QUARK-HADRON DUALITY AND HIGH EXCITATIONS

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QCD is under intense scrutiny for a long time.
The issue of duality:

- Late 1960's ← Before QCD
- Mid 1970s-'80s ← DIS, QCD SR, heavy quarkonium,…
- Mid 1990s ←Heavy quark mass expansion. *Advent of precise data in B mesons.*

☺ My 1999 review summarizes theoretical understanding of quark-hadron duality by the year 1999.

✓ Revival of interest:

- ☺ Precision data slowly accumulating (e.g. BELLE);
- ☺ "AdS/QCD" (Strassler et al., Stephanov et al.)
- ☺ Chiral symmetry restoration for high excitations (L. Glozman et al.)

- $\Pi(Q^2)$ calculated through α_s^a and $1/Q^{2b}$, while terms α_s^{a+1} and $1/Q^{2(b+1)}$ are dropped.
- Duality: theoretical (truncated) quark-gluon spectral density is to coincide with experiment with the uncertainty of order $O[\alpha_s^{a+1}]$ and $O[1/s^{b+1}]$. The uncertainty of this order of magnitude is "natural" ⇒ Terms of this order are neglected in theoretical calculation of $\Pi(Q^2)$.
- Deviations going beyond the natural uncertainty are referred to as violations of duality.
- Once our calculation of $\Pi(Q^2)$ becomes more precise, the definition of the ``natural uncertainty" in Im Π changes accordingly.
- In Euclidean, we have asymptotic expansion in α_s and $1/Q^2$. Duality is perfect ⇒ there can be NO violation!
- Exponentially suppressed terms e^{-Q} in Euclidean become more important than $1/s^b$ in Minkowski. Violations of duality. Necessarily OSCILLATING.

☺ Duality ⟺ Quasiclassics

$$\sigma_{hadr} = \sigma_{quark\text{-}gl} = E^{a}\,(1 + (\ln E)^{-1} + \ldots (\ln E)^{-b} + \ldots$$

$$E^{-c} + \ldots + E^{-d}$$ ~~+ power-suppressed~~

~~& oscillating corrections)~~

$$\boxed{\text{New} \overset{?}{=} \text{well-forgotten OLD}}$$

- The problem is hard as it is essentially Minkowskian; Euclidean explorations (e.g. lattices) are of little help!

- Analytic ideas are badly needed!

- AdS/QCD™ ← an example of a very interesting "failure"

OUTLINE of my talk:

- Basics of quasiclassics;
- Chiral symmetry restoration;
- AdS/QCD™ and VMD & universality.

Quasiclassics & level spacing

$$\int p\, dx \sim E\, L \sim E^2/\sigma \sim n$$

$$\Downarrow$$

Equidistant spectrum:

$$M_n^2 \sim \sigma\, n$$

Chiral Symmetry restoration

- Multiple works of Glozman et al; see also NSVZ '81
- Question: (say)

  $\delta M_n = M_{nV} - M_{nA}$ at $n >> 1$, where n is the (radial) excitation number?

- **To make the problem well-defined we have to go to $N \longrightarrow \infty$; otherwise high excitations $\longrightarrow$ continuum**

Quasiclassically

$$\Gamma_n = (B/N)\, M_n \,, \quad M_n \sim n^{1/2}$$

B ~ 0.5 phenomenologically & from 2D model ('t Hooft model); Hence,

$$\Gamma_n / M_n \sim B/N$$

The limits $N \longrightarrow \infty$ and $n \longrightarrow \infty$ do NOT commute!

Consider:

$$\Pi\ (Q^2) = <T\{J(x), J(0)\}>_q$$

Moreover, for the parity pair, say,

$$\delta\ \Pi\ (Q^2) = \Pi_S(Q^2) - \Pi_P(Q^2)$$

we have

$$\boxed{\delta\Pi\ (Q^2)=\Sigma_n\{f_{nS}(Q^2+M_{nS}{}^2)^{-1}-f_{nP}(Q^2+M_{nP}{}^2)^{-1}\}}$$

sum over res

at large
Euclidean
Q^2 we get

$\sim (1/N)<\overline{\Psi}\,\Psi>^2 Q^{-4}$

$^{\text{mod}}$ logs

To begin with, assume that f_{nP} **and** f_{nS} are degenerate.

- These constants can be normalized from

$$\Pi\,(Q^2) \sim N\,Q^2 \log\,Q^2$$

$$\Updownarrow$$

$$f_n \sim N\,\Lambda^2\,M_n^{\;2} \sim N\,\Lambda^4\,n$$

Then, equating OPE to the resonannce representation,

we get, with satisfaction,

$$\Pi_s(Q) - \Pi_p(Q) = \begin{cases} \frac{1}{N}\,\frac{\langle\bar{\Psi}\Psi\rangle^2}{Q^4} \\ \\ N\,\Lambda^2\,Q^2\sum_n \frac{\delta M_n^2}{\left(Q^2+M_n^2\right)^2} \end{cases}$$

✶ If δM_n^2 is NOT sign-alternating, NO MATCHING !!!!!!!!

✶✶ The minimal solution:

$M_n^2\,\delta M_n^2 \to$ sign-alternating constant

- ☆ and, then,

$$|\delta\, M_n| \sim \frac{\Lambda}{n^{3/2}}$$

(or faster!)

✶✶✶ The impact of $\delta\, f_n \neq 0$:

$$\boxed{\frac{\delta\, f_n}{f_n} \sim \frac{1}{n^2}}$$

Part II: "AdS/QCD"

Inspiration: AdS/CFT duality.

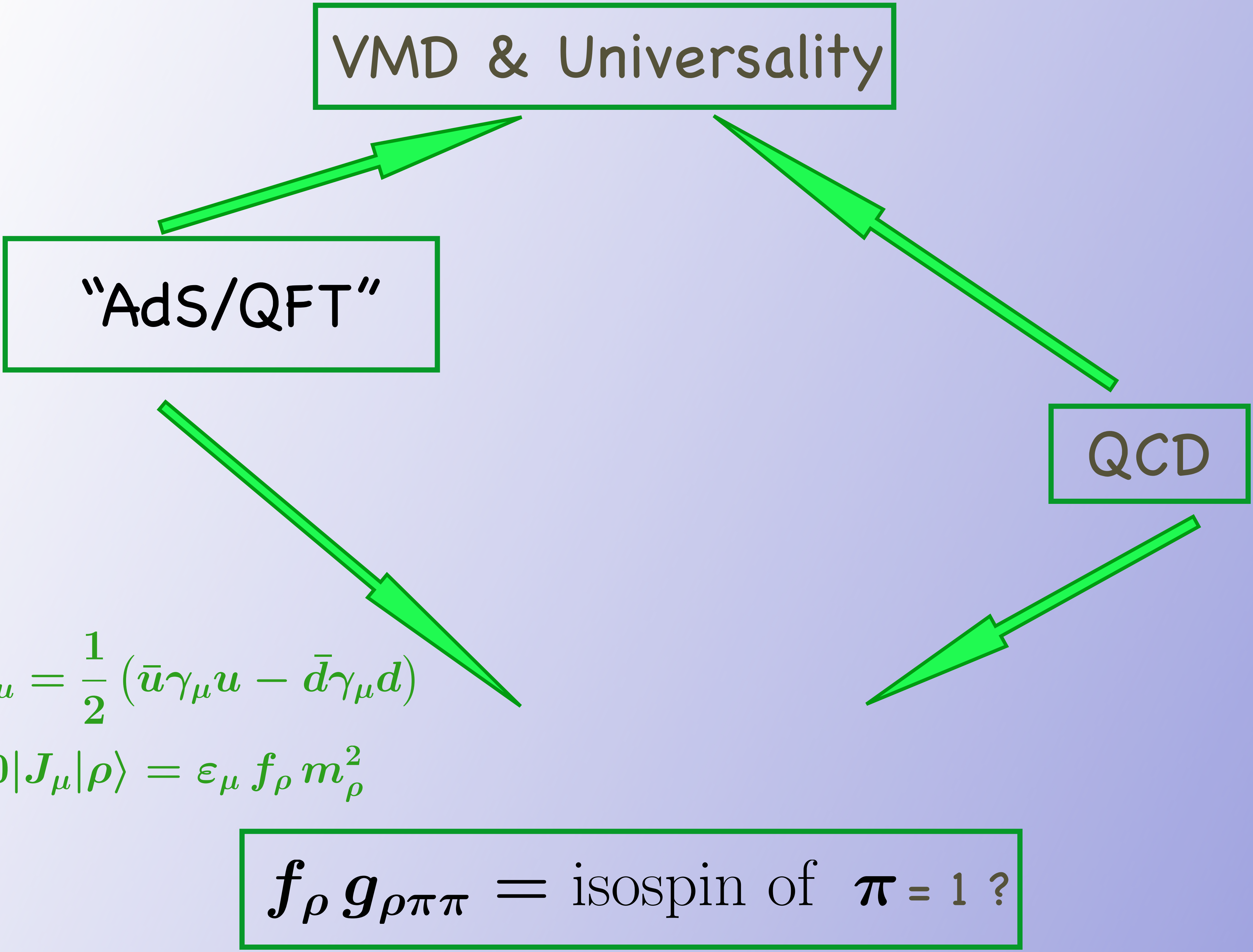
Confinement OK,

Chiral symmetry breaking OK.

BUT: !!!!!!!

- Strong coupling (NO ASY FREEDOM),
- No linear trajectories, $M_n \sim \cancel{\sqrt{n}} \sim n$
- VMD & Universality unnatural!

"AdS/QFT": one starts from QCD and attempts to construct its 5D holographic dual. Simple holographic models of QCD emerge and are being studied (Strassler,.....)



VMD & Universality

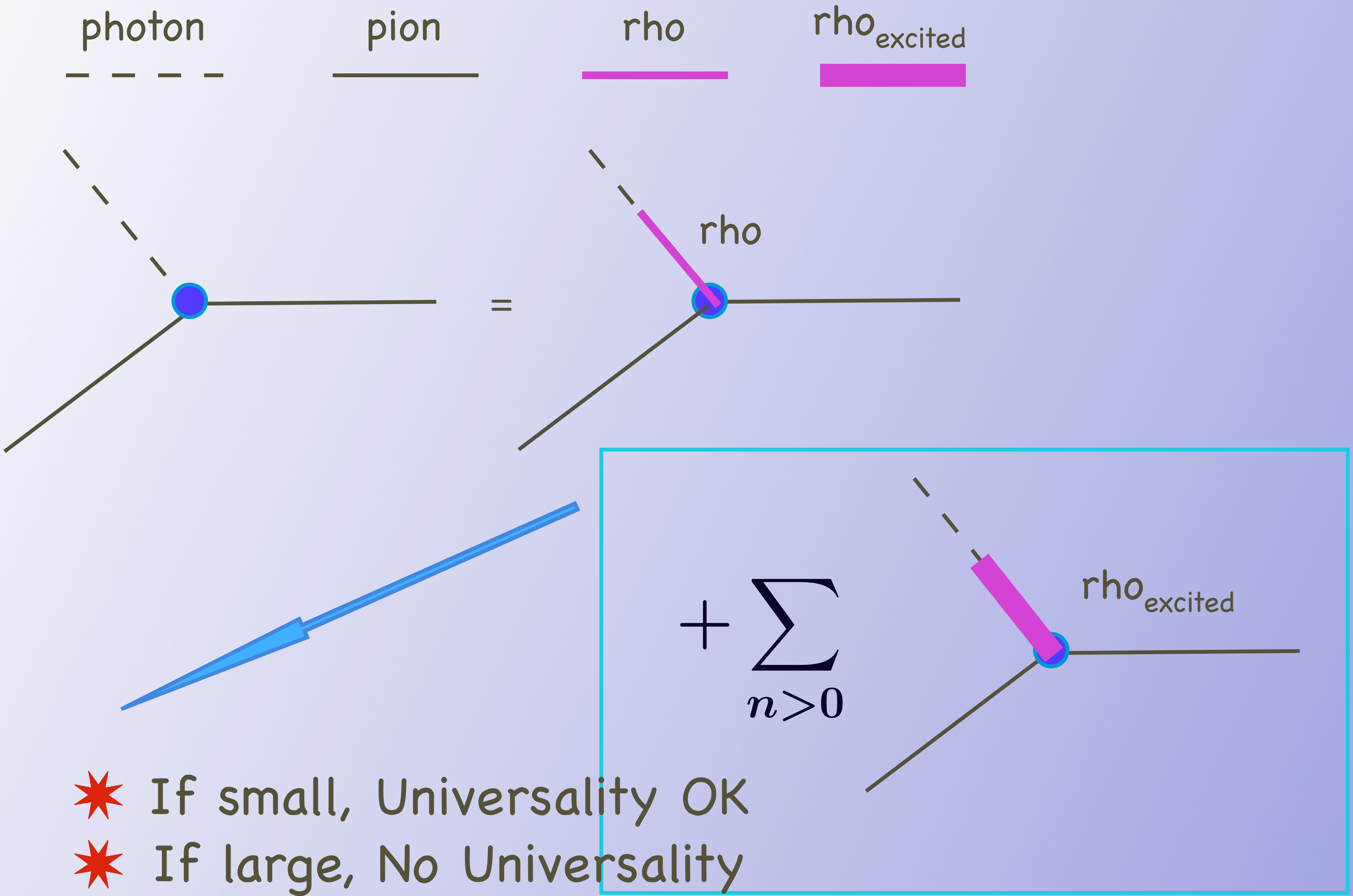
“AdS/QFT”

QCD

$$J_\mu = \frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)$$

$$\langle 0|J_\mu|\rho\rangle = \varepsilon_\mu\, f_\rho\, m_\rho^2$$

$f_\rho\, g_{\rho\pi\pi} =$ isospin of π = 1 ?



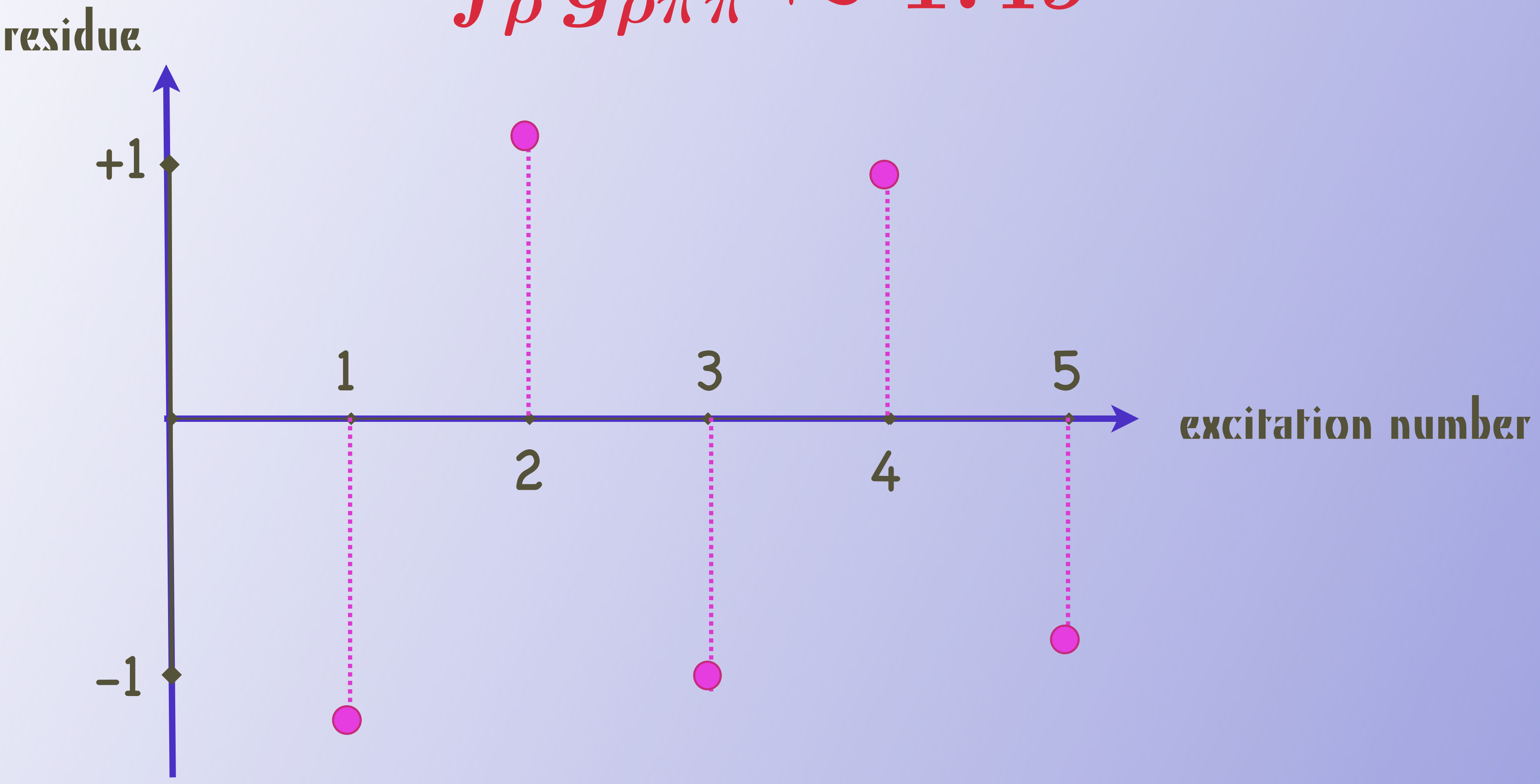
$$+\sum_{n>0}$$

- If small, Universality OK
- If large, No Universality

"AdS/QCD"

Hoang et al. (typical of all "AdS/QCD" models):

$$f_\rho \, g_{\rho\pi\pi} \approx 1.49$$

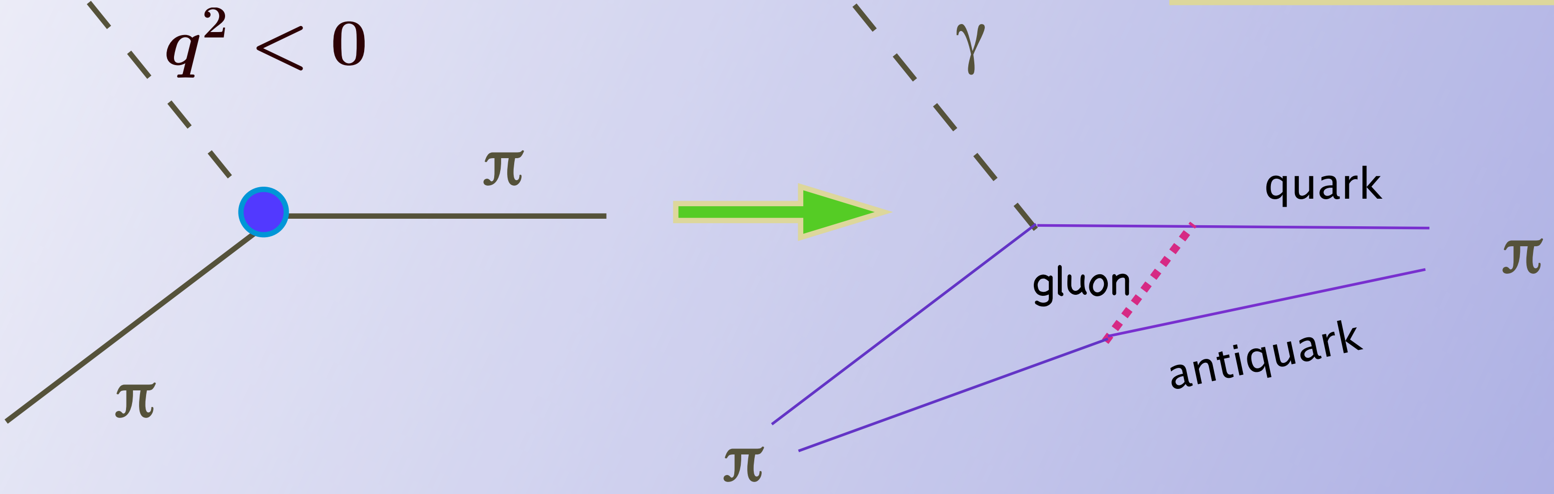


How about "Just QCD"?

- ✶ Sign-alternating regime ... YES
- ✶✶ No suppression of $f_{\rho_n} g_{\rho_n \pi\pi}$ at $n \gg 1$ ... NO !!!!

Sign-alternating regime

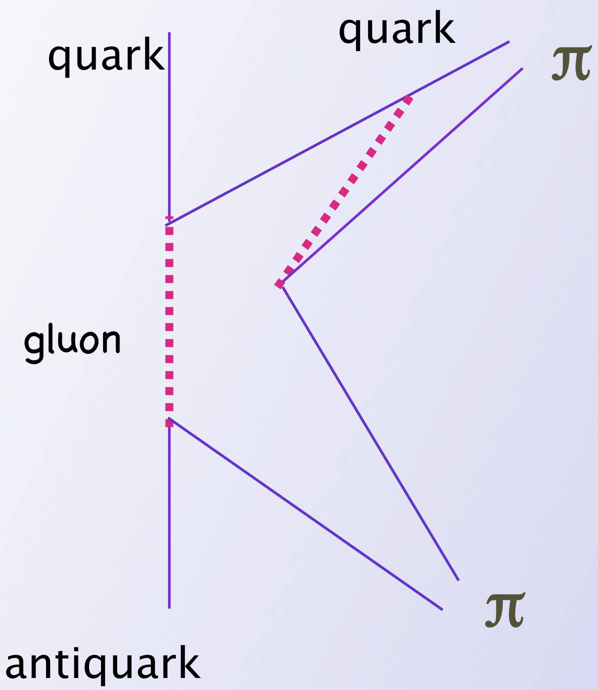
$$F_\pi(Q^2) \sim \frac{1}{Q^2 \ln Q^2}$$



F_π falls off faster than $1/Q^2$ ... Sign alteration INEVITABLE !!!

What suppression of the residue is expected in QCD?

A typical graph converting a highly excited rho into two pions is



$$g_{\rho_n\pi\pi} \sim \frac{f_{\rho_n} f_\pi^2}{M_n^2} \frac{1}{(\ln M_n^2)^2}$$

$$f_{\rho_n} g_{\rho_n\pi\pi} \sim \frac{f_{\rho_n}^2 f_\pi^2}{M_n^2} \frac{1}{(\ln M_n^2)^2}$$

$$|f_{\rho_n} g_{\rho_n\pi\pi}| \sim \frac{1}{n^2} \frac{1}{(\ln n)^2} \quad \text{at large} \;\; n$$

✳ My experience: n=2 is already large!

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Conclusions:

- Chiral symmetry restoration at large n occurs at the rate $|\delta M_n| \sim \Lambda\, n^{-3/2}$ (related to the QC $<\bar{\psi}\psi>^2$);
- "AdS/QCD" so far gives WRONG n dependence of M_n (was known!);
- "AdS/QCD" also gives WRONG n dependence of $f_{\rho_n} g_{\rho_n \pi\pi}$ $(\sim n^0$ vs. n^{-2} in QCD)

new!