Nearly Conformal QCD and AdS/CFT

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First Workshop on Quark-Hadron Duality and the Transition to pQCD

GdT and S. J. Brodsky, PRL **94**, 201601 (2005)

Outline

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- ADS/CFT and QCD
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The Holographic Correspondence

- Original correspondence between $\mathcal{N} = 4$ SYM at large N_C and the low energy supergravity approximation to Type IIB string on $AdS_5 \times S^5$:

  Maldacena, hep-th/9711200.

$$\begin{array}{ccc} \text{Warped higher dim space} & & \text{Conformal d} = 4 \text{ spacetime boundary} \\ \text{Type IIB } \ (\mathrm{AdS}_5 \times \mathrm{S}^5) & \leftrightarrow & \mathcal{N} = 4 \,\mathrm{SYM}\, (\mathrm{SO}(4,2) \otimes \mathrm{SU}(4)) \\ ? & \leftrightarrow & \mathrm{QCD} \end{array}$$

$SO(4,2)$ is isomorphic with the isometries of AdS^5, and $SU(4) \sim SO(6)$ with S^5.

- Description of strongly coupled gauge theory using a dual gravity description!

- QCD is fundamentally different from SYM theories where all matter is in the adjoint rep of $SU(N_C)$. Introduction of quarks in the fundamental rep is dual to an open string sector:

  Gross and Ooguri, hep-th/9805129; E. Witten, hep-th/9805112.

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0209080; hep-th/0212207; de Téramond and Brodsky, hep-th/0409074; hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218.

- Gluonium spectrum (top-bottom):

Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115.

- D3/D7 branes (top-bottom):

Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and J. Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nunez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and S. Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164.

- Other aspects of high energy scattering in warped spaces:

S. B. Giddings, hep-th/0203004; Andreev and Siegel, arXiv:hep-th/0410131; Kang and Nastase, hep-th/0410173; Nastase, hep-th/0501039; hep-th/0501068.

- Branes in Minkowski space:

Siopsis, hep-th/0503245.

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^μ, D, K^μ, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops (running coupling). For $\beta = d\alpha_s(Q^2)/dlnQ^2 = 0$ (fixed point theory), PQCD is a conformal theory: Parisi, Phys. Lett. B **39**, 643 (1972).

- Phenomenological success of dimensional scaling laws for exclusive processes $d\sigma/dt \sim 1/s^{n-2}$ (n total number of constituents), implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies (PQCD predicts powers of α_s and logs).

- Theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point (constant in the IR): Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Brodsky, Menke, Merino and Rathsman, hep-ph/0212078; Deur, this conference.

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu,\nu} dx^\mu dx^\nu - dz^2) : \; x^\mu \to \lambda x^\mu, \; z \to \lambda z,$$

maps scale transformations into the holographic coordinate $r = R^2/z$.

- String mode in r is the extension of the hadron wf into the fifth dimension.
- Different values of r correspond to different scales at which the hadron is examined.
- Invariant separation between quarks:

$$x_\mu x^\mu \to \lambda^2 x^2, \quad r \to \frac{r}{\lambda}.$$

- The AdS boundary at $r \to \infty$ correspond to the $Q \to \infty$, UV zero separation limit.

Confinement

- QCD is a confining theory in the infrared with mass gap Λ_{QCD} and a well-defined spectrum of color-singlet states.
- There is a maximum separation of quarks and a minimum value of r.
- AdS space should end at a finite value $r_o = \Lambda_{QCD} R^2 = 1/z_o$.
- Cutoff at r_o breaks conformal invariance and allows the introduction of the QCD scale.
- Non-conformal metric dual to a confining gauge theory:

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) + ds_X^2,$$

  where $A(z) \to 0$ as $z \to 0$. Asymptotically $AdS \times X$, where X is a 5-dim compact manifold if 4-dim gauge theory is dual to a critical (10 dim) string (Polchinski and Strassler).
- Simplified model: metric factor $e^{2A(z)}$ is a step function. Analog to the MIT bag model, but with boundary conditions on the holographic coordinate.
- Conformal behavior at short distances and confinement at large distances $\Longrightarrow$ Truncated AdS/CFT model.

Classical Correspondence and Interpolating Operators

- Duality between string in AdS_{d+1} and $N_C \to \infty$ limit of a conformal gauge theory given by the generating functionals of the string and CFT theories at the AdS d-dim boundary.

- d-dim generating functional in presence of external source Φ_o:

$$Z_{QCD} = [\Phi_o(x)] = \int \mathcal{D}[\psi, \overline{\psi}, A] \exp\left\{ iS_{QCD} + i \int d^d x \Phi_o \mathcal{O} \right\}.$$

- $d+1$-dim string partition function:

$$Z_{grav}[\Phi(x,z)] = \int \mathcal{D}[\Phi] e^{iS_{grav}[\Phi]} \,.$$

- Boundary condition:

$$Z_{grav}\left[\Phi(x,z)_{z=0} = \Phi_o(x)\right] = Z_{QCD}\left[\Phi_o\right].$$

Gubser, Klebanov and Polyakov, hep-th/9802109; Witten, hep-th/9802150

- Near the boundary of AdS_{d+1} space $z \to 0$:

$$\Phi(x,z) \to z^{\Delta}\Phi_+(x) + z^{d-\Delta}\Phi_-(x).$$

- $\Phi_-(x)$ is the boundary source: $\Phi_- = \Phi_o$ (Non-normalizable mode)

- $\Phi_+(x)$ is the response function (normalizable mode): $\langle \mathcal{O} \rangle_{\Phi_o} = (2\Delta - d)\, \Phi_+(x)$

  Klebanov and Witten, hep-th/9905104

- Dimensions: $[\Phi] = 0, \ [\Phi_-] = [\Phi_o] = d - \Delta, \ [\Phi_+] = [\mathcal{O}] = \Delta$

- The AdS/QCD correspondence is interpreted as a classical duality between the valence state of a hadron in the asymptotic 4-dim boundary and the lightest mass string mode in $AdS_5 \times S^5$.

- The physical string modes $\Phi(x,z) \sim e^{-iP\cdot x} f(r)$, are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.

- For large-r $f(r) \sim r^{-\Delta}$. The dimension Δ of the string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

- Quarks in the fundamental representation are introduced at the AdS boundary and we compute their wavefunctions as they propagate into the bulk.

- Interpolating operators $\mathcal{O}(x)$ at the boundary: $\mathcal{O} = G^a_{\mu\nu} G^{a\mu\nu}, \mathcal{O} = \overline{\psi}\gamma_5\psi, \ldots$
- QCD/gravity duality dictionary:

QCD (4-dim)	Gravity (5-dim)
Hadron int. op. $\mathcal{O}$	Normalizable mode $\Phi(x, z)$
Hadron mass $\mathcal{M}$	Eigenvalues of 5-dim WF
Conformal dim Δ	5-dim mass μ
Large Coupling	Small coupling
Large Q	Small z
Mass gap Λ_{QCD}	Cutoff $z = z_o$

Quantum Fluctuations and Boundary Excitations

- Higher Fock components of a hadron wf and states with non-zero orbital angular momentum are manifestations of quantum fluctuations of QCD.
- Metric fluctuations of the bulk geometry about the the fixed AdS background should correspond to quantum fluctuations of Fock states above the valence state.
- For large Lorentz spin, orbital excitations in the boundary correspond to quantum fluctuations in the AdS sector. See: Gubser, Klebanov and Polyakov, hep-th/0204051.
- Higher orbital excitations are matched quanta to quanta with fluctuations around the spin $0, \frac{1}{2}, 1, \frac{3}{2}$ string solutions on AdS_5. Identification avoids huge string dimensions $\Delta \sim (g_s N_C)^{\frac{1}{4}}$ at large N_C for spin > 2.
- The large-r asymptotic behavior of each string mode is matched with the conformal dimension of the boundary interpolating operators for each hadron state, maintaining conformal invariance: an L quantum excitation corresponds to a five dimensional mass μ in the bulk.
- Allowed values of μ determined asymptotically requiring that the dimensions are spaced by integers: spectral relation $(\mu R)^2 = \Delta(\Delta - 4)$. For large L: $\mu \simeq L/R$ (string results).

Glueball Spectrum

- AdS wave function with effective mass μ:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

where $\Phi(x,z) = e^{-iP\cdot x}\, f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

- Glueball interpolating operator with twist -dimension minus spin- two, and conformal dimension $\Delta = 4 + L$

$$\mathcal{O}_{4+L} = F D_{\{\ell_1} \dots D_{\ell_m\}} F,$$

where $L = \sum_{i=1}^m \ell_i$ is the total internal space-time orbital momentum.

- Normalizable scalar AdS mode (d = 4):

$$\Phi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,a}\Lambda_{QCD}\right)$$

with $\alpha = 2 + L$ and scaling dimension $\Delta = 4 + L$.

- 4-d mass spectrum from boundary conditions on the normalizable string mode at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$

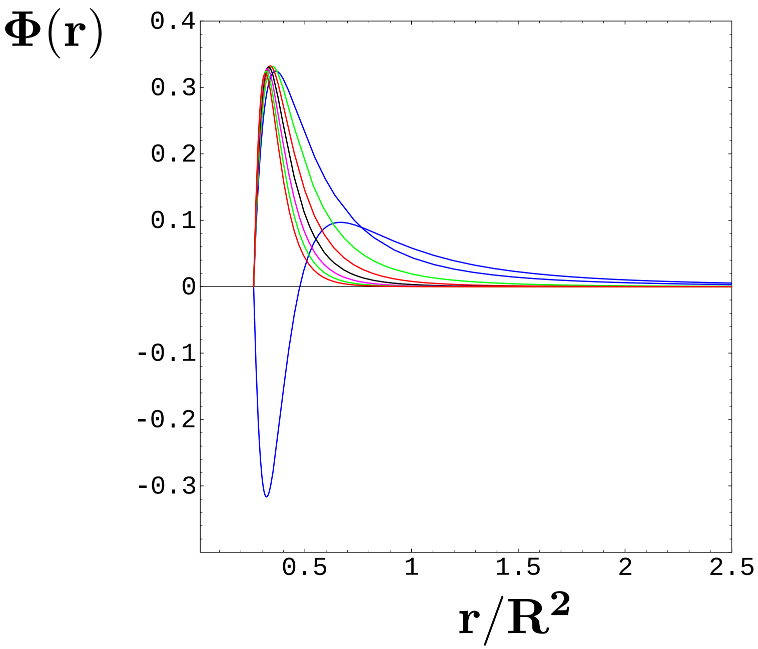


Fig: Gluonium orbital string modes for $\Lambda_{QCD} = r_o/R^2$ = 0.26 GeV.

- $L = 0$ lowest glueball state Θ^{++}: $\mathcal{M} = 1.34\,\mathrm{GeV}, \quad \Lambda_{QCD} = 0.26\,\mathrm{GeV}$.
- Lattice results: $N_C = 3, \;\; \mathcal{M} = 1.47 - 1.64\,\mathrm{GeV}$

Meson Spectrum

- Wave eq. in AdS for a vector field Φ_μ with polarization along Poincaré coordinates:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0,$$

where $\Phi_\mu(x, z) = e^{-iP\cdot x}\, f_\mu(z)$ and $P_\mu P^\mu = \mathcal{M}^2$ ($\Phi_z = 0$ gauge).

- Vector meson: twist-two, dimension $\Delta = 3 + L$

$$\mathcal{O}^\mu_{3+L} = \overline{\psi}\gamma^\mu D_{\{\ell_1} \ldots D_{\ell_m\}}\psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Normalizable AdS vector mode:

$$\Phi^\mu_{\alpha,k}(x, z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,k}\Lambda_{QCD}\right)\epsilon^\mu,$$

with $\alpha = 1 + L$ and $\Delta = 3 + L$.

- 4-d mass spectrum $\Phi^\mu(x, z_o) = 0$: $\mathcal{M}_{\nu,n} = \alpha_{\nu,n}\Lambda_{QCD}$.
- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \ldots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).

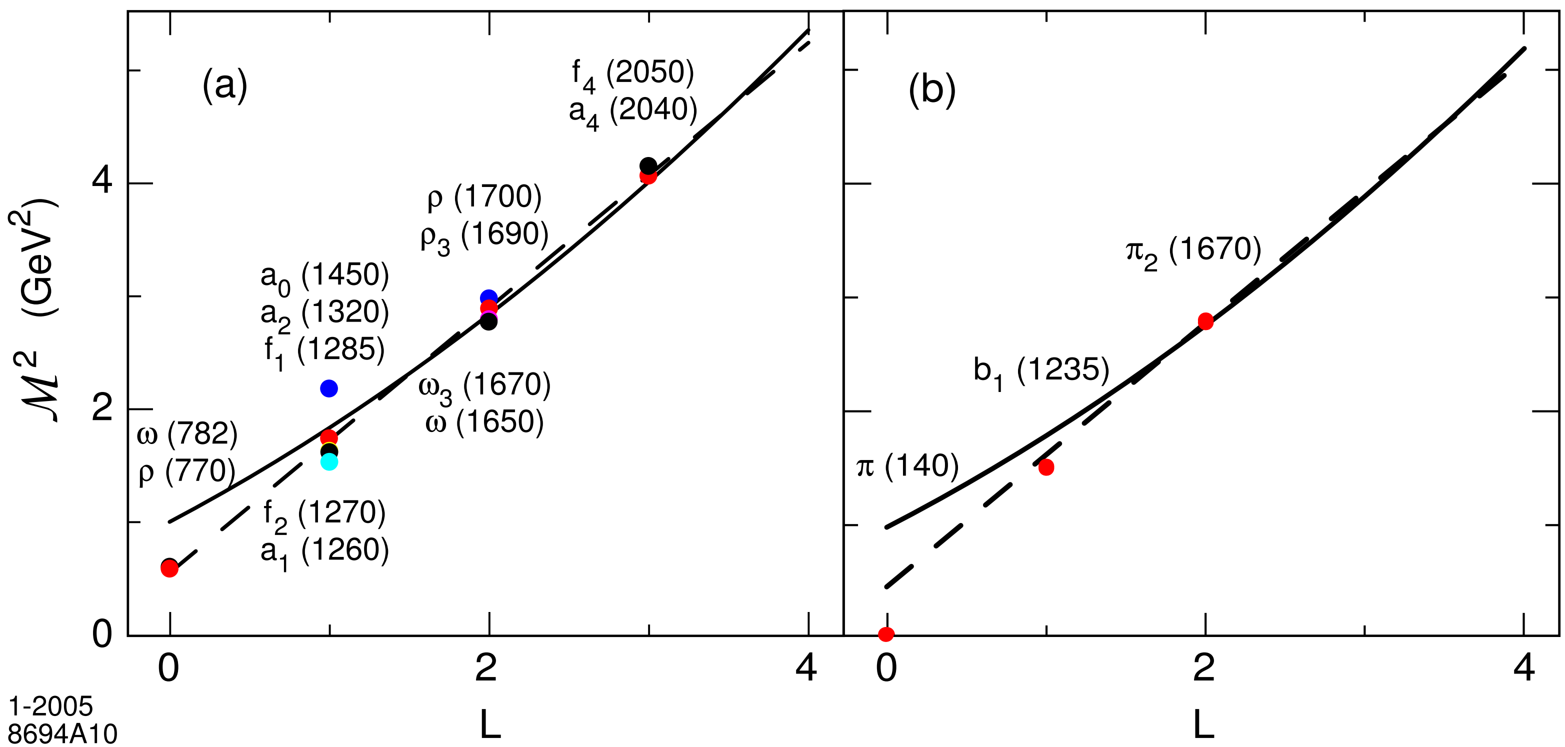


Fig: Light meson orbital spectrum: 4-dim states dual to vector fields in the bulk, $\Lambda_{QCD} = 0.26 \text{ GeV}$

Baryon Spectrum

- Solve the full 10-dim Dirac, $\not{D}\hat{\Psi} = 0$, since baryons are charged under the $SU(4) \sim SO(6)$ R-symmetry of S^5 (string y-junction) - baryon number conservation?
- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on the compact space X with eigenvalues λ_κ:

$$\hat{\Psi}(x,z,y) = \sum_\kappa \Psi_\kappa(x,z)\eta_\kappa(y)$$

- From the 10-dim Dirac equation, $\not{D}\hat{\Psi} = 0$:

$$\left[z^2\,\partial_z^2 - d\,z\,\partial_z + z^2\mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2}\left(\frac{d}{2}+1\right) + (\lambda_\kappa + \mu)R\,\hat{\Gamma}\right] f(z) = 0,$$

$$i\not{D}_X \eta(y) = \lambda\,\eta(y),$$

where $\Psi(x,z) = e^{-iP\cdot x}\,f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and $\hat{\Gamma}u_\pm = \pm u_\pm$. For AdS_5, $\hat{\Gamma}$ is the four-dim chirality operator γ_5.

Henningson and Sfetsos; Muck and Viswanathan

- μ determined asymptotically by spectral comparison with orbital excitations in the boundary: $\mu = L/R$ and λ are the eigenvalues of the Dirac equation on S^{d+1}:

$$\lambda_\kappa R = \pm \left(\kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2...$$

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i .$$

- Normalizable AdS fermion mode (lowest KK-mode $\kappa = 0$:

$$\Psi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^{\frac{5}{2}} \Big[J_\alpha(z\beta_{\alpha,k}\Lambda_{QCD})\, \mu_+(P) + J_{\alpha+1}(z\beta_{\alpha,k}\Lambda_{QCD})\, \mu_-(P) \Big].$$

where $\mu^- = \frac{\gamma^\mu P_\mu}{P} \mu^+$, $\alpha = 2 + L$ and $\Delta = \frac{9}{2} + L$.

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\nu,n} = \alpha_{\nu,n} \Lambda_{QCD}, \qquad \mathcal{M}^-_{\nu,n} = \alpha_{\nu+1,n} \Lambda_{QCD}$$

- Spin-$\frac{3}{2}$ Rarita-Schwinger eq. in AdS similar to spin-$\frac{1}{2}$ in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, Ψ_μ. See: Volovich, hep-th/9809009.

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

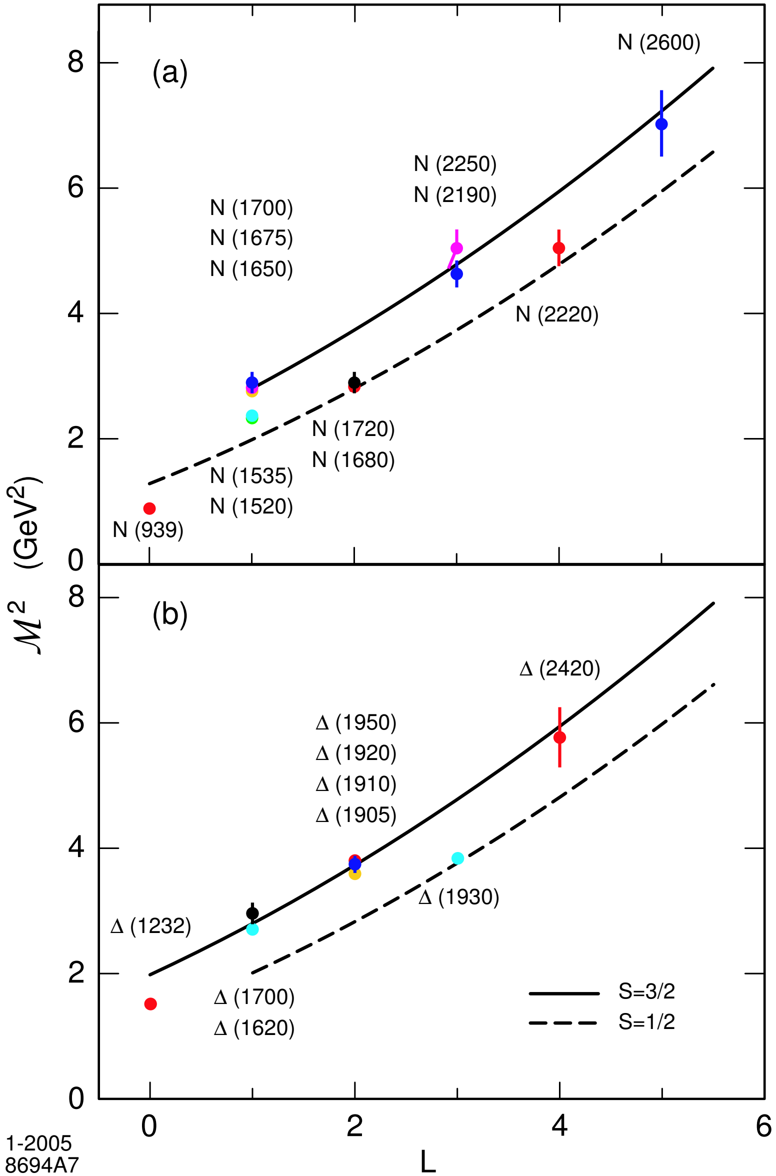


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.22 GeV

Hadronic Form Factor in Space and Time-Like Regions

SJB and GdT in preparation

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source (hadron spin σ):

$$
\begin{aligned}
F(Q^2)_{I\to F} &= R^{3+2\sigma}\int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)}\Phi_F(z)\, J(Q,z)\, \Phi_I(z) \\
&\simeq R^{3+2\sigma}\int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z),
\end{aligned}
$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$
\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.
$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

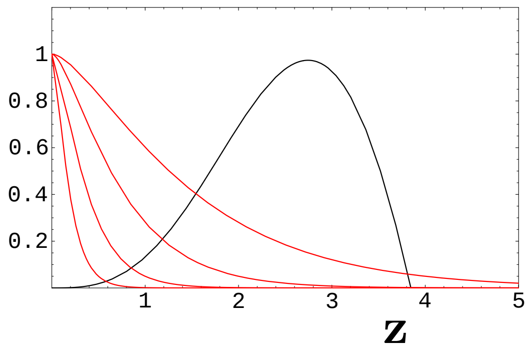
$$
J(Q,z) = zQK_1(zQ).
$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- Propagation of external perturbation suppressed inside AdS.

- At large enough $Q \sim r/R^2$, the interaction occurs in the large-r conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$

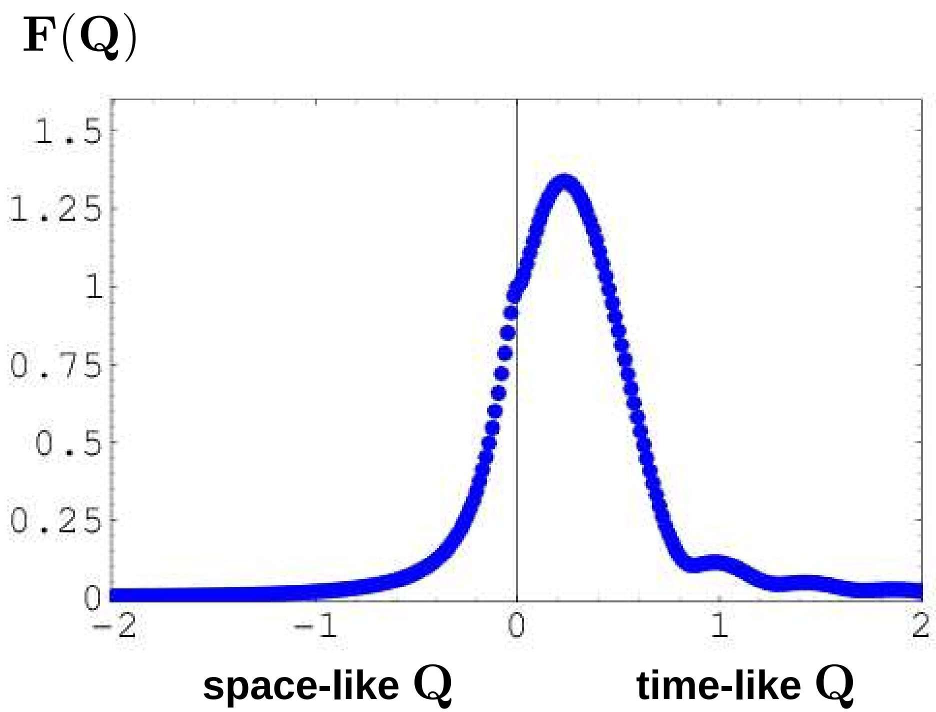


- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

- Prediction for the pion form factor in the holographic model (numerical analysis):



$\mathrm{s} \leftrightarrow \mathrm{t}$ **duality!**

Holographic Model for Light-Front Wavefunctions

SJB and GdT in preparation

- Define the transverse center of momentum $\vec{R}_\perp$ of a hadron in terms of the energy momentum tensor $T^{\mu\nu}$

$$\vec{R}_\perp = \frac{1}{P^+} \int dx^- \int d^2 \vec{r}_\perp T^{++} \vec{r}_\perp .$$

- In terms of partonic variables:

$$x_i \vec{r}_{\perp i} = \vec{R}_\perp + \vec{b}_{\perp i},$$

where $\vec{r}_{\perp i}$ are the physical coordinates and $\vec{b}_{\perp i}$ are frame-independent internal coordinates:

$$\vec{R}_\perp = \sum_i x_i \vec{r}_{\perp i}, \quad \sum_i \vec{b}_{\perp i} = 0.$$

- The normalizable string modes Φ_α obey the completeness relation:

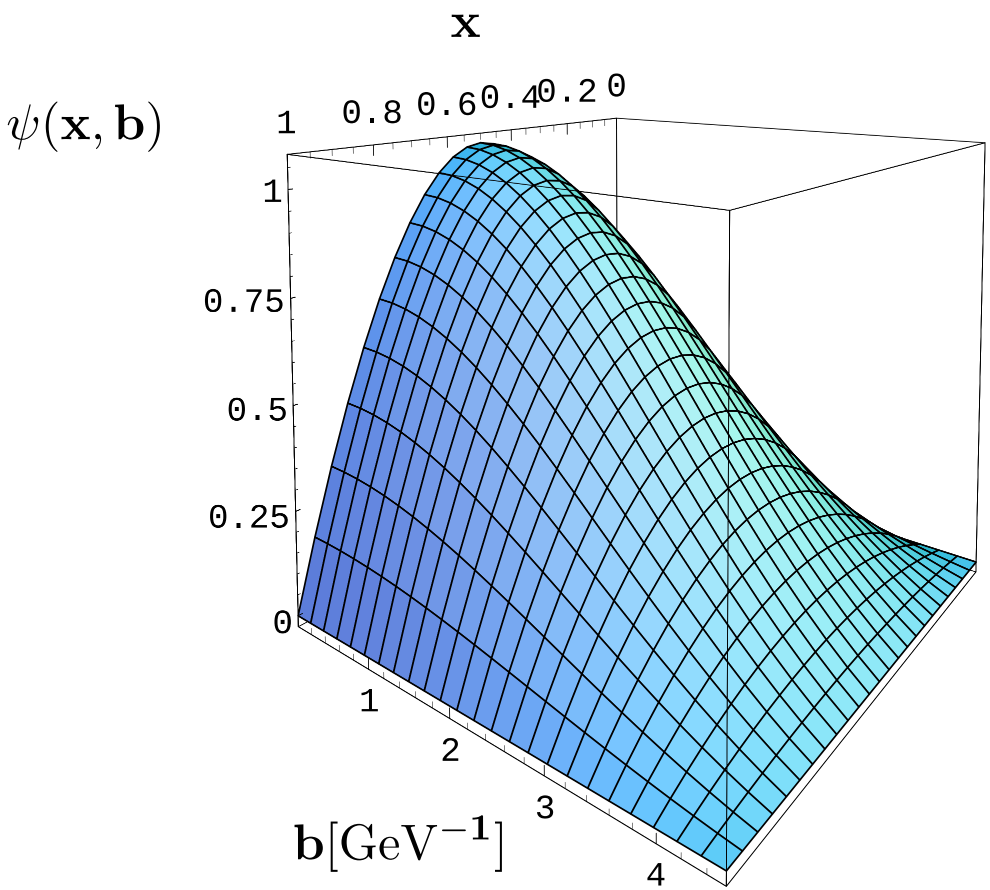
$$\sum_\alpha \Phi_\alpha(z)\Phi_\alpha(z') = \left(\frac{R e^{A(z)}}{z}\right)\delta(z-z').$$

- Mapping of string modes to impact space representation of LFWF, which also span a complete basis.

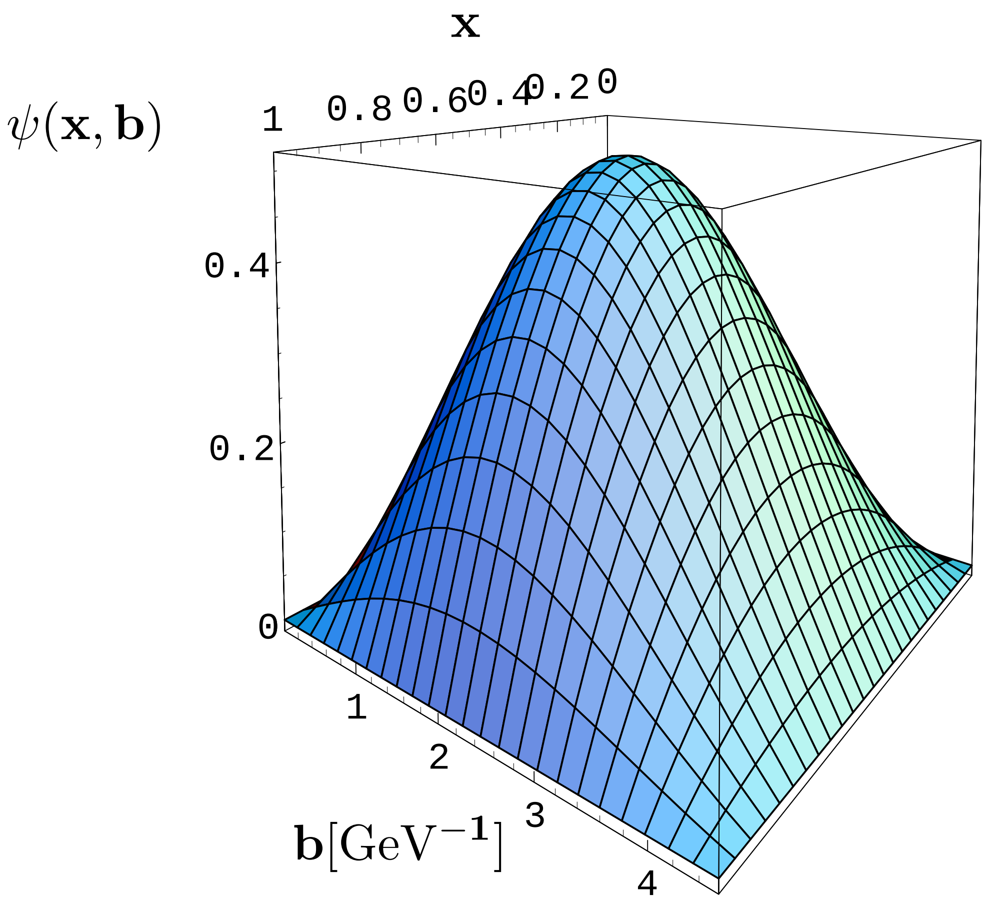
- Two-parton $n = 2$ LFWF including orbital angular momentum $\ell = 0, 1, 2\ldots$ and radial modes $k = 1, 2, 3, \ldots$ is to first approximation:

$$\psi_{n,\ell,k}(x,b) = B_{n,\ell,k}\, x(1-x)\frac{J_{n+\ell-1}\left(b\beta_{n-1,k}\Lambda_{QCD}\right)}{b}, \tag{1}$$

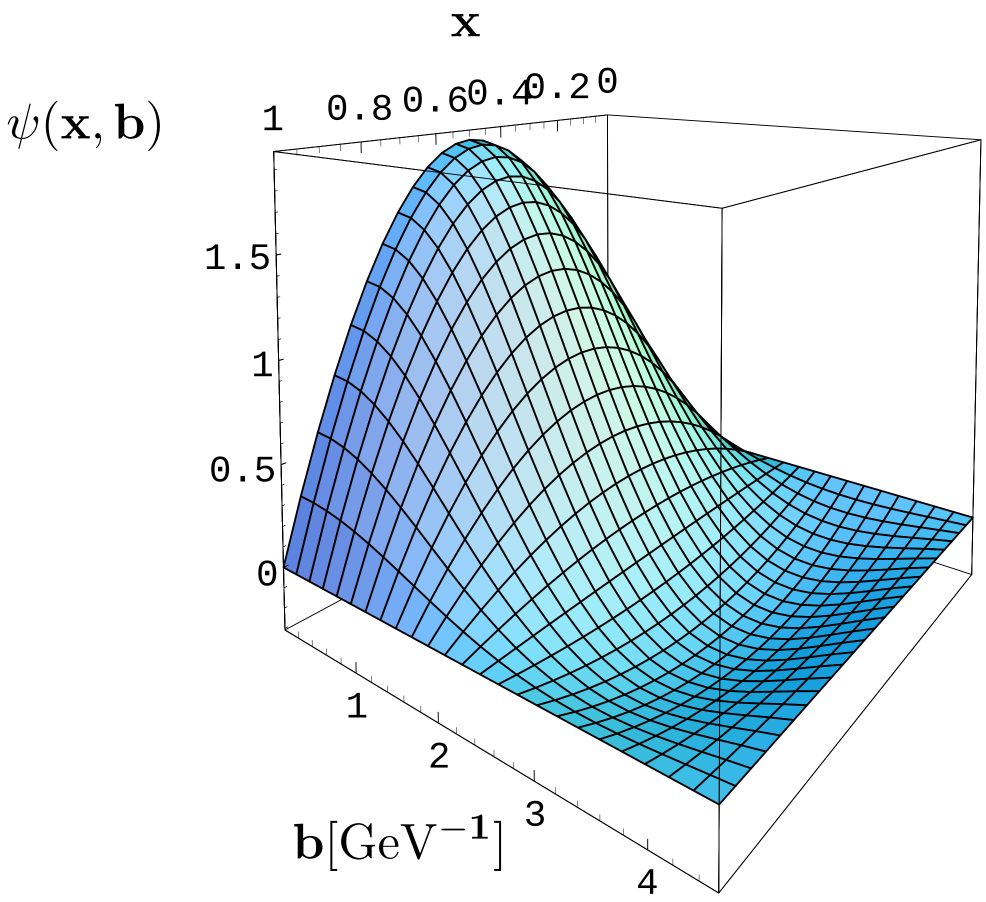
where $b = |\vec{b}_\perp|$.



Two-parton ground state LFWF in impact space $\psi(x,b)$ for a for $n=2, \ell=0, k=1$.



Two-parton first orbital exited state in impact space $\psi(x, b)$ for a for $n = 2, \ell = 1, k = 1$.



Two-parton first radial exited state LFWF in impact space $\psi(x, b)$ for $n = 2, \ell = 0, k = 2$.

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons).
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- String modes dual to baryons extrapolate to three fermion fields at zero separation in the AdS boundary.
- Only dimension $3, \frac{9}{2}$ and 4 states $\overline{q}q$, qqq, and gg appear in the duality at the classical level!
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Simple description of space and time-like structure of hadronic form factors.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model. Modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.