Status of Polarized and Unpolarized Parton Distributions

Johannes Blümlein

DESY

1. Introduction
2. Unpolarized Parton Distributions
3. Polarized Parton Distributions
4. $\alpha_s$ and $\Lambda_{QCD}$
5. Future Avenues
1. Introduction

When is a Parton?

S. DRELL:

Infinite Momentum Frame: $P - \text{large}$

\[ \tau_{\text{int}} \ll \tau_{\text{life}} \]

\[ \tau_{\text{int}} \sim \frac{1}{q_0} = \frac{4P}{Q^2(1-x)} \]

\[ \tau_{\text{life}} \sim \frac{1}{\sum_i (E_i - E)} = \frac{2P}{\sum_i (k_{\perp i}^2 + M_i^2)/x_i - M^2} \approx \frac{2Px(1-x)}{k_{\perp}^2} \]

\[ \frac{\tau_{\text{int}}}{\tau_{\text{life}}} = \frac{2k_{\perp}^2}{Q^2(1-x)^2} \]

Stay away from $x \to 0$, since $xP$ becomes too small.

Stay away from $x \to 1$.

\[ Q^2 \gg k_{\perp}^2. \]
Main Research Objectives:

☞ Precise Measurement of $\alpha_s(M^2_Z)$
☞ Reveal polarized and unpolarized parton densities at highest precision
☞ Precision tests of QCD
☞ Find novel sub-structures

$\Rightarrow$ Perturbative QCD:
NNLO calculations using new technologies
$\Rightarrow$ Lattice QCD:
Calculation of certain non-perturbative quantities a priori
Theory of DIS

'69, QCD: '72

Parton Model

Light Cone Expansion [f]

Twist 2

polarized unpolarized

Fixed Order PT: QCD

Twist 3 Twist 4 ...

Higher Twist

\( \alpha_s \)

\( O(\alpha_s^4) \) '73, '74, '80, '97

\( O(\alpha_s^3) \) '73, '82, 2004

\( O(\alpha_s^3) \) '82, '92, 2004

Splitting functions

Coefficient functions

Sum Rules

'60ies - now

Diffractive Scattering

More General View

Non-forw. scattering

Angular Momentum: q, G

Resummations?

Higher Orders ➞ New Algorithms

Novel Mathematics

Special Kinematics

Domain: Small x

'75, '86

'90 - '98

J. Blümlein

Duality 05, Frascati, June 2005
3 Loop Splitting Functions

\[ \gamma_{qq}(N) \]

\[ \gamma_{gg}(N) \]

- LO
- NLO
- NNLO

\( \alpha_s = 0.2, N_f = 4 \)

Moch, Vermasern, Vogt, 2004
3 Loop Coefficient Functions

\[ x(c_{L,q} \otimes q_S) \]

- LO
- NLO
- NNLO

\[ x(c_{L,g} \otimes g) \]

\[ \alpha_s = 0.2, \ n_f = 4 \]

input (13)

\[ \frac{d \ln F_{2,NS}}{d \ln Q^2} \]

\[ \mu_r = Q \]

Moch, Vermasern, Vogt, 2004/05
Unpolarized Parton Distributions

Kinematic Domain

---

J. Blümlein
Duality 05, Frascati, June 2005
H1, ZEUS + fixed target data

\[ F_2 \cdot 2^i \]

- **H1 e^+p**
- **ZEUS e^+p**
- **BCDMS**
- **NMC**

H1 Collaboration

J. Blümlein

**J. Blümlein**  
*Duality 05, Frascati, June 2005*  
13
Scaling violations of $F_2(x, Q^2)$. 

J. Blümlein

Duality 05, Frascati, June 2005
$W^2 > 12.5 \text{ GeV}^2$, $Q^2 > 4 \text{ GeV}^2$

NNLO:

$$\alpha_s(M_Z^2) = 0.1139^{+0.0026}_{-0.0028}$$

J.B., H. Böttcher, A. Guffanti, 2004
# The World Data on $F_2$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$x$</th>
<th>$Q^2, \text{GeV}^2$</th>
<th>$F_2$</th>
<th>Norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCDMS (100)</td>
<td>0.35 – 0.75</td>
<td>11.75 – 75.00</td>
<td>51</td>
<td>1.018</td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.35 – 0.75</td>
<td>13.25 – 75.00</td>
<td>59</td>
<td>1.011</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.35 – 0.75</td>
<td>32.50 – 137.50</td>
<td>50</td>
<td>1.017</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.35 – 0.75</td>
<td>43.00 – 230.00</td>
<td>49</td>
<td>1.018</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.35 – 0.50</td>
<td>7.00 – 65.00</td>
<td>15</td>
<td>1.003</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.30 – 0.62</td>
<td>7.30 – 21.39</td>
<td>57</td>
<td>1.003</td>
</tr>
<tr>
<td>H1 (hQ2)</td>
<td>0.40 – 0.65</td>
<td>200 – 30000</td>
<td>26</td>
<td>1.018</td>
</tr>
<tr>
<td>ZEUS (hQ2)</td>
<td>0.40 – 0.65</td>
<td>650 – 30000</td>
<td>15</td>
<td>1.001</td>
</tr>
<tr>
<td><strong>proton</strong></td>
<td></td>
<td></td>
<td>322</td>
<td></td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.35 – 0.75</td>
<td>13.25 – 99.00</td>
<td>59</td>
<td>0.992</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.35 – 0.75</td>
<td>32.50 – 137.50</td>
<td>50</td>
<td>0.993</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.35 – 0.75</td>
<td>43.00 – 230.00</td>
<td>49</td>
<td>0.993</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.35 – 0.50</td>
<td>7.00 – 65.00</td>
<td>15</td>
<td>0.980</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.30 – 0.62</td>
<td>10.00 – 21.40</td>
<td>59</td>
<td>0.980</td>
</tr>
<tr>
<td><strong>deuteron</strong></td>
<td></td>
<td></td>
<td>232</td>
<td></td>
</tr>
<tr>
<td>BCDMS (120)</td>
<td>0.070 – 0.275</td>
<td>8.75 – 43.00</td>
<td>36</td>
<td>1.000</td>
</tr>
<tr>
<td>BCDMS (200)</td>
<td>0.070 – 0.275</td>
<td>17.00 – 75.00</td>
<td>29</td>
<td>1.000</td>
</tr>
<tr>
<td>BCDMS (280)</td>
<td>0.100 – 0.275</td>
<td>32.50 – 115.50</td>
<td>27</td>
<td>1.000</td>
</tr>
<tr>
<td>NMC (comb)</td>
<td>0.013 – 0.275</td>
<td>4.50 – 65.00</td>
<td>88</td>
<td>1.000</td>
</tr>
<tr>
<td>SLAC (comb)</td>
<td>0.153 – 0.293</td>
<td>4.18 – 5.50</td>
<td>28</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>non – singlet</strong></td>
<td></td>
<td></td>
<td>208</td>
<td></td>
</tr>
<tr>
<td><strong>total</strong></td>
<td></td>
<td></td>
<td>762</td>
<td></td>
</tr>
</tbody>
</table>

- **CUTS**: $0.3 < x < 1.0$ for $F_2^p$ and $F_2^d$
  - $0.0 < x < 0.3$ for $F_2^{n.s.} = 2(F_2^p - F_2^d)$
  - $4.0 < Q^2 < 30000 \text{GeV}^2$, $W^2 > 12.5 \text{GeV}^2$
**Fully Correlated Error Calculation**

- The fully correlated 1σ error for the parton density \( f_q \) as given by Gaussian error propagation is

\[
\sigma(f_q(x))^2 = \sum_{i,j=1}^{n_p} \left( \frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \text{cov}(p_i, p_j),
\]

where the \( \partial f_q/\partial p_i \) are the derivatives of \( f_q \) w.r.t. the parameters \( p_i \) and the \( \text{cov}(p_i, p_j) \) are the elements of the covariance matrix as determined in the fit.

- The derivatives \( \partial f_q/\partial p_i \) at the input scale \( Q_0^2 \) can be calculated analytically. Their values at \( Q^2 \) are given by evolution.

- The derivatives evolved in MELLIN-N space are transformed back to \( x\)-space and can then be used according to the error propagation formula above.

\[ \Rightarrow \] As an example the derivative of \( f(x, a, b) \) w.r.t. parameter \( a \) in MELLIN–N space reads:
Fit Results

- Parameter values and Covariance Matrix at the input scale \( Q_0^2 = 4.0 \text{ GeV}^2 \)

\[
x q_i(x, Q_0^2) = A_i x^{a_i} (1-x)^{b_i} (1 + \rho_i x^{\frac{1}{2}} + \gamma_i x)
\]

<table>
<thead>
<tr>
<th>( u_v )</th>
<th>( a )</th>
<th>0.299 ± 0.007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b )</td>
<td>4.157 ± 0.031</td>
</tr>
<tr>
<td></td>
<td>( \rho )</td>
<td>0.751</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>28.833</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( d_v )</th>
<th>( a )</th>
<th>0.488 ± 0.048</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b )</td>
<td>6.609 ± 0.332</td>
</tr>
<tr>
<td></td>
<td>( \rho )</td>
<td>-1.690</td>
</tr>
<tr>
<td></td>
<td>( \gamma )</td>
<td>17.247</td>
</tr>
</tbody>
</table>

\( \Lambda_{QCD}^{(4)} = 233 \pm 34 \text{ MeV} \)

\( \chi^2/ndf = 630/757 = 0.83 \)

- Covariance Matrix at the input scale \( Q_0^2 = 4.0 \text{ GeV}^2 \)

<table>
<thead>
<tr>
<th></th>
<th>( \Lambda_{QCD}^{(4)} )</th>
<th>( a_{u_v} )</th>
<th>( b_{u_v} )</th>
<th>( a_{d_v} )</th>
<th>( b_{d_v} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_{QCD}^{(4)} )</td>
<td>1.15E-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_{u_v} )</td>
<td>1.03E-4</td>
<td>5.40E-5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_{u_v} )</td>
<td>-8.45E-5</td>
<td>1.71E-4</td>
<td>9.59E-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_{d_v} )</td>
<td>4.17E-4</td>
<td>8.84E-6</td>
<td>-4.35E-4</td>
<td>2.32E-3</td>
<td></td>
</tr>
<tr>
<td>( b_{d_v} )</td>
<td>2.32E-3</td>
<td>4.21E-4</td>
<td>-2.28E-3</td>
<td>1.48E-2</td>
<td>1.10E-1</td>
</tr>
</tbody>
</table>
Heavy Flavor NS-contributions

\[ \frac{F_2^{W_{cc}}}{(F_2^{NS} + F_2^{W_{cc}})} \]

- \( Q^2 = 10 \text{ GeV}^2 \)
- \( Q^2 = 100 \text{ GeV}^2 \)
- \( Q^2 = 1000 \text{ GeV}^2 \)
- \( Q^2 = 10000 \text{ GeV}^2 \)

\( Q^2 \) range of data: \( 4 \text{ GeV}^2 < Q^2 < 116 \text{ GeV}^2 \)
Non-Singlet 3-Loop QCD Analysis

\[ F_2^{\text{mp}}(x, Q^2) + C \]

- BCDMS
- NMC
- SLAC
- This Fit

For proton, deuteron:

\[ x > 0.3 \]
Higher Twist Contributions:

\[ 4 < W^2 < 12.5 \text{ GeV}^2, \quad Q^2 > 4 \text{ GeV}^2 \]
Moments and Lattice Results

<table>
<thead>
<tr>
<th>$f$</th>
<th>$n$</th>
<th>This Fit</th>
<th>MRST04</th>
<th>A02</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_v$</td>
<td>2</td>
<td>0.288 ± 0.003</td>
<td>0.285</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.084 ± 0.001</td>
<td>0.082</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0319 ± 0.0004</td>
<td>0.032</td>
<td>0.033</td>
</tr>
<tr>
<td>$d_v$</td>
<td>2</td>
<td>0.113 ± 0.004</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.026 ± 0.001</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0078 ± 0.0004</td>
<td>0.009</td>
<td>0.010</td>
</tr>
<tr>
<td>$u_v - d_v$</td>
<td>2</td>
<td>0.175 ± 0.004</td>
<td>0.171</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.058 ± 0.001</td>
<td>0.055</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0241 ± 0.0005</td>
<td>0.022</td>
<td>0.024</td>
</tr>
</tbody>
</table>

First lattice results on $u_v - d_v, N = 2$ yield promising values using overlap-fermions (QCDSF).

More results also are upcoming.
The Singlet Sector

Parton Densities: Relative Size

![Graph showing parton densities for MRST2001 with Q^2 = 10 GeV^2]
Ratios of Unpolarized PDFs

\[ f_{PDF}(x, Q^2) \]
\[ Q^2 = (100 \text{ GeV})^2 \]

MRST/CTEQ

\[ f_{PDF}(x, Q^2) \]
\[ Q^2 = (100 \text{ GeV})^2 \]

Alekhin/CTEQ

Parton Distributions

H1 PDF 2000: $Q^2 = 4$ GeV$^2$

Fit to H1 data
- experimental errors
- model uncertainties

Fit to H1 + BCDMS data

parton distribution

H1 Collaboration
Slope of $F_2$ at low $x$

Very likely, that the $\overline{\text{MS}}$–gluon is remains positive!

J. Blümlein

DIS: achievements and needs

DIS05, April 2005

– p.11
PILE–UP EFFECTS:
Iterative vs Exact Solution of Evolution Equations

Blümlein, Riemersma, van Neerven, Vogt, 1996
Gluon Density

QCD Fits
- (H1+BQMS) total uncertainty
- (H1+BQMS) exp. + $\alpha_s$ uncert.
- (H1+BQMS) exp. uncertainty
- (H1)

$Q^2=5 \text{ GeV}^2$

$Q^2=20 \text{ GeV}^2$

$Q^2=200 \text{ GeV}^2$

MRST2002NLO
- CTEQ6.1M

$Q^2 = 2 \text{ GeV}^2$

$\times g(x,Q^2)$

$10^{-4}$ $10^{-3}$ $10^{-2}$ $10^{-1}$

$10^0$ $10^1$ $10^2$ $10^3$ $10^4$

$0$ $1$ $2$ $3$

H1 Collaboration
M. Klein, 2004: Projection for a possible measurement at HERA of central importance to study the small $x$ behaviour of the gluon distribution
\( \bar{d} - \bar{u} \)

\[ x(\bar{d}(x) - \bar{u}(x)) = 1.195 x^{1.24} (1 - x)^{9.10} (1 + 14.05x - 45.52x^2) \]

\[ Q^2 = 1 \text{ GeV}^2 \]
Strange quark distribution

- CCFR: iron target, EMC effect. How large?

**Can HERMES measure** \( s(x, Q^2) \) ?
$c\bar{c}$ Structure Function $F_2$
Mellin-space representation:

\[ F_{NLO}^{2,q}(x,Q^2) \]

S. Alekhin and J.B., 2004

- necessary for scheme-invariant evolution.
- fast and accurate access to heavy flavor Wilson coefficients.
Polarized Nucleons

How is the nucleon spin distributed over the partons?

\[ S_n = \frac{1}{2} \left[ \Delta(u + \bar{u}) + \Delta(d + \bar{d}) + \Delta(s + \bar{s}) \right] + \Delta G + L_q + L_g \]

\[ S_n = \frac{1}{2} \]

\[ \Delta \Sigma = 0.138 \pm 0.082, \quad (0.150 \pm 0.061) \]
\[ \Delta G = 1.026 \pm 0.554, \quad (0.931 \pm 0.679) \]

EMC, 1987: The nucleon spin is not the sum of the light quark spins.

Measure:

Polarized parton densities: \( \Delta q_i, \Delta G \)

How can one access the parton angular momentum?

Polarized heavy flavor contributions.

- Polarized structure functions contain also twist 3 contributions.

How to unfold these terms?
Polarized Parton Densities:

pioneering work: Dortmund GRSV, 1996, 2001

Analysis by other groups:
AAC (Japan), 2000, 2004
J.B., H. Böttcher, 2002
Leader et al., 2002
Altarelli et al., 1997

NLO : \( \alpha_s(M_z^2) = 0.113^{+0.10}_{-0.08} \)

J.B., H. Böttcher, 2002
Currently slight move towards lower values.
Figure 12: Model fit to potential power corrections in $g_1(x, Q^2)$ as extracted from the world polarization asymmetry data in the present analysis (see text). Dashed line: model I, Eq. (70); dotted line: model II, Eq. (71). The full lines correspond to the parameterization (ISET=4) in the present analysis, to which the corresponding power correction model induces a perturbation. The shaded area corresponds to the $1\sigma$ correlated error.
Comparison with Lattice Moments:

<table>
<thead>
<tr>
<th>Moment</th>
<th>BB, NLO</th>
<th>QCDSF</th>
<th>LHPC/SESAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u_v$</td>
<td>0 0.926</td>
<td>0.889 ± 0.029</td>
<td>0.860 ± 0.069</td>
</tr>
<tr>
<td></td>
<td>1 0.163 ± 0.014</td>
<td>0.198 ± 0.008</td>
<td>0.242 ± 0.022</td>
</tr>
<tr>
<td></td>
<td>2 0.055 ± 0.006</td>
<td>0.041 ± 0.009</td>
<td>0.116 ± 0.042</td>
</tr>
<tr>
<td>$\Delta d_v$</td>
<td>0 −0.341</td>
<td>−0.236 ± 0.027</td>
<td>−0.171 ± 0.043</td>
</tr>
<tr>
<td></td>
<td>1 −0.047 ± 0.021</td>
<td>−0.048 ± 0.003</td>
<td>−0.029 ± 0.013</td>
</tr>
<tr>
<td></td>
<td>2 −0.015 ± 0.009</td>
<td>−0.028 ± 0.002</td>
<td>0.001 ± 0.025</td>
</tr>
<tr>
<td>$\Delta u_v - \Delta d_v$</td>
<td>0 1.267</td>
<td>1.14 ± 0.03</td>
<td>1.031 ± 0.081</td>
</tr>
<tr>
<td></td>
<td>1 0.210 ± 0.025</td>
<td>0.245 ± 0.009</td>
<td>0.271 ± 0.025</td>
</tr>
<tr>
<td></td>
<td>2 0.070 ± 0.011</td>
<td>0.069 ± 0.009</td>
<td>0.115 ± 0.049</td>
</tr>
</tbody>
</table>

1st moments: Still problematic.
Heavy flavor:

$g_1$: Watson, 1982; Vogelsang, 1990

$g_2$: J.B., Ravindran, van Neerven, 2003
**Sum Rules and Integral Relations:**

**Twist 2:**

\[ g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \]

Wandzura, Wilczek, 1977;


\[ g_3(x, Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4(y, Q^2) \]


**Twist 3:**

**Include nucleon mass effects.**

J.B., A. Tkabladze, 1998

\[
\begin{align*}
g_1(x, Q^2) &= \frac{4M^2 x^2}{Q^2} \left[ g_2(x, Q^2) - 2 \int_x^1 \frac{dy}{y} g_2(y, Q^2) \right] \\
\frac{4M^2 x^2}{Q^2} g_3(x, Q^2) &= g_4(x, Q^2) \left( 1 + \frac{4M^2 x^2}{Q^2} \right) + 3 \int_x^1 \frac{dy}{y} g_4(y, Q^2) \\
2x g_5(x, Q^2) &= -\int_x^1 \frac{dy}{y} g_4(y, Q^2)
\end{align*}
\]
Quark Helicity Distributions

HERMES Experiment, hep-ex/0407032
(→ A. Miller’s talk)

\[ x \cdot \Delta u \]
\[ x \cdot \Delta d \]
\[ x \cdot \overline{\Delta u} \]
\[ x \cdot \overline{\Delta d} \]
\[ x \cdot \Delta s \]

\[ Q^2 = 2.5 \text{ GeV}^2 \]

HERMES 2003

Statistical PDFs: C. Bourrely, J. Soffer
and F. Buccella, hep-ph/0109160

HERMES Experiment, hep-ex/0407032
(→ A. Miller’s talk)

\[ x \cdot \Delta u \]
\[ x \cdot \Delta d \]
\[ x \cdot \overline{\Delta u} \]
\[ x \cdot \overline{\Delta d} \]
\[ x \cdot \Delta s \]

\[ Q^2 = 2.5 \text{ GeV}^2 \]

HERMES 2003

Statistical PDFs: C. Bourrely, J. Soffer
and F. Buccella, hep-ph/0109160

H. Böttcher

Phenomenology of PDF’s ...

Seattle/USA, 18 – 20 Nov., 2004

– p.41
Comparison with $\Delta q$ from Semi-Incl. Data

$\Delta(u+\bar{u})(x)$

$\triangle$ HERMES

$\bullet$ BB-LO distribution (1)

$\Rightarrow z$-range in the Semi-Incl. Analysis: $0.2 < z < 0.7$
Inclusive + Semi-inclusive Analysis


Parton densities at $Q^2 = 10 GeV^2$; error bands: $\Delta \chi^2 = 1; 2\%$. 
.... allows very precise measurements

Example: Flavor Separation of polarized PDF's

![Graph showing Flavor Separation of polarized PDF's](image)
### $\Lambda_{\text{QCD}}$ and $\alpha_s(M_Z^2)$

<table>
<thead>
<tr>
<th>NLO</th>
<th>$\alpha_s(M_Z^2)$</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTEQ6</td>
<td>0.1165</td>
<td>±0.0065</td>
<td></td>
<td>[1]</td>
</tr>
<tr>
<td>MRST03</td>
<td>0.1165</td>
<td>±0.0020</td>
<td>±0.0030</td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1171</td>
<td>±0.0015</td>
<td>±0.0033</td>
<td>[3]</td>
</tr>
<tr>
<td>ZEUS</td>
<td>0.1166</td>
<td>±0.0049</td>
<td></td>
<td>[4]</td>
</tr>
<tr>
<td>H1</td>
<td>0.1150</td>
<td>±0.0017</td>
<td>±0.0050</td>
<td>[5]</td>
</tr>
<tr>
<td>BCDMS</td>
<td>0.1110</td>
<td>±0.006</td>
<td></td>
<td>[6]</td>
</tr>
<tr>
<td>BB (pol)</td>
<td>0.113</td>
<td>±0.004</td>
<td>±0.009</td>
<td>[7]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NNLO</th>
<th>$\alpha_s(M_Z^2)$</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRST03</td>
<td>0.1153</td>
<td>±0.0020</td>
<td>±0.0030</td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1143</td>
<td>±0.0014</td>
<td>±0.0009</td>
<td>[3]</td>
</tr>
<tr>
<td>SY01(ep)</td>
<td>0.1166</td>
<td>±0.0013</td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>SY01(νN)</td>
<td>0.1153</td>
<td>±0.0063</td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>BBG</td>
<td>0.1139</td>
<td>+0.0026/−0.0028</td>
<td></td>
<td>[9]</td>
</tr>
</tbody>
</table>

**BBG**: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^3)$:

$$\Lambda = 233 \pm 30 \text{ MeV}$$

**Alpha Collab**: $N_f = 2$ Lattice; non-pert. renormalization

$$\Lambda = 245 \pm 16 \pm 16 \text{ MeV}$$

**QCDSF Collab**: $N_f = 2$ Lattice, pert. reno.

$$\Lambda = 249 + 13 + 13/−8−17 \text{ MeV}$$  also other collab., (cf. PDG).
DIS: $\alpha_s(M_Z^2)$

\[ \begin{array}{c|c|c|c|c|c|c|c}
0.1 & 0.105 & 0.11 & 0.115 & 0.12 & 0.125 & 0.13 \\
CTEQ6 & MRST03 & A02 & ZEUS & H1 & BCDMS & MRST03 & A02 & SY01(ep) & SY01(\nu) & BBG & BB & SMC & ABFR \\
\end{array} \]
Future Avenues

HERA:

- Collect high luminosity for $F_2(x, Q^2)$, $F_2^{cc}(x, Q^2)$, $g_2^{cc}(x, Q^2)$, and measure $h_1(x, Q^2)$.

- Measure: $F_L(x, Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized.

JLAB:

- High precision measurements in the large $x$ domain at unpolarized and polarized targets; supplements HERA’s high precision measurements at small $x$. 
ELIC:

- High precision measurements in the medium $x$ domain; both unpolarized and polarized

**The quest for large luminosity!**
• What is the correct value of $\alpha_s(M^2_Z)$? $\overline{\text{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non-singlet and singlet analysis; careful treatment of heavy flavor.[Theory & Experiment]

• Flavor Structure of Sea-Quarks: More studies needed.[All Experiments]

• Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed.[Theory]

• QCD at Twist 3: $g_2(x, Q^2)$, semi-exclusive Reactions [High Precision polarized experiments, JLAB, EIC]

• Comparison with Lattice Results: $\alpha_s$, Moments of Parton Distributions, Angular Momentum.

• Calculation of more hard scattering reactions at the 3-loop level: ILC, LHC

• Further perfection of the mathematical tools:
  $\Rightarrow$ Algorithmic simplification of Perturbation theory in higher orders.

• Even higher order corrections needed?