Microbunching from Shot Noise in Beam Delivery Systems for x-FELs:

Modelling by Vlasov Solver Methods

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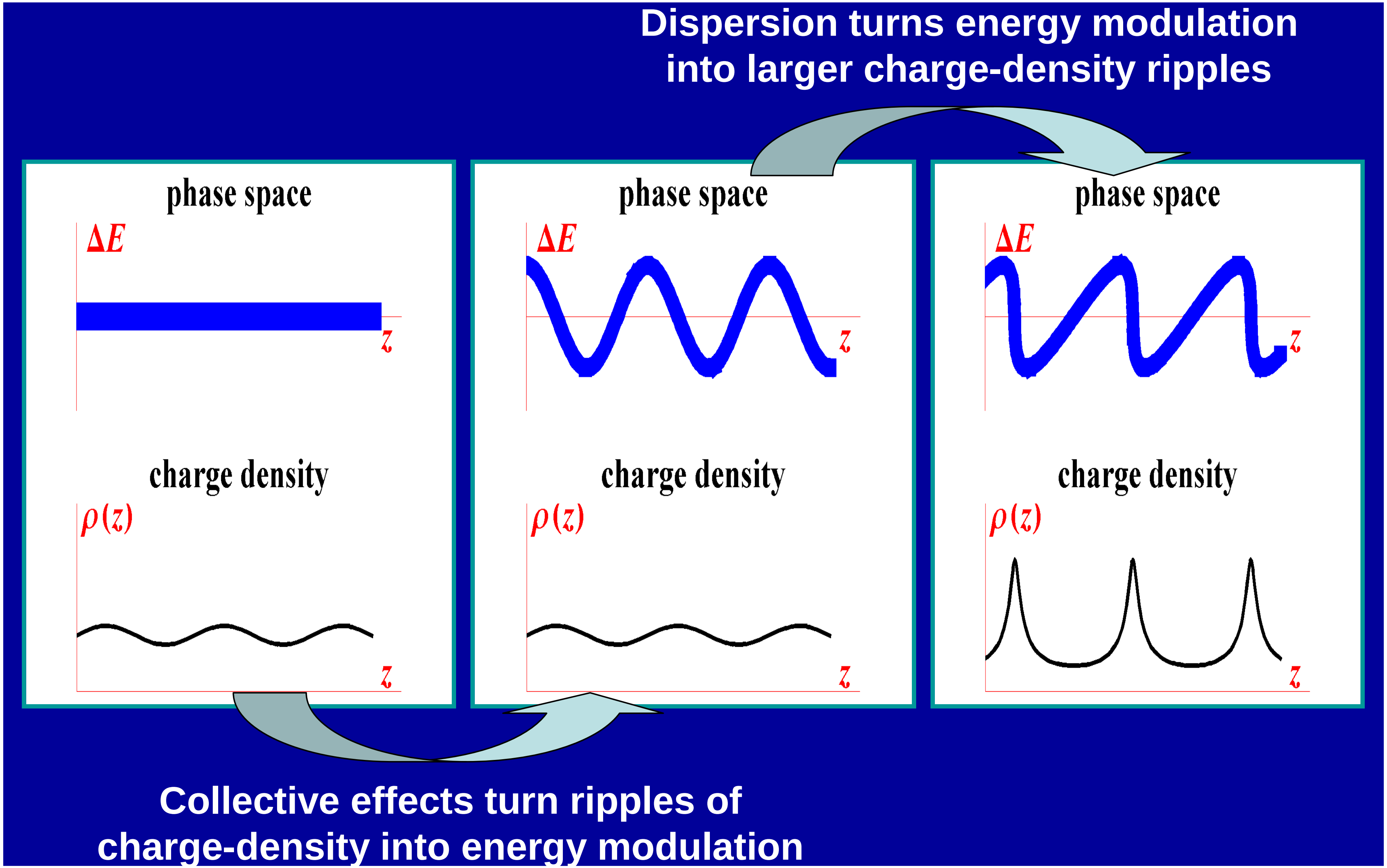
Isola d'Elba, September 9, 2007

What is the microbunching instability?



Motivations

- The microbunching instability can cause unacceptable degradation of beam quality in the longitudinal phase space
- Controlling the instability is important for x-rays FEL design
  - Has consequences on design choices/hardware (*e.g.* 'laser heater')
- It is an issue in particular for FERMI@Elettra
  - 150 keV max. uncorrelated energy spread desired in undulators
- Shot noise is the most fundamental (and unavoidable) source of charge fluctuations seeding the instability
  - Other sources may be important but are not considered here.

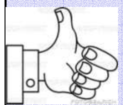
Simulating the micrubunching instability is challenging

- The instability is by its nature sensitive to small fluctuations in phase space density
- Good resolution of phase space needed

- **Three distinct methods are currently being used:**
  - **Linear analysis**
  - **Macro-particle simulations**
  - **Vlasov solvers**

Pros & cons of Vlasov solvers

- **Pros**:

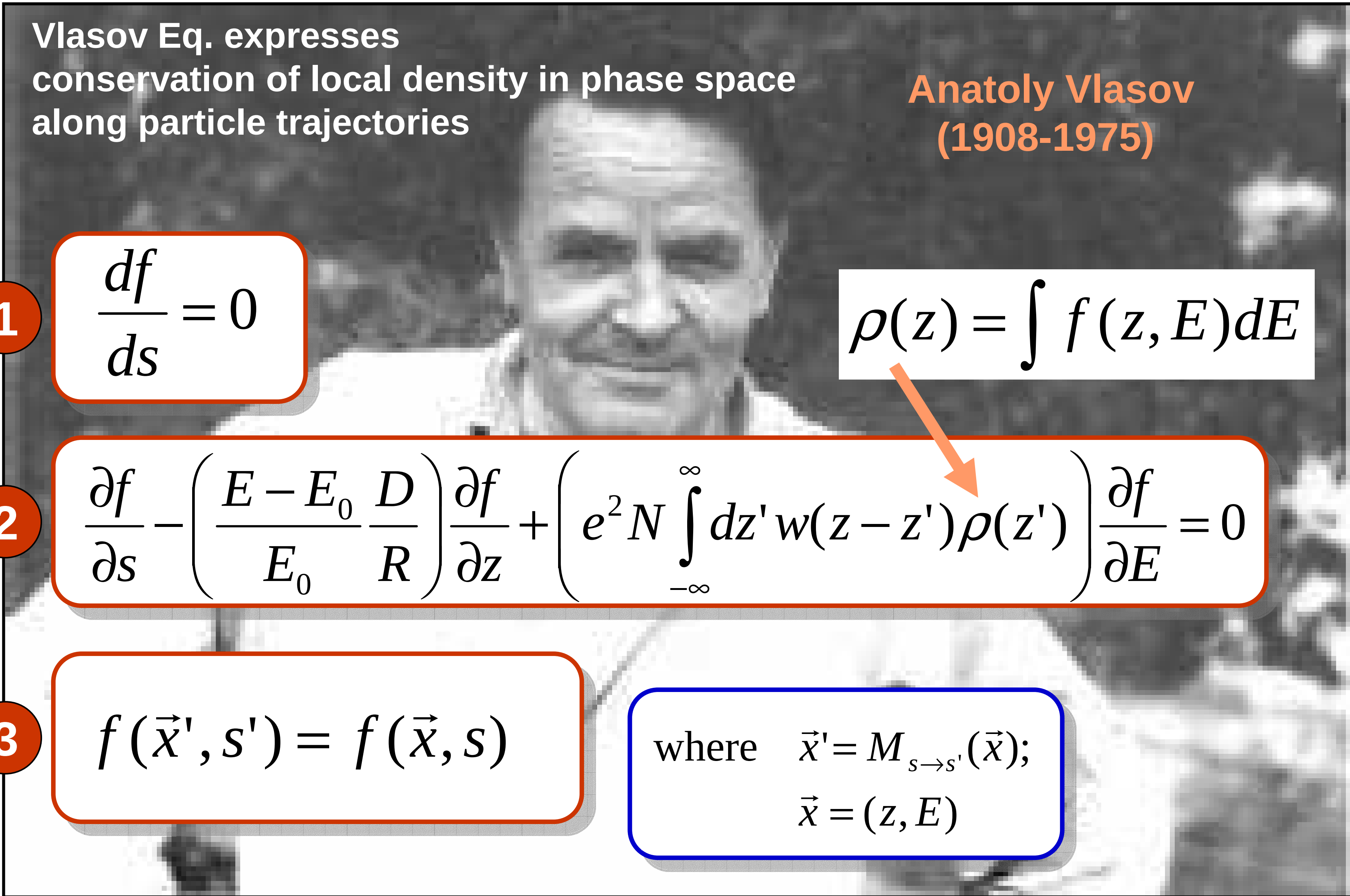


  - Avoids spurious fluctuations caused by finite number of macroparticles
  - Can resolve fine structures in low-populated regions of phase space
  - More accurate detection of instability

- **Cons**:

  - Computationally more intensive in higher dimension
  - Requires simplified modelling of collective forces in low dimension
  - Density representation on a grid introduces spurious smoothing.

Three ways of writing the Vlasov equation



Vlasov Eq. expresses conservation of local density in phase space along particle trajectories

Anatoly Vlasov (1908-1975)

1

$$\frac{df}{ds} = 0$$

$$\rho(z) = \int f(z, E)\, dE$$

2

$$\frac{\partial f}{\partial s} - \left(\frac{E - E_0}{E_0} \frac{D}{R} \right) \frac{\partial f}{\partial z} + \left(e^2 N \int_{-\infty}^{\infty} dz'\, w(z - z') \rho(z') \right) \frac{\partial f}{\partial E} = 0$$

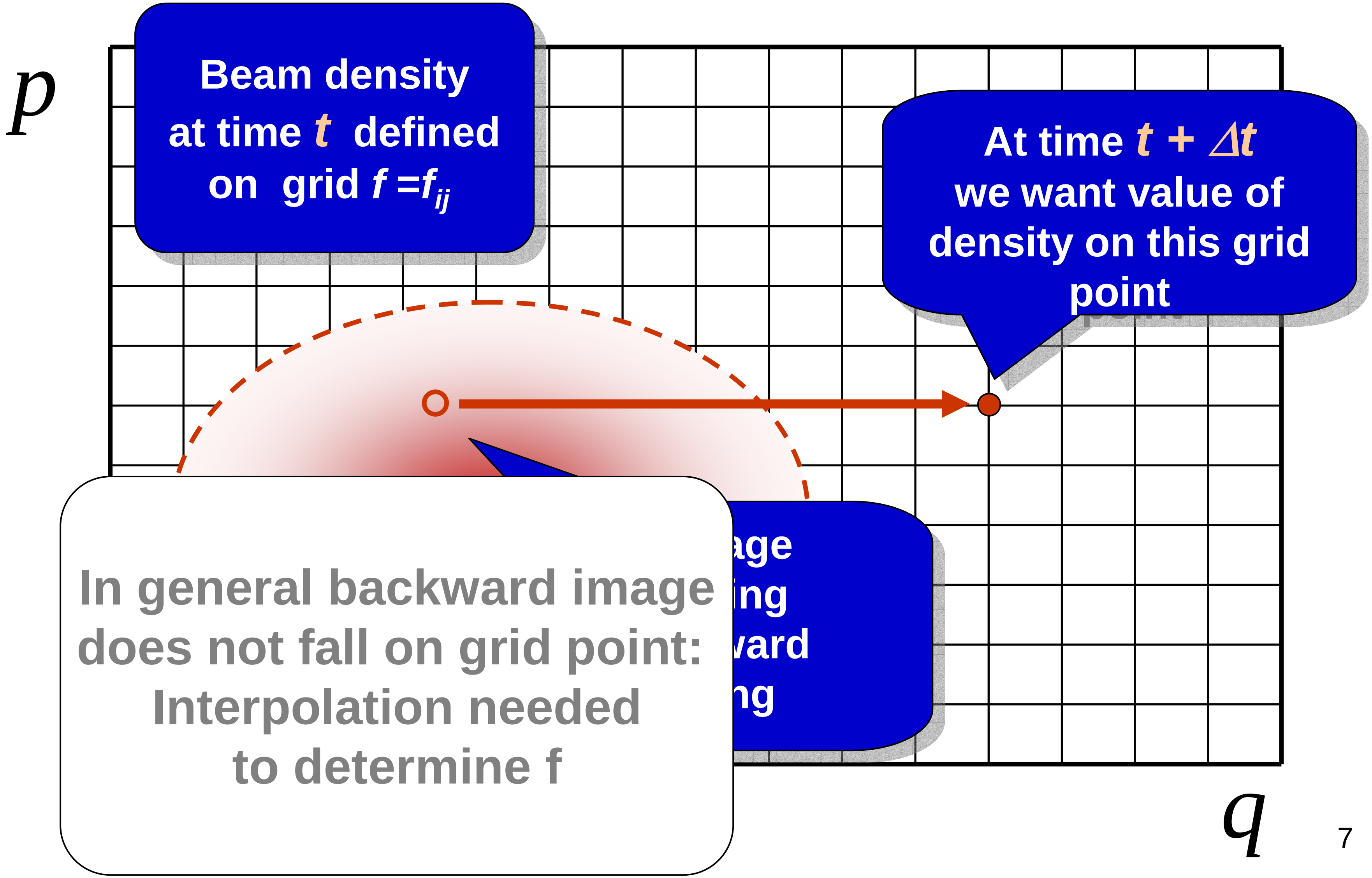
3

$$f(\vec{x}', s') = f(\vec{x}, s)$$

where $\vec{x}' = M_{s \to s'}(\vec{x});$

$\vec{x} = (z, E)$

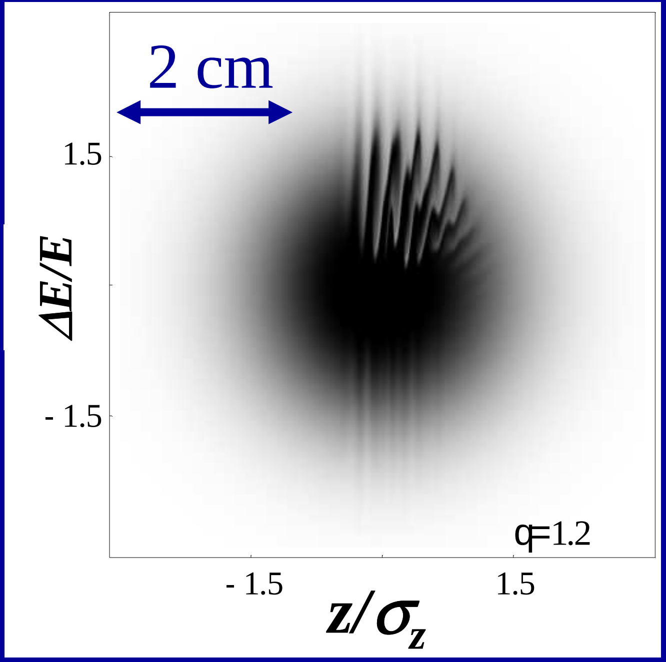
Propagate density one-step forward: *e.g.* drift



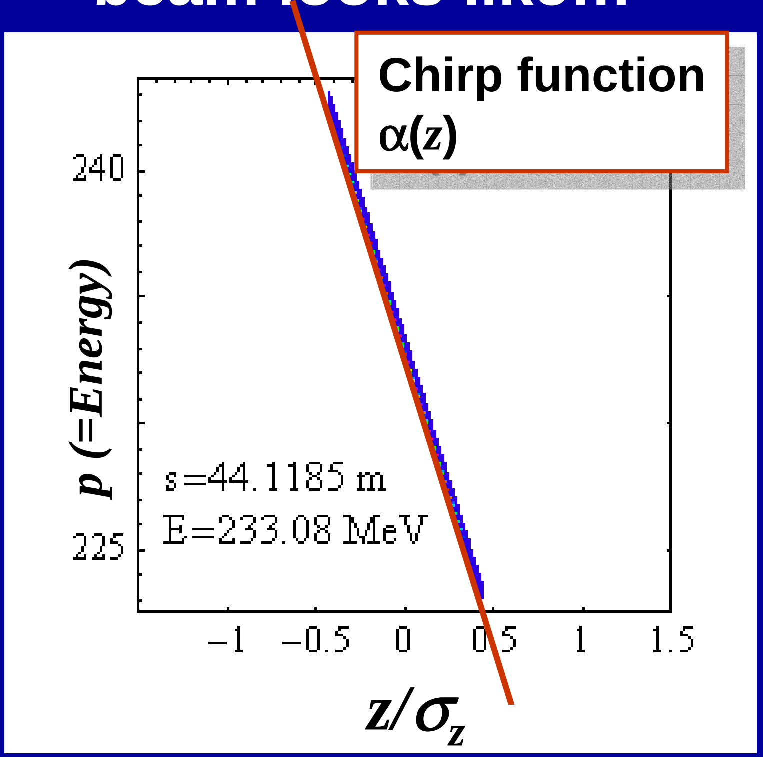
Chirped beams pose some technical problems …

We would like to use a rectangular grid ….

This is the beam we like…



This is what a chirped beam looks like…



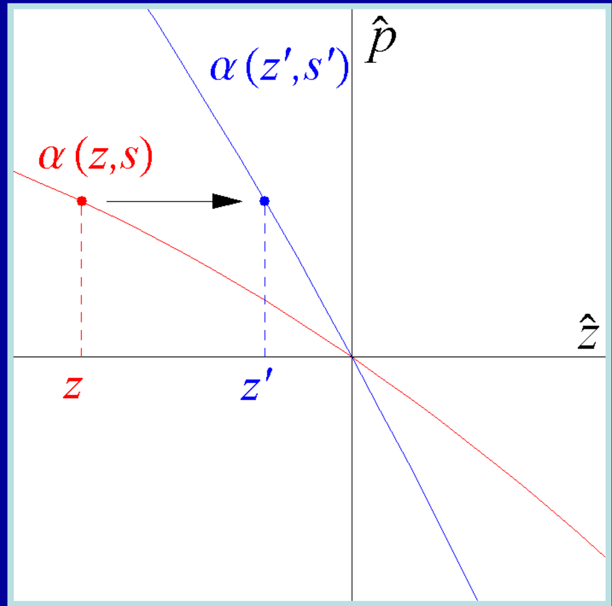
Transform away the *z/E* correlation

Transformation to "capped" coordinates

$$\hat{z} = z$$
$$\hat{p} = p - \alpha(z, s).$$

Chirp (correlation) function

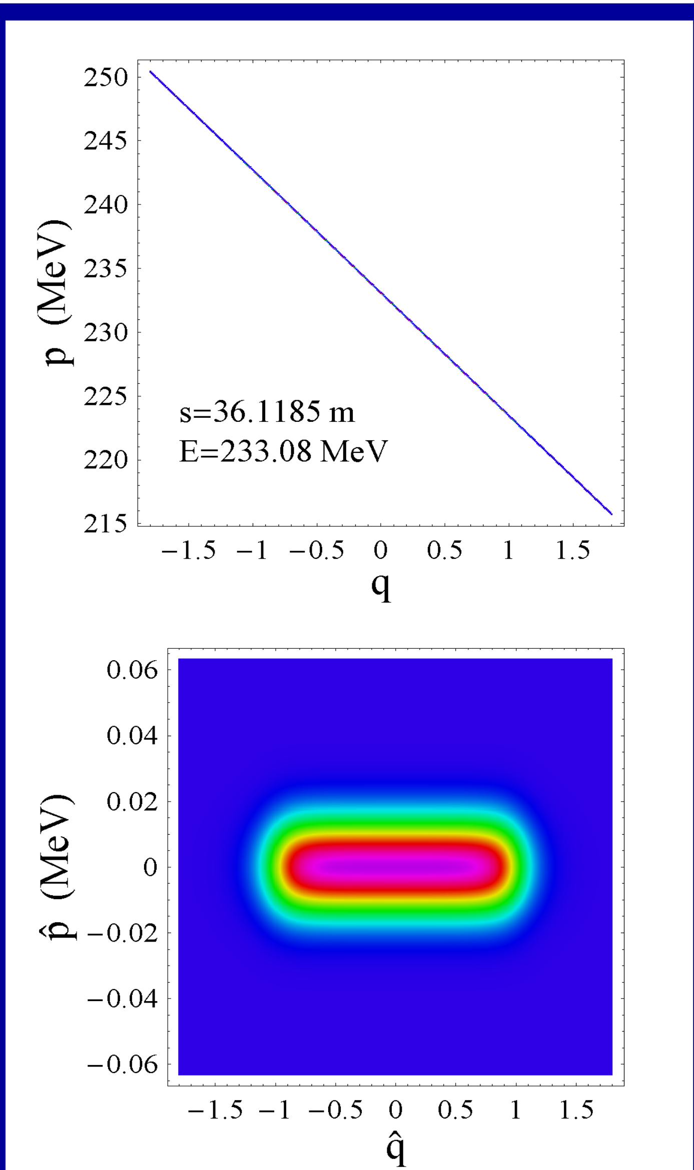
Chirp function evolves like the support of a beam with zero uncorrelated energy spread



Initial chirp function:

$$\alpha(z, s_0) = \int_{-\infty}^{\infty} dp p f(z, p; s_0) / \int_{-\infty}^{\infty} dp f(z, p; s_0)$$

In transformed coordinates
beam density looks good (= it nicely fills the grid)



Beam density in
z/E **coordinates**

Beam density in the transformed coordinates

Here we can use
a rectangular grid efficiently

Collective effects & account of effect of a finite emittance on longitudinal slippage

Starting from the 4D Vlasov equation make some ansatz on form of density function and average over transverse coordinates

$$F_{\rm sm}(z) = -e^2 Nc \int_{-\infty}^{\infty} dk\ \hat{Z}(k)\hat{\rho}(k)e^{ikz}e^{-k^2\underbrace{\sigma_\perp^2}/2}$$

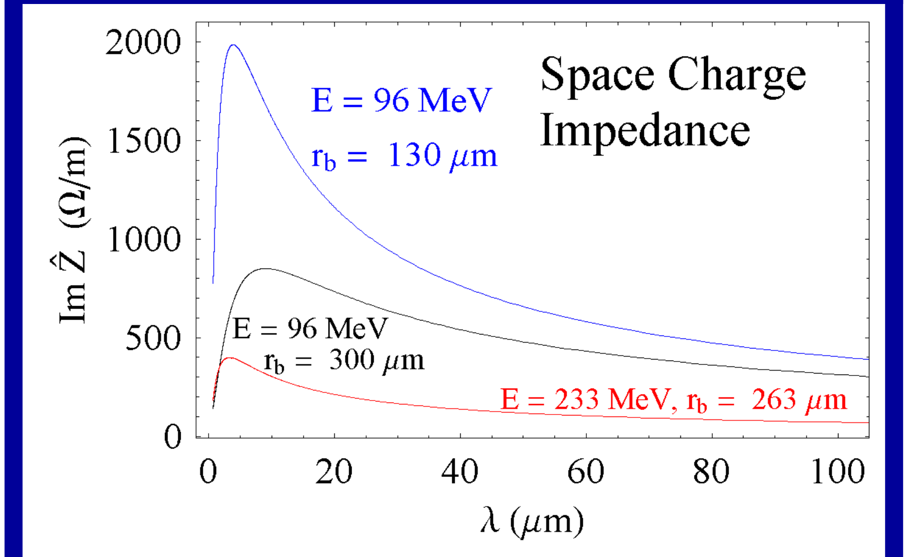
$$\sigma_\perp = \sqrt{2\varepsilon_x \mathcal{H}}$$

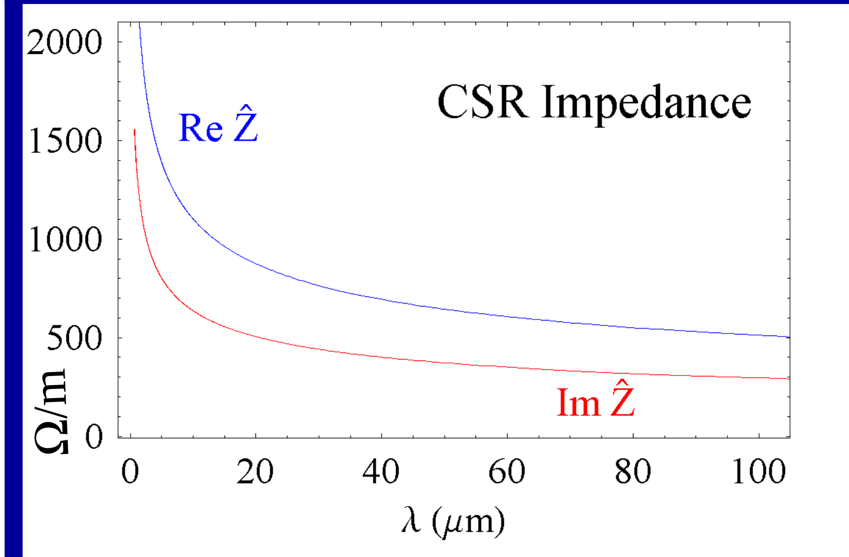
$$\mathcal{H} = \gamma_x D^2 + 2\alpha_x DD' + \beta_x (D')^2$$

Length-scale for emittance-induced slippage in z

Model of finite transverse emittance equivalent to low pass-filter

Collective effects are evaluated using impedance models





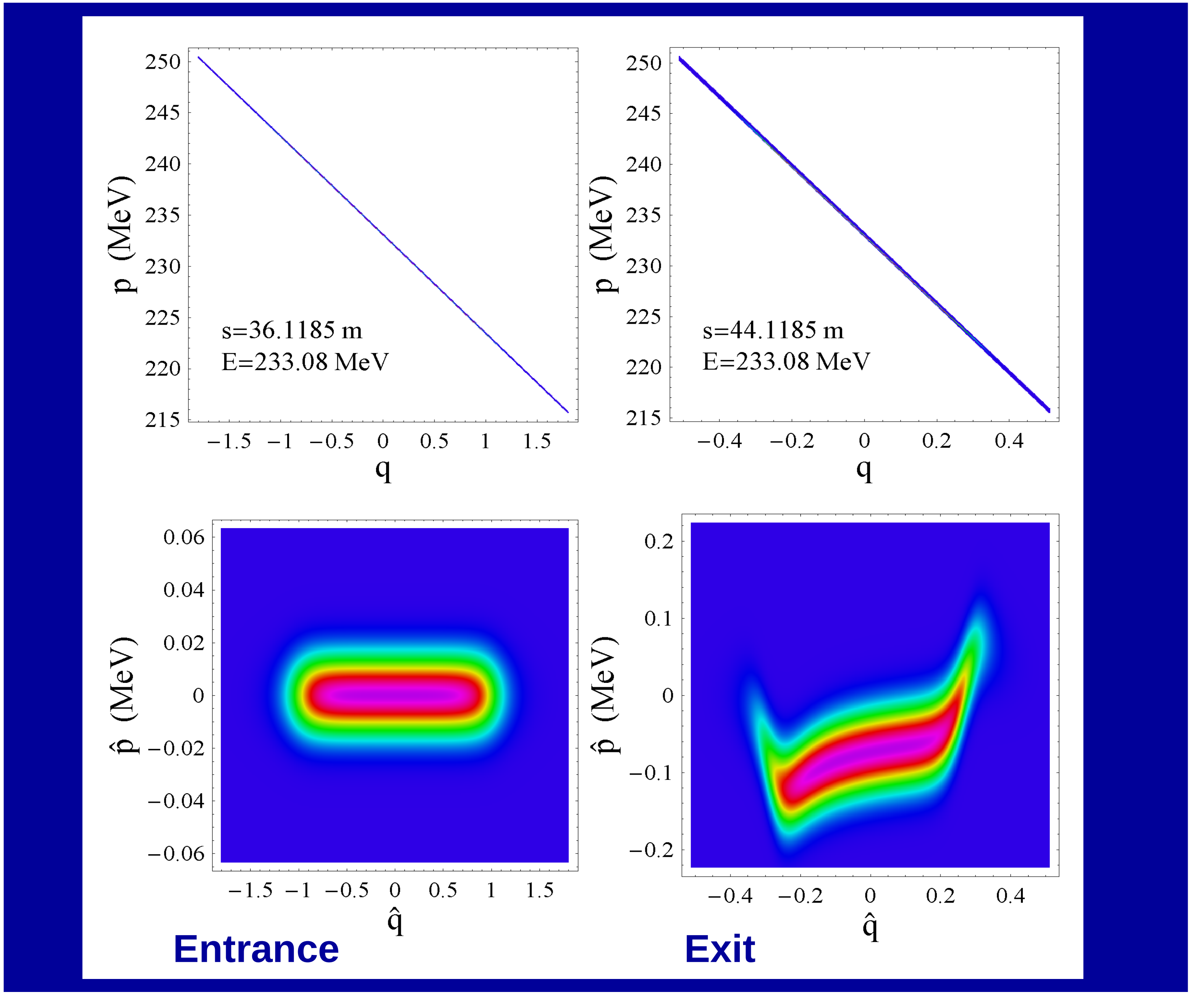
$$\hat{Z}(k) = \frac{iZ_0}{\pi\gamma r_b^2} \frac{1 - xK_1(x)}{x}\bigg|_{x=kr_b/\gamma},$$

- On-axis field from transversely uniform charge density with circular cross-section
- Free space

$$\hat{Z}(k) = Z_0 \frac{\Gamma(2/3)}{4\pi R 3^{1/3}}[\sqrt{3} + i](kR)^{1/3}$$

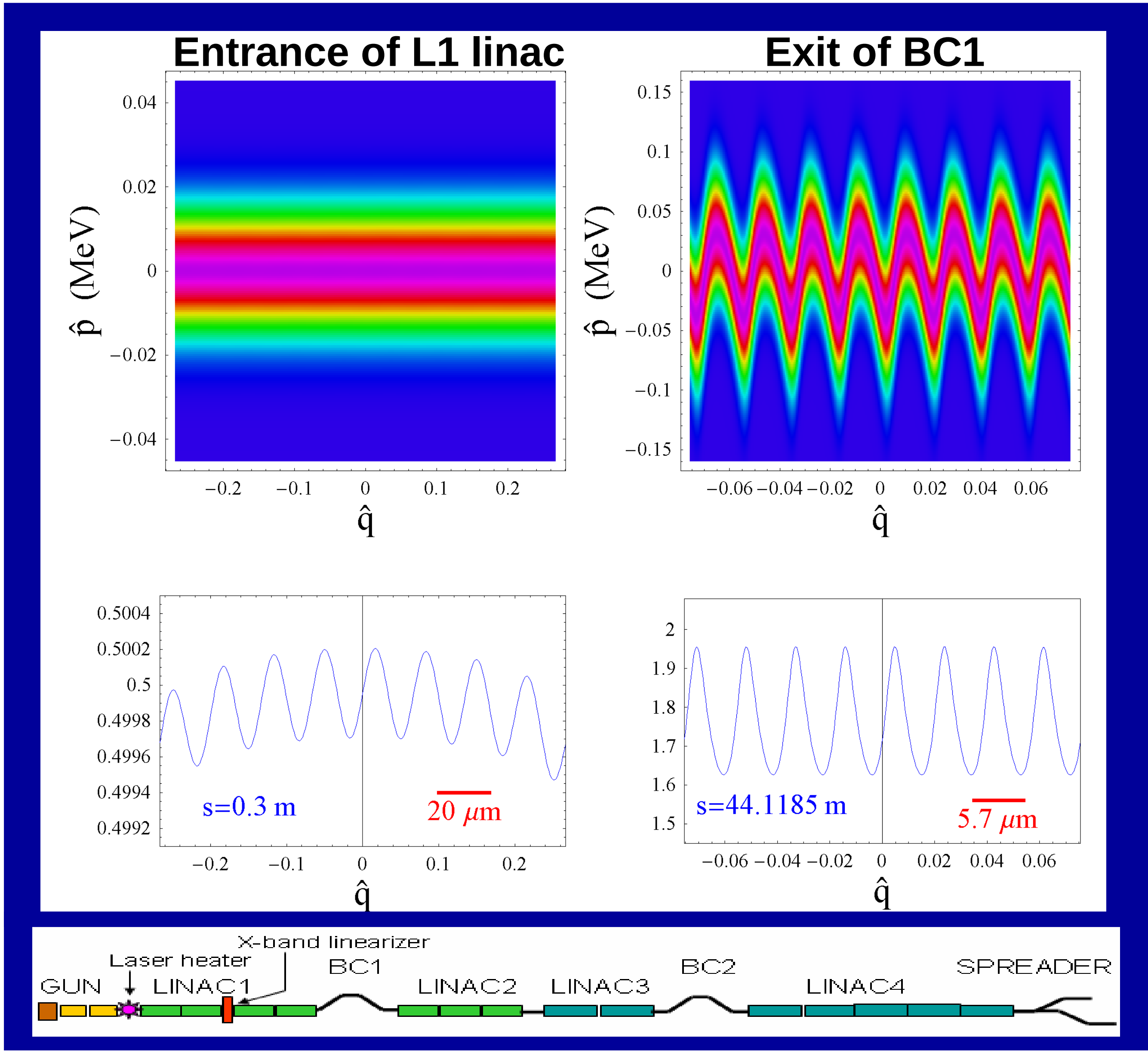
- Model of beam in uniform motion on circular orbit
- Free space

Example of beam propagation through a bunch compressor



CSR only

Microbunching instability: determining the small-amplitude gain function



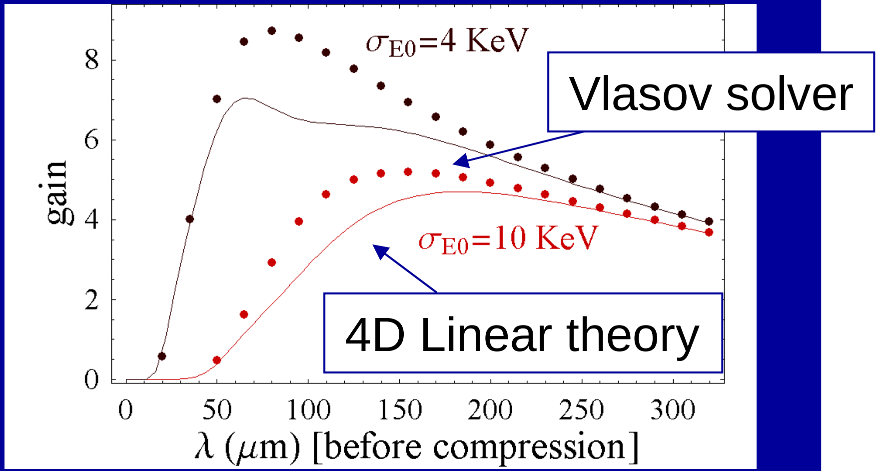
Space charge + CSR

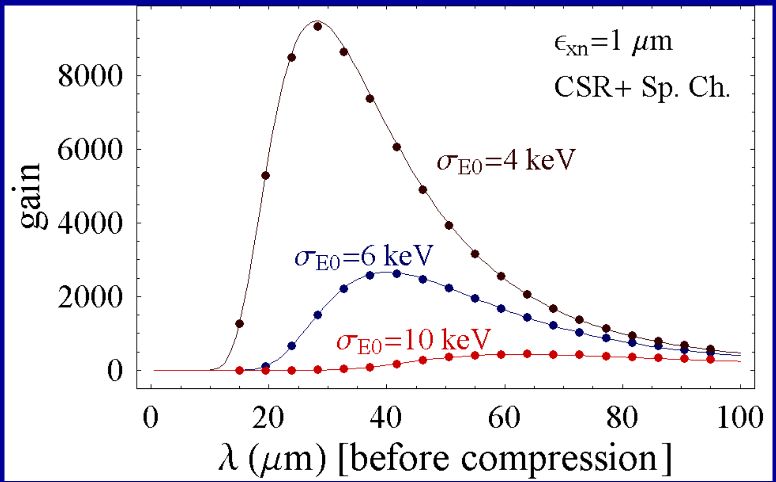
initial E = 95 MeV
σ_E= 10 KeV
peak curr. = 95.5 A
compr. factor = 3.52

Gain factor is about 170

Contact with linear theory validates solver

Small Amplitude Gain Function (L1 through BC2)





CSR only (space charge turned off)

Discrepancy between 4D linear theory and Vlasov solver due to approximate account of transverse dynamics by solver

CSR + space charge

FERMI Two-BC lattice

How do we model shot noise?

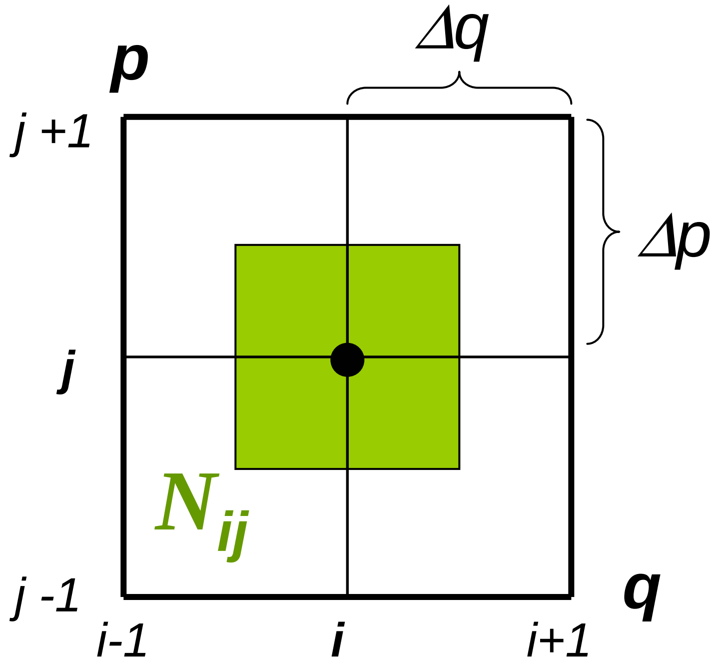
- Place a random perturbation on top of initial smooth distribution on grid

- **No. of electrons in phase-space cell obey the Poisson statistics:**

$$N_{ij} = \langle N_{ij} \rangle + \langle N_{ij} \rangle^{1/2} \xi_{ij}$$

$$\langle N_{ij} \rangle = N f_{ij}^{(0)} \Delta q \Delta p$$

Normal stochastic process: average=0 variance=1



$$f_{ij} = f_{ij}^{(0)} \left(1 + \frac{\xi_{ij}}{\langle N_{ij} \rangle^{1/2}} \right)$$

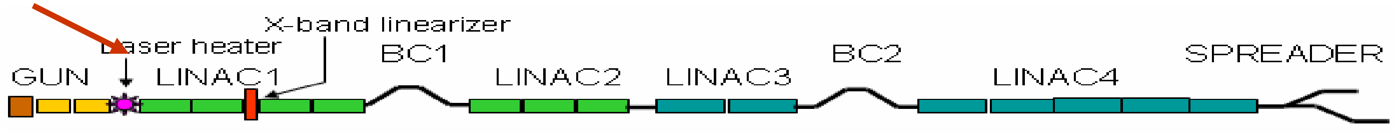
$f_{ij}^{(0)}$ normalized to unity

Beam dynamics with shot noise for Two-BC FERMI@Elettra Lattice

- Initial phase-space beam has
  - uniform z-density, gaussian energy density
  - + random perturbation to model shot noise
- Peak current at extraction $I_f = 1kA$

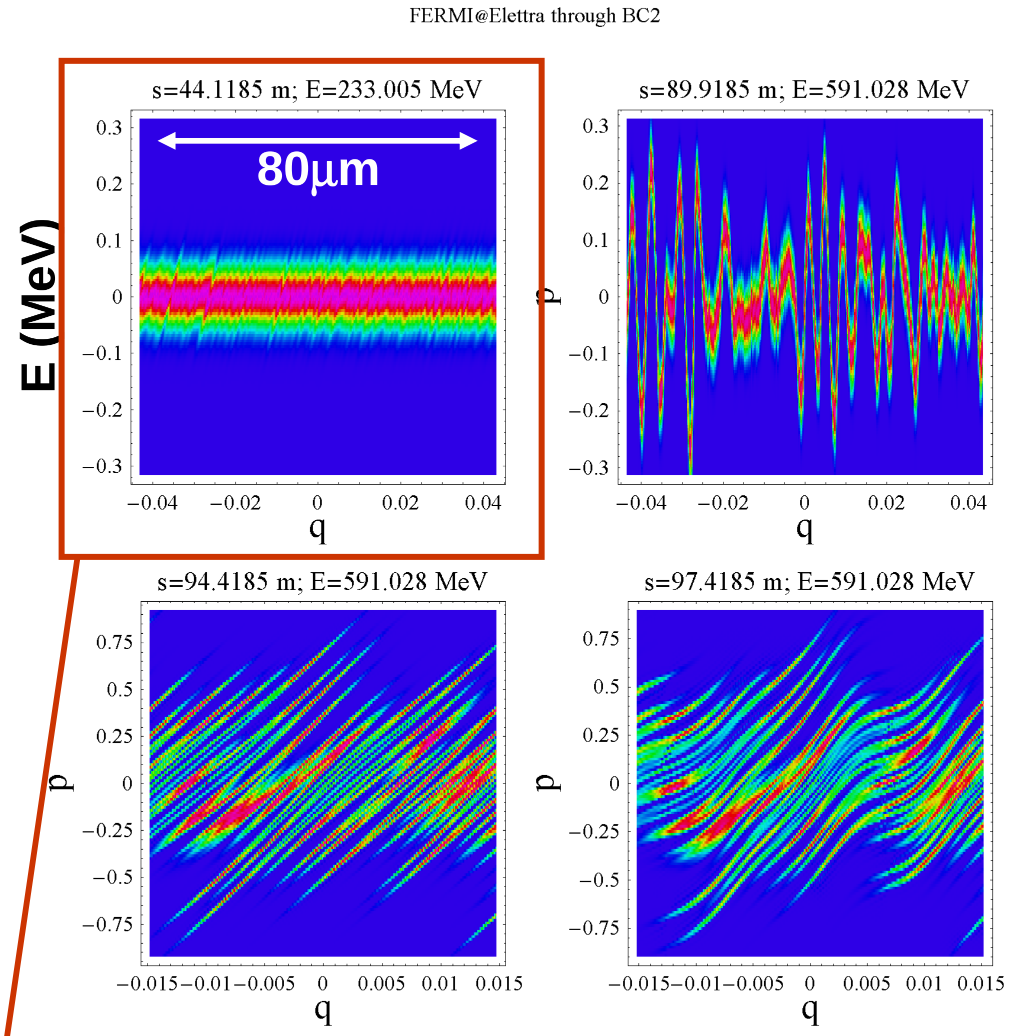
Simulation starts here:

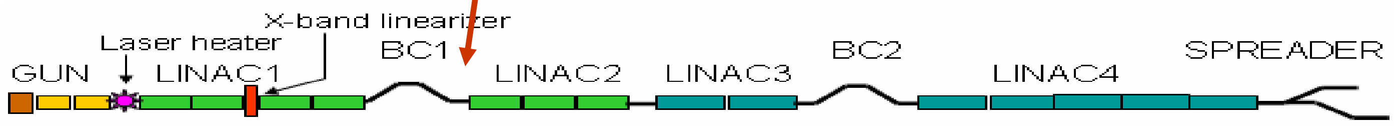
$E = 96$ MeV, $\sigma_{E0} = 10$ keV, $\varepsilon = 1 \mu m$



…beam at exit of BC1

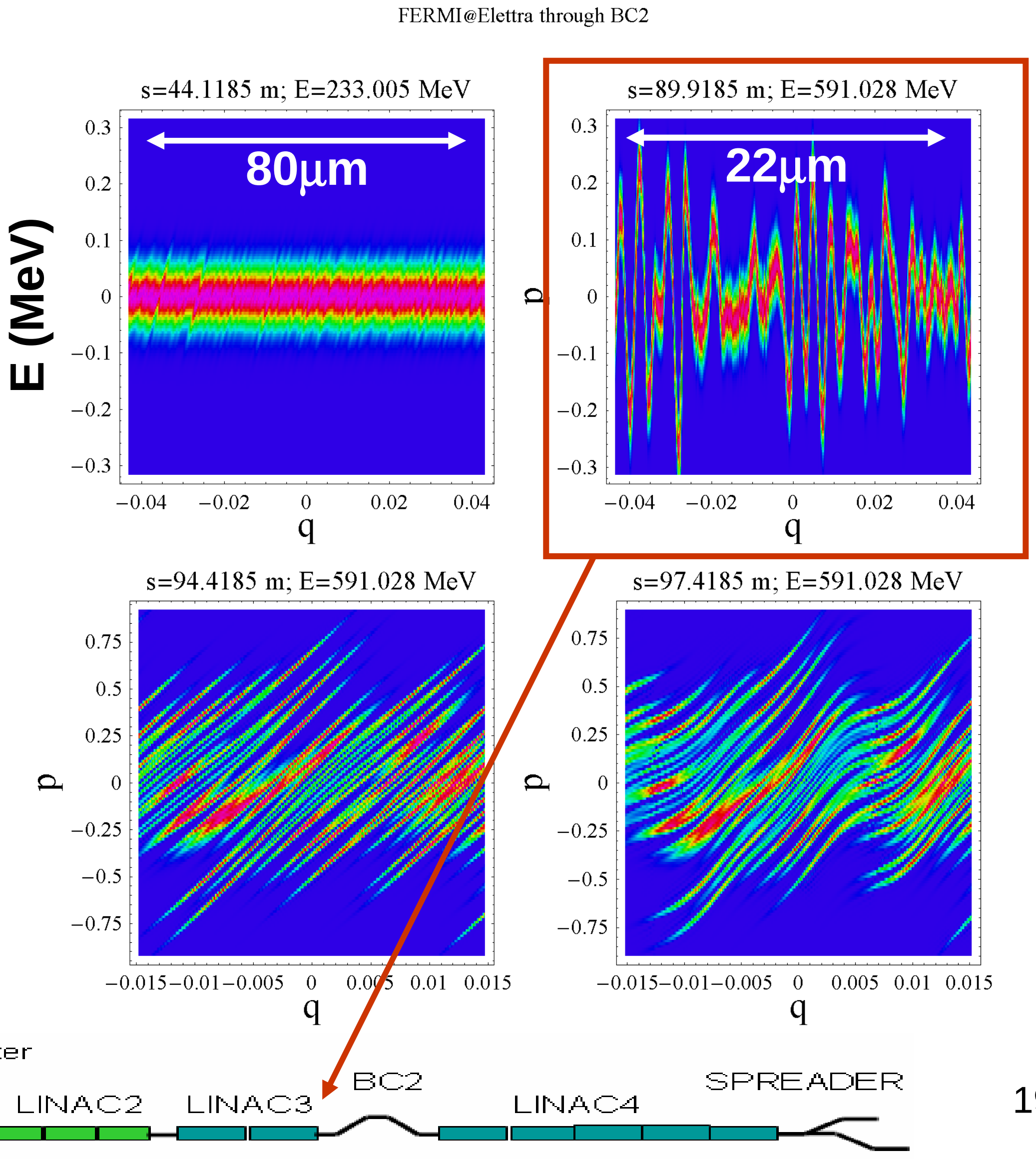
- Energy modulation induced by collective effect starts to become visible





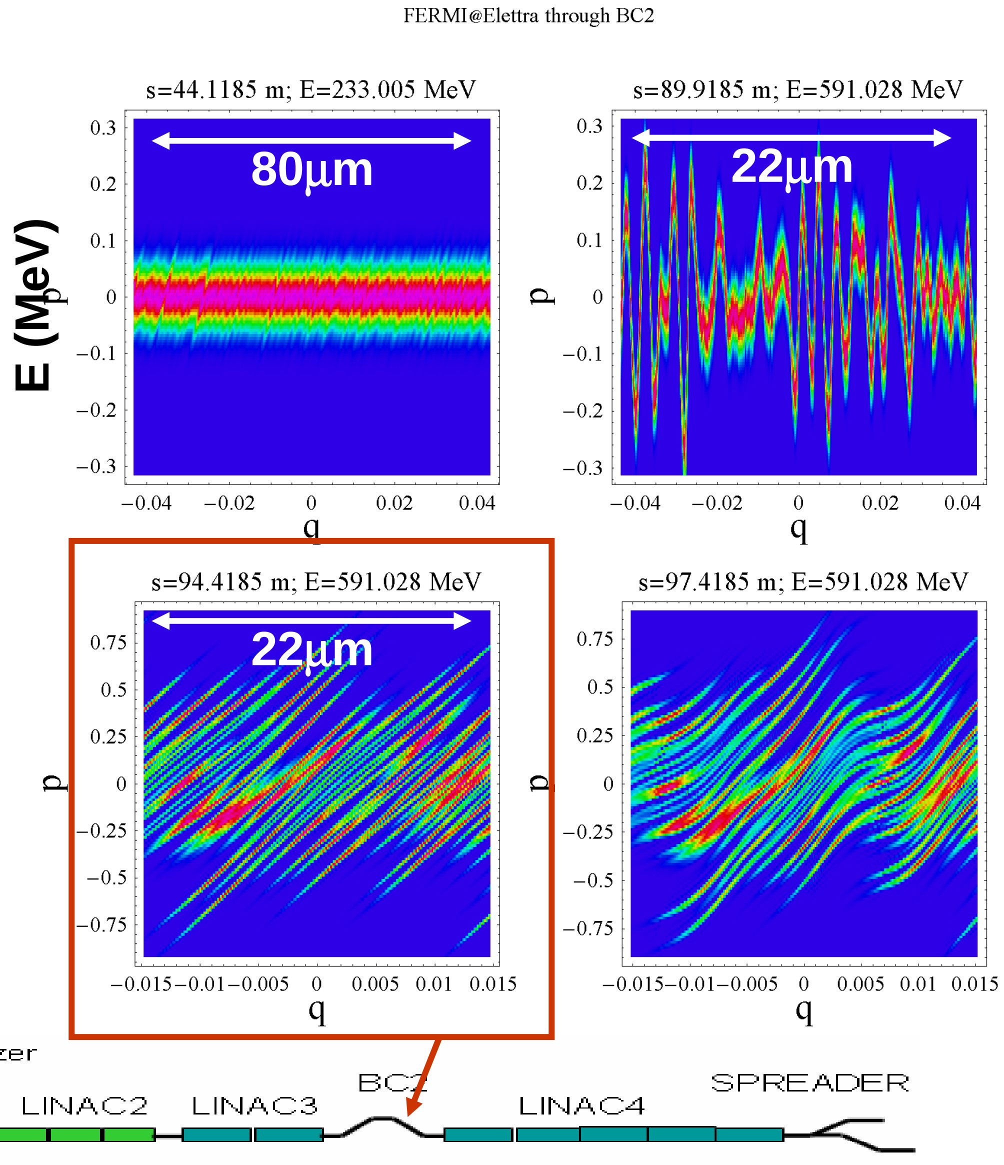
…beam at entry of BC2

- Charge density fluctuations in the few %s range by the end of BC1 seed a large energy modulation by the time beam enters BC2.



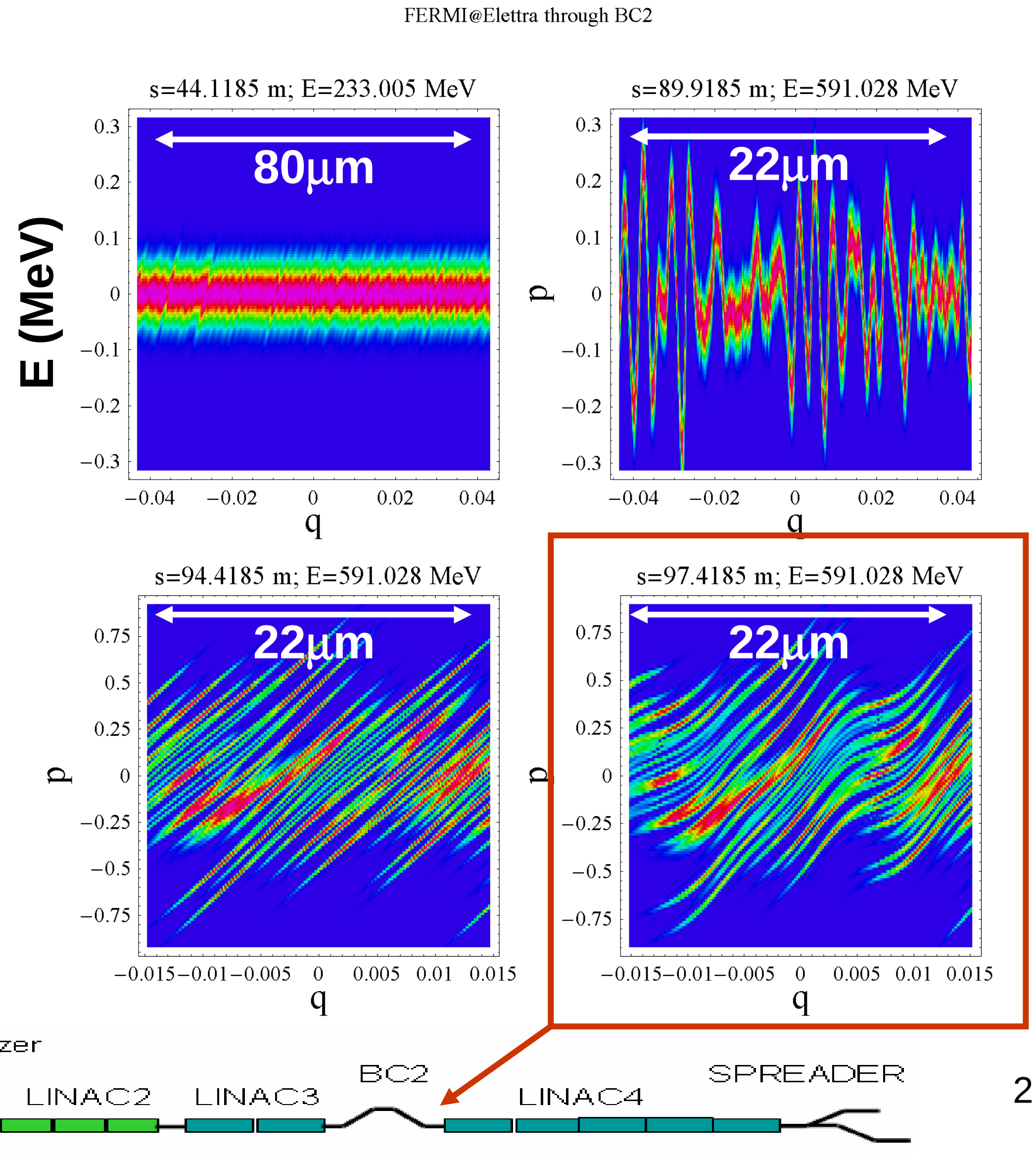
…beam after 3rd bend of BC2

- Evidence of saturation by the exit of the 3rd bend in BC2



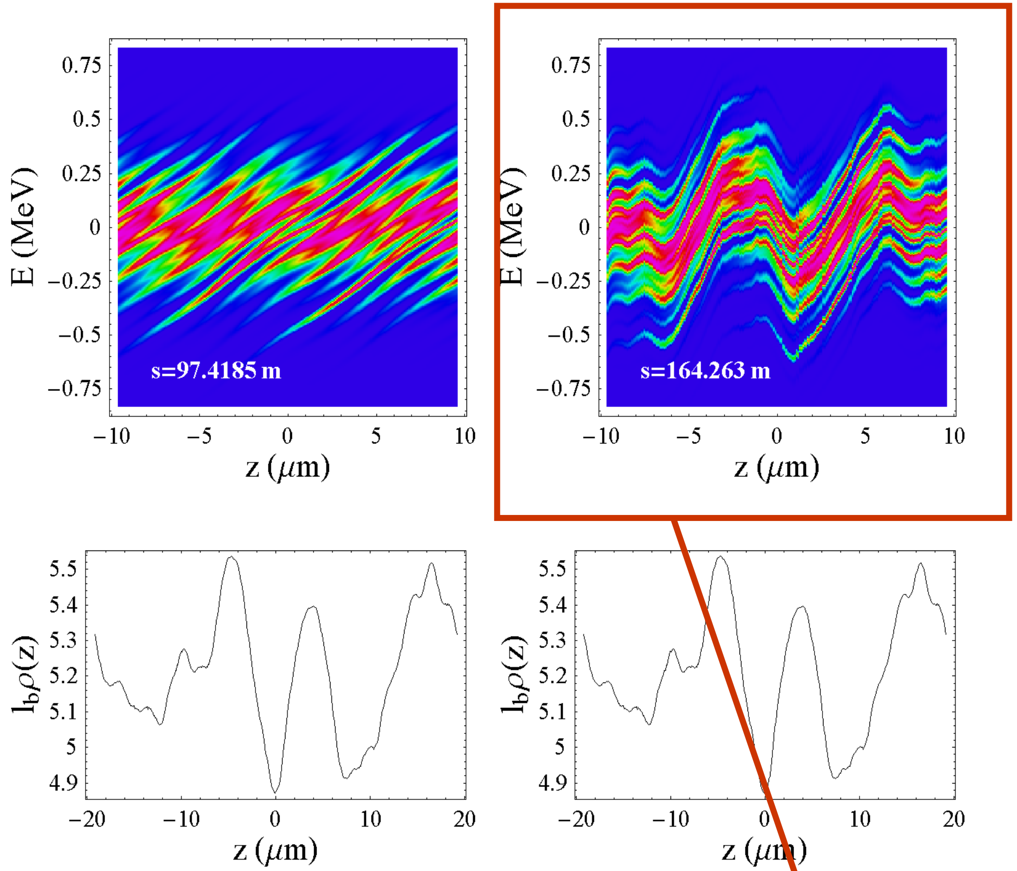
…beam at exit of BC2

- Last bend of BC2 has modest impact.

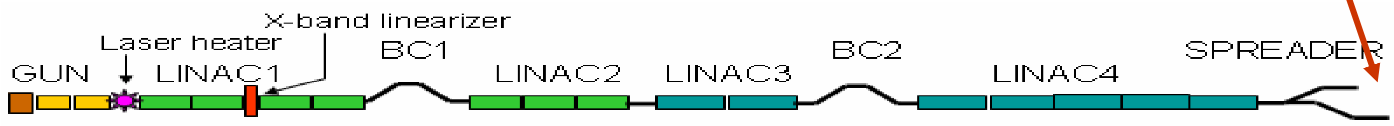


… beam after spreader

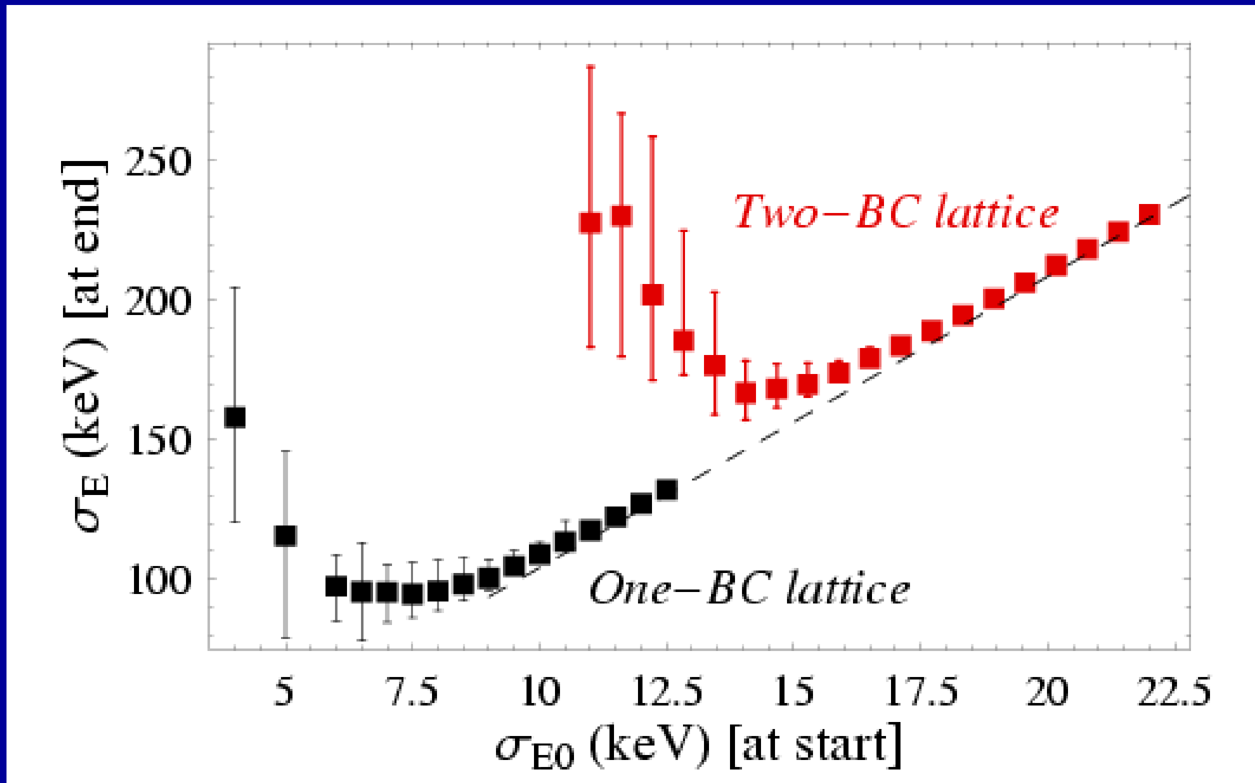
- Space charge adds further energy modulation in the linac after BC2



$(\ \sigma_{E0} = 13\ keV\ )$

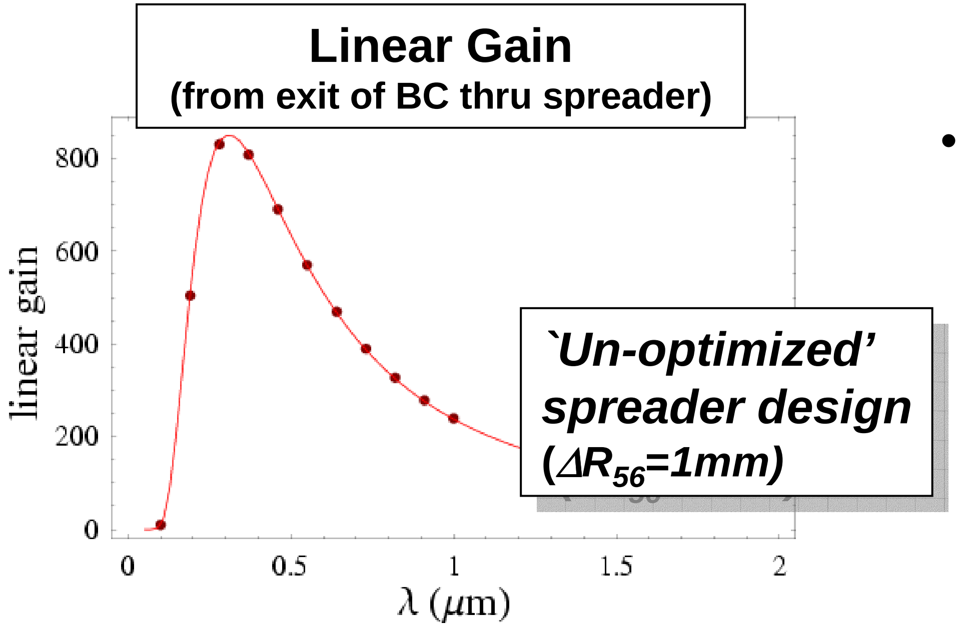


What the minimum achievable uncorrelated energy spread at extraction for FERMI ?

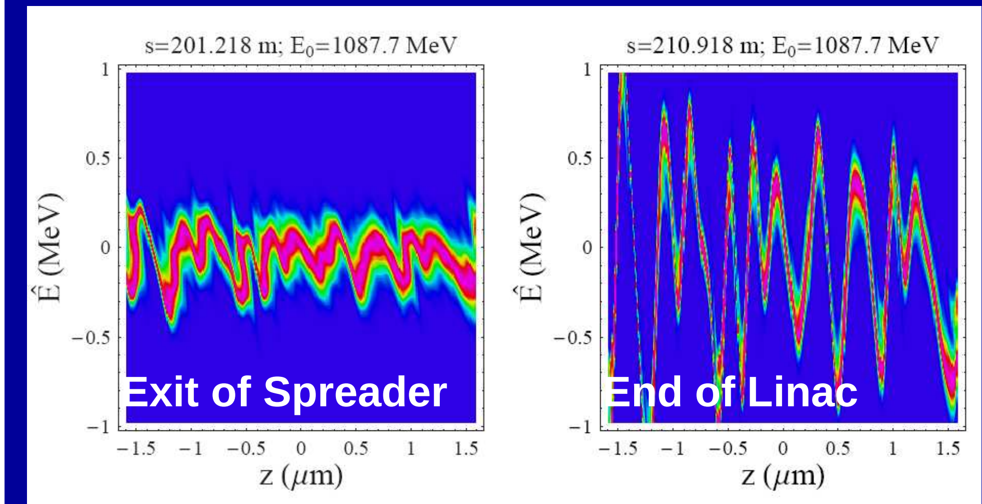


- The One-BC lattice found to meet specifications for beam energy spread.

Design of spreader affects microbunching

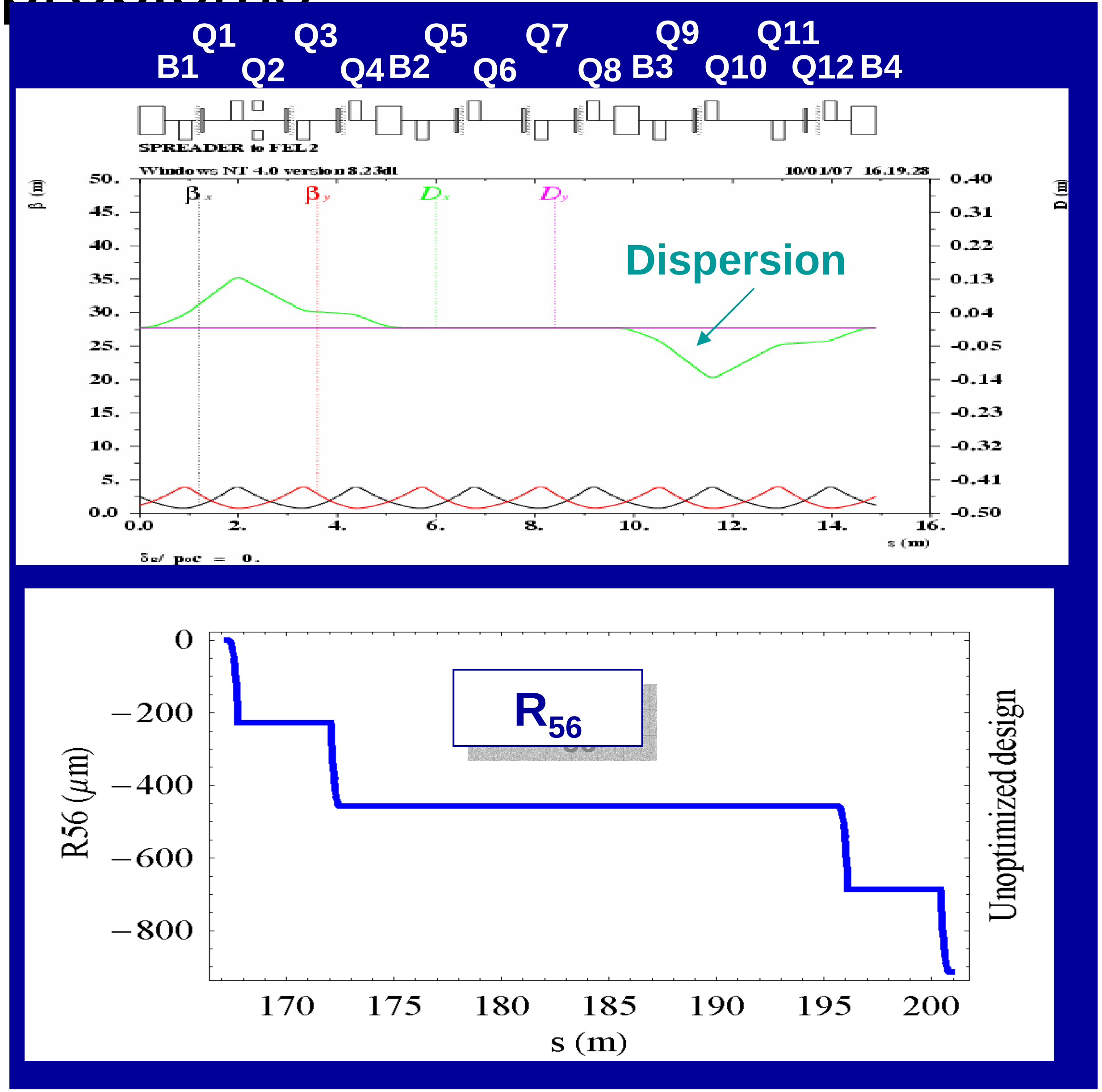


- The 1D space-charge model predict that a fairly small ΔR_{56} (~ mm) can result into a large gain in the sub μm wavelength range



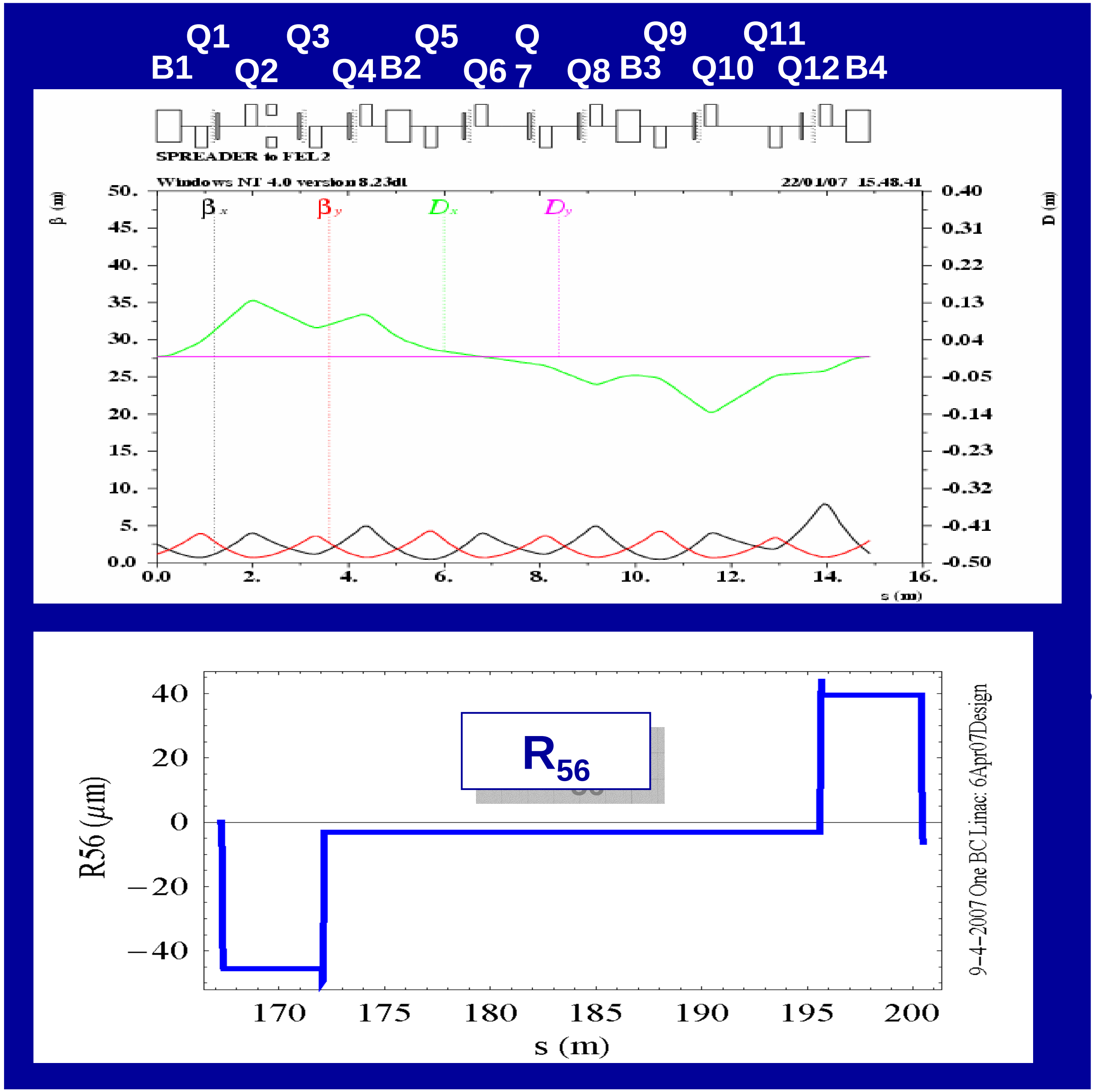
- **The Vlasov solver shows energy modulations of almost 1MeV at exit of Linac for 'un-optimized' spreader design**

Spreader with large ΔR_{56} ('un-optimized' design) causing problems



- Two pairs of bends (dogleg); 100 mrad bending
- Each pair is a perfect achromat

Spreader design with reduced ΔR_{56} works OK



Quadrupoles Q2 and Q10 are tuned to provide a closed dispersion bump and adjust R_{56} to zero. (A. Zholents)

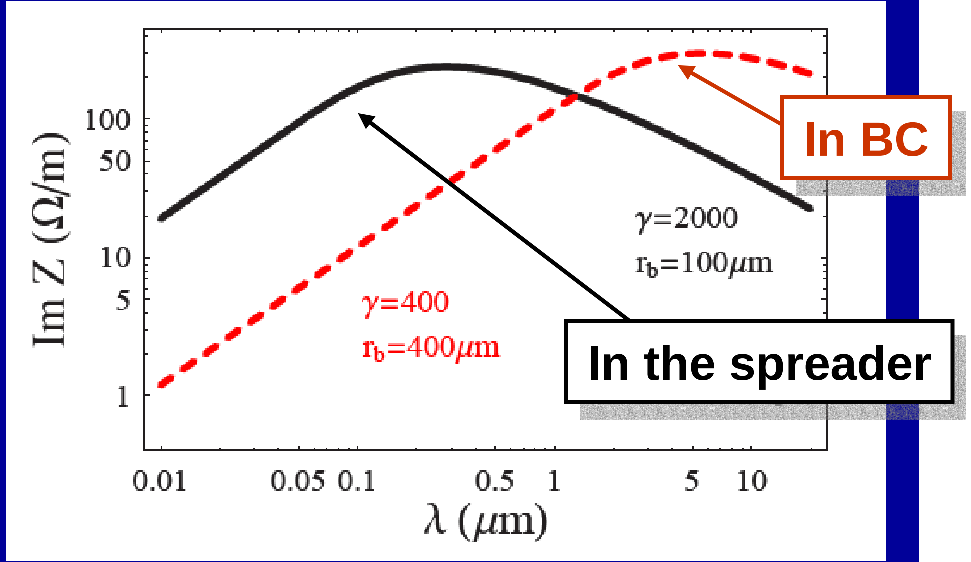
Simulations for lattice with this design show no noticeable rms energy spread increase because of the spreader

Is the predicted effect in un-optimized spreader real?

- **The peak of gain (~0.3μm) corresponds to maximum of space-charge impedance for beam in the spreader region**
- **Maximum occurs at**

  $\lambda = 2\pi r_b/\gamma$

1D Space-Charge Long. Impedance



Caveat:

- Validity of 1D model of SC impedance breaks down for $\lambda <\sim 2\pi r_b/\gamma$

Conclusions

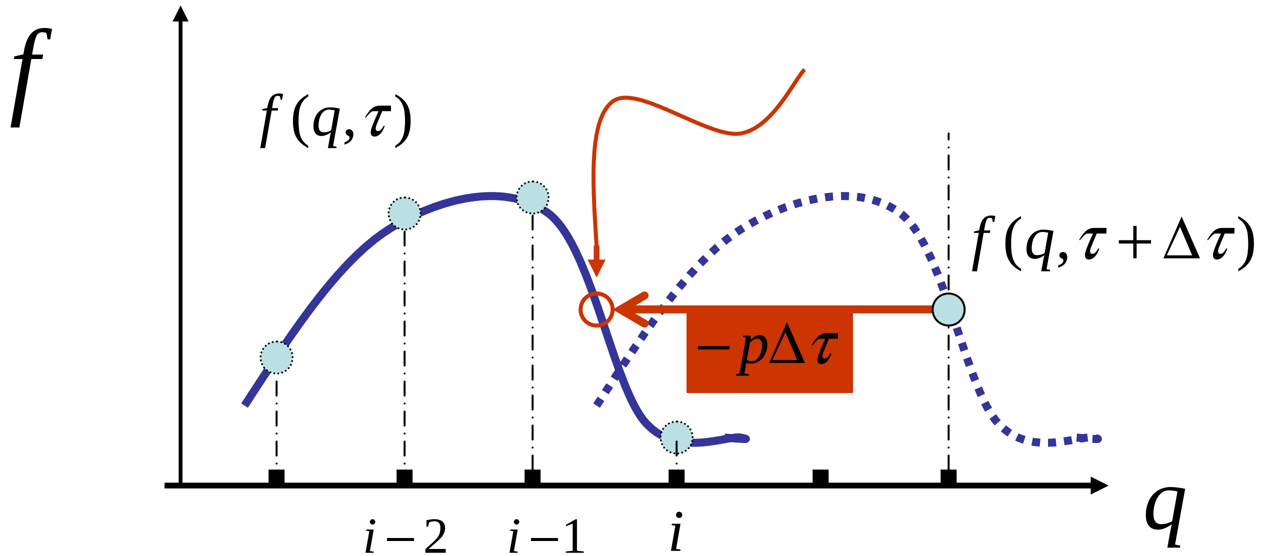
- A 2D Vlasov Solver as an effective tool for studying the microbunching instability
- Simulations show that shot-noise alone would cause an energy spread larger than the desired 150 keV in the Two-BC Lattice for FERMI
  - One-BC lattice OK
- Results consistent with 1B macroparticle simulations (J.Qiang) -- preliminary comparisons
- Further studies needed to better delimit use of 1D model of space-charge (*e.g* for dynamics through spreader)

Example of interpolation between adjacent grid-points for 1D case

$$f(q,\tau+\Delta\tau) = f(q-p\Delta\tau,\tau)$$

Beam density At later time = **Beam density At present time**

Value of f is determined by interpolation using *e.g.* values of f on adjacent grid points



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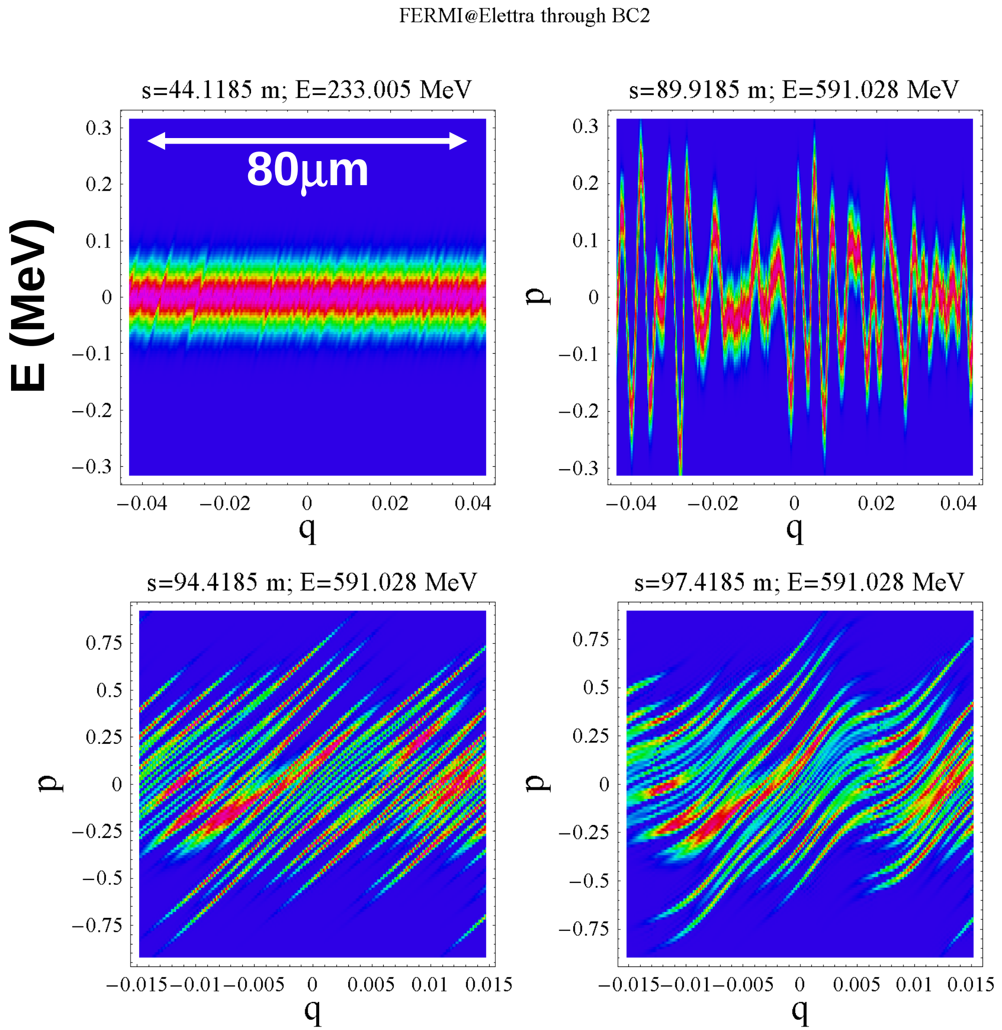
- Modest energy modulation visible after BC1



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