Universally Scaled Analysis of Harmonic Generation in an FEL Amplifier With Multiple Wigglers

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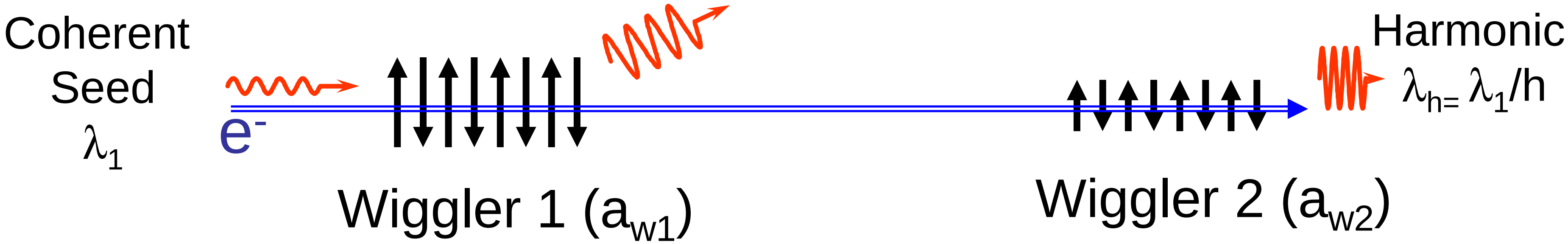
1. Introduction

Undesirable feature of SASE radiation

– large random fluctuations i.e. poor temporal coherence

Alternative to SASE

- Amplification of harmonics in multiple wigglers
- Coherence of initial seed should improve coherence of generated field relative to SASE
- Several studies, but not using universally scaled approach



Assume wigglers have same wiggler period, l_w

2. Model

Universally scaled FEL equations (steady-state):

$$\frac{d\theta_j}{d\bar{z}} = \bar{p}_j$$

$$\frac{d\bar{p}_j}{d\bar{z}} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{dA}{d\bar{z}} = \left\langle e^{-i\theta} \right\rangle$$

where $\theta_j = (k_w + k)z - \omega t_j$ $\qquad \bar{p}_j = \frac{\gamma_j - \gamma_r}{\rho \gamma_r}$

$$\rho |\mathrm{A}|^2 = \frac{\text{Radiation power}}{\text{Beam power}}$$

$$\bar{z} = \frac{z}{L_g} = \frac{4\pi\rho z}{\lambda_w}$$

FEL parameter : $\rho = \frac{1}{\gamma_r}\left(\frac{a_w \omega_p}{4ck_w}\right)^{2/3} \propto a_w^{2/3}$

2. Model - Harmonics

FEL resonance condition :

$$\lambda_h = \lambda_w \frac{1+a_w^2}{2h\gamma^2}$$

h = harmonic number

$a_w = \frac{eB_w\lambda_w}{mc}$ = wiggler parameter

In wiggler 1 :

$$\lambda_1 = \lambda_w \frac{1+a_{w1}^2}{2\gamma^2}$$

In wiggler 2 :

$$\lambda_2 = \lambda_w \frac{1+a_{w2}^2}{2\gamma^2}$$

If $\lambda_2 = \frac{\lambda_1}{h}$ ⇒ $a_{w2} = \sqrt{\frac{1+a_{w1}^2}{h} - 1}$

so $a_{w1} > \sqrt{h-1}$ (e.g. for h=10, a_{w1}>3)

When a_{w1}, a_{w2} >>1 : $a_{w2} = \frac{a_{w1}}{\sqrt{h}}$

2. Model - Harmonics

When $a_{w1}, a_{w2} >>1$:

Wiggler 1 :

$$\frac{d\theta_{1j}}{d\bar{z}_1} = \bar{p}_{1j}$$

$$\frac{d\bar{p}_{1j}}{d\bar{z}_1} = -(Ae^{i\theta_{1j}} + c.c.)$$

$$\frac{dA}{d\bar{z}_1} = \left\langle e^{-i\theta_1} \right\rangle$$

Wiggler 2 :

$$\frac{d\theta_{1j}}{d\bar{z}_1} = \frac{\bar{p}_{1j}}{h}$$

$$\frac{d\bar{p}_{1j}}{d\bar{z}_1} = -\frac{1}{\sqrt{h}}(Ae^{ih\theta_{1j}} + c.c.)$$

$$\frac{dA}{d\bar{z}_1} = \frac{1}{\sqrt{h}}\left\langle e^{-ih\theta_1} \right\rangle$$

2. Model - Harmonics

If we now define new electron & field variables:

$$\theta_{hj} = h\,\theta_{1j} \qquad \bar{p}_{hj} = h^{1/3}\,\bar{p}_{1j} \qquad A_h = h^{1/6} A_1$$

$$\bar{z}_h = \frac{\bar{z}_1}{h^{1/3}}$$

We obtain a single set of universally scaled equations for any wiggler tuned to harmonic h :

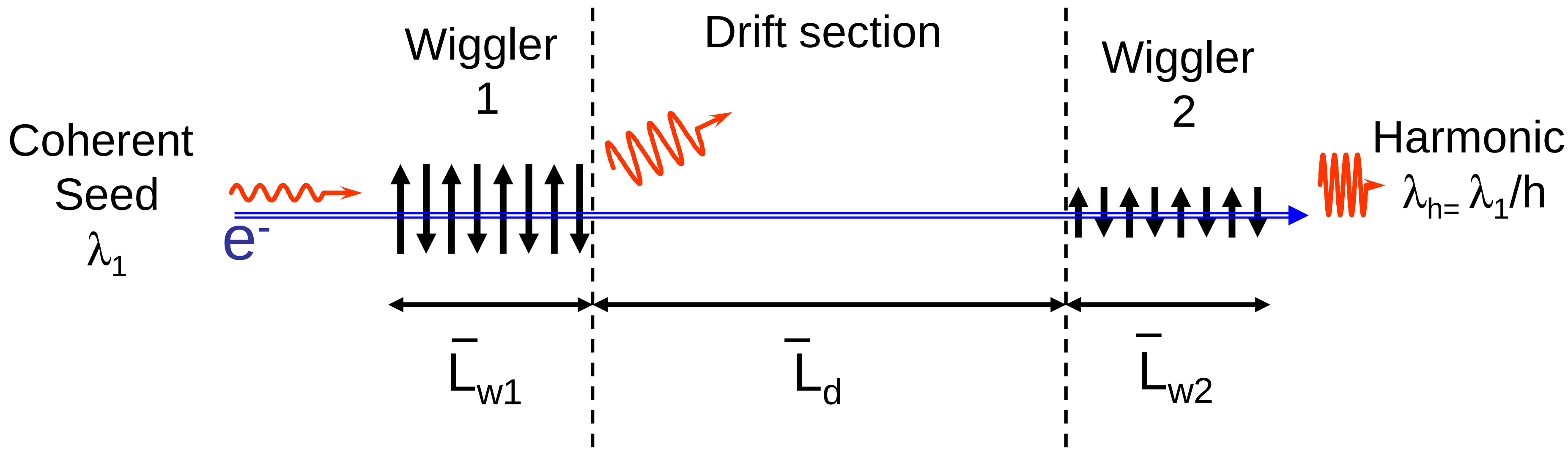
$$\frac{d\theta_{hj}}{d\bar{z}_h} = \bar{p}_{hj}$$

$$\frac{d\bar{p}_{hj}}{d\bar{z}_h} = -(Ae^{i\theta_{hj}} + c.c.)$$

$$\frac{dA}{d\bar{z}_h} = \left\langle e^{-i\theta_h} \right\rangle$$

Allows simple analysis of wide range of parameter space

2. Model – Implementation



- Integrate FEL equations from $0 \rightarrow \bar{L}_{w1}$

- In drift section, A=0, so

  $\bar{p}_j$=constant, $\theta_j \rightarrow \theta_j + \bar{p}_j \bar{L}_d$

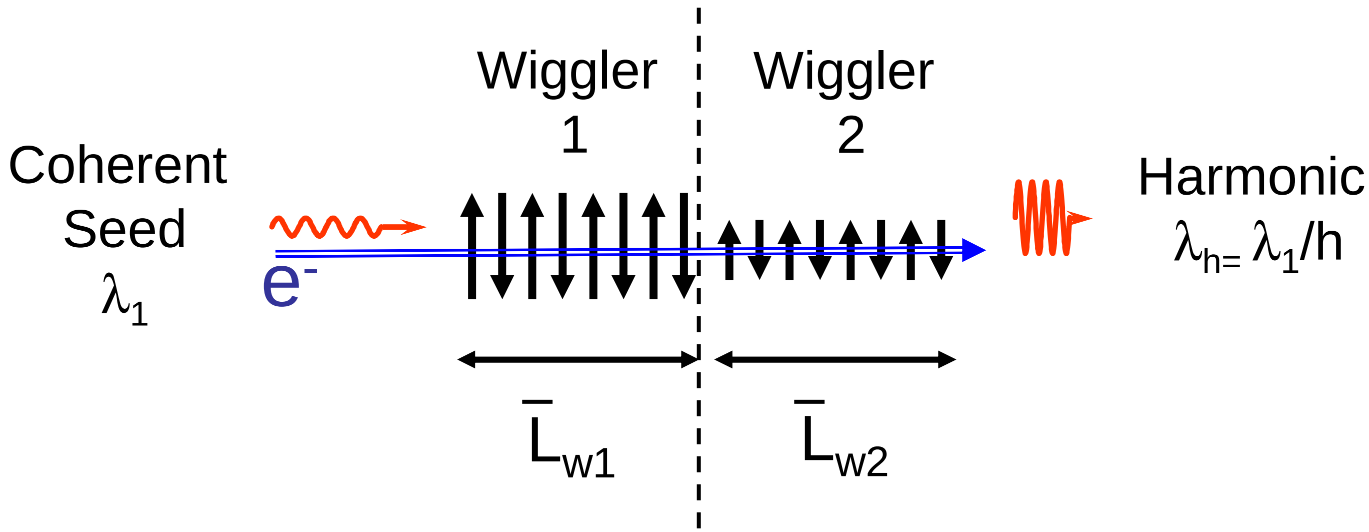
- At entrance to wiggler 2 :

  A=0, $\theta_j \rightarrow h\,\theta_j$, $\bar{p}_j \rightarrow h^{1/3}\,\bar{p}_j$

- Integrate FEL equations from $\bar{L}_{w1}+\bar{L}_d \rightarrow \bar{L}_{w1}+\bar{L}_d+\bar{L}_{w2}$

3. Results – 2 wigglers

First, consider 2 wigglers with no drift section ($\bar{L}_d$=0) :



Proposed for frequency tripling by nonlinear harmonic generation in

Bonifacio, De Salvo & Pierini NIMA **293**, 627 (1990)
Bonifacio, De Salvo & Scharlemann NIMA **296**, 787 (1990)

First wiggler produces bunching at fundamental & harmonic
- harmonic bunching radiates in second wiggler

3. Results – 3rd harmonic

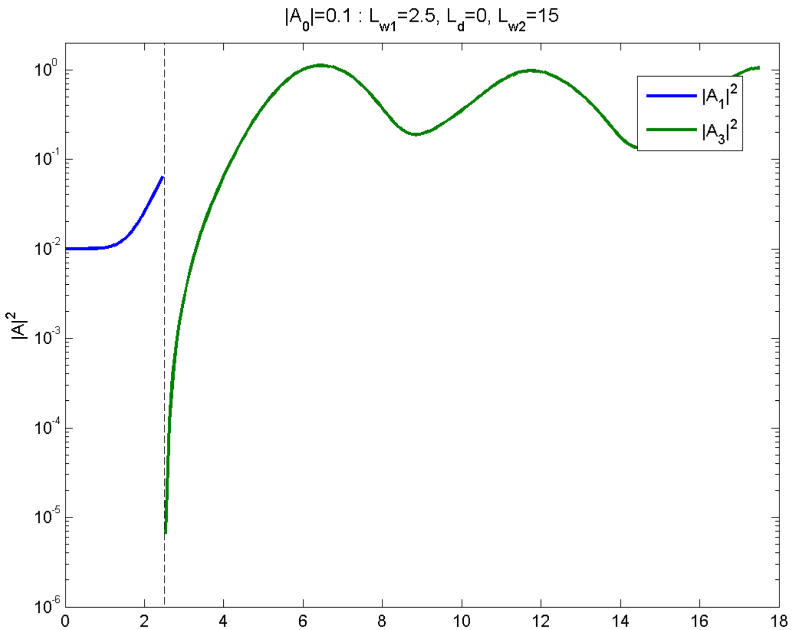
Cold beam ($\bar{\sigma}=0$) :

$h=3$, $\bar{L}_{w1}=2.5$, $\bar{L}_{w2}=15$

Field intensity :

Fundamental – blue

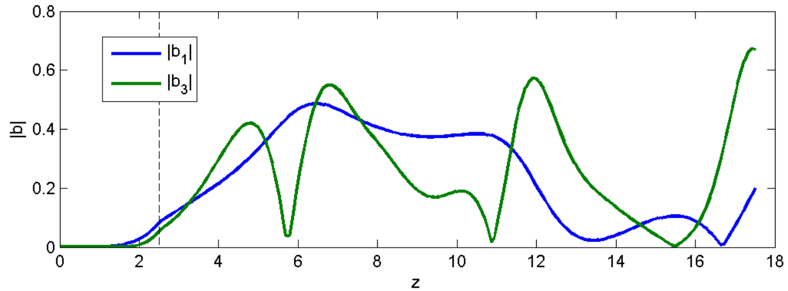
Harmonic - green



Bunching :

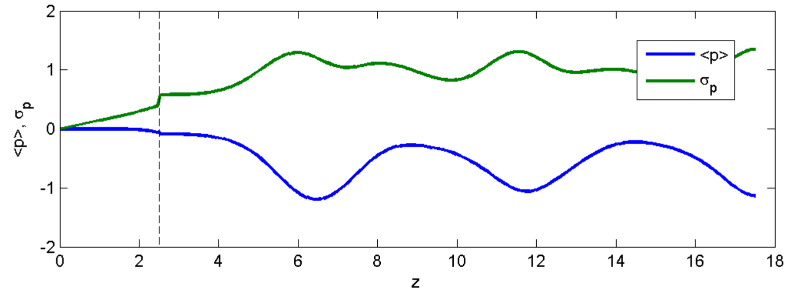
Fundamental – blue

3rd harmonic - green



Average energy <p> – blue

Energy spread (σ) - green



3. Results – 3rd harmonic

Increasing length of wiggler 1(L_{w1}), then bunching increases BUT

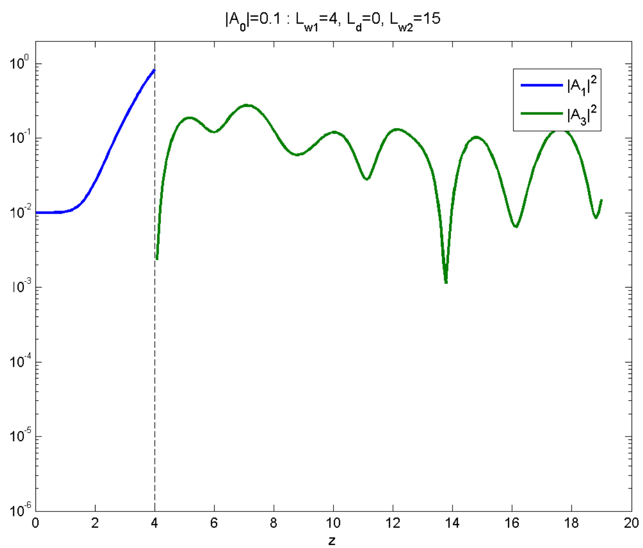
- FEL process in wiggler 1 increases energy spread
- no exponential growth of harmonic in wiggler 2

if $\bar{\sigma}_h > 1$ i.e.

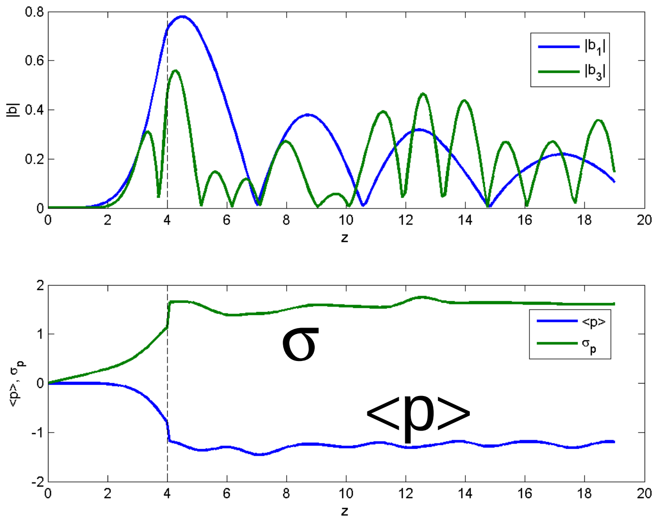
$$\frac{\Delta\gamma}{\gamma} < \frac{\rho}{h^{1/3}}$$

e.g. for L_{w1} = 4 (cold beam)

|A|²



|b|

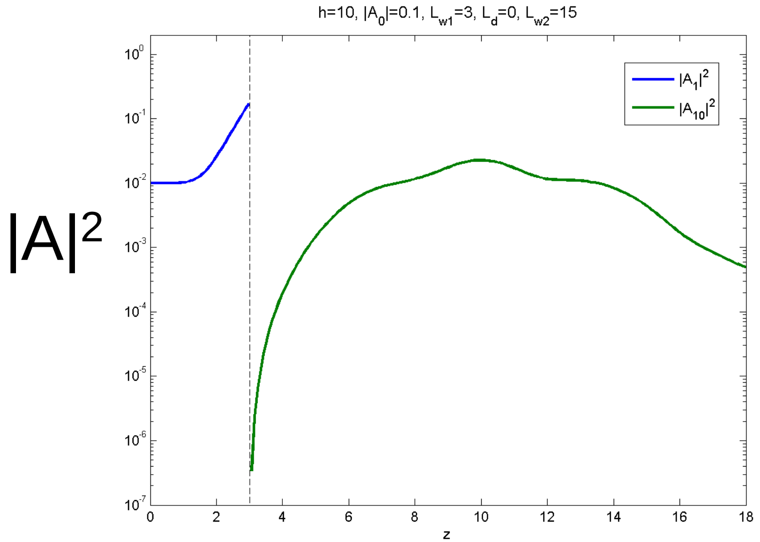


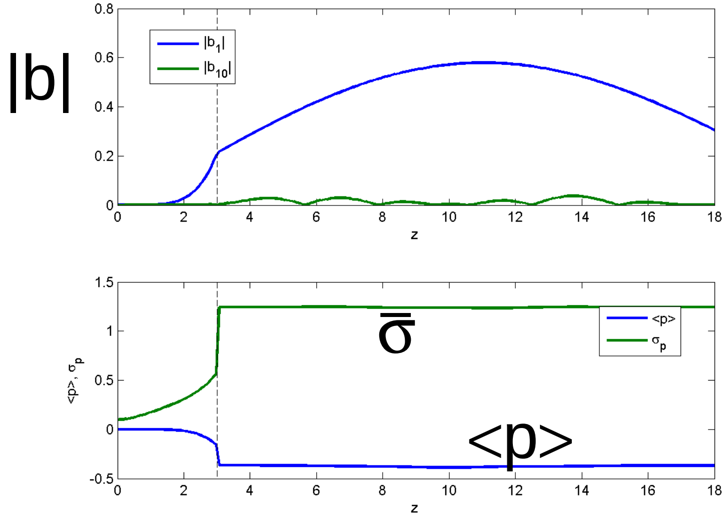
Harmonic intensity (L_{w1} = 4) << Harmonic intensity (L_{w1} = 2.5)

3. Results – 10th harmonic

As h increases, induced energy spread becomes restrictive as $\propto h^{1/3}$

e.g. for h =10 and $\bar{\sigma}$=0.1

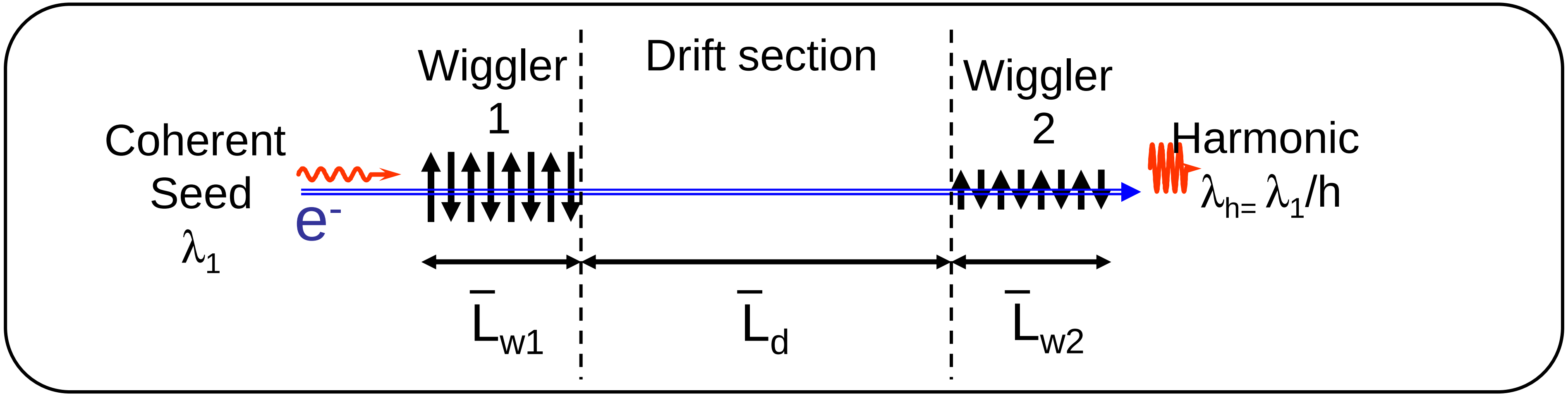


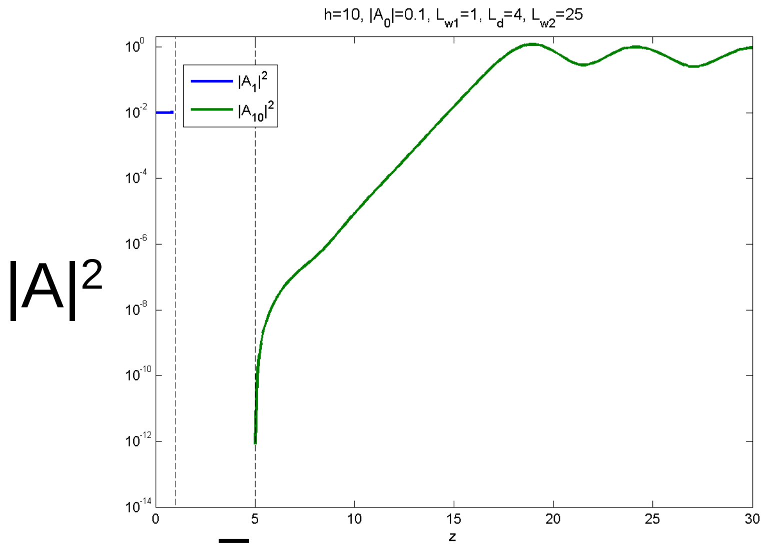


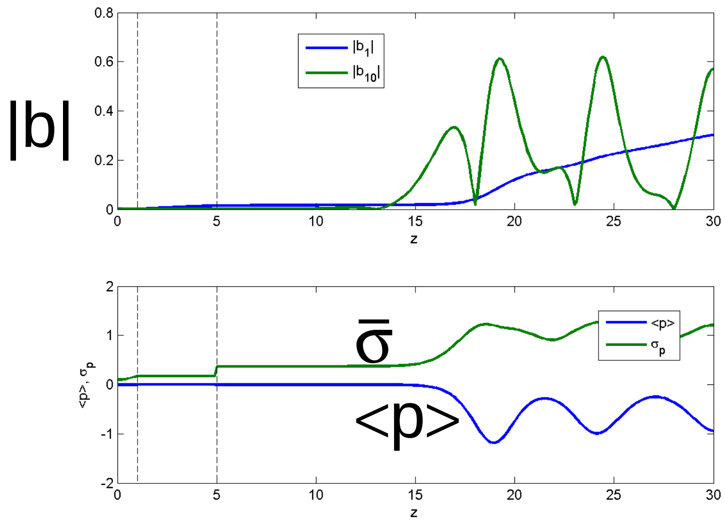
3. Results – 10^{th} harmonic with drift section

Drift section – allows bunching to increase due to electron inertia without increase of energy spread

L.Yu et al. PRA **44**, 5158 (1991)
Bonifacio, Corsini & Pierini, PRA **45**, 4091 (1992)





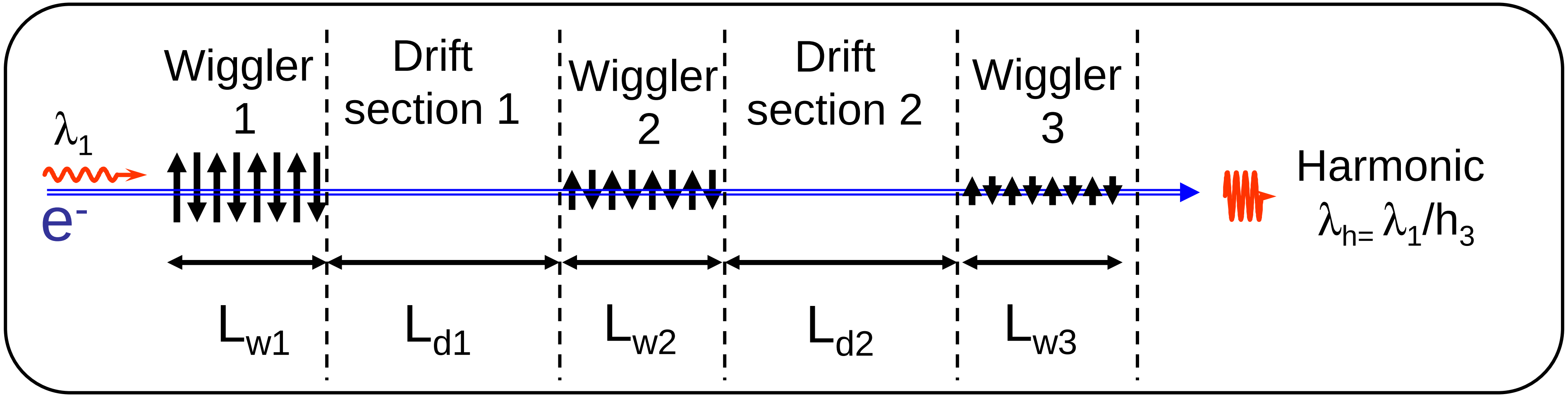


Drift section ($\bar{L}_d$=4) has increased harmonic intensity by 2 orders of magnitude

3. Results – Higher harmonics

For higher harmonics (e.g. h>10),

it may be possible to use 3 wigglers
separated by drift sections

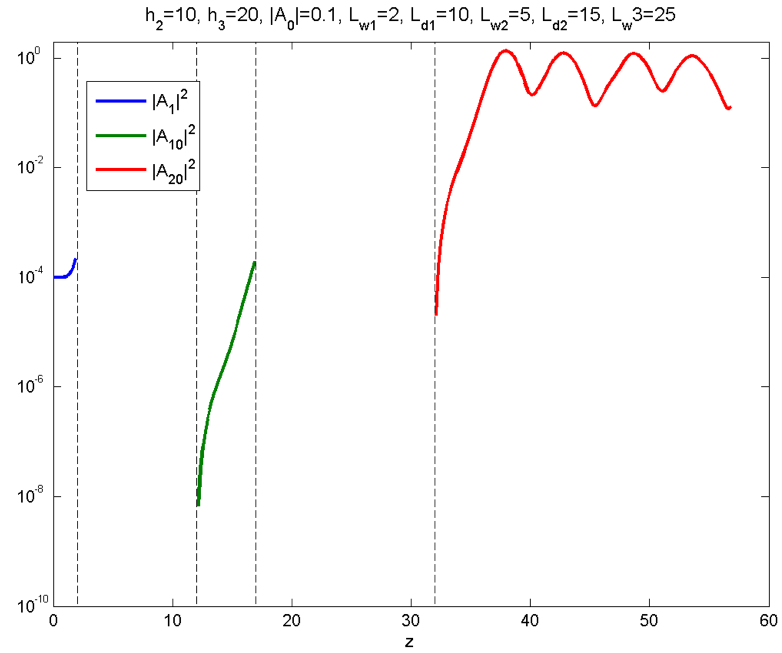


3. Results – Higher harmonics

20th harmonic : h_2=10, h_3=20

Cold beam (σ=0)

$|A|^2$



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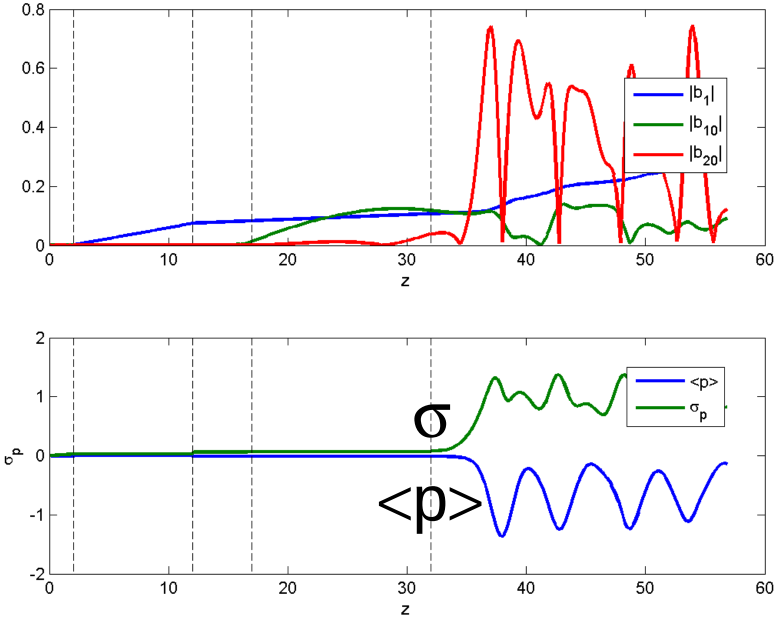


Blue – fundamental

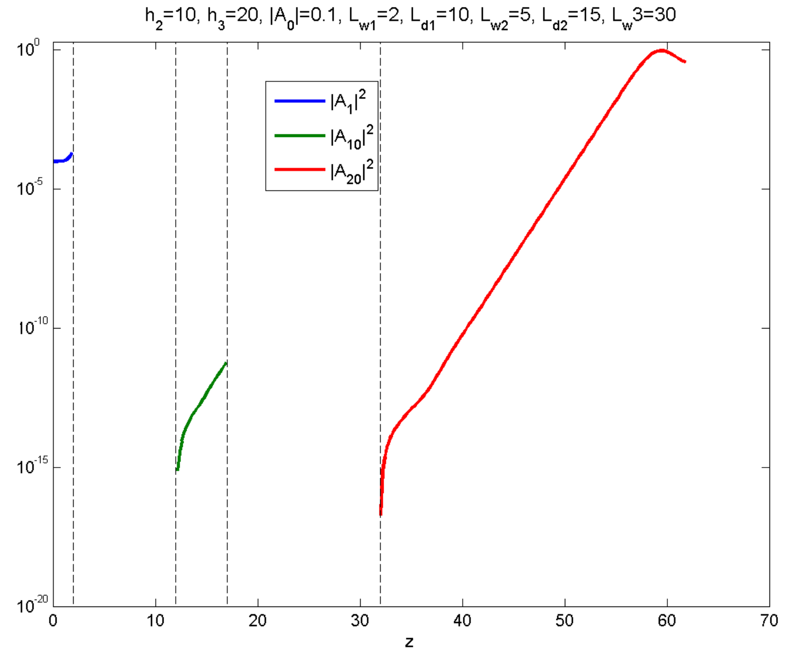
Green – h=10

Red – h=20

3. Results – Higher harmonics

Warmer beam ($\bar{\sigma}$=0.2) :
Still produces significant intensity at 20th harmonic

$|A|^2$



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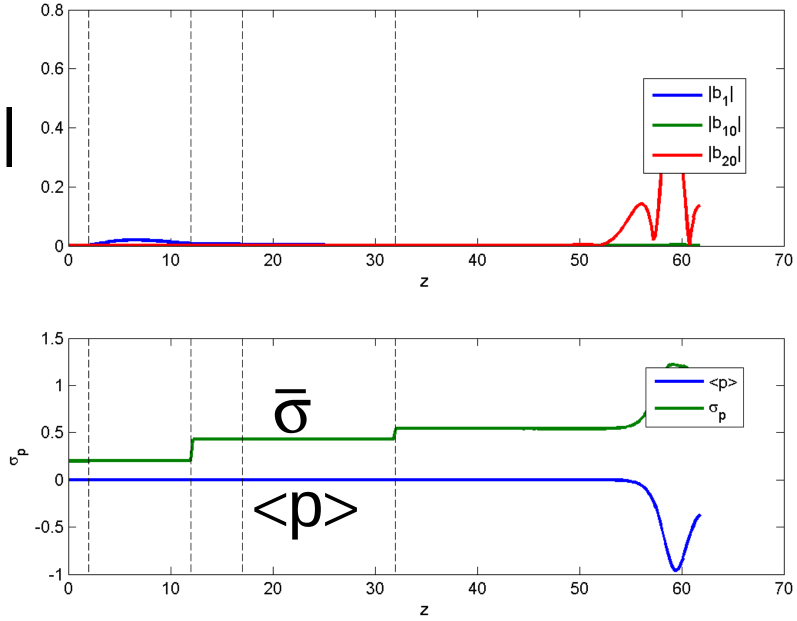

Blue – fundamental
Green – h=10
Red – h=20

4. Conclusions

Developed a simple 1-D **universally scaled** model
of harmonic generation in multiple wigglers
valid for large $a_{w.}$

Robustness to :
Shot noise?
Pulse/slippage effects?
3D effects?

See talk by Marinelli (Mon, 12.40)
"HGHG schemes for short wavelengths"