Controlling chaos in the wave/particle interaction

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Outline

- Chaotization of the dynamics to stabilize a wave
  - Using a monochromatic Hamiltonian model of the FEL
  - Control based upon a linear stability analysis of periodic orbits
  - A control technique allowing to find the apt values of the parameters

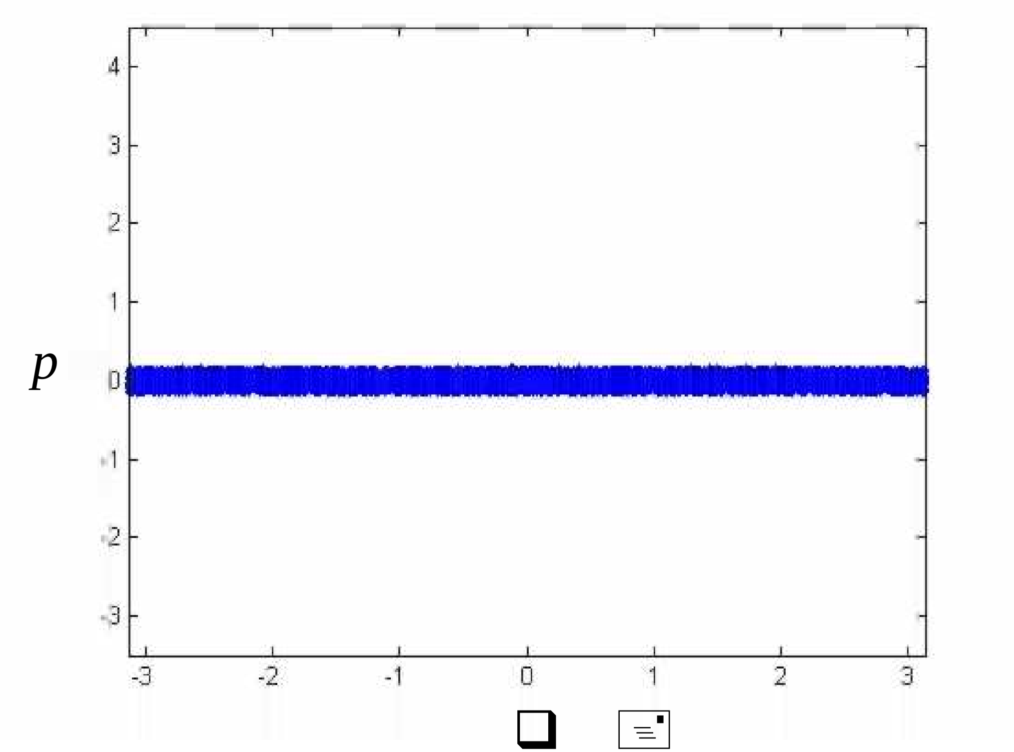
- Regularization of the dynamics to reduce energy spread
  - Using a model of electrons interacting with test-waves
  - Method providing a control term which creates barriers preventing diffusion in phase-space
  - Experimental check

Hamiltonian model with N bodies and one wave

$$H_N = \underbrace{\sum_{j=1}^{N} \frac{p_j^2}{2}}_{\text{kinetic part}} - \underbrace{2\sqrt{\frac{I}{N}} \sum_{j=1}^{N} \cos(\theta_j - \varphi)}_{\text{interaction term}}$$

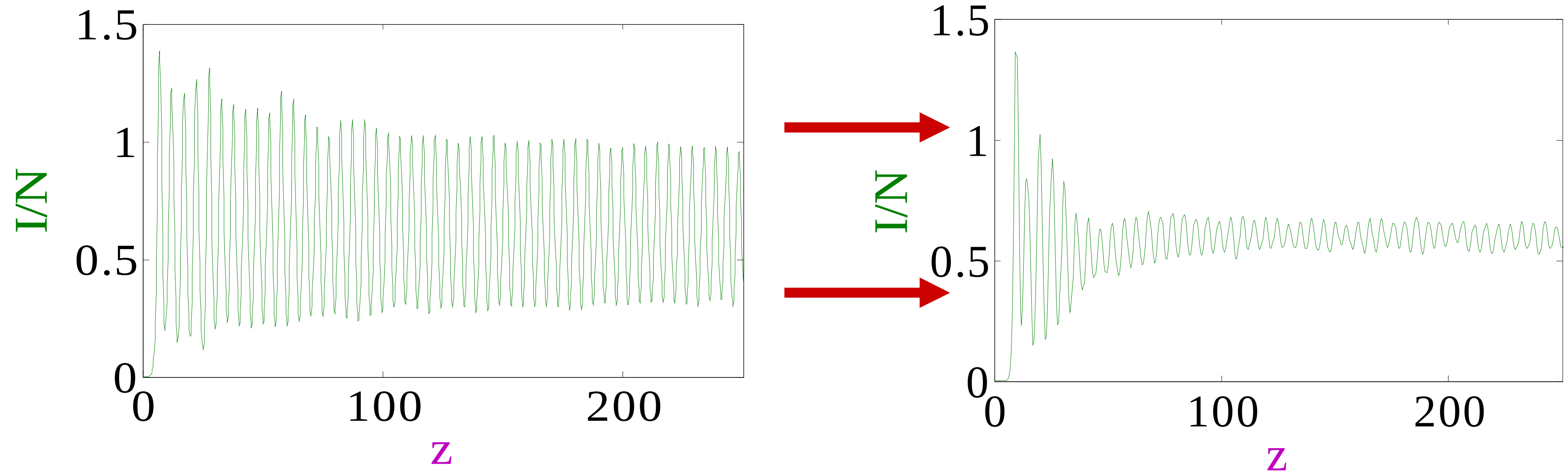
(q_j, p_j) longitudinal position/ momentum of the particles

(j, I) phase/ intensity of the wave



R. Bonifacio *et al.*, Rivista del Nuovo Cimento **3**, 1 (1990)

Reducing oscillations



A key structure : the macro-particle

- An aggregate of particles (bunching)
- Its oscillations in phase-space is responsible for the wave's fluctuations

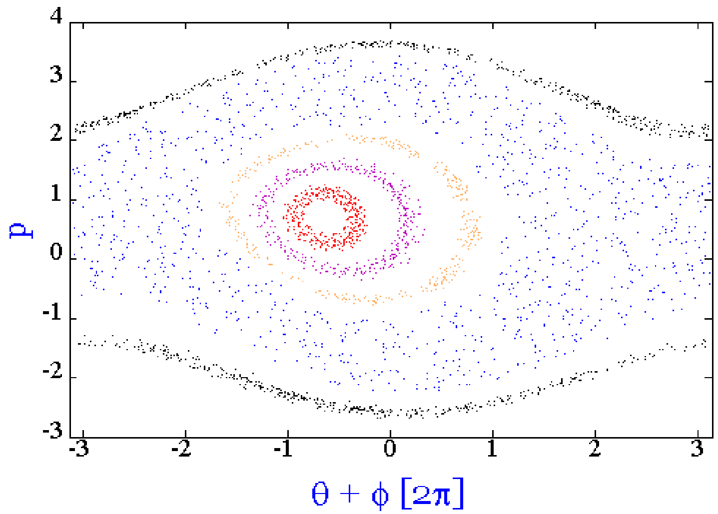
R. Bachelard *et al.*, Eur. Phys. J. D, **42**, 125 (2007)

A mean-field Hamiltonian

Self-consistent

$$H_N = \sum_{j=1}^{N} \frac{p_j^2}{2} - 2\sqrt{\frac{I}{N}} \sum_{j=1}^{N} \cos(\theta_j - \phi)$$

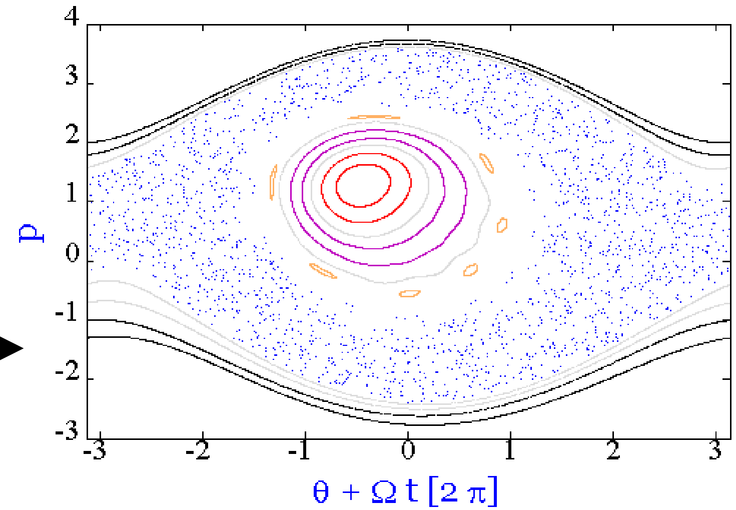
Poincaré sections at $dI/dt = 0$



Mean-field

$$H_{mf} = \frac{p^2}{2} - 2\sqrt{\frac{\tilde{I}(t)}{N}} \cos(\theta - \tilde{\phi}(t))$$

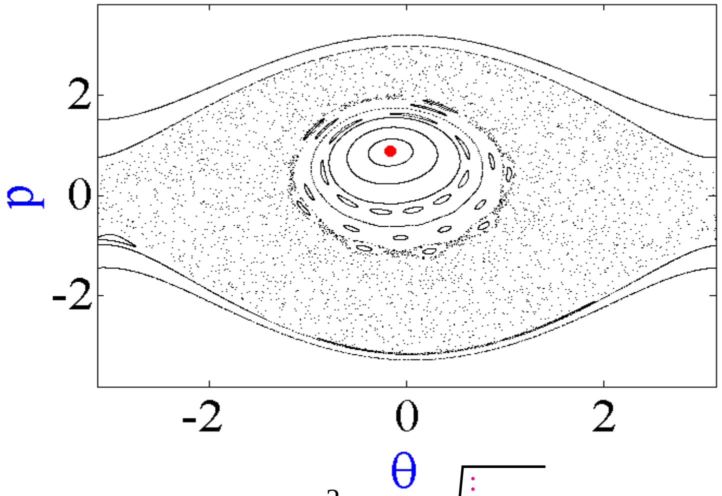
Poincaré sections at $d\tilde{I}(t)/dt = 0$

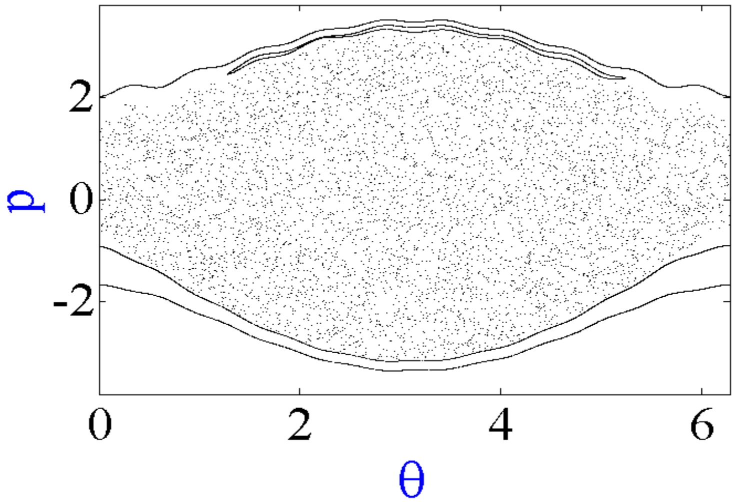


Mean-field model :

- The macro-particle seen as an invariant structure of phase-space
- Methods of « control of chaos » to reshape this invariant structure

The residue method



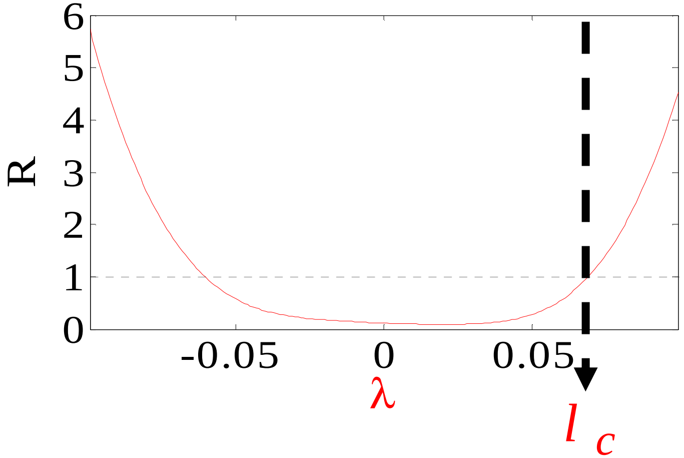


$$H_{mf} = \frac{p^2}{2} - 2\sqrt{\frac{I(t)}{N}}\cos\left(q - \varphi(t)\right)$$

Mean-field model

$$- 2l\cos(k(q - w_1 t))$$

+ test-wave

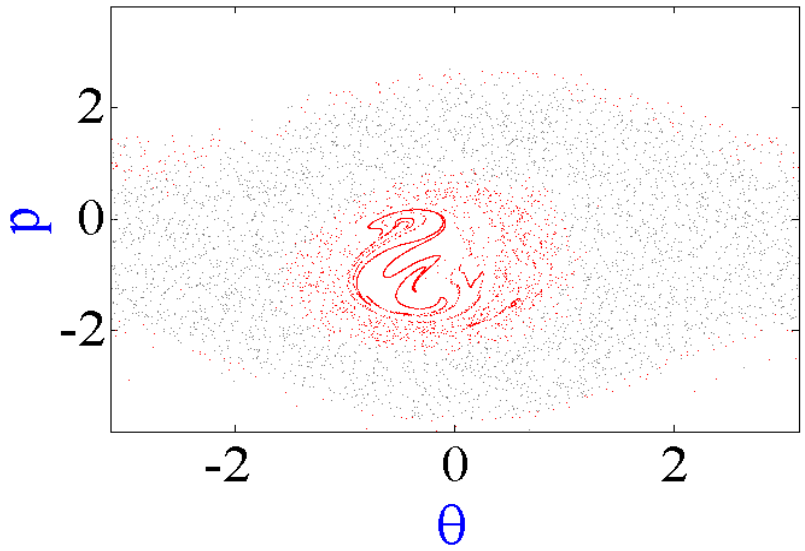


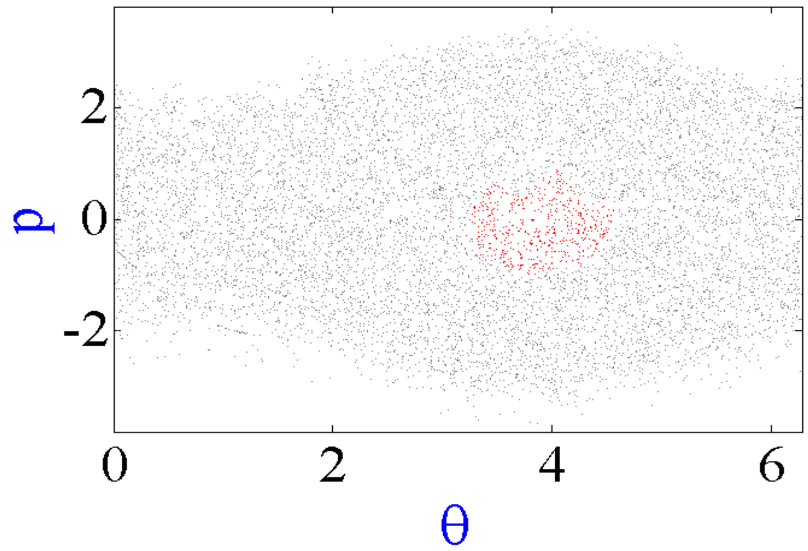
Linear Stability Analysis
of the Periodic Orbit

⇒ Destruction of the macro-particle

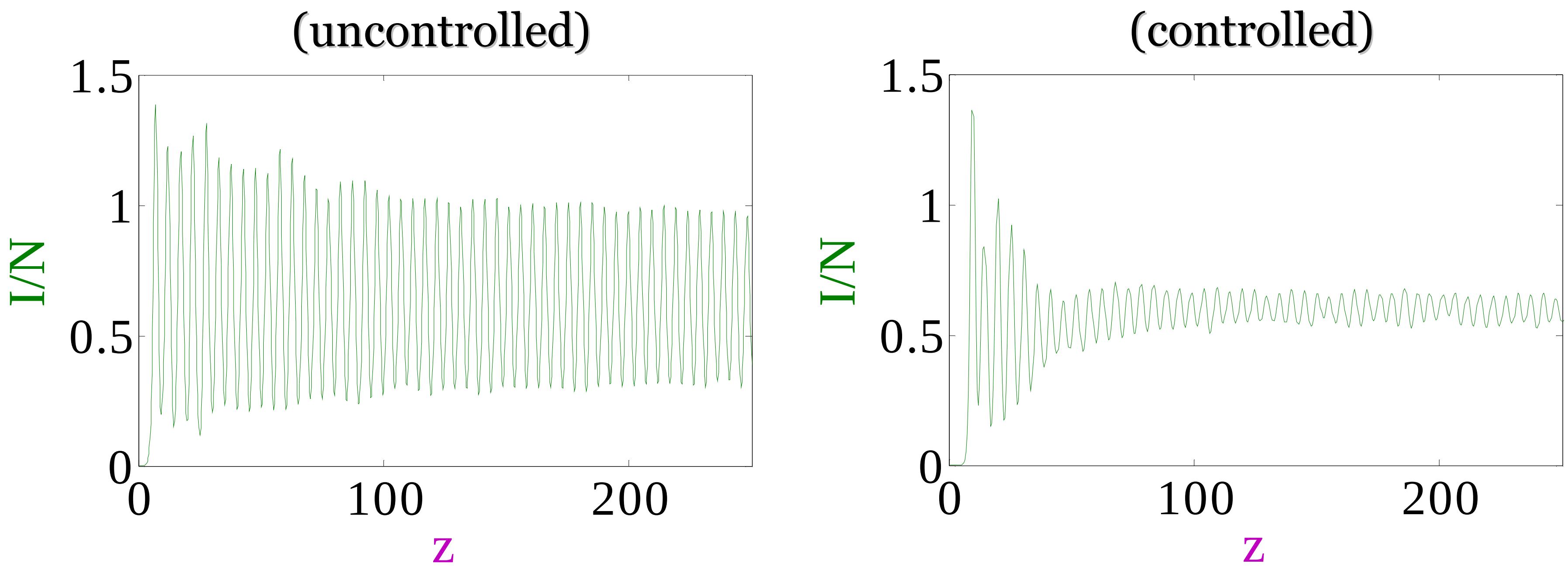
Control of the self-consistent dynamics

$$H_N = \sum_{j=1}^{N} \frac{p_j^2}{2} - 2\sqrt{\frac{I}{N}} \sum_{j=1}^{N} \cos(\theta_j + \varphi) \qquad - 2\lambda_c \sum_j \cos(k(q_j - \omega_1 t))$$

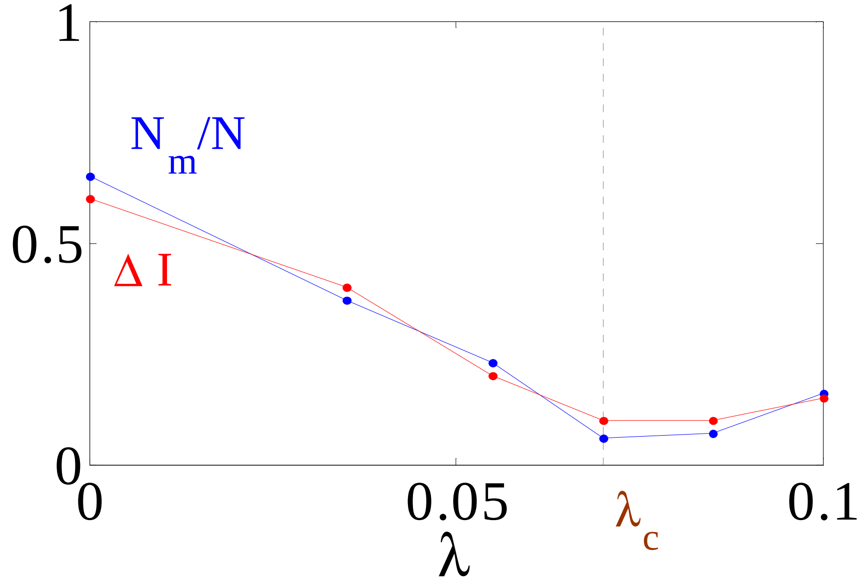




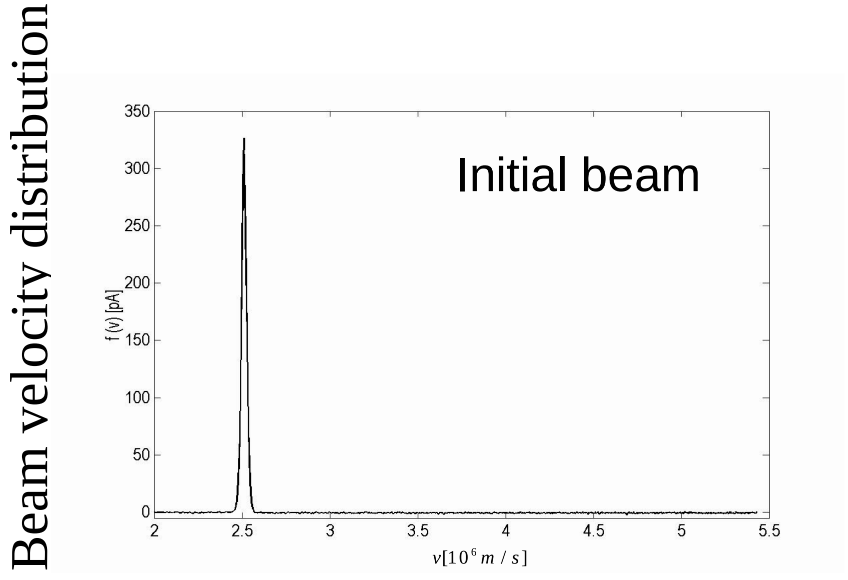
As for the intensity...

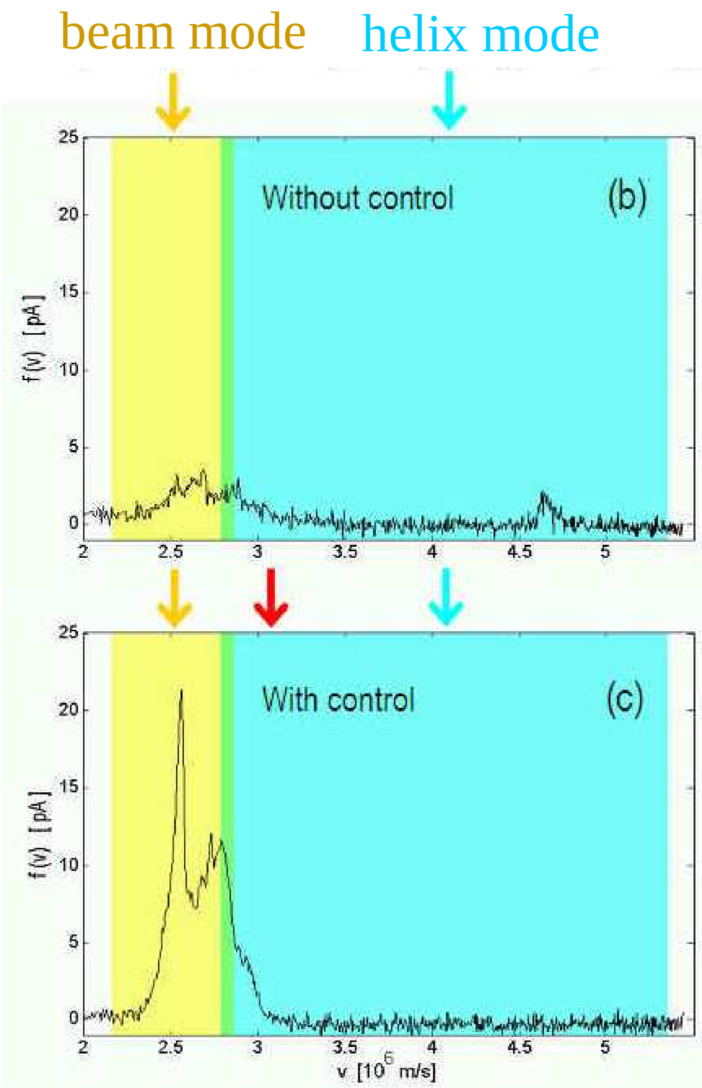


Optimality of the control



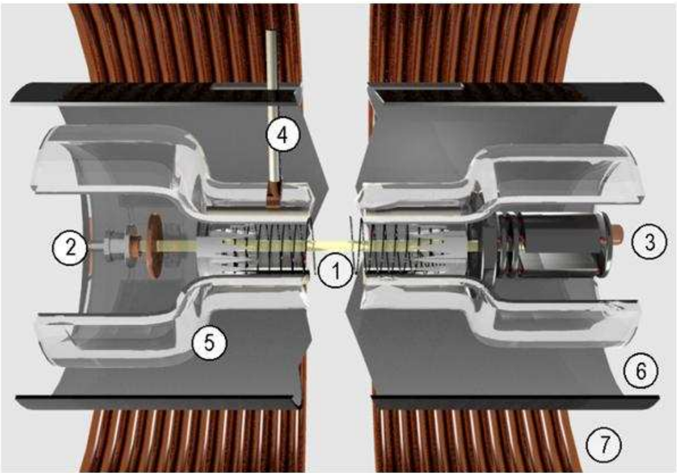
Experiment on a Travelling Wave Tube



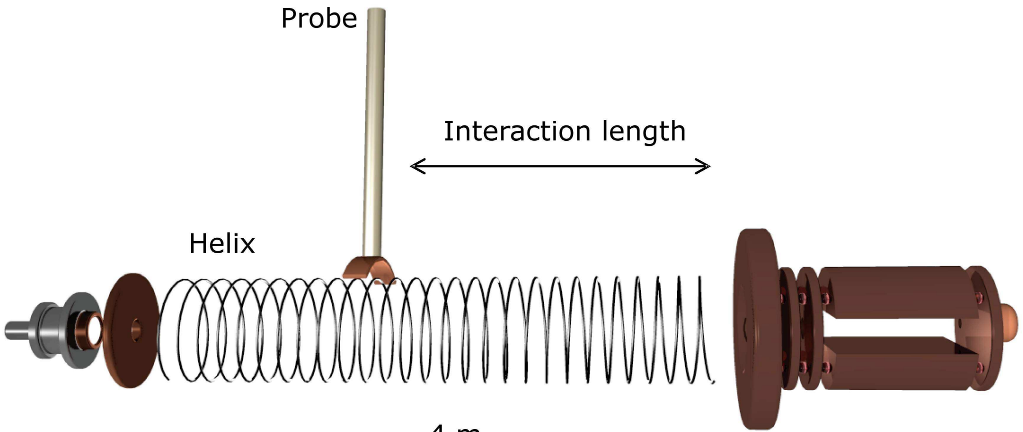


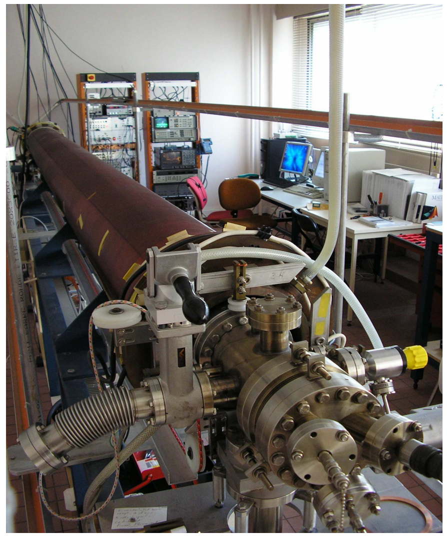
C. Chandre *et al.*, Phys. Rev. Lett. **94**, 074101 (2005)

Experiment on a Travelling Wave Tube



1) helix
2) electron gun
3) trochoïdal analyzer
4) antenna
5) vacuum chamber



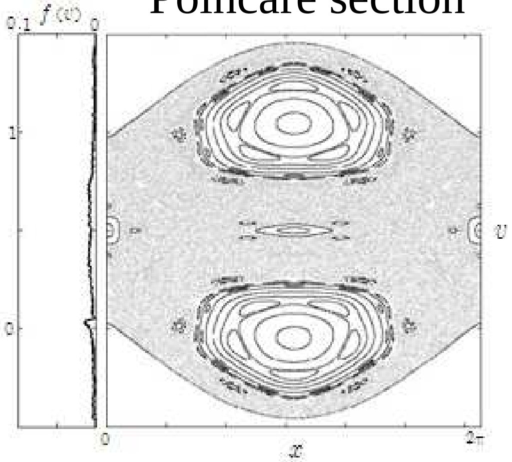


Experiment realized by F. Doveil, Y. Elskens, A. Macor
at PIIM, Université de Provence

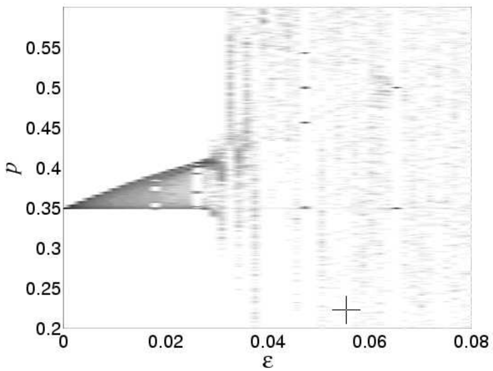
$$H_c = \frac{p^2}{2} + \varepsilon_1 \cos(k_1 x - \omega_1 t + \varphi_1) + \varepsilon_2 \cos(k_2 x - \omega_2 t + \varphi_2)$$

Poincaré section

PDF of the velocity



> PDF of the momenta of a trajectory

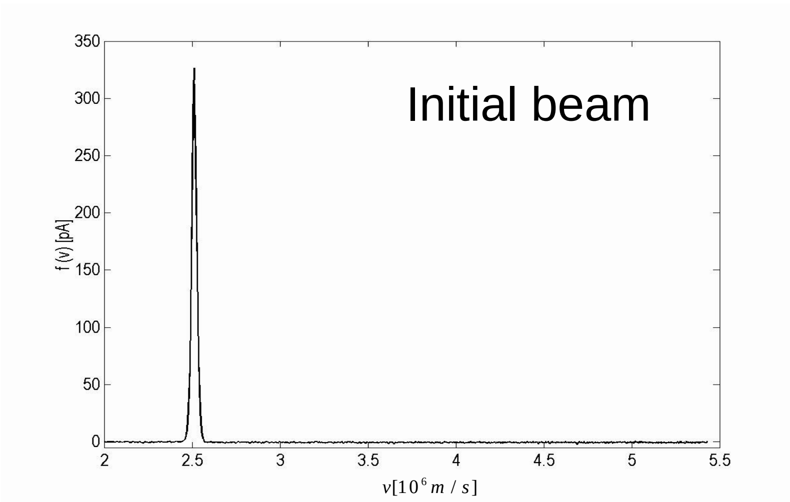


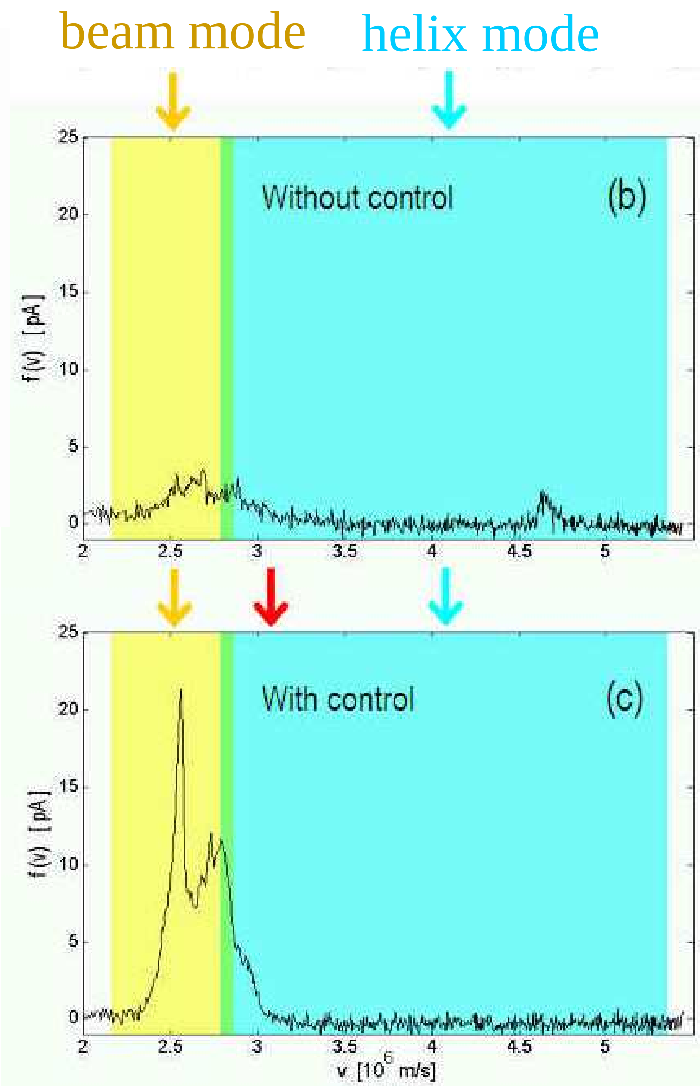
The new barriers prevent diffusion of electrons in position-momentum space

C. Chandre *et al.*, Phys. Rev. Lett. **94**, 074101 (2005)

Experiment on a Travelling Wave Tube

Beam velocity distribution





$$\text{Energy} : \boxed{\frac{E_{contr}}{E_{syst}} \approx 0{,}1\%}$$

C. Chandre *et al.*, Phys. Rev. Lett. **94**, 074101 (2005)

Experiment on a Travelling Wave Tube

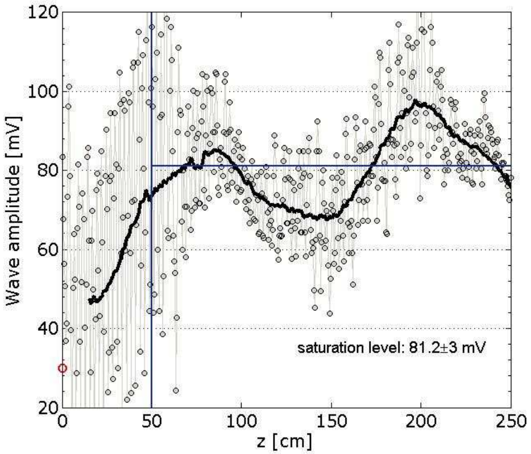
TWT with one wave at low intensity : test-particle regime

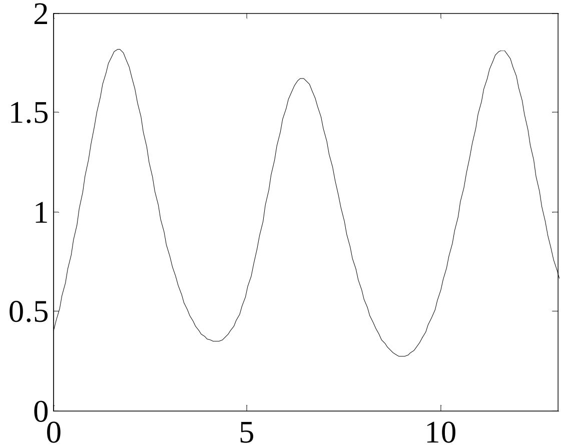
$$H = \sum_j \left(\frac{p_j^2}{2} + \varepsilon_1 \cos(k_1 x_j - \omega_1 t + \varphi_1) \right)$$

TWT with one wave at larger intensity : self-consistent regime

$$H = \sum_j \left(\frac{p_j^2}{2} + \sqrt{I} \cos(k x_j + \varphi) \right)$$

→ The TWT is a nice test-bed for FEL experiments





- Methods of control of invariant structures of phase-space
  - cutting down aggregation of particles by destroying invariant structures
  - reducing diffusion of particles in phase-space by building barriers

- Perspectives/in progress
  - a method of control to limit sensitivity to initial conditions (jittering), based on a Lie algebra formalism
  - derivation of a Hamiltonian model for a beam of particles interacting with an electromagnetic field