The Quantum Free Electron Laser (QFEL)

R. Bonifacio
INFN-Milano and CBPF [*]– RJ – Brazil

N. Piovella, M.M. Cola, L. Volpe
Dipartimento di Fisica & INFN-Milano

G.R.M. Robb
University of Strathclyde, Glasgow, UK

A. Schiavi
Università di Roma "La Sapienza"

[*] Centro Brasileiro de Pesquisas Fisicas (CBPF), Rio de Janeiro, Brazil

OUTLINE

Classical SASE and spiking

Quantum theory and QFEL parameter

Linear quantum theory

Quantum purification and narrow linewidth

Why a laser wiggler? Emittance criteria,

possible experimental set up and parameters

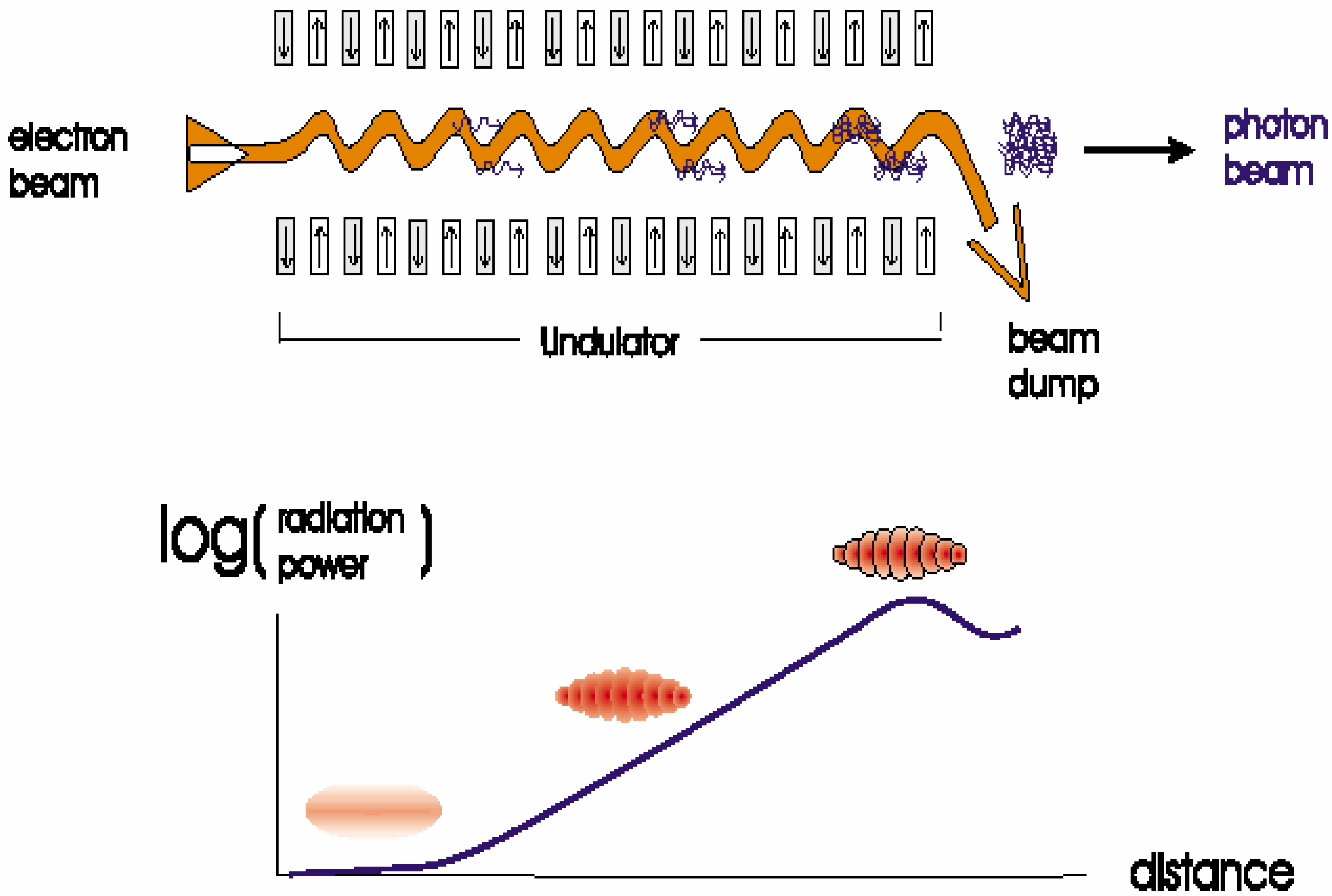
Conclusions

what is QFEL?

QFEL is a novel macroscopic quantum coherent effect: collective Compton backscattering of a high-power laser wiggler by a low-energy electron beam.

The QFEL linewidth can be four orders of magnitude smaller than that of the classical SASE FEL

SASE high-gain regime

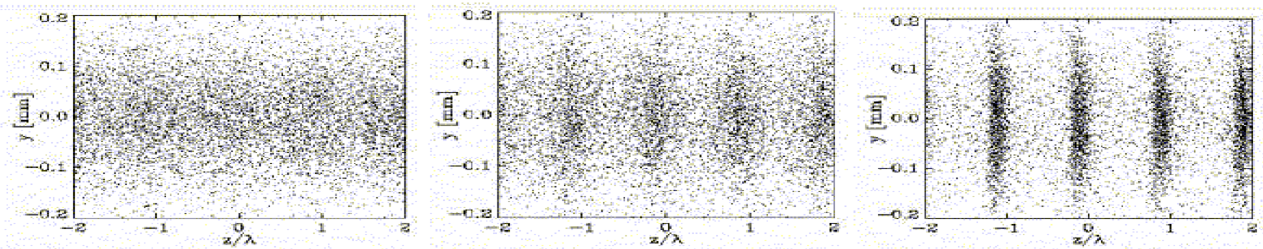


SELF-BUNCHING

- start up from noise
- exponential growth of intensity and bunching
- saturation $(P_{rad} \sim \rho\, P_{beam})$ after several L_g

b~0 → b~0.8

bunching:



$$b = \left| \frac{1}{N} \sum_{j=1}^{N} e^{-i\theta_j} \right|$$

wiggler length (several L_g)

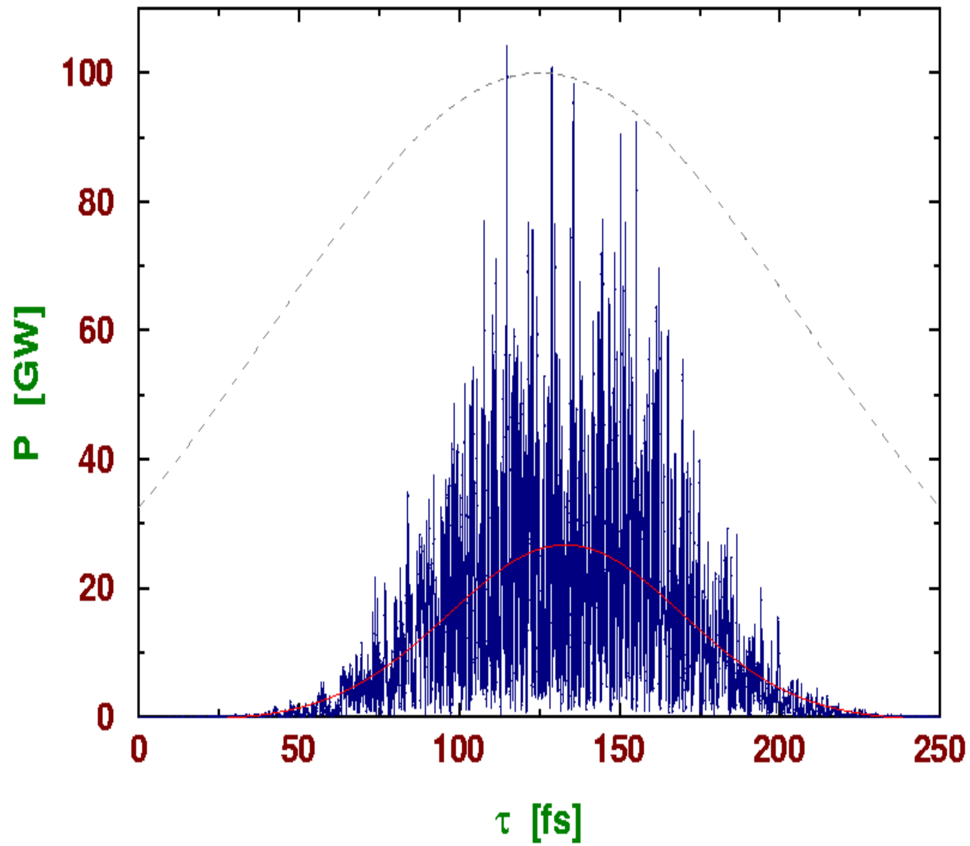
DRAWBACKS OF 'CLASSICAL' SASE

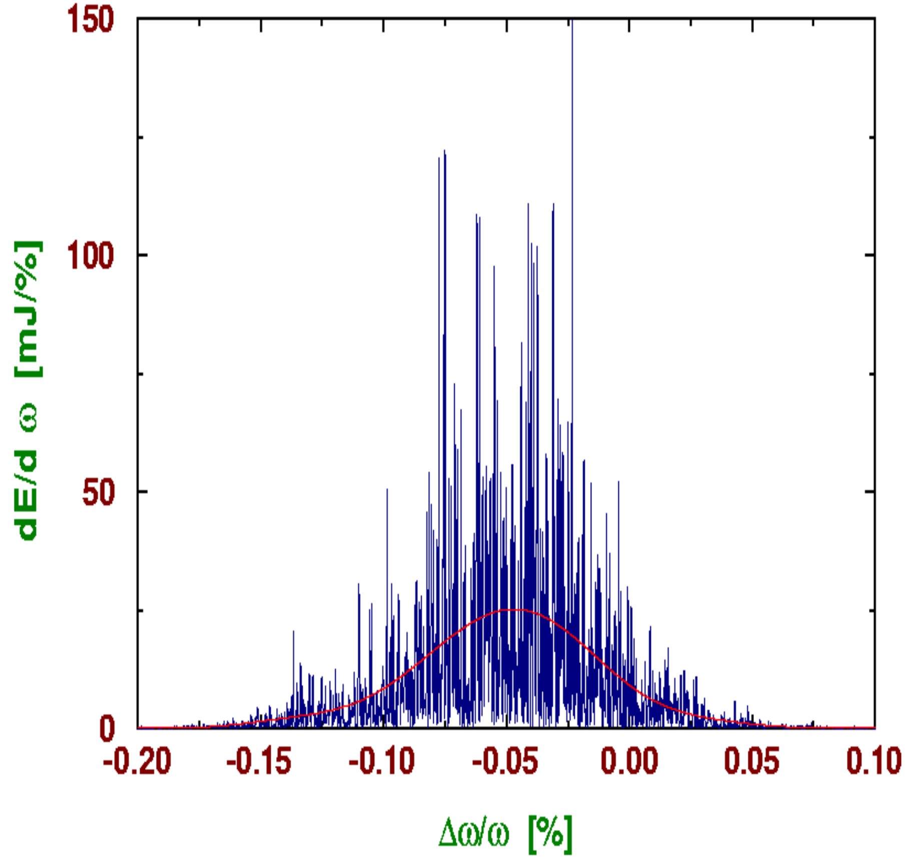
R.Bonifacio, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL (1994)

Time profile has many random spikes ($n = L_b/L_c$)

Broad and noisy spectrum at short wavelengths (x-ray FELs)

simulations from DESY for the SASE experiment





QUANTUM FEL MODEL

Procedure :

Describe N particle system as a Quantum Mechanical ensemble

↓

Write a Schrödinger equation for macroscopic wavefunction: Ψ

or equivalently ..

the equation for the Wigner function (quantum distribution): W

Include slippage z_1
(i.e. propagation)

→

$\Psi(\theta, \bar{z}, z_1)$
or
$W(\theta, p, \bar{z}, z_1)$

R.Bonifacio, N.Piovella, G.Robb, A. Schiavi, PRST-AB (2006)

1D QUANTUM FEL MODEL

$$i\frac{\partial\Psi}{\partial\bar{z}} = -\frac{1}{2\bar{\rho}^{3/2}}\frac{\partial^2\Psi}{\partial\theta^2} - i\left\{A(z_1,\bar{z})e^{i\theta} - c.c.\right\}\Psi$$

$$\frac{\partial A}{\partial\bar{z}} + \frac{\partial A}{\partial z_1} = \int_0^{2\pi} d\theta\,|\Psi(\theta,z_1,\bar{z})|^2\,e^{-i\theta}$$

$$\bar{z} = \frac{z}{L_g}$$

$$z_1 = \frac{z - v_z t}{L_c}$$

$$L_c \approx \left(\frac{\lambda}{\lambda_L}\right)L_g$$

A : normalized FEL amplitude

$\bar{\rho} = \rho_{FEL}\left(\frac{mc\gamma}{\hbar k}\right)$: QUANTUM FEL parameter

the classical model is valid when $\bar{\rho} >> 1$

$\frac{\partial A}{\partial z_1} = 0$ ⟶ G. Preparata from QFT, PRA (1988)

LINEAR THEORY

$$\left(A \propto e^{i\lambda\bar{z}}\right)$$

Classical theory

1) $(\lambda - \Delta)\lambda^2 + 1 = 0$

R.Bonifacio, C.Pellegrini, L.Narducci, Opt. Commun. (1985)

Quantum theory

Previous approach

2) $(\lambda - \Delta')\lambda^2 + 1 = 0 \quad \Delta' = \Delta - \frac{1}{2\bar{\rho}} \quad \Delta_{max} = \frac{1}{2\bar{\rho}}$

C.B.Schroeder, C.Pellegrini, P.Chen, PRE (2001)

Mistake of equation (2): incorrect quantization $pe^{-i\theta} \rightarrow \hat{p}e^{-i\hat{\theta}}$

Correct quantization $\left[e^{-i\hat{\theta}}, \hat{p}\right] = e^{-i\hat{\theta}}$

$$pe^{-i\theta} \rightarrow \frac{1}{2}\left(\hat{p}e^{-i\hat{\theta}} + e^{-i\hat{\theta}}\hat{p}\right)$$

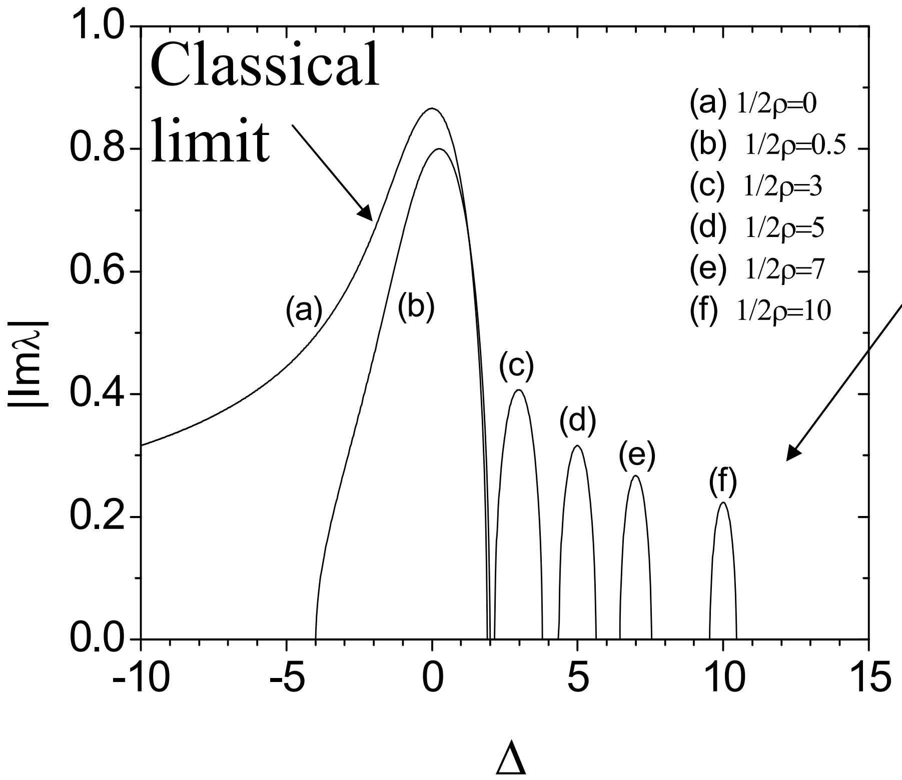
Weyl symmetrization rule

3) $(\lambda - \Delta)\left(\lambda^2 - \frac{1}{4\bar{\rho}^2}\right) + 1 = 0$

R.Bonifacio, N.Piovella, G.Robb, A. Schiavi, PRST-AB (2006)

Quantum Linear Theory $\left(A \propto e^{i\lambda\bar{z}}\right)$

$$(\lambda - \Delta)\left(\lambda^2 - \frac{1}{4\bar{\rho}^2}\right) + 1 = 0$$



Quantum regime for $\bar{\rho}$<1

Resonance: $\Delta = \frac{1}{2\bar{\rho}}$

width $\propto \sqrt{\bar{\rho}}$

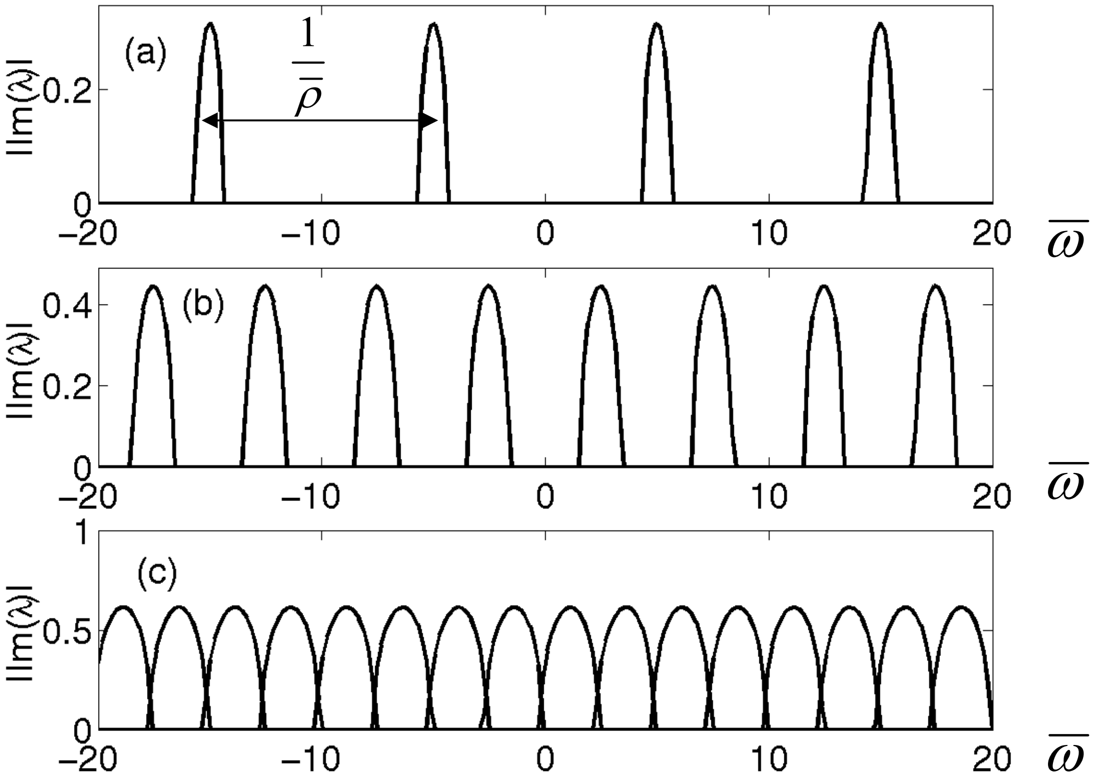
discrete frequencies as in a cavity

$$(\lambda-\Delta)\left(\lambda^2-\frac{1}{4\bar{\rho}^2}\right)+1=0 \qquad \left(\Delta=\frac{n}{2\bar{\rho}}-\bar{\omega}\right) \qquad \left(\bar{\omega}=\frac{\omega-\omega_{sp}}{2\rho\omega_{sp}}\right)$$

$\bar{\rho}=0.1$

$\bar{\rho}=0.2$

$\bar{\rho}=0.4$



max for $\Delta=1/2\bar{\rho}$

$$\bar{\omega}_n=\frac{1}{2\bar{\rho}}(2n-1)$$

width $4\sqrt{\bar{\rho}}$

Continuous limit $\quad 4\sqrt{\bar{\rho}}\geq 1/\bar{\rho}\rightarrow\bar{\rho}\geq 0.4$

The physics of Quantum FEL

$$\bar{\rho} = \rho \frac{mc\gamma}{\hbar k} = \frac{\sigma(p_z)}{\hbar k}$$

Momentum-energy levels:
($p_z = n\hbar k$, $E_n \propto p_z^2 \propto n^2$)

$n\ \hbar k$

$\hbar k$ ↓

$(n-1)\ \hbar k$

$$\omega_n = E_n - E_{n-1} \propto \left[n^2 - (n-1)^2\right] \propto 2n - 1 \qquad (n = 0, -1, \ldots\ldots)$$

Equally spaced frequencies as in a cavity

CLASSICAL REGIME: $\bar{\rho} >> 1$
many momentum level transitions
⇒ many spikes

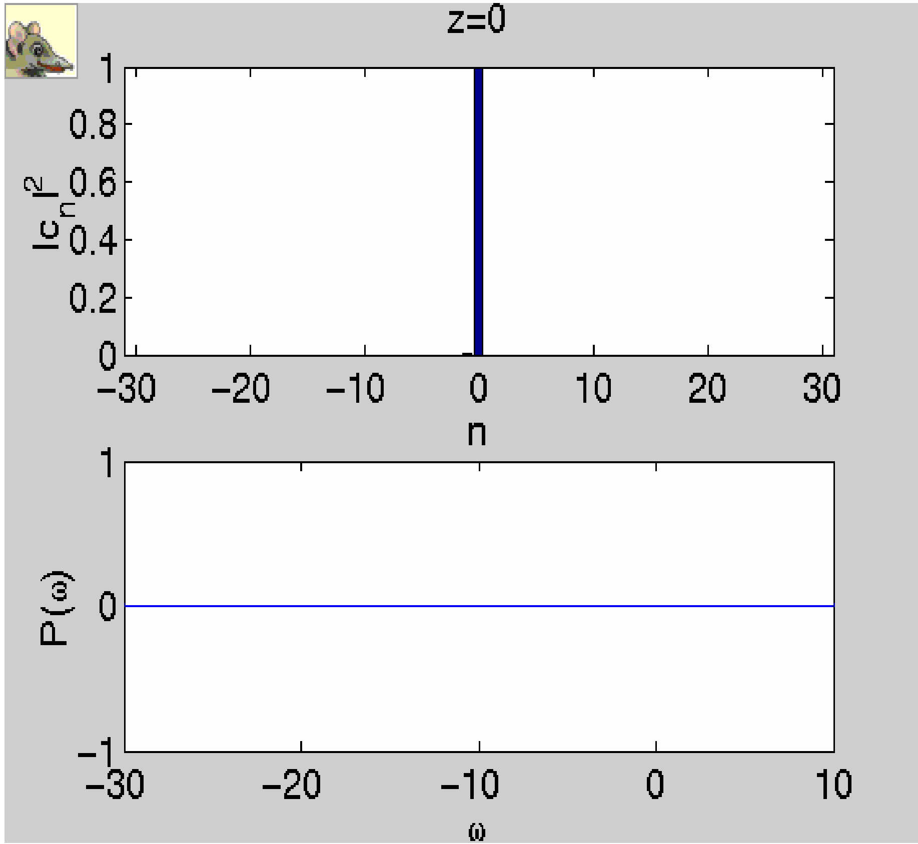
QUANTUM REGIME: $\bar{\rho} \leq 1$
a single momentum level transition
⇒ single spike

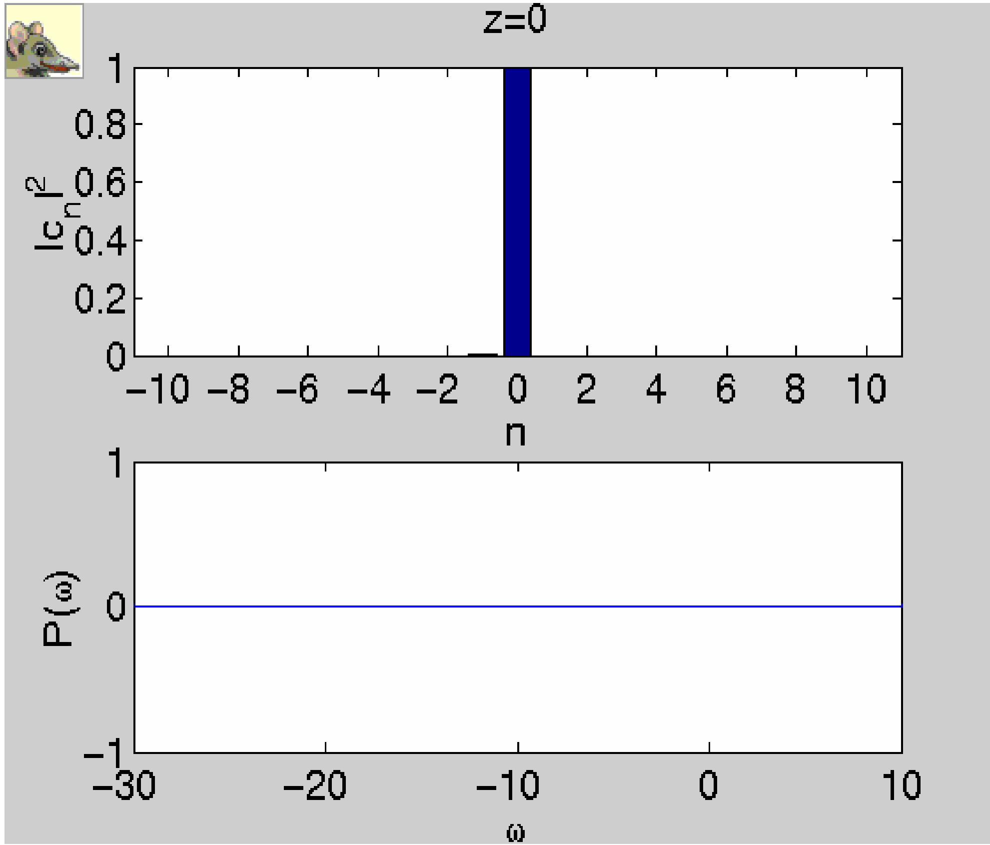
In classical regime with universal scaling no dependence on $\bar{\rho}$

momentum distribution for SASE

CLASSICAL REGIME: $\bar{\rho}=5$

QUANTUM REGIME: $\bar{\rho}=0.1$





Classical regime:
both n<0 and n>0 occupied

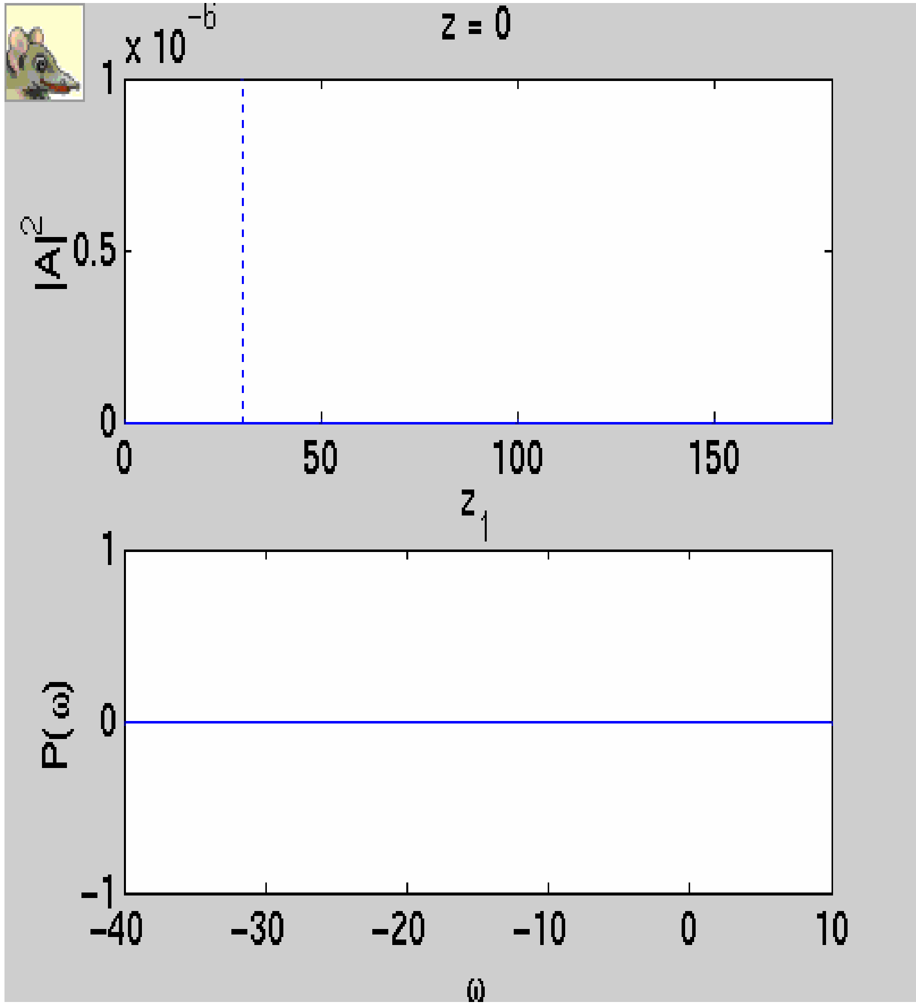
Quantum regime:
sequential SR decay, only n<0

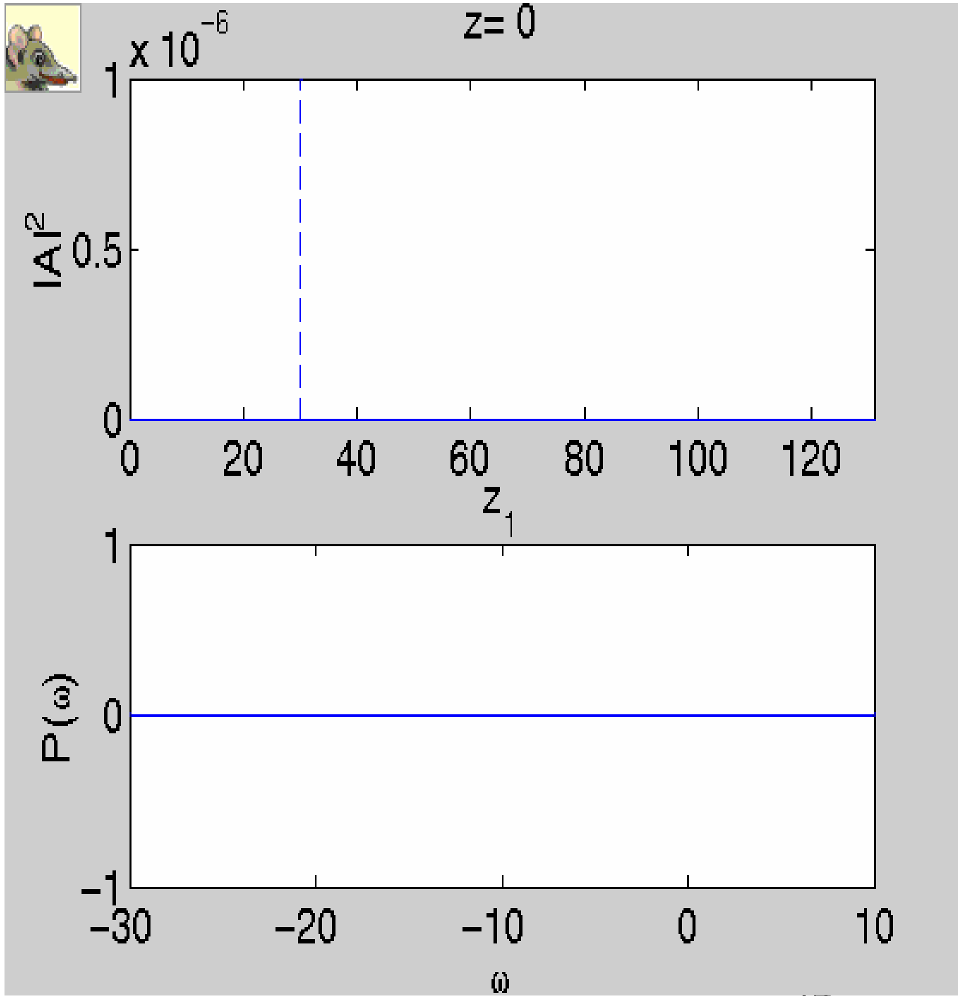
SASE mode operation

R.Bonifacio, N.Piovella, G.Robb, NIMA(2005)

quantum regime $\left(\bar{\rho} = 0.05\right)$

classical regime $\left(\bar{\rho} = 5\right)$

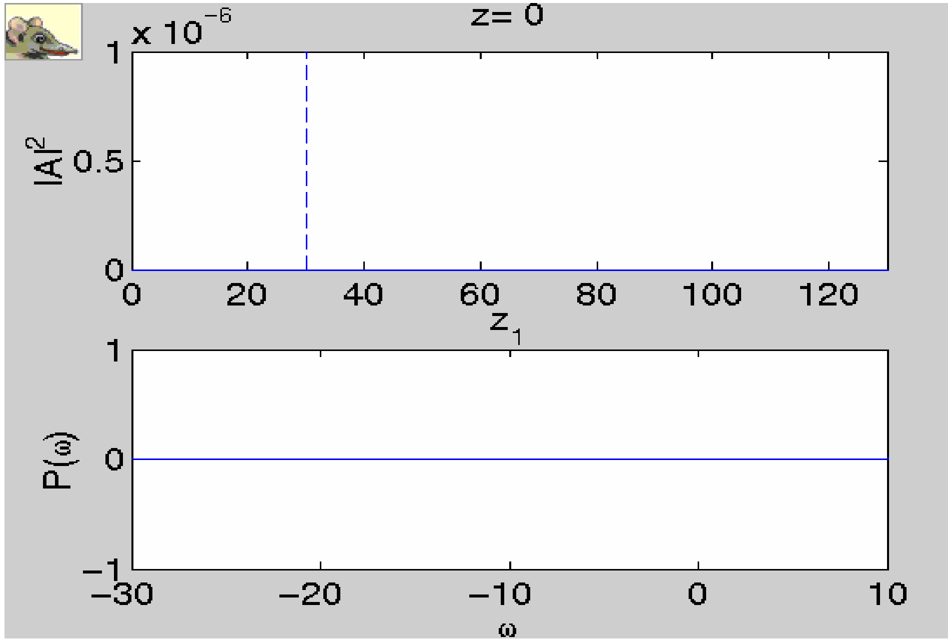




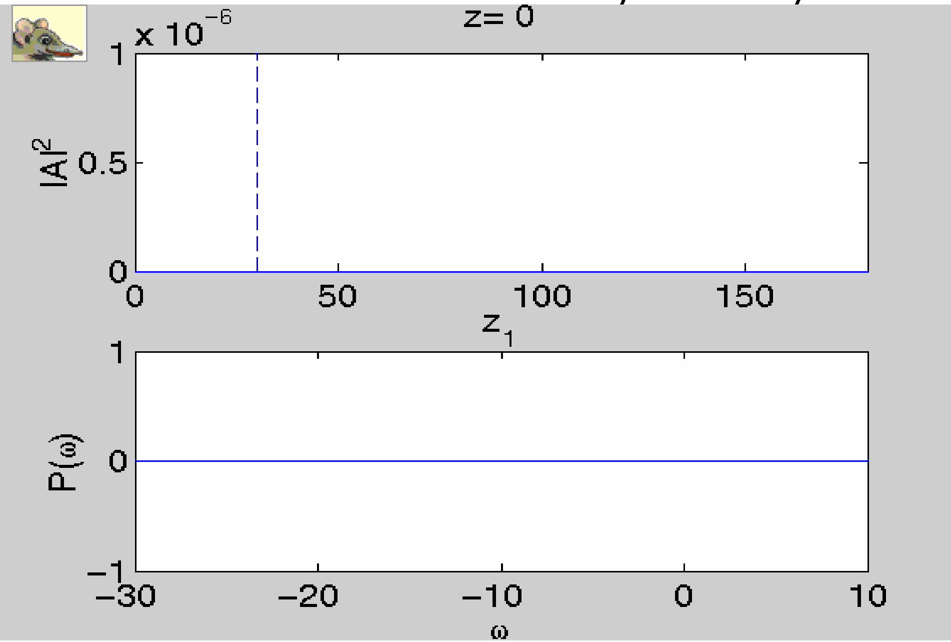
$L / L_c = 30$

$$\overline{\omega}_n = (2n-1)/2\overline{\rho} \quad [n = 0,-1,..]$$

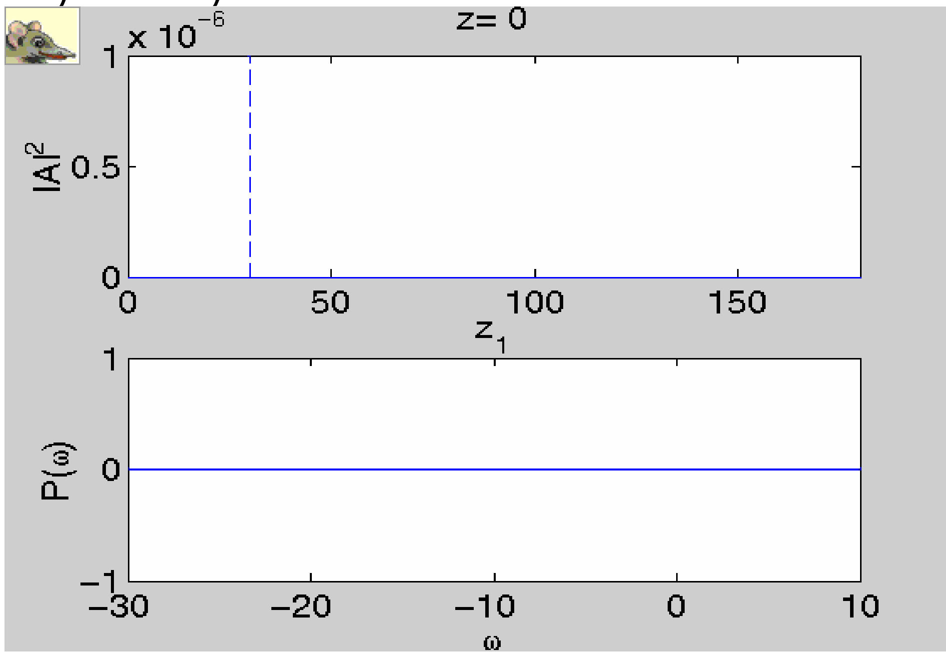
$\bar{\rho} = 0.1 \;\; 1/\bar{\rho} = 10$



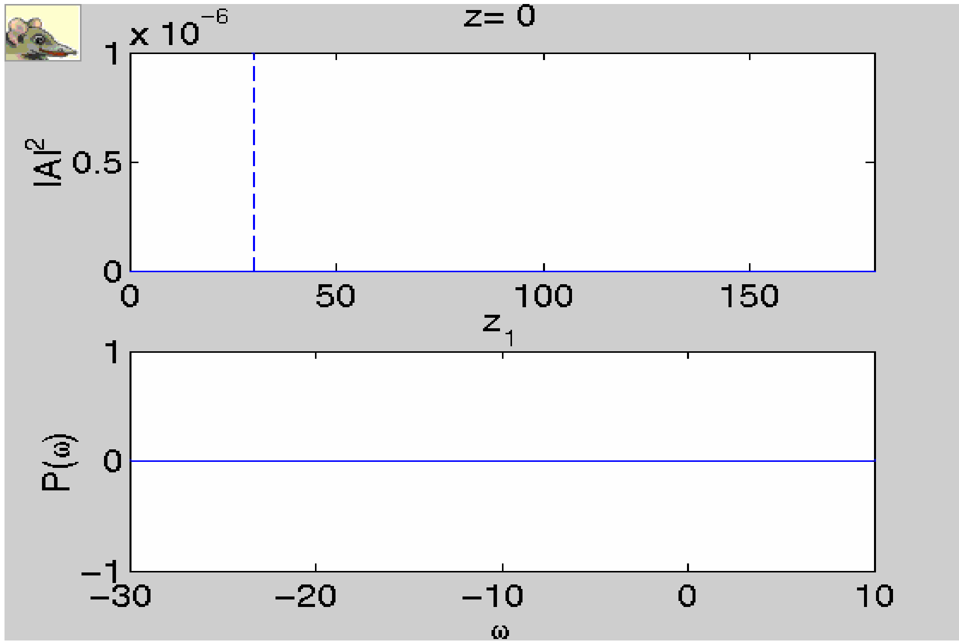
$\bar{\rho} = 0.2 \;\; 1/\bar{\rho} = 5$



$\bar{\rho} = 0.3 \;\; 1/\bar{\rho} = 3.3$



$\bar{\rho} = 0.4 \;\; 1/\bar{\rho} = 2.5$

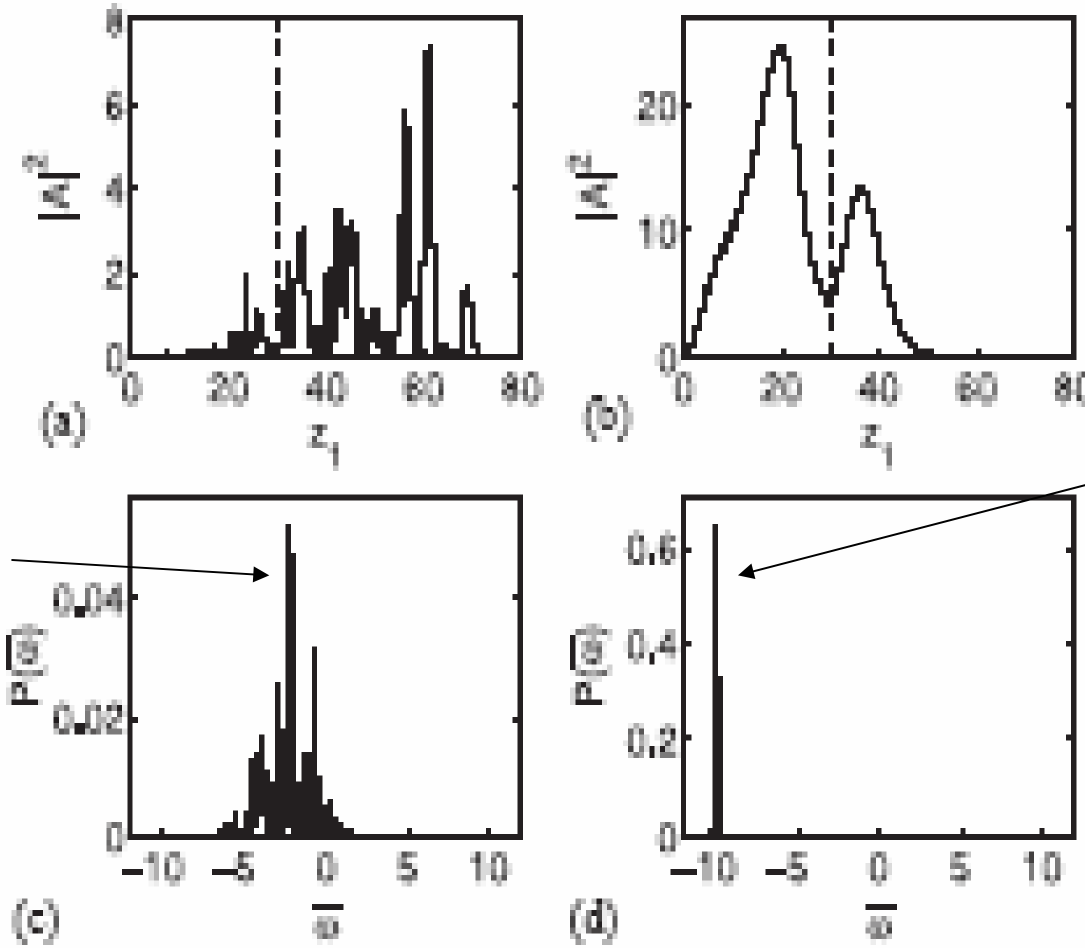


CLASSICAL AND QUANTUM SASE

classical regime, $\bar{\rho}=5$ quantum regime, $\bar{\rho}=0.1$

$$\frac{\Delta\omega}{\omega} \approx 2\rho \approx 1$$

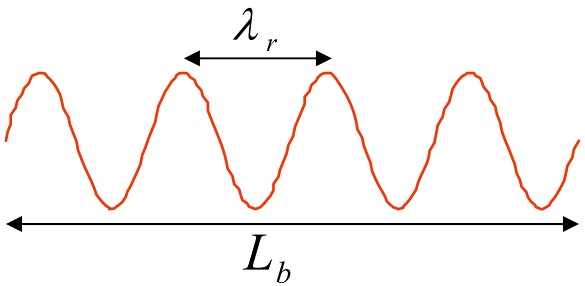
many spikes under the gain curve $\approx 2\pi(L_b/L_c)$



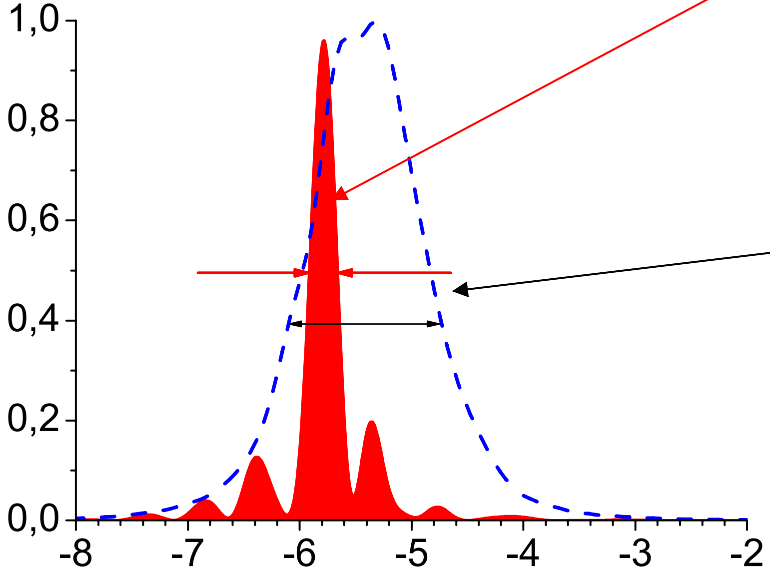
$$\frac{\Delta\omega}{\omega} \approx \frac{\lambda}{L_b} \approx 10^{-7}$$

only one spike!

LINEWIDTH OF THE SPIKE IN THE QUANTUM REGIME



$$\left(\frac{\Delta\omega}{\omega}\right)_{QFEL} \approx \frac{\lambda_r}{L_b}$$



$$\frac{\Delta\omega}{\omega} \approx \frac{\lambda}{L_b}$$

QUANTUM SINGLE SPIKE
($\sim 10^{-7}$)

$$\frac{\Delta\omega}{\omega} \approx 2\rho$$

CLASSICAL ENVELOPE
(10^{-3} - 10^{-4})

why QFEL requires a LASER WIGGLER?

$$\bar{\rho} = \rho \frac{mc\gamma}{\hbar k_r} = \rho\gamma \frac{\lambda_r}{\lambda_C} \quad \left(\lambda_C = \frac{h}{mc} \right) \qquad \gamma = \sqrt{\frac{\lambda_w (1 + a_W^2)}{2\lambda_r}}$$

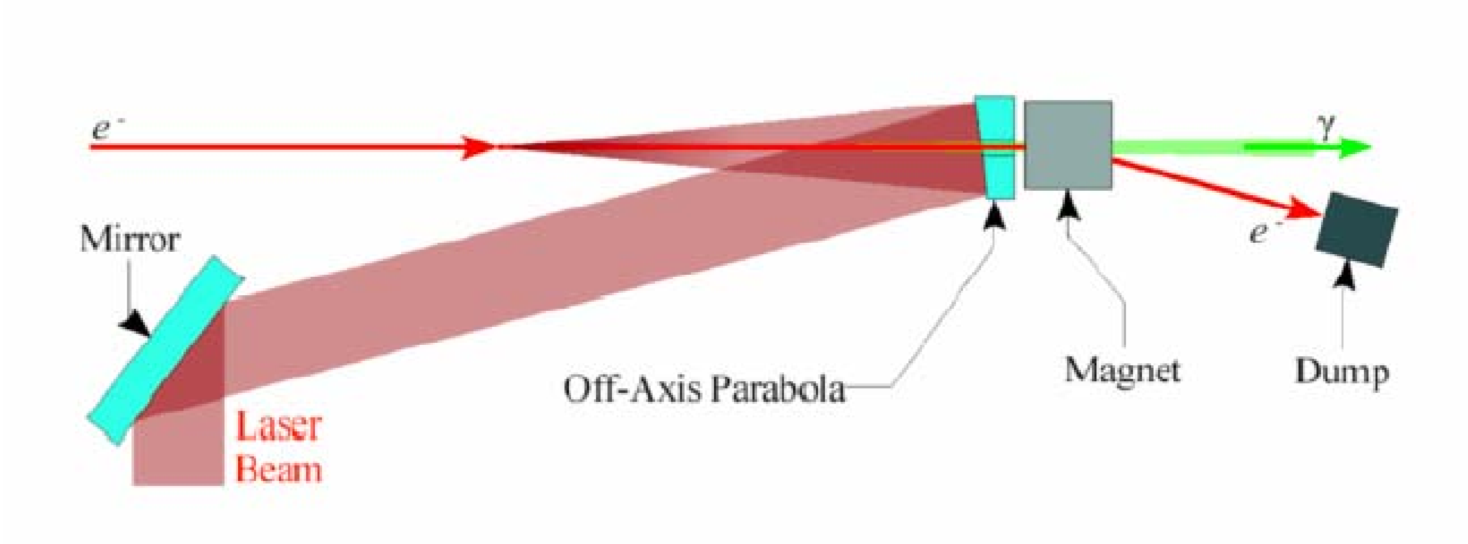
$$\bar{\rho} \le 1 \Rightarrow \rho \le \frac{\sqrt{2}\lambda_C}{\sqrt{\lambda_r \lambda_W (1 + a_W^2)}}$$ and $$L_W \approx \frac{\lambda_W}{\rho} \ge \frac{\sqrt{\lambda_r \lambda_W^3 (1 + a_W^2)}}{\sqrt{2}\lambda_C}$$

for a laser wiggler $\lambda_W \to \lambda_L / 2$

to lase at λ_r=1 A:

MAGNETIC WIGGLER:	LASER WIGGLER
$\lambda_W \sim$ 1cm, E ~3.5 GeV	$\lambda_W \sim$ 1 μm, E ~25 MeV
$\rho \sim 10^{-6}$, $L_W \sim$ 3Km	$\rho \sim 10^{-4}$, $L_W \sim$ 2 mm

typical parameters for QFEL



Electron beam

E [MeV]	21
I [A]	420
ε_n [mm mrad]	0.1-1
$\delta\gamma/\gamma$ [%]	0.01

Laser beam

λ_L [μm]	1
P_L [TW]	2
τ [ps]	40
w_0 [μm]	30
Z_r [mm]	3

QFEL beam

λ_r [A]	1.5
P_r [MW]	3.5
$\Delta\omega/\omega$	10^{-7}
Brillance	10^{28}

EMITTANCE CRITERIA FOR A LASER WIGGLER

1) e-beam contained in the laser beam:

$$Z_R = \frac{4\pi R^2}{\lambda_L} \quad , \quad \beta^* = \frac{\sigma_b^2 \gamma}{\varepsilon_n}$$

$$Z_R \leq \beta^* \quad \Rightarrow \quad \boxed{\varepsilon_n \leq \frac{\gamma \lambda_L}{4\pi} \left(\frac{\sigma}{R} \right)^2}$$

for instance γ=50, λ_L=1 μm, $\varepsilon_n \leq 4 \left(\frac{\sigma}{R} \right)^2$ **If R~2σ , ε_n<1 mm mrad !!**

---

2) Resonance broadening due to off-axis emission smaller than the natural linewidth

$$\boxed{\begin{aligned} \varepsilon_n &\leq \frac{\gamma \lambda_r}{2\pi} \sqrt{\frac{Z_{rad}}{L_{gain}}} \\ &\approx 0.06\, \lambda_L \frac{\sigma}{R} \sqrt{(1 + a_w^2)(2Z_L / L_g)} \end{aligned}}$$

$$Z_r = \frac{4\pi\sigma^2}{\lambda_r}$$

SCALING LAWS FOR A LASER WIGGLER

independent parameters: $\lambda_r(\dot{A}),\ \lambda_L(\mu m),\ a_w,\ \overline{\rho}$

$$\rho \approx 5\cdot 10^{-4}\frac{\overline{\rho}}{\sqrt{\lambda_r\lambda_L(1+a_w^2)}}$$

$$L_g(mm) \approx 0.1\sqrt{\lambda_r\lambda_L^3(1+a_w^2)\frac{1+\overline{\rho}}{\overline{\rho}^3}}$$

e-beam:

$$\gamma \approx 50\sqrt{\frac{\lambda_L}{\lambda_r}\left(1+a_w^2\right)}$$

$$I(A) \approx \frac{5\cdot 10^3}{a_w^2}\sqrt{\frac{\lambda_L}{\lambda_r^5}(1+a_w^2)\overline{\rho}^3(1+\overline{\rho})}$$

$$\varepsilon_n(mm\,rad) \approx 0.13\cdot\lambda_L\sqrt{1+a_w^2}$$

$$\sigma(\mu m) \approx 4\left[\lambda_r\lambda_L^5(1+a_w^2)\frac{1+\overline{\rho}}{\overline{\rho}^3}\right]^{1/4}$$

laser beam:

$$P_L(TW) \approx 3\cdot a_w^2\sqrt{\lambda_r\lambda_L(1+a_w^2)\frac{1+\overline{\rho}}{\overline{\rho}^3}}$$

$$\tau(ps) \approx 1.35\cdot\sqrt{\lambda_r\lambda_L^3(1+a_w^2)\frac{1+\overline{\rho}}{\overline{\rho}^3}}$$

R. Bonifacio, N.Piovella, M.Cola, L.Volpe, NIMA (2007)

$$2Z_L = 5L_g,\ R = 1.5\sigma$$

QFEL parameter	$\bar{\rho}$	0.2	5	0.2
Laser wave length	λ_L (μm)	1	1	10
Radiation length	λ_r (Å)	1.5	1.5	1.5
Wiggler parameter	a_0	0.3	0.8	0.3
FEL parameter	ρ	$7.5\cdot10^{-5}$	$1.5\cdot10^{-3}$	$2.4\cdot10^{-5}$
Gain length	L_g (mm)	1.2	0.03	37.5
Laser Rayleigh range	Z_L (mm)	3	0.07	94
Laser radius	R (μm)	15.4	2.4	274
Laser power	P_L (TW)	2	0.3	6
Laser duration	τ_L (ps)	40	1	1200
Interaction length	L_{int} (mm)	6	0.14	187
E–beam energy	γ	42.6	52.3	135
E-beam radius	σ (μm)	10.3	1.6	183
Peak current	I (kA)	0.4	22	1.3
Emittance limit	ε_n (mm mrad)	0.1	0.1	0.9
Gain band width	$\Delta\gamma/\gamma$	$3.4\cdot10^{-5}$	$1.5\cdot10^{-3}$	$1.1\cdot10^{-5}$
FEL line width	$\Delta\omega/\omega$	$2.1\cdot10^{-7}$	$1.5\cdot10^{-3}$	$6.6\cdot10^{-7}$
Number of spikes	N_s	1	278	1
FEL power	(MW)	3.5	900	11
Photons' number	N_{ph}	$6\cdot10^{9}$	$3\cdot10^{10}$	$6\cdot10^{9}$
Peak Brilliance	$B^{(*)}$	10^{28}	$1.6\cdot10^{26}$	$1.2\cdot10^{23}$

Classical versus quantum SASE

Classical SASE FEL X-ray experiments (DESY,LCLS):
- require very long Linac (~GeV, Km) and undulators (~100 m)
- Generate cahotic radiation with broad and spiky spectrum ($\Delta\omega/\omega$~10^{-3}).
- Have very high cost (10^9 U$) and large size

a QFEL experiment
- will generates a single spike almost monochromatic X-ray radiation ($\Delta\omega/\omega$~10^{-7}).
- Needs a laser wiggler
- Reduces cost (~ 10^6 U$)
- Very compact apparatus (~ m)

Summary

Classical regime : $\overline{\rho} >> 1$

Quantum regime: $\overline{\rho} \leq 1$: discreteness of momentum exchange relevant=> quantum effects.

The system is prepared in a defined momentum state p_0, making transition to the lower state

The system radiates a monocromatic train wave lambda, whose length is L_b. Hence one has a single line with linewidth

$$\lambda_r / L_b \approx 10^{-7}$$

In the opposite case random transition from many momentum states. Each transition gives a spike with different frequency. Total bandwidth: $\rho \approx 10^{-3}$

QFEL has a linewidth 4 orders of magnitude smaller than the classical

The dimensions and cost are three order of magnitude smaller