The Theory of

Bubble Acceleration:

numerical and analytical results

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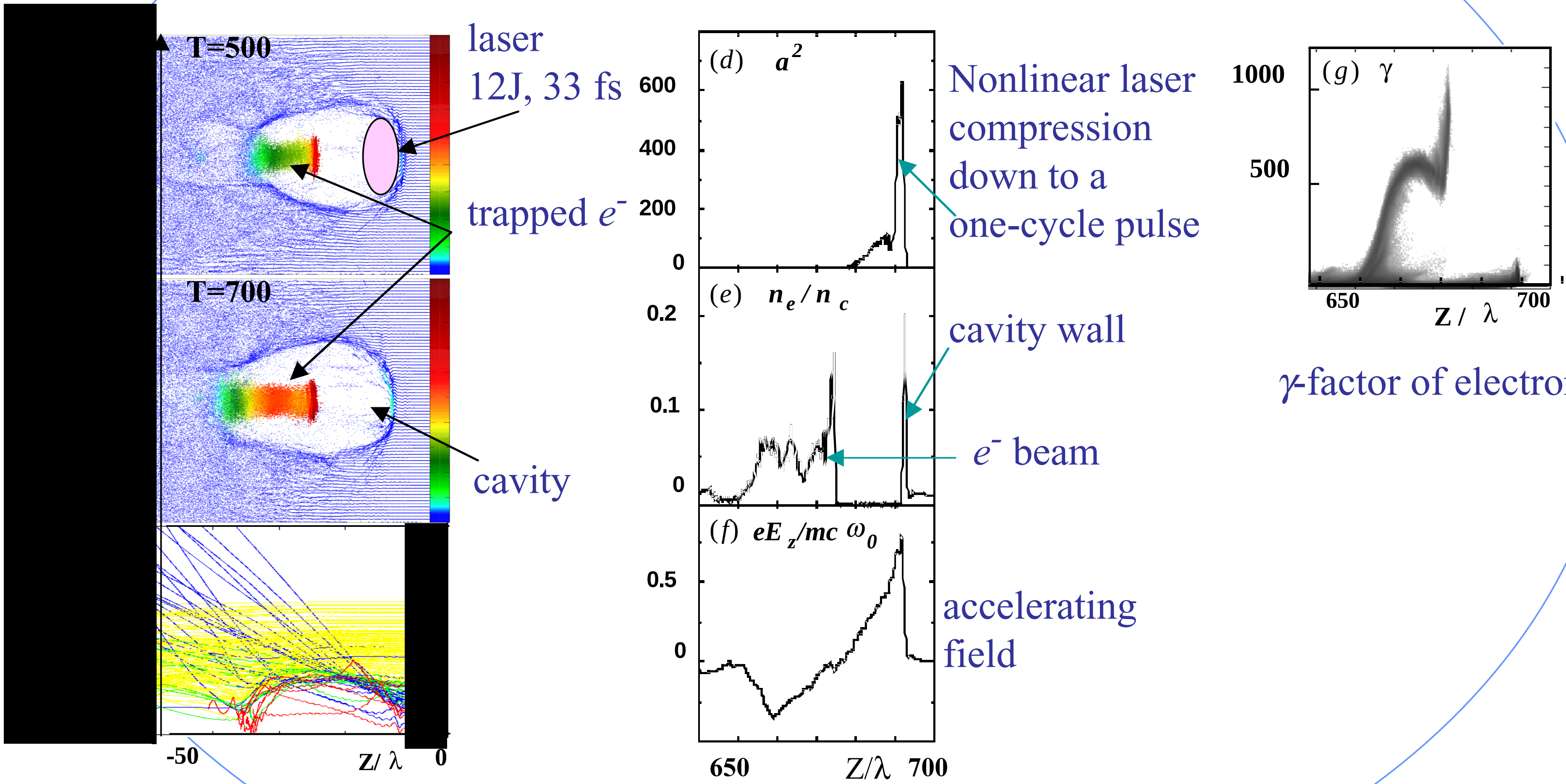
Outline

- Numerical results
- Analytical theory
- Conclusions

Numerical Results

A.Pukhov & J.Meyer-ter-Vehn, Appl. Phys. B, 74, p.355 (2002)



γ-factor of electrons

- A short laser pulse expels electrons and produces a cavity (**the bubble**).

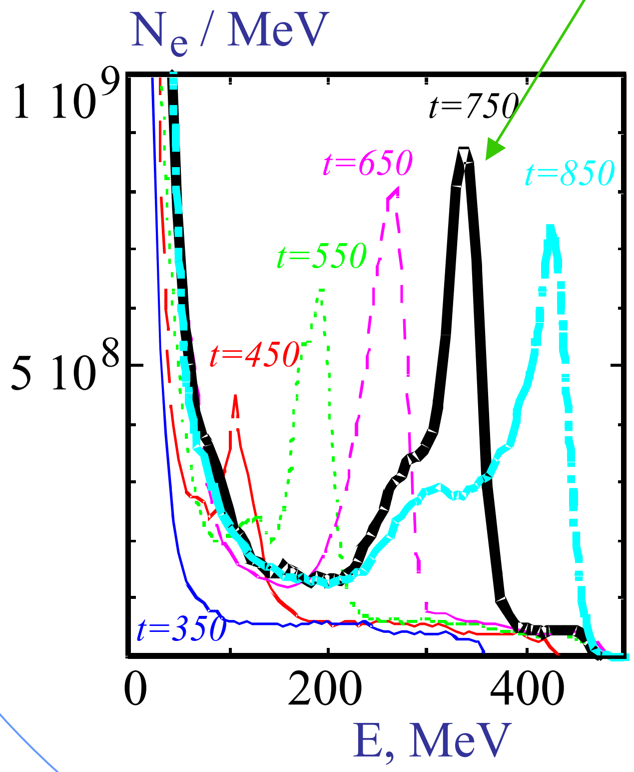
- The uncompensated charge of ions inside the bubble attracts and accelerates electrons at the rear side of the cavity.

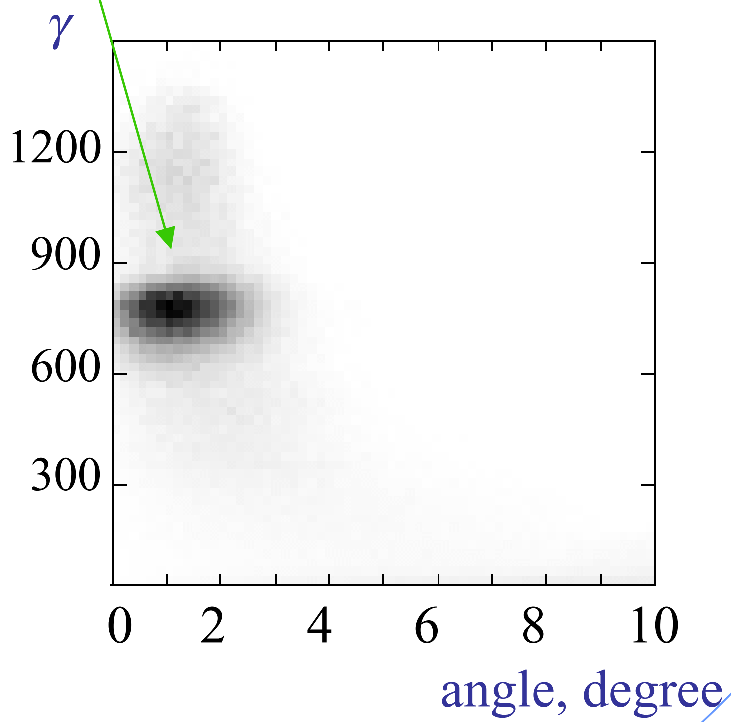
A.Pukhov & J.Meyer-ter-Vehn, Appl. Phys. B, 74, p.355 (2002)

Time evolution of the electron spectrum

monoenergetic electron beam

The *normalized* transverse emittance of the beam
ε = 5π μm mrad





The Key Result

The bubble acceleration scheme generates a quasi-monochromatic electron jet.

Analytical Theory

Solved problems

- Physics of the cavity
- The relativistic _-factors of the bubble $\_{p}$ and the trapped electrons $\_{tr}$
- Bubble stability

Theoretical Model

- Unmovable ions
- Cold hydrodynamics for electrons

$$\partial_t \vec{p} + \left(\vec{v} \quad \right)\vec{p} = -q\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{H}\right)$$

- Maxwell’s equations

The most important property of the theoretical model

The ultrarelativistic bubble regime can be described with the linear theory.

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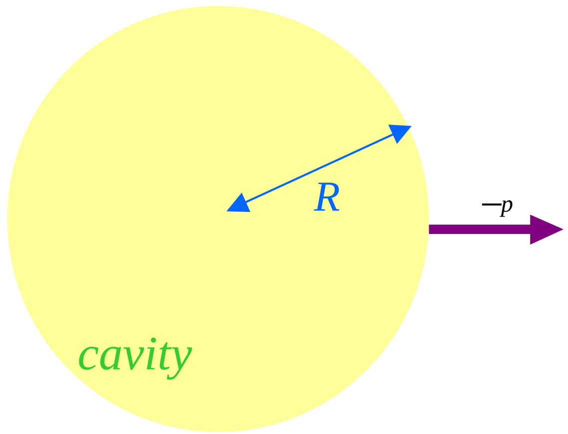
Theoretical Outcome

- We obtained a 3D analytical solution in the ultrarelativistic limit.

- We present the main properties of this solution.

Physics of the cavity

$$\left(\frac{2\pi R}{\lambda_p}\right)^2 = 2.6\gamma_p\sqrt{\ln(2\gamma_p)}$$



$$\lambda_p = \frac{2\pi c}{\omega_{pe}}$$

The boundary of the bubble is where the relativistic factor of electrons coincides with the relativisticfactor of the bubble (the resonance particle-wave interaction).

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The largest relativistic factor of the trapped electrons

$$\gamma_{tr} = 11\gamma_p^{3}\sqrt{\ln(2\gamma_p)}$$

Bubble Stability

• The perturbations of the longitudinal momentum component decay as $p_x \sim 1/\tau$

• The perturbations of the transversal momentum components decay as $p_y, p_z \sim 1/\tau^2$

• The stability is due to 3D-effects: the perturbations run away before they are amplified (convective stability)

Basic Equations

Charge Conservation

$$\frac{\partial \rho}{\partial t} + div\left(\vec{u}\rho\right) = 0$$

Linear Dynamical Equations

$$\left(-\partial_t \partial_x - {}^2\right)P_x = -\left(1 + \rho\right)$$

$$\left(\partial_t{}^2 - \partial_x{}^2 - \partial_z{}^2\right)P_y + \partial_y \partial_z P_z = \partial_y\left(\partial_x + \partial_t\right)P_x$$

$$\partial_y \partial_z P_y + \left(\partial_t{}^2 - \partial_x{}^2 - \partial_y{}^2\right)P_z = \partial_y\left(\partial_x + \partial_t\right)P_x$$

Conclusions

- **The bubble accelerationregime is the only known stable linear regime of ultra-relativistic acceleration freed of chaos and non-linear instabilities.**

- **The bubble acceleration regime is able to generate a quasi-monochromatic electron spectrum.**