The transition from pQCD to npQCD

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First Workshop on Quark-Hadron Duality and the Transition to pQCD
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- Introduction
- Overview of Data
- Different approaches for duality and choice of the method
- Transition from pQCD to npQCD
- Studies of HT
- Conclusions & Outlook
Introduction (1)

High energy reaction: cross section factors into

**Long distance**: measurable part

$1/\Lambda_{QCD} \approx \text{hadronic size}$

Related to quarks and gluons distribution inside the nucleon: hadronic observables

**Low energy**: confinement, npQCD

**Short distance**: pert. calculable part

$1/Q \ll \text{hadronic size}$

Parton interaction negligible: asymptotically free quarks

**High energy**: regime of perturbative QCD

⇒ Transition from soft to hard QCD

The mechanism of transformation of parton into hadron (and vice versa) modifies the final state: partons get transformed but not the cross section

Hadronic cross sections

(averaged over appropriate energy range)

$\Sigma_{\text{hadrons}} = \Sigma_{\text{quarks+gluons}}$

Partonic cross sections

(from perturbative quark-gluon theory)

Complementarity between Parton and Hadron description of observables

Relation to nature and transition from non-perturbative to pQCD
Introduction (2)

Present in Nature in different aspects:

- \(e^+ - e^- \rightarrow \text{hadrons} \equiv \sum_q (e^+e^- \rightarrow q\bar{q}) \Rightarrow \sigma_{\text{hadrons}} \equiv \sum_q \hat{\sigma}_q\)

- \(ep \rightarrow eX \Rightarrow d\sigma \approx \sum_q \int dx q(x, Q^2) d\hat{\sigma}_q\)

- \(ep \rightarrow ehX \Rightarrow d\sigma \approx \sum_q \int dx q(x, Q^2) D_h(z, Q^2) d\hat{\sigma}_q\)

- \(e^-p \rightarrow e^- X\)

- \(eA \rightarrow eX\)

- \(\tau \rightarrow \nu + \text{hadrons}\)

- semi-leptonic decay of heavy quarks

- \(\gamma p \rightarrow \pi^+ + n\)
\[ \tau \rightarrow \nu + \text{hadrons} \]

M. Shifman, hep-th/0009131

\[ \gamma p \rightarrow \pi^+ n \]

L.Y. Zhu et al., PRL 91 (2003) 022003,
L.Y. Zhu et al., PRC 71 (2005) 044603

\[ eA \rightarrow eX \]

J. Arrington et al. (submitted)
Data (2)

\[ e^+ - e^- \rightarrow \text{hadrons} \]

\[ e p \rightarrow e X \]

I. Niculescu et al., PRL 85 (2000) 1182,
I. Niculescu et al., PRL 85 (2000) 1186
A. Airapetian et al., PRL 90 (2003) 092002

\[ e \rightarrow p^{\pm} \rightarrow e \rightarrow X \]

R. Fatemi et al., PRL 91 (2003) 222002

\[ e \rightarrow p^{\pm} \rightarrow e \rightarrow X \]

\[ \frac{A_1^{\text{res}}}{A_1^{\text{DIS}}} = 1.11 \pm 0.16 \pm 0.18 \]

for \( Q^2 > 1.6 \text{ GeV}^2 \)

\[ \frac{A_1^{\text{res}}}{A_1^{\text{DIS}}} > 1 \quad \text{for} \quad Q^2 < 1.1 \text{ GeV}^2 \]

Strong violation of duality

Preliminary Eg1 data

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Remarks and Questions

- Breakdown of Duality at sufficiently low $Q^2$:
  1. Which value of $Q^2$?
  2. Same value for unpolarised and polarised structure functions?

- Duality expected to be isospin dependent:
  1. $p$ behavior
  2. $n$ behavior

Close & Isgur, PL B509 (2001) 81; Isgur et al., PRD 64 (2001) 054005

*Global duality* $\implies$ average over large $W^2$ range (whole resonance region)

*Local duality* $\implies$ average over small $W^2$ range (single resonances)

Important: passage from **qualitatively** to **quantitatively** picture
Kinematical variables

\[ x' = 1/\omega' \quad \omega' = 1/x + M^2/Q^2 \quad \text{B.G.} \]

\[ \xi = 2x/(1 + (1 + 4x^2M^2/Q^2)^{1/2}) \quad \text{Jlab} \]

\[ x_w = Q^2 + B/(Q^2 + W^2 - M^2 + A) \quad \text{B.Y.} \]

\[ x', \xi \text{ rescale S.F. to lower } x \text{ with } Q^2 \text{ dep.} \]

Rescaling larger at lower \( Q^2 \)

Use of \( x \) to avoid ambiguities associated to usage other variables
3 approaches (1)

a) Mellin moments:

\[ M_n(Q^2) = \int_0^1 dx x^{n-2} F_2(x, Q^2) \]

elastic contribution should be included
elastic contribution dominant for \( Q^2 \leq 1 \text{ GeV}^2 \)
need of experimental values of SF outside resonance region

b) Point by point comparison: SF vs \( Q^2 \) at specific \( x \) values
elastic contribution excluded by kinematic
ok for unpolarised SF because lot of data, NOT ok for polarised SF

c) Comparison between SF integrals in RES & DIS regions, in the same \( x \) interval
elastic contribution excluded by kinematic
3 approaches (2)

a) Mellin moments

\[ \mathcal{I}_n, \mathcal{M}_n \]

\[ n=2 \]

\[ n=4 \]

\[ n=6 \]

\[ n=8 \]

\[ Q^2 (\text{GeV}^2) \]

b) Point by point comparison

S. Liuti et. al, PRL 89 (2002) 162001
c) Comparison between SF integrals in RES & DIS regions, in the same $x$ interval

\[
I^{res}(Q^2) = \int_{x_m}^{x_M} F_2^{Res}(x, Q^2) \, dx
\]

\[
I^{DIS}(Q^2) = \int_{x_m}^{x_M} F_2^{DIS}(x, Q^2) \, dx
\]

\[
\tilde{\Gamma}^{res}_1(Q^2) = \int_{x_m}^{x_M} g_1^{Res}(x, Q^2) \, dx
\]

\[
\tilde{\Gamma}^{DIS}_1(Q^2) = \int_{x_m}^{x_M} g_1^{DIS}(x, Q^2) \, dx
\]

\[
g_1 = A_1 \cdot \frac{F_2}{2x(1+R)}
\]

\[
(x_M \div x_m) \iff W_m^2 \div W_M^2 \simeq 1 \div 4 \text{ GeV}^2 \forall Q^2
\]

\[
R = \frac{I^{Res}}{I^{DIS}} = 1 \iff \text{Duality fulfilled} \implies R = \frac{\tilde{\Gamma}^{Res}_1}{\tilde{\Gamma}^{DIS}_1} = 1
\]

- Resonance region can be described in terms of quark degrees of freedom
- Distinction between resonance & DIS region is somehow artificial
- Duality provides access to large $x$ where DIS data suffer for low statistic
Transition from pQCD to npQCD

Problem of continuation of the pQCD curve into the resonance region

Theoretically based on the idea that partonic d.o.f are dominant in the RES region

Starting point: NLO PDF for the unpolarised structure function $F_2$

Practically - even under this assumption - corrections to the NLO analysis arise from:

- Target Mass Corrections (TMC) $\Rightarrow O(1/Q^2)$
- Large $x$ Resummation effects (LxR) $\Rightarrow$ Leading Twist
- NNLO $\Rightarrow$ Leading Twist
- Dynamical Higher Twist (HT) $\Rightarrow O(1/Q^2)$
- For the neutron: nuclear effects $\Rightarrow$ Leading Twist
- Anything else $\Rightarrow$ beyond twist expansion

Corrections have to be applied consistently to ALL observables to guarantee universality.
$F_2^{\text{DIS}}$ from PDF (LO & NLO)

PDFs: MRST99, CTEQ5, GRV94 (LO & NLO), GRV98 (LO & NLO)

Quark-Hadron Duality NOT fulfilled by PDFs at LO or NLO

NLO PDF unable to reproduce large $x$ region
From Phenomenological Parameterisations

Phen. Parameterisations: ALLM97, NMC95, BY (GRV94mod)

Obtained by fitting DIS data even at low $Q^2$

$\Rightarrow$ implicitly include non-perturbative effects
Non-perturbative Contributions

- Starting point: NLO PDF at $Q^2 = Q_0^2$
- Evaluation of Target Mass Correction
- Evaluation of Large $x$ Resummation

Quantitative analysis:

$\Rightarrow$ Disentangle Non Perturbative Contributions
Target Mass Corrections (TMC)

\[ F_2(x, Q^2) = F_{2LT}(x, Q^2) + \frac{H(x, Q^2)}{Q^2} + \mathcal{O}(1/Q^4) \]

\[ F_{2LT,TMC}^c(x, Q^2) = \frac{x^2}{\xi^2 \gamma^3} F_2^\infty(\xi, Q^2) + 6 \frac{x^3 M^2}{Q^2 \gamma^4} \int_\xi^1 \frac{d\xi'}{\xi'^2} F_2(\xi', Q^2) \]

\[ F_2^\infty = F_2 \text{ without TMC} \]

Limit of validity: \( x^2 M^2 / Q^2 < 1 \)

Applied in a similar way to

\[ g_1 = A_1 \cdot \frac{F_2}{2x(1+R)} \]
Large $x$ Resummation (1)

- First observed by Brodsky and Lepage, SLAC-REP224 (1979)

- Recently reconsidered by:
  2. S. Liuti et al. PRL 89 (2002) 162001

Scattering from off-shell quark:

\[ k^2_{\mu} = x \left[ M^2 - \frac{k^2 \perp + M_X^2}{1-x} - \frac{k^2 \perp}{x} \right] \neq m^2 \]
Large $x$ Resummation (2)

Consequence:

Phase space for the parton’s $k_T$

limited by

\[ k^2_{T(MAX)} = Q^2(1 - z)/z \]

instead of

\[ k^2_{T(MAX)} \approx Q^2 \]

LxR terms arise from terms containing power of $\ln(1-z)$ terms in $C_{NS}(z)$

\[ F_2^{NS}(x, Q^2) = \frac{\alpha_s}{2\pi} \sum_q \int_x^1 dz \ C_{NS}(z) \ q_{NS}(x/z, Q^2) \]

- $z$ longitudinal variable in evolution equations; $C(z)$ Wilson coefficient functions
- only valence quark distributions relevant in this kinematic $\rightarrow F_2^{NS}$

\[ x \gg \Rightarrow C_{NS} \gg \Rightarrow Q^2 \rightarrow Q^2(1 - z)/z \ \text{and} \ \alpha_s(Q^2) \rightarrow \alpha_s(Q^2(1 - z)/z) \]
Effects of Target Mass Correction (TMC) and Large $x$ Resummation (LxR)

Duality seems satisfied within $\approx 10\%$ for $Q^2 \geq 1.5$ GeV$^2$

⇒ Investigation of this 10% effect
Polarised case and data from Jlab

\[ R_{LT} = \frac{\tilde{\Gamma}_1 \text{ res}}{\tilde{\Gamma}_1} \]

\[ Q^2 \text{ [GeV}^2\text{]} \]

\[ R. \text{ Fatemi } et \text{ al.}, \text{ PRL } 91 \text{ (2003) 222002} \]

E94110-data supplied by V. Tvaskis

\[ \Gamma \sim 1 \text{ res} / \Gamma \sim 1 \]

\[ 10^{-1} \]

\[ 1 \]

\[ 10 \]

\[ 10 \]

\[ Q^2 \text{ [GeV}^2\text{]} \]
$x$ dependence of HT

- NLO + TMC + LxR analysis → very small HT in whole $x$ region
- Extracted values consistent with different method & more precise
- Different behaviour for HT at low $Q^2$
HT contributions

\[ H(x, Q^2) = Q^2 (F^\text{res}_2(x, Q^2) - F^\text{LT}_2); \quad C_{HT} = \frac{H(x, Q^2)}{F^\text{pQCD}_2} \equiv Q^2 \frac{F^\text{res}_2(x, Q^2) - F^\text{LT}_2}{F^\text{LT}_2} \]

Comparison of HT from RES and from DIS (old analyses) at same \( x \) values

Low \( Q^2 \): \( H_{\text{pol}} \) large and negative
\[ F_{2}^{LT+HT} = F_{2}^{LT} \cdot \left(1 + \frac{C}{Q^2}\right) \]

high \( x \): \( C_{res}(x) \neq C_{DIS}(x) \)

\[ C(x) = \frac{H(x)}{F_{2}^{LT}} \]

No \( Q^2 \) dependence in \( C(x) \) and \( H(x) \)

Different behavior for unpolarised and polarised HT
Conclusions

• Quantitative analysis of Unpolarised and Polarised data compared with:
  - pQCD analyses using global PDF (GRV94, GRV98, CTEQ5, MRST99)
  - phenomenological fits with non-perturbative contributions (ALLM97, NMC95, BY (GRV94mod))

• Non perturbative contributions, TMC and LxR disentangled

• Duality seems satisfied within 10%

• Extraction of HT:
  1. Polarised $\neq$ Unpolarised
  2. RES $\neq$ DIS
Outlook

- Open questions:
  1. Are we unraveling new degrees of freedom more pertinent to the scale of the hadronization phase?
  2. Do we understand the $Q^2$ dependence in terms of a “standard” pQCD based scheme?
  3. Are we witnessing a breakdown on factorization?
  4. How are the smooth curves compared to the data? What are the best statistical estimators to be used?

- Many data from different reactions on proton, neutron, GDH, nuclei, semi-inclusive, photoproduction ... are available

- More $e^+e^-$, $\tau$ decays...to be explored

- Many new results and approaches seen in this workshop
\[ R_{LT}^{\text{pol}} = \frac{\Gamma_{LT}}{\Gamma_1} \]

- PDF uncertainty
- Exp. uncertainty

\( g_1^{\text{GRSV}} \) (2000)
\( g_1^{\text{BB}} \) (2002)