994).

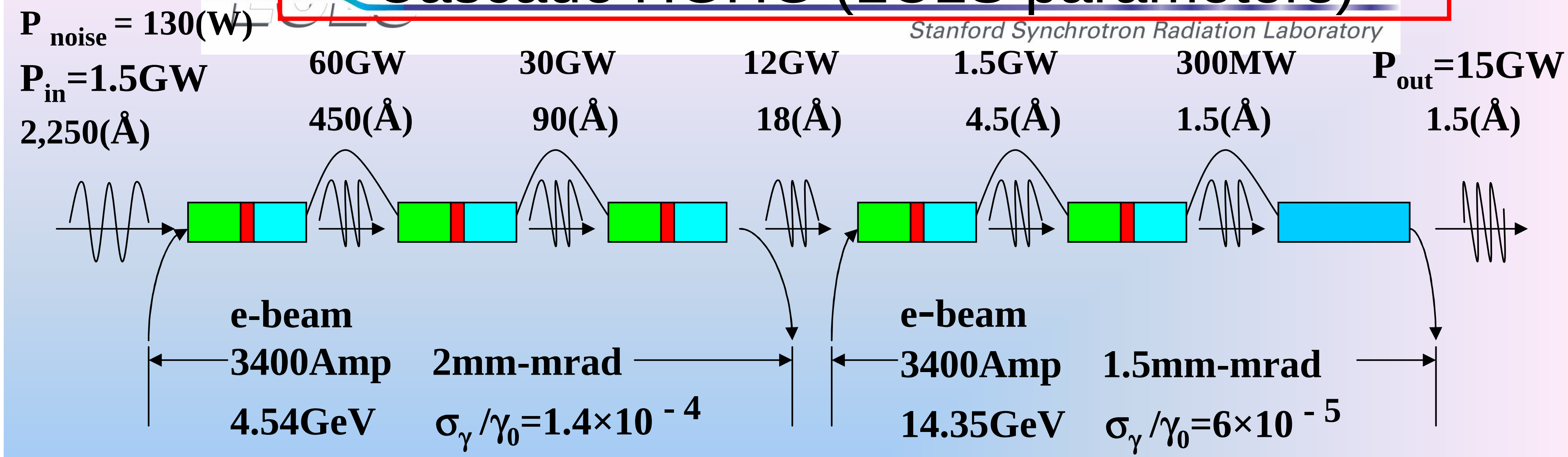
[16] L. Giannessi, P. Musumeci, & S. Spampinati, J. Appl. Phys. 98, 043110 (2005).

[17] B.W.J. McNeil, *et al.* , New J. of Phys. 9, 82 (2007).

[18] P. Emma *et al.*, Phys. Rev. Lett. 92, 074801 (2004); A.A. Zholents and W.M. Fawley, Phys. Rev. Lett. 92, 224801 (2004), E.L. Saldin *et al.*, Phys. Rev. ST –Accel. Beam 9, 050702 (2006).

Stanford Linear Accelerator Center
Stanford Synchrotron Radiation Laboratory

Cascade HGHG (LCLS parameters)



	1^{st} Stage		2^{nd} Stage		3^{rd} Stage		4^{th} Stage		5^{th} Stage		Amplifier
λ(Å)	2250	450	450	90	90	18	18	4.5	4.5	1.5	1.5
λ_w(cm)	10.7	6.9	6.9	4.6	4.6	3.2	5.5	3.9	3.9	3	3
$d\psi/d\gamma$	0.53		0.845		0.59		0.13		0.16		
σ_γ / γ	1.43×10^{-4}		1.46×10^{-4}		1.48×10^{-4}		8.52×10^{-5}		1.13×10^{-4}		1.38×10^{-4}
L_w(m)	2	8	0.2	8.5	0.3	12	4	5.5	8	9	37.5
L_G(m)	1.1	1.0	1.0	1.0	1.0	1.4	2.6	3.3	3.3	5.4	5.4

L_{total} = 95m to reach 15 GW at 1.5 Å using 5^{th} and 3^{rd} harmonics

NSLS
Brookhaven National Laboratory

Juhao Wu
jhwu@SLAC.Stanford.EDU