Status of Polarized and Unpolarized Parton Distributions

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DESY

1. Introduction
2. Unpolarized Parton Distributions
3. Polarized Parton Distributions
4. α_s and Λ_{QCD}
5. Future Avenues

1. Introduction

When is a Parton ?

S. DRELL: **Infinite Momentum Frame:** P - large

$$\tau_{\rm int} \ll \tau_{\rm life}$$

$$\tau_{\rm int} \sim \frac{1}{q_0} = \frac{4Px}{Q^2(1-x)}$$

$$\tau_{\rm life} \sim \frac{1}{\sum_i E_i - E} = \frac{2P}{\sum_i (k_{\perp i}^2 + M_i^2)/x_i - M^2} \simeq \frac{2Px(1-x)}{k_\perp^2}$$

$$\frac{\tau_{\rm int}}{\tau_{\rm life}} = \frac{2k_\perp^2}{Q^2(1-x)^2}$$

Stay away from $x \rightarrow 0$, since xP becomes too small.

Stay away from $x \rightarrow 1$.

$$Q^2 \gg k_\perp^2.$$

MAIN RESEARCH OBJECTIVES :

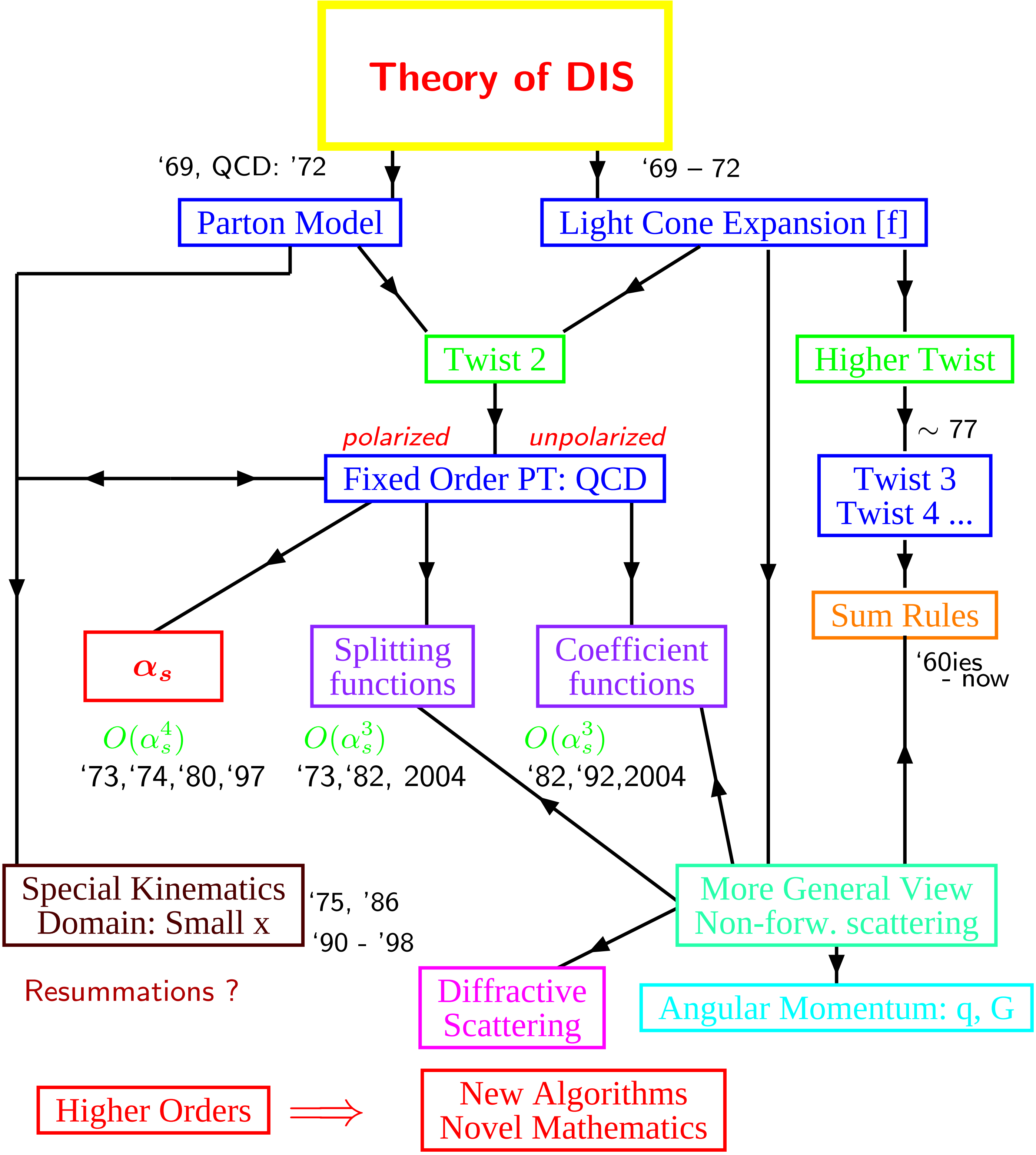
- ☞ Precise Measurement of $\alpha_s(M_Z^2)$
- ☞ Reveal polarized and unpolarized parton densities at highest precision
- ☞ Precision tests of QCD
- ☞ Find novel sub-structures

$\Longrightarrow$ Perturbative QCD :

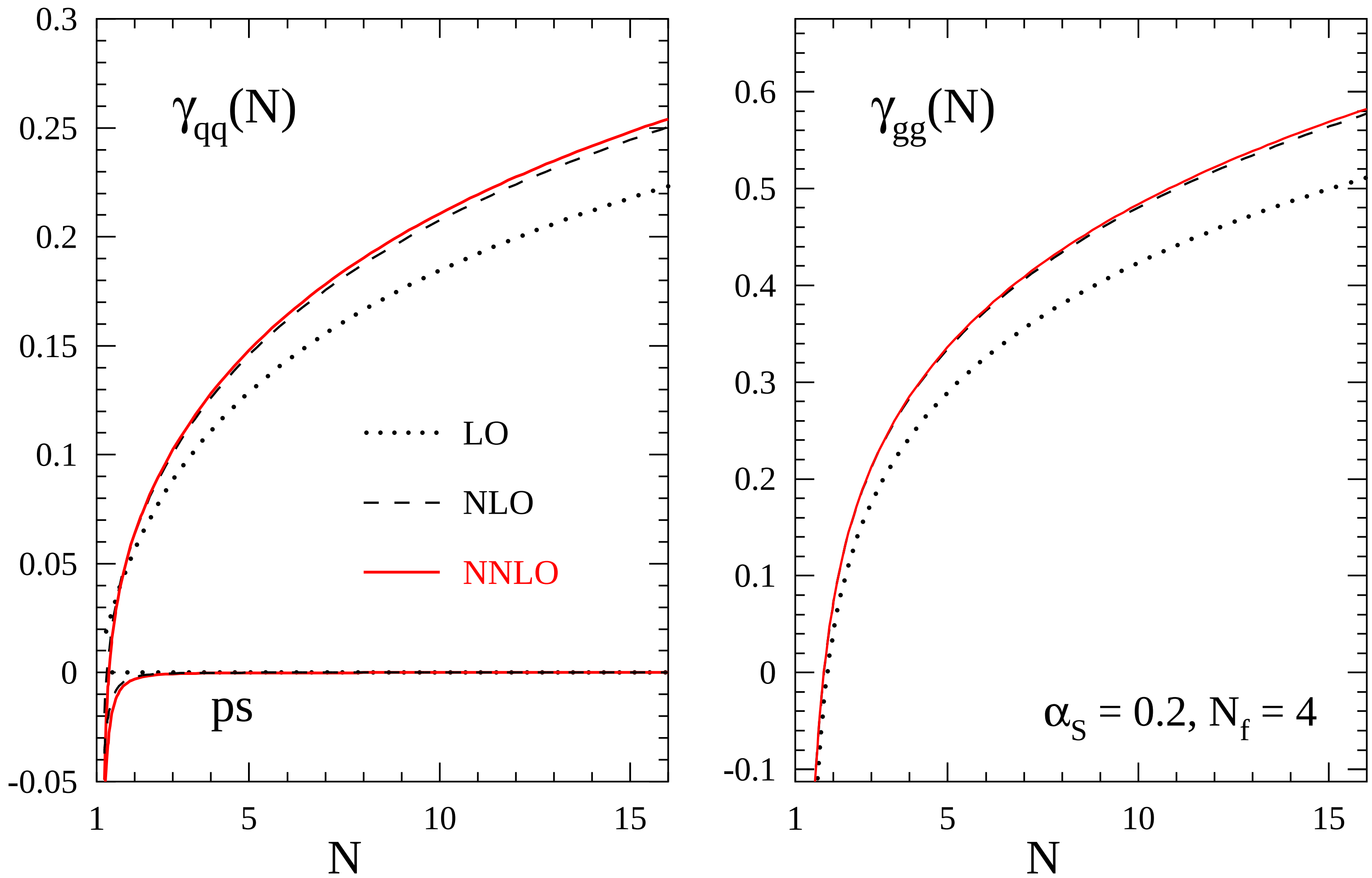
NNLO calculations using new technologies

$\Longrightarrow$ Lattice QCD :

Calculation of certain non-perturbative quantities a priori

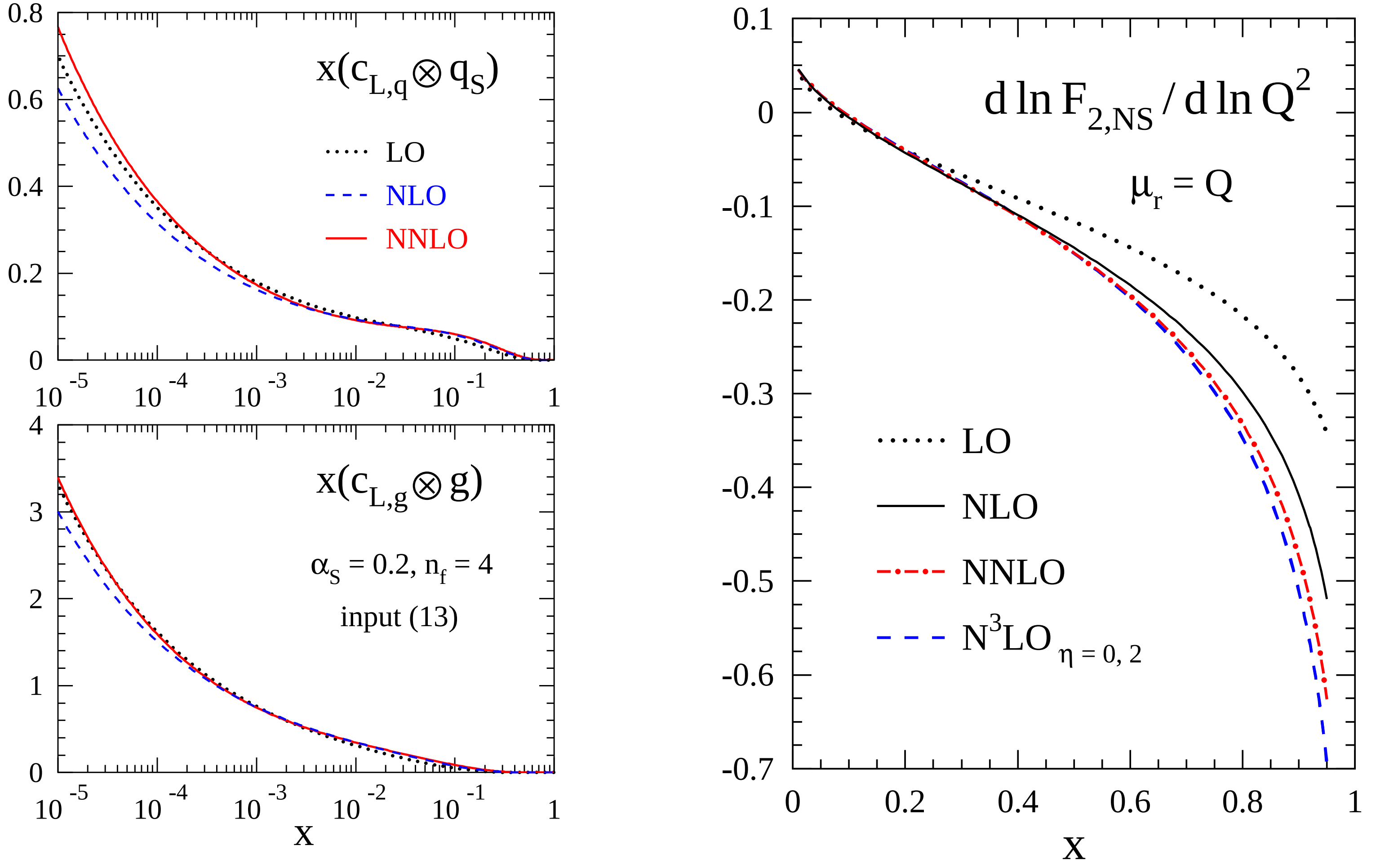

Theory of DIS
'69, QCD: '72
'69 – 72
Parton Model
Light Cone Expansion [f]
Twist 2
Higher Twist
polarized
unpolarized
~ 77
Fixed Order PT: QCD
Twist 3
Twist 4 ...
Sum Rules
α_s
Splitting functions
Coefficient functions
'60ies - now
$O(\alpha_s^4)$
'73,'74,'80,'97
$O(\alpha_s^3)$
'73,'82, 2004
$O(\alpha_s^3)$
'82,'92,2004
Special Kinematics
Domain: Small x
'75, '86
'90 - '98
More General View
Non-forw. scattering
Resummations ?
Diffractive Scattering
Angular Momentum: q, G
Higher Orders
$\Longrightarrow$
New Algorithms
Novel Mathematics

3 Loop Splitting Functions



Moch, Vermasern, Vogt, 2004

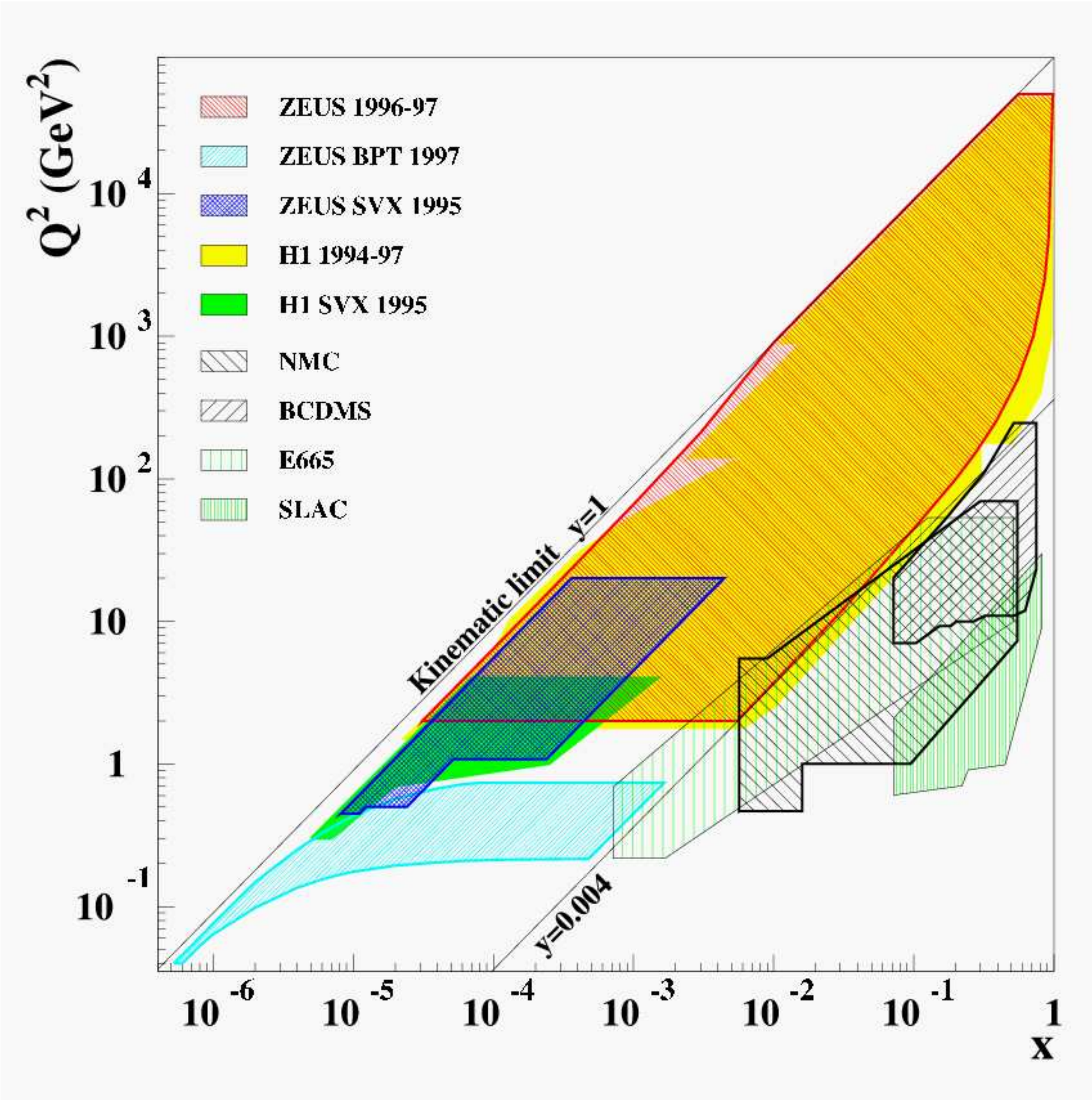
3 Loop Coefficient Functions



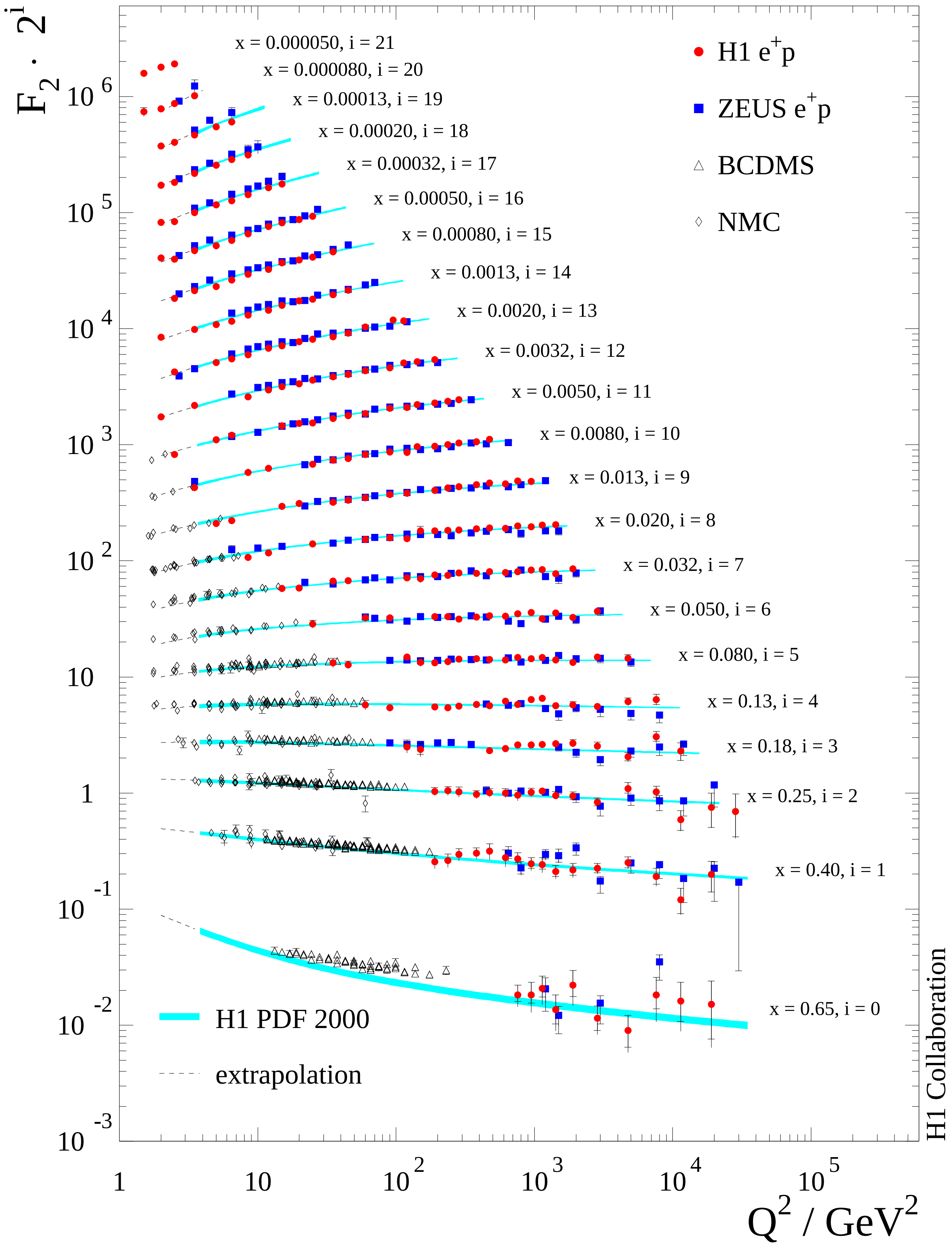
Moch, Vermasern, Vogt, 2004/05

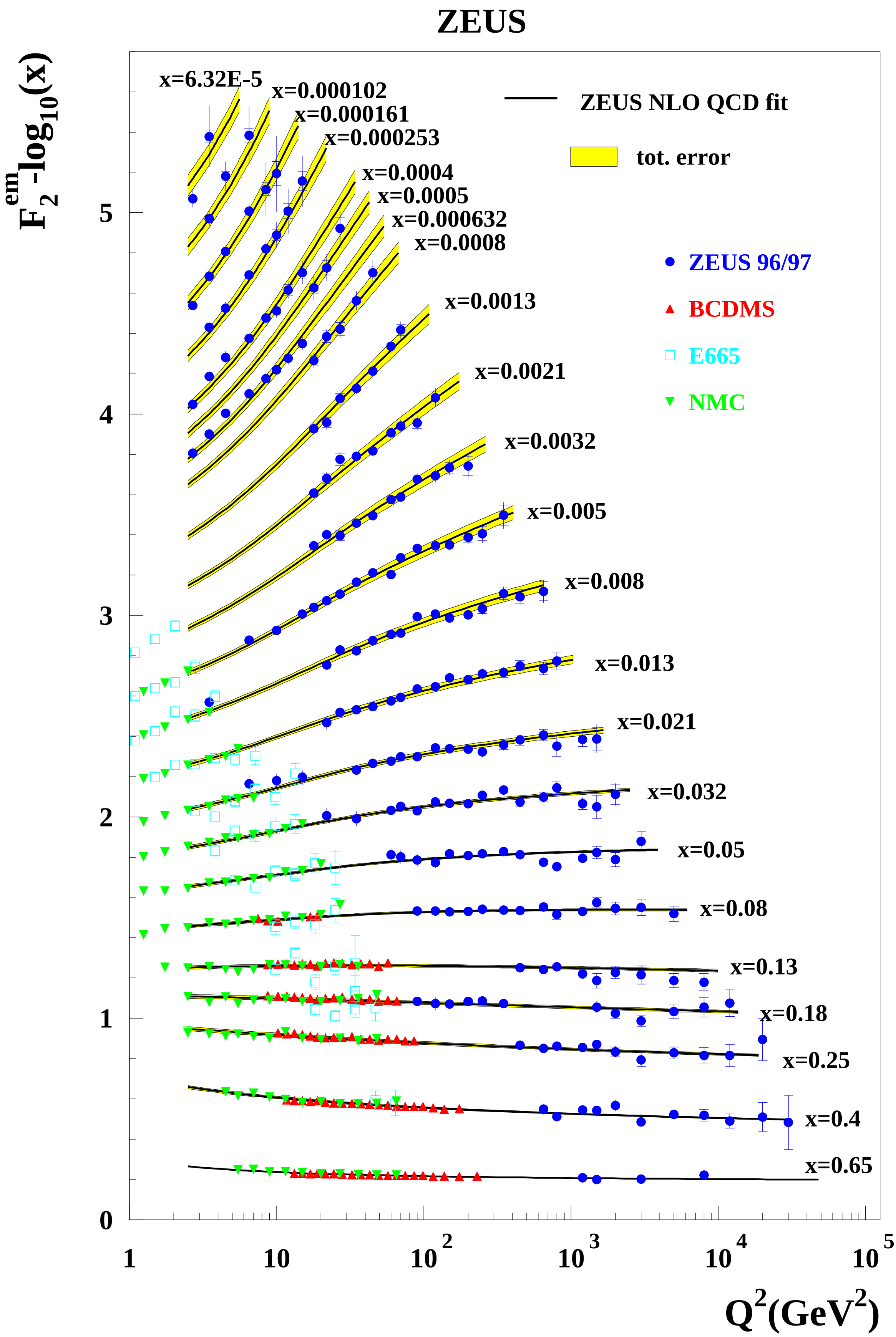
Unpolarized Parton Distributions

Kinematic Domain



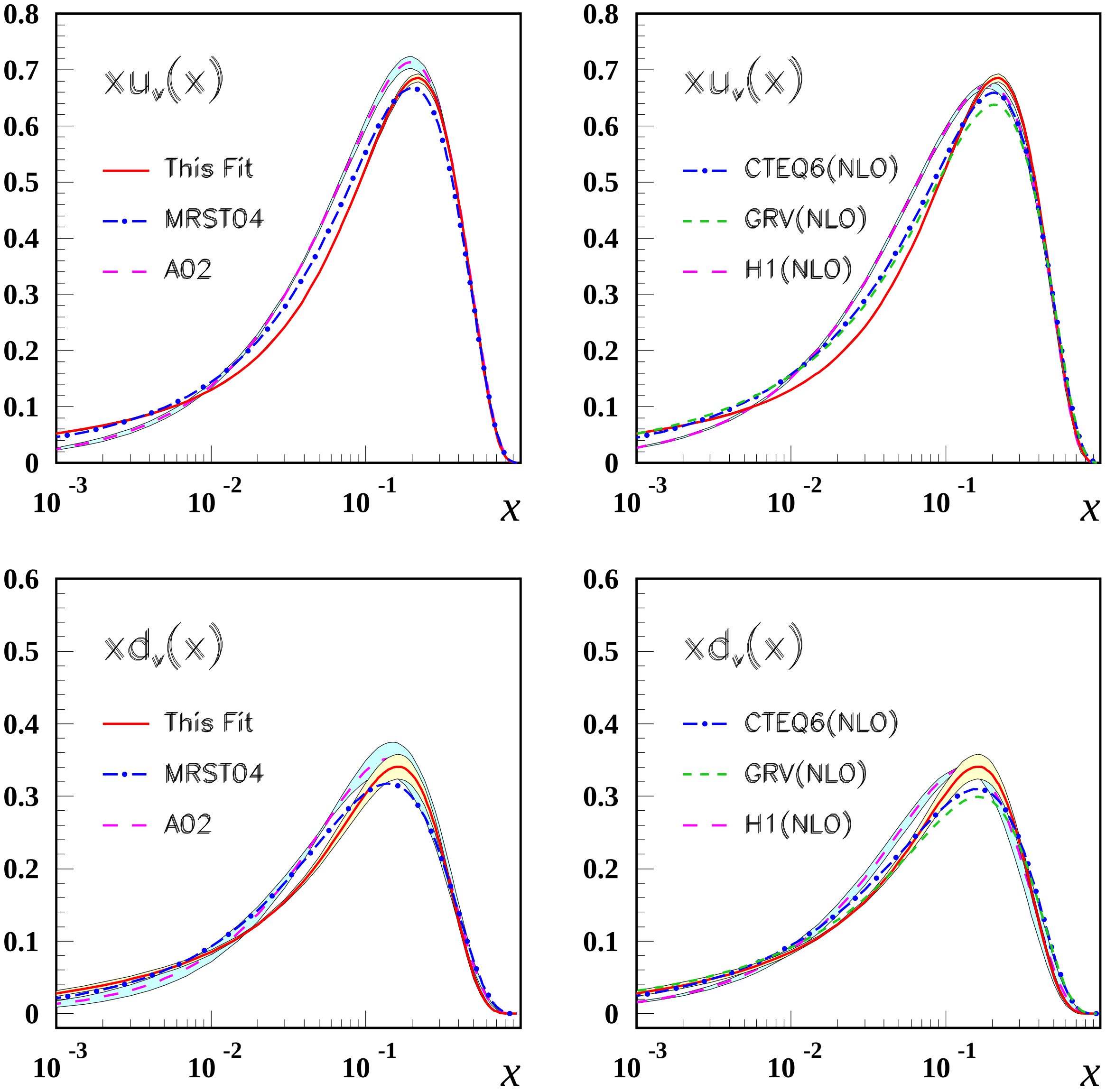
H1, ZEUS + fixed target data





Scaling violations of $F_2(x, Q^2)$.

QCD NS-Analysis to 3 Loops



$W^2 > 12.5\,\text{GeV}^2, Q^2 > 4\,\text{GeV}^2$

NNLO :

$\alpha_s(M_Z^2) = 0.1139^{+0.0026}_{-0.0028}$

J.B., H. Böttcher, A. Guffanti, 2004

The World Data on F_2

$Experiment$	x	Q^2, GeV^2	F_2	$Norm$
BCDMS (100)	0.35 – 0.75	11.75 – 75.00	51	1.018
BCDMS (120)	0.35 – 0.75	13.25 – 75.00	59	1.011
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	1.017
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	1.018
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	1.003
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	1.003
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	1.018
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	1.001
$proton$			322	
BCDMS (120)	0.35 – 0.75	13.25 – 99.00	59	0.992
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	0.993
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	0.993
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	0.980
SLAC (comb)	0.30 – 0.62	10.00 – 21.40	59	0.980
$deuteron$			232	
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	36	1.000
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	29	1.000
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	27	1.000
NMC (comb)	0.013 – 0.275	4.50 – 65.00	88	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	1.000
$non - singlet$			208	
$total$			762	

- **CUTS**: $0.3 < x < 1.0$ for F_2^p and F_2^d

  $0.0 < x < 0.3$ for $F_2^{ns} = 2(F_2^p - F_2^d)$

  $4.0 < Q^2 < 30000\, GeV^2$, $W^2 > 12.5\, GeV^2$

Fully Correlated Error Calculation

- The fully correlated 1σ error for the parton density f_q as given by Gaussian error propagation is

$$\sigma(f_q(x)^2 = \sum_{i,j=1}^{n_p} \left(\frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \mathrm{cov}(p_i, p_j) \ , \qquad (1)$$

  where the $\partial f_q / \partial p_i$ are the derivatives of f_q w.r.t. the parameters p_i and the $\mathrm{cov}(p_i, p_j)$ are the elements of the covariance matrix as determined in the fit.

- The derivatives $\partial f_q / \partial p_i$ at the input scale Q_0^2 can be calculated analytically. Their values at Q^2 are given by evolution.

- The derivatives evolved in $\mathrm{MELLIN\text{-}N}$ space are transformed back to x–space and can then be used according to the error propagation formula above.

$\Longrightarrow$ As an example the derivative of $f(x, a, b)$ w.r.t. parameter a in $\mathrm{MELLIN{-}N}$ space reads:

Fit Results

- Parameter values and Covariance Matrix at the input scale $Q_0^2 = 4.0\,GeV^2$

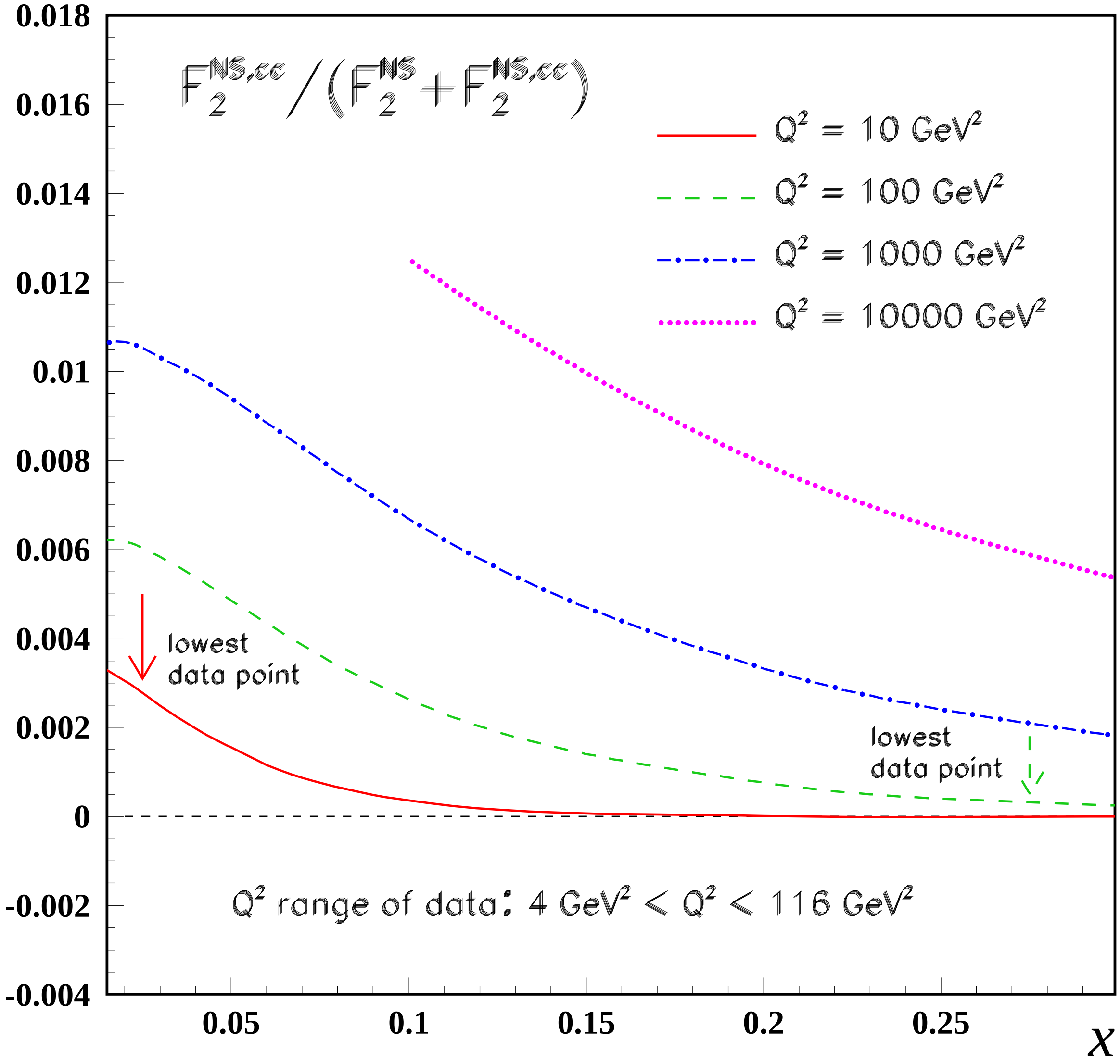
$$xq_i(x, Q_0^2) = A_i x^{a_i} (1-x)^{b_i} (1 + \rho_i x^{\frac{1}{2}} + \gamma_i x)$$

u_v	a	0.299 $\pm$ 0.007
	b	4.157 $\pm$ 0.031
	ρ	0.751
	γ	28.833
d_v	a	0.488 $\pm$ 0.048
	b	6.609 $\pm$ 0.332
	ρ	-1.690
	γ	17.247
$\Lambda_{QCD}^{(4)}$		233 $\pm$ 34 MeV
χ^2/ndf		= 630/757 = 0.83

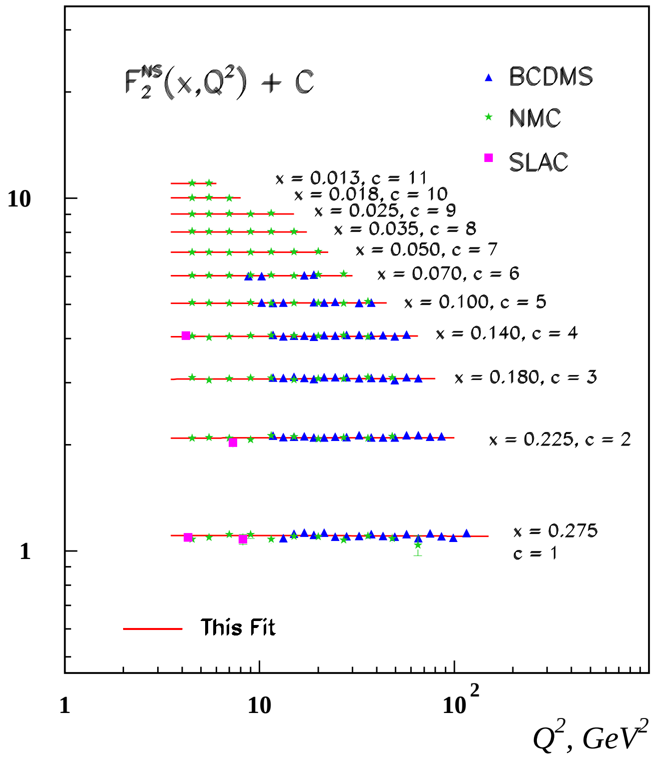
- Covariance Matrix at the input scale $Q_0^2 = 4.0\,GeV^2$

	$\Lambda_{QCD}^{(4)}$	a_{u_v}	b_{u_v}	a_{d_v}	b_{d_v}
$\Lambda_{QCD}^{(4)}$	**1.15E-3**				
a_{u_v}	1.03E-4	**5.40E-5**			
b_{u_v}	-8.45E-5	1.71E-4	**9.59E-4**		
a_{d_v}	4.17E-4	8.84E-6	-4.35E-4	**2.32E-3**	
b_{d_v}	2.32E-3	4.21E-4	-2.28E-3	1.48E-2	**1.10E-1**

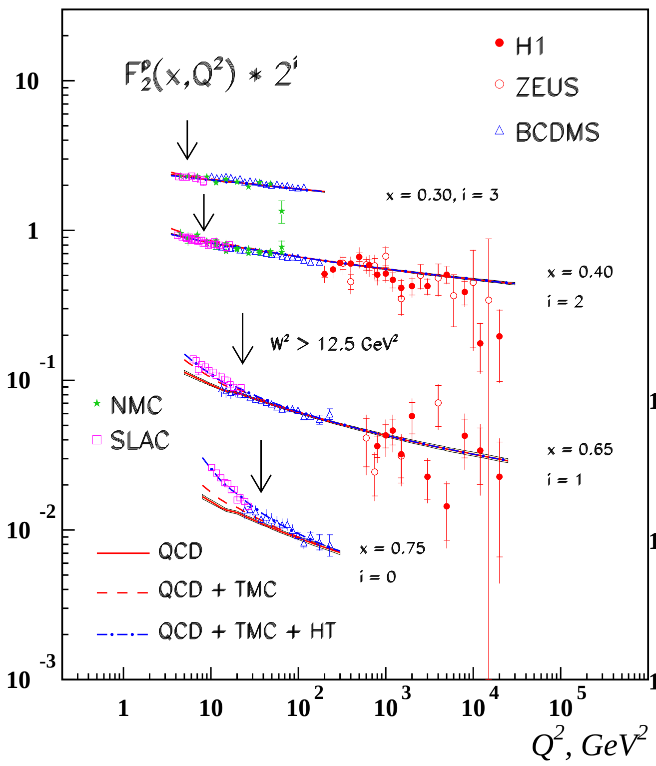
Heavy Flavor NS-contributions



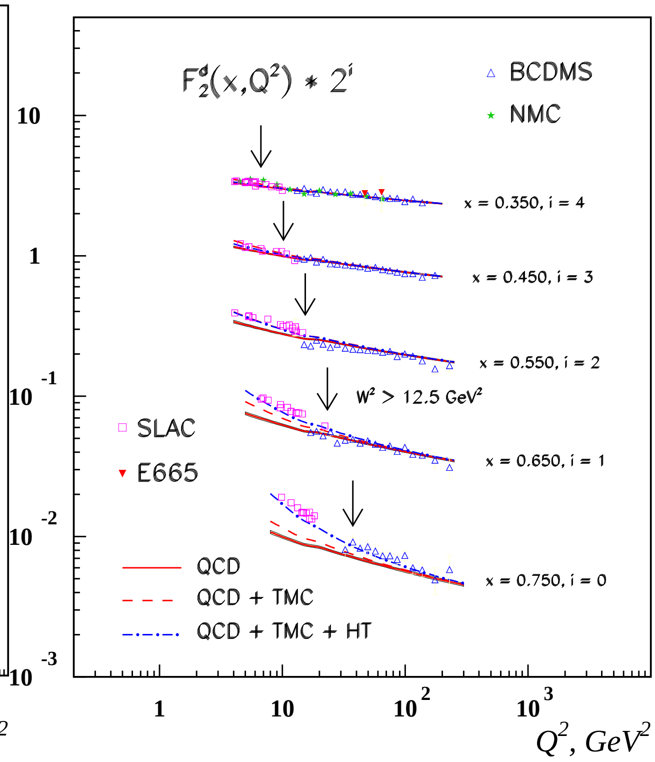
Non-Singlet 3-Loop QCD Analysis



$x > 0.3$



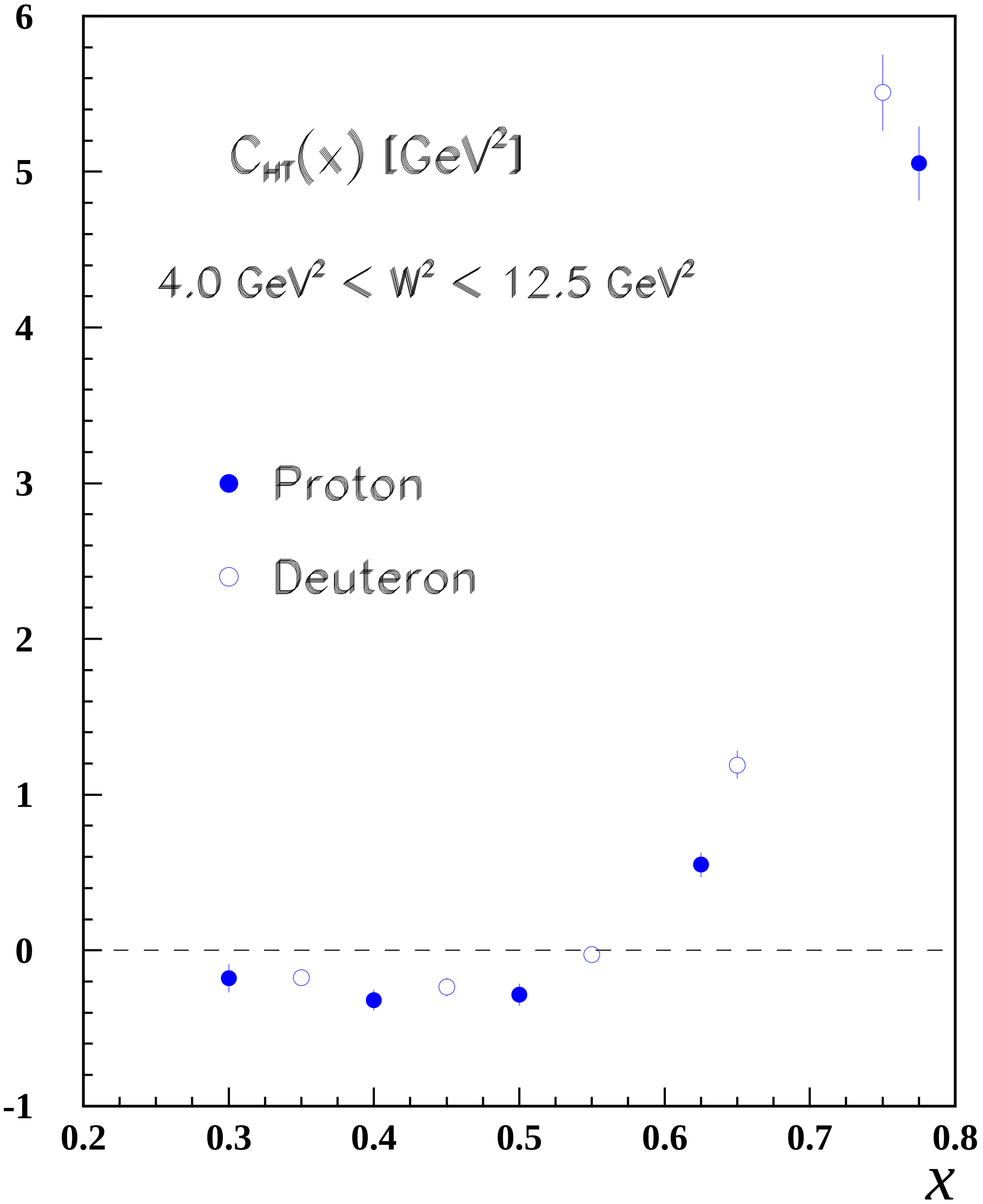
proton



deuteron

Higher Twist Contributions:

$$4 < W^2 < 12.5\,\mathrm{GeV}^2, Q^2 > 4\,\mathrm{GeV}^2$$



Moments and Lattice Results

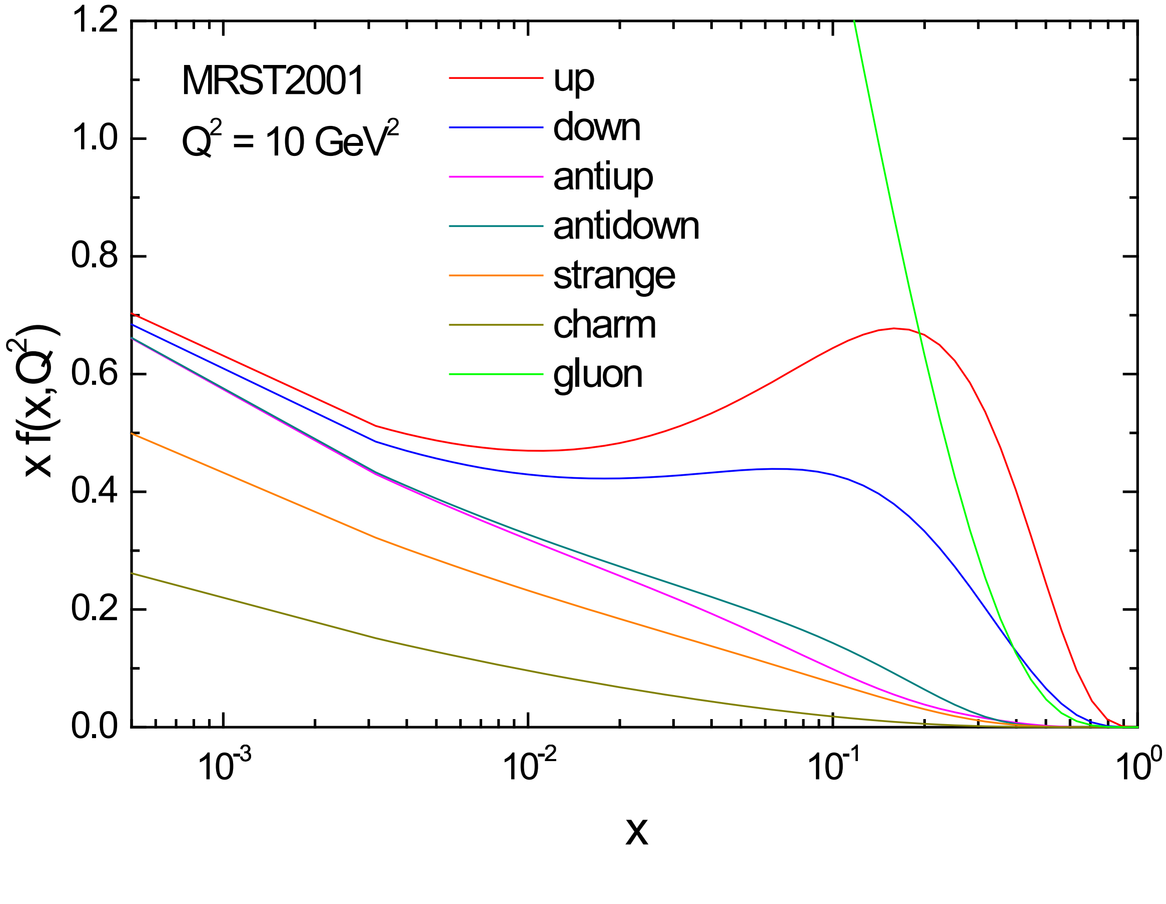
f	n	This Fit	MRST04	A02
u_v	2	0.288 ± 0.003	0.285	0.304
	3	0.084 ± 0.001	0.082	0.087
	4	0.0319 ± 0.0004	0.032	0.033
d_v	2	0.113 ± 0.004	0.115	0.120
	3	0.026 ± 0.001	0.028	0.028
	4	0.0078 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.175 ± 0.004	0.171	0.184
	3	0.058 ± 0.001	0.055	0.059
	4	0.0241 ± 0.0005	0.022	0.024

First lattice results on $u_v - d_v, N = 2$ yield promising values using overlap-fermions (QCDSF).

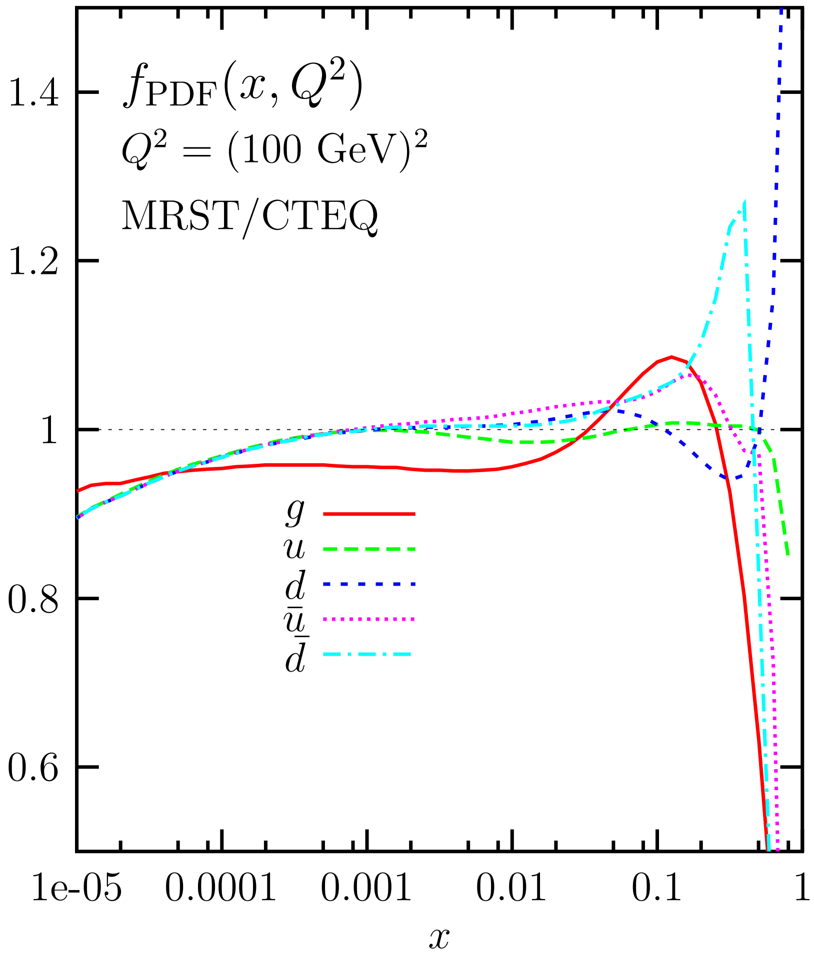
More results also are upcoming.

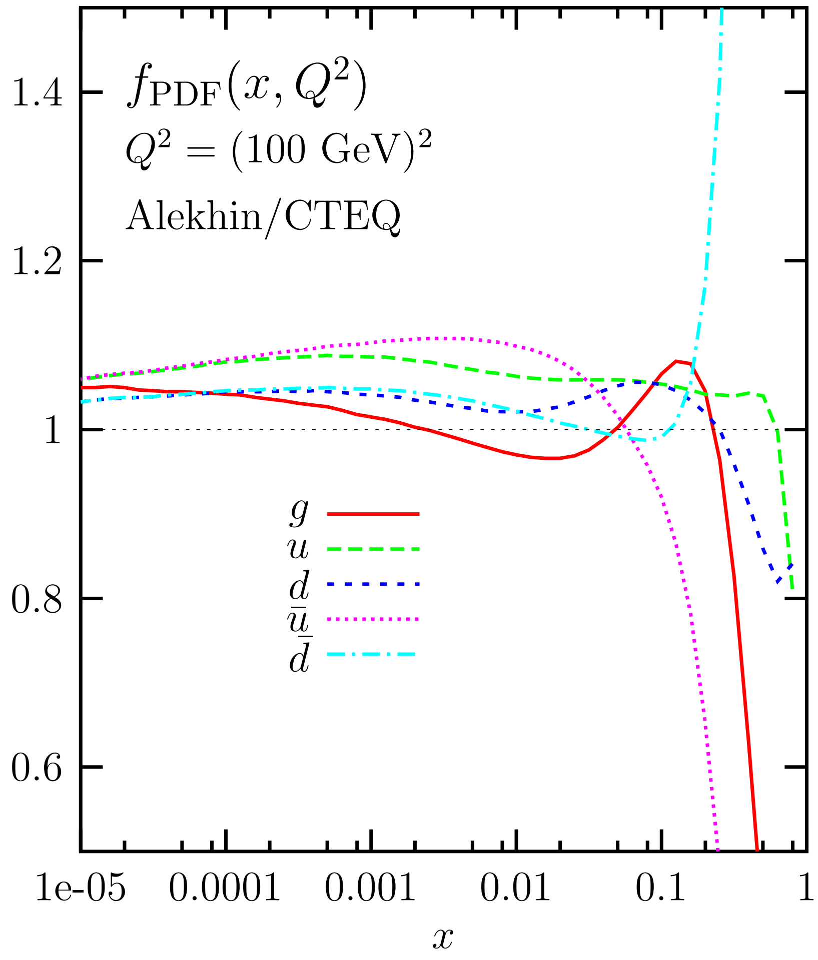
The Singlet Sector

Parton Densities: Relative Size



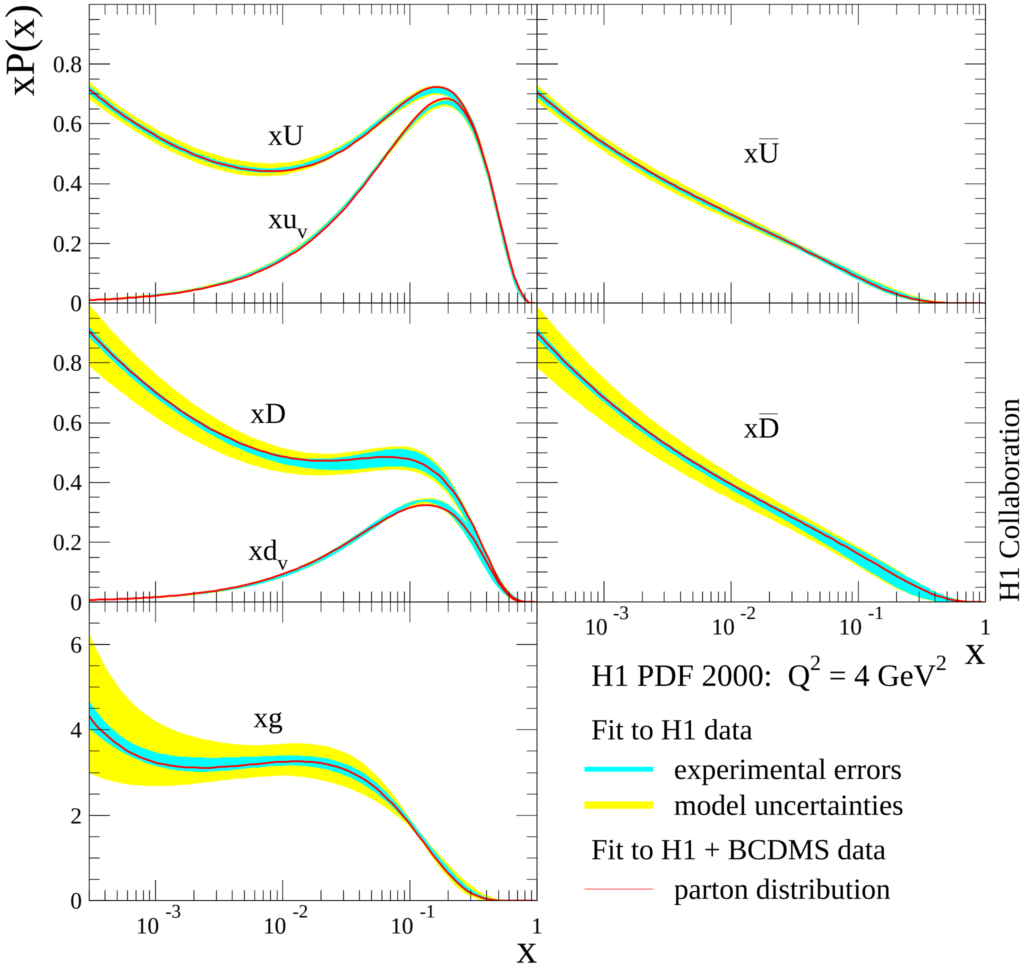
Ratios of Unpolarized PDFs





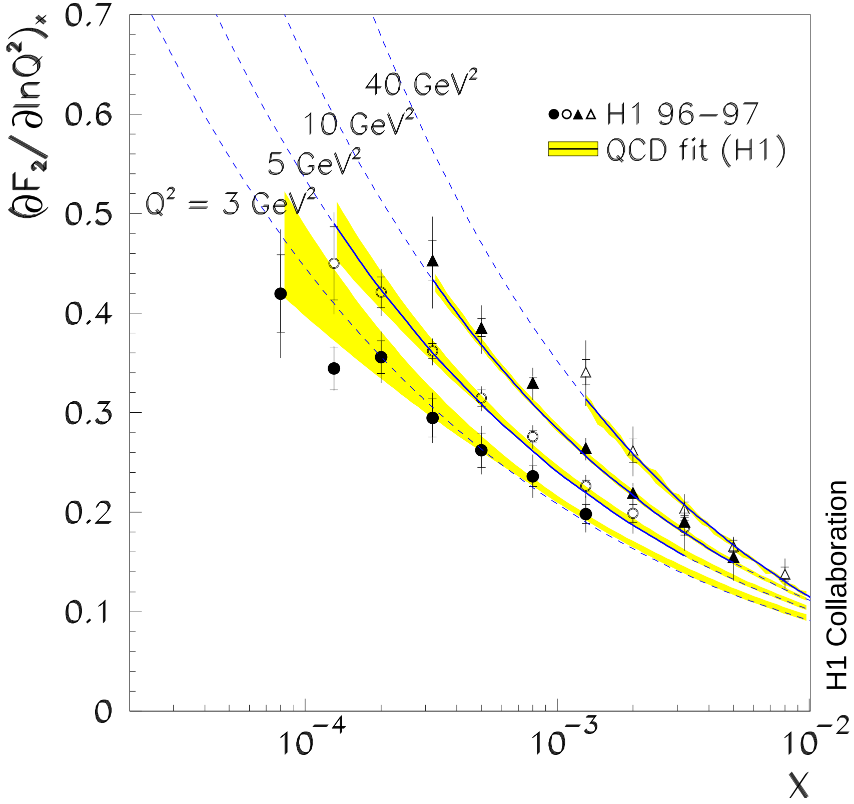
Ref.: A. Djouadi and S. Ferrag, hep-ph/0310209

Parton Distributions

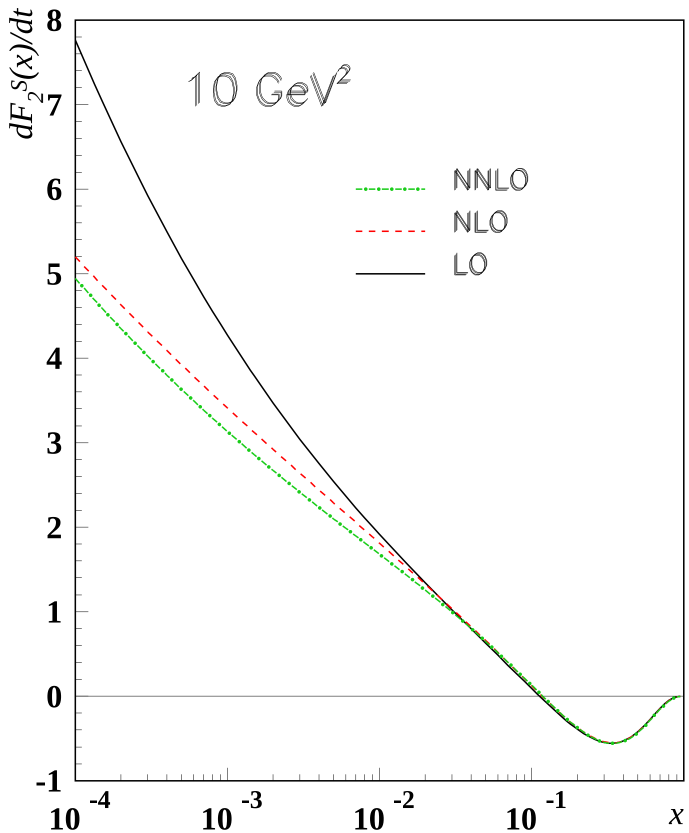


H1

Slope of F_2 at low x



H1

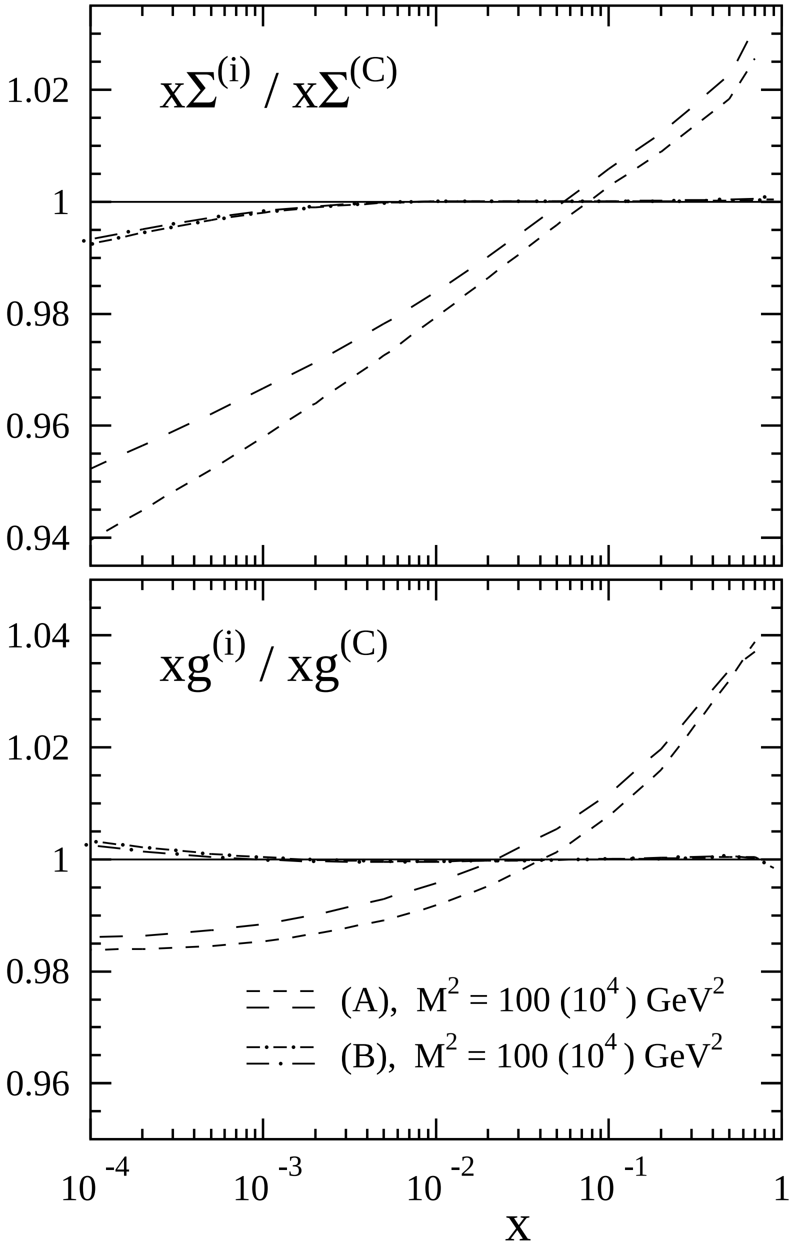


J.B., A. Guffanti 2005

Very likely, that the $\overline{\mathrm{MS}}$–gluon is remains positive!

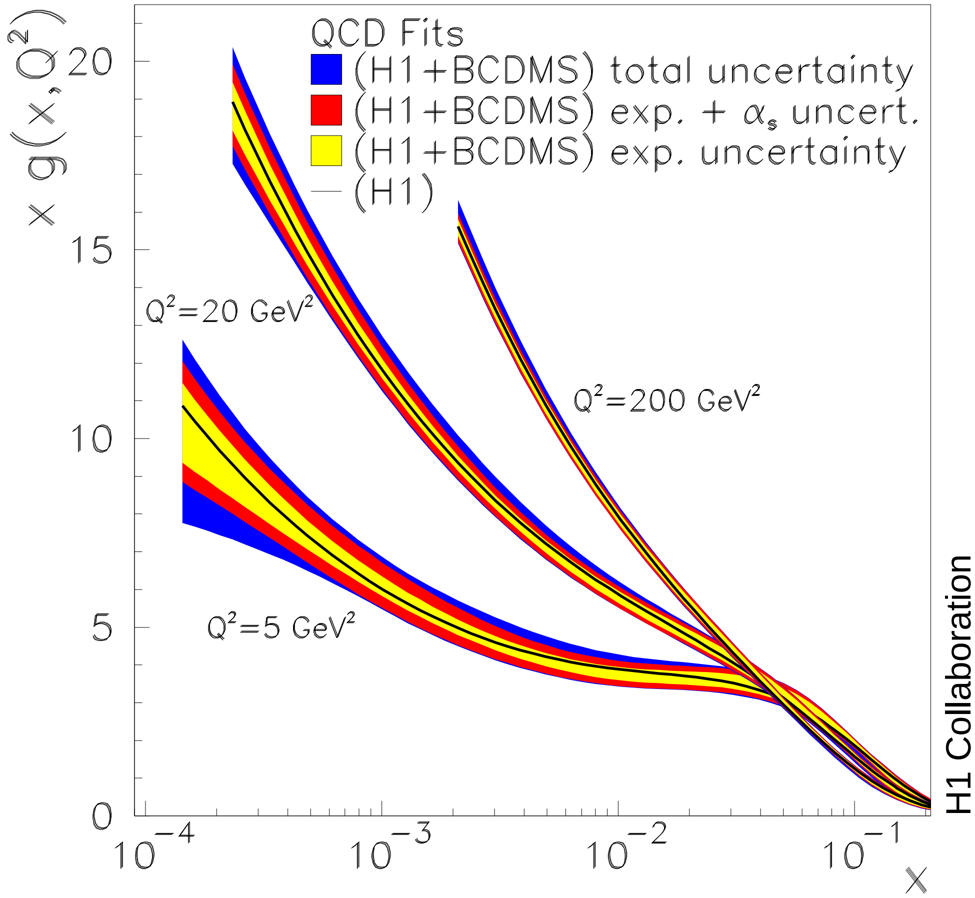
Pile–Up Effects:

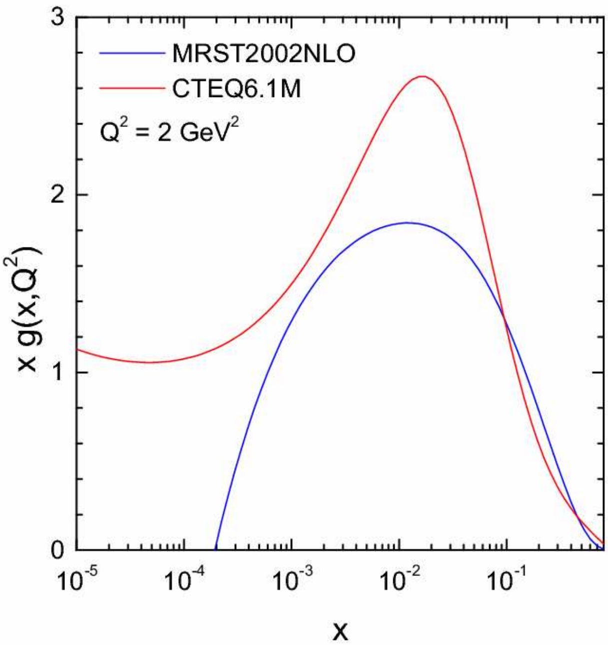
Iterative vs Exact Solution of Evolution Equations



Blümlein, Riemersma, van Neerven, Vogt, 1996

Gluon Density

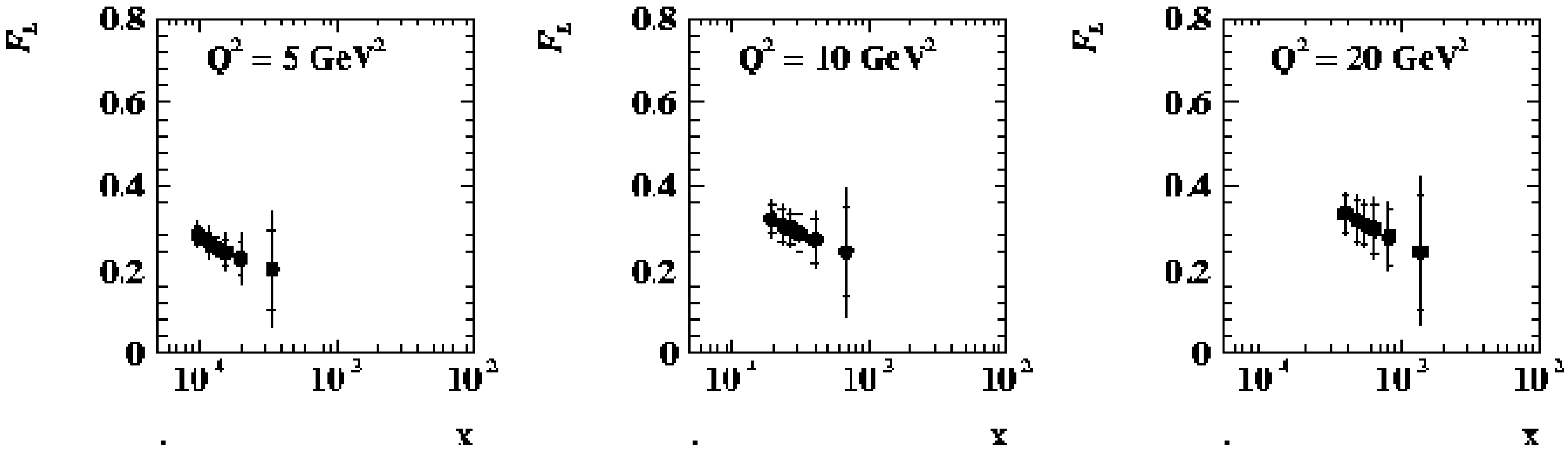




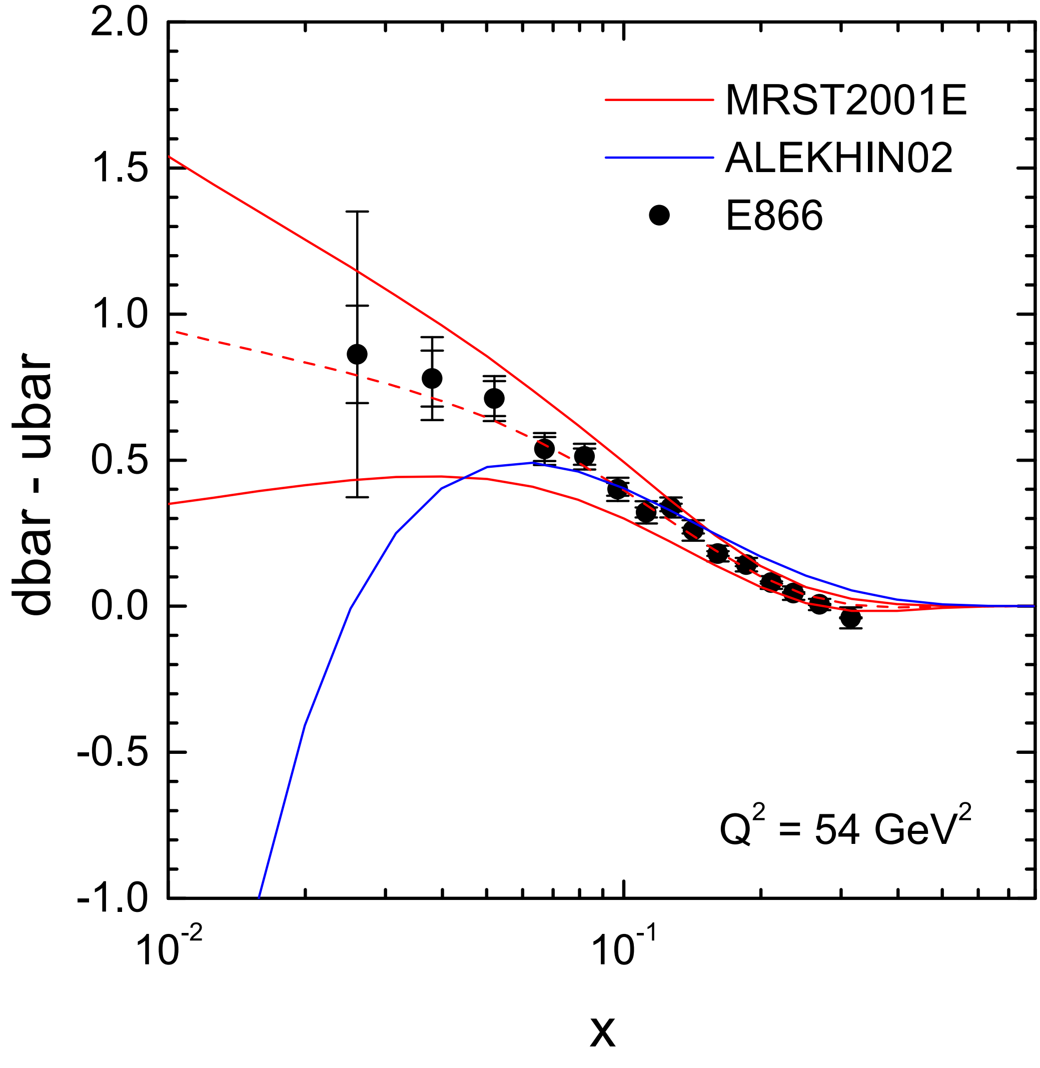
$F_L(x, Q^2)$

M. Klein, 2004: Projection for a possible measurement at HERA

$\Longrightarrow$ of central importance to study the small x behaviour of the gluon distribution



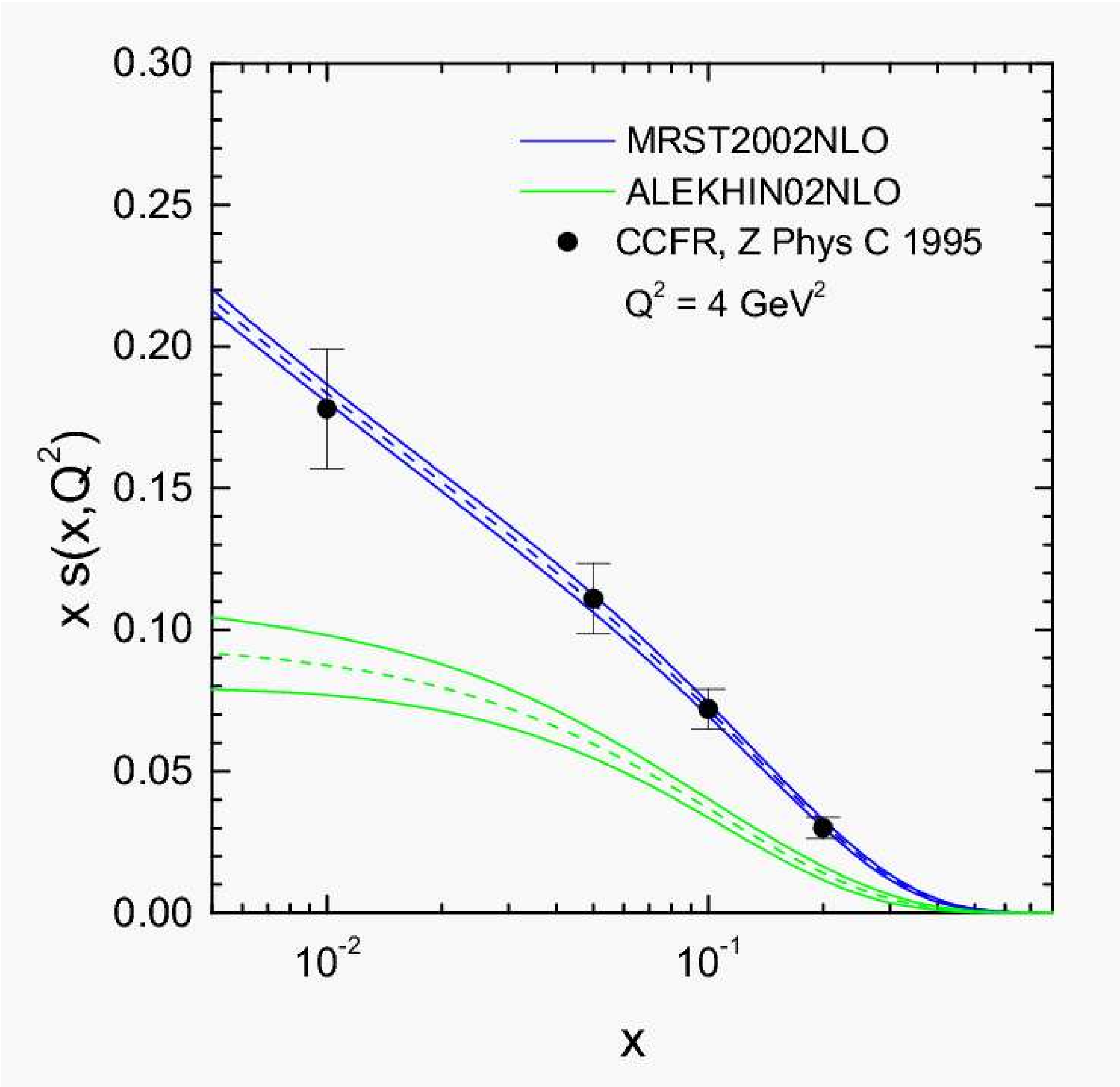
$\overline{d} - \overline{u}$



$\times(\overline{d}(x) - \overline{u}(x)) = 1.195x^{1.24}(1-x)^{9.10}(1 + 14.05x - 45.52x^2)$

$Q^2 = 1$ GeV^2

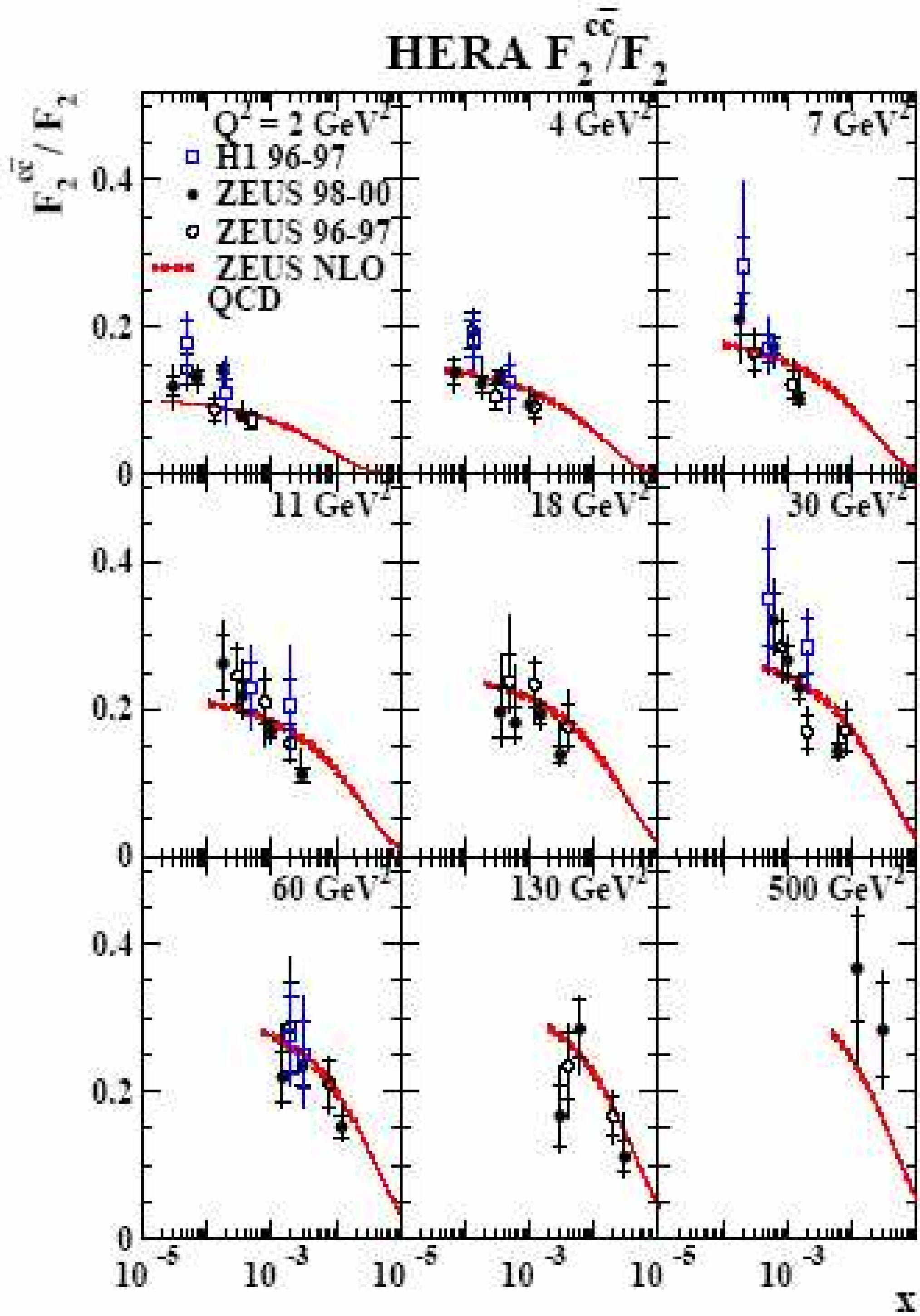
Strange quark distribution



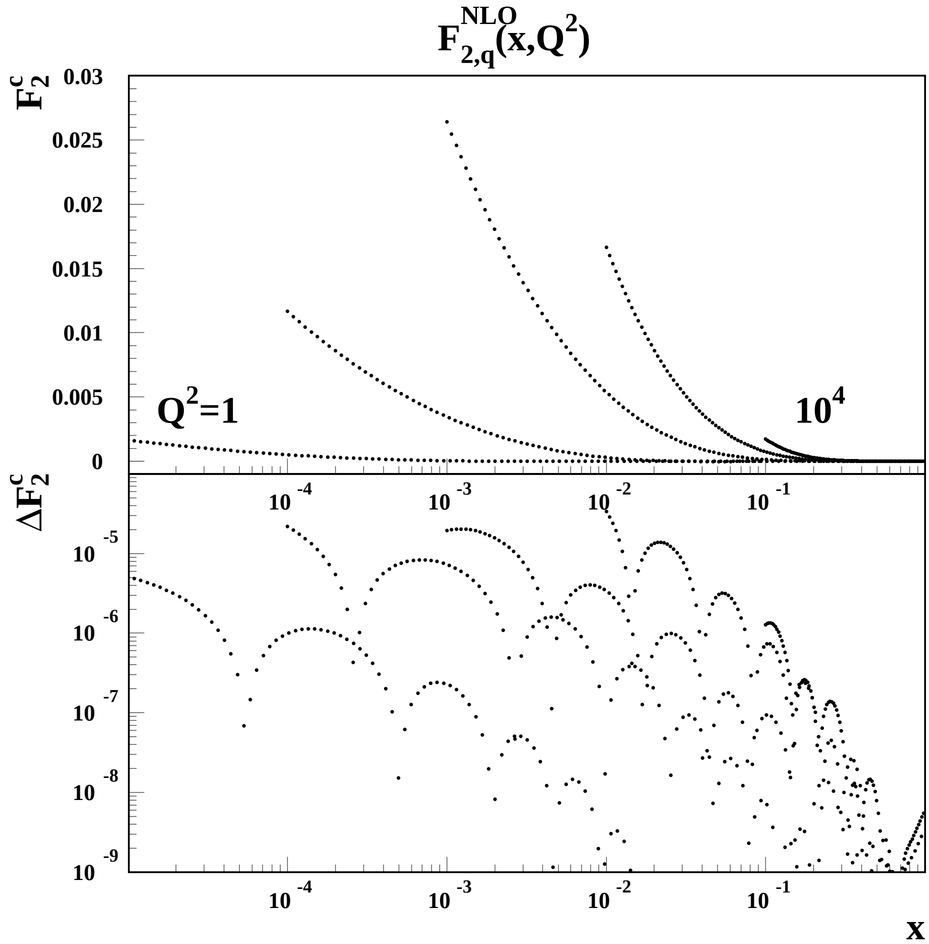
- CCFR : iron target, EMC effect. How large ?

CAN HERMES MEASURE $s(x,Q^2)$?

$c\bar{c}$ Structure Function F_2



Mellin-space representation :



S. Alekhin and J.B., 2004

- necessary for scheme-invariant evolution.
- fast and accurate access to heavy flavor Wilson coefficients.

Polarized Nucleons

HOW IS THE NUCLEON SPIN DISTRIBUTED OVER THE PARTONS?

$$S_n = \frac{1}{2}\left[\Delta(u+\bar{u}) + \Delta(d+\bar{d}) + \Delta(s+\bar{s})\right] + \Delta G + L_q + L_g$$

$$S_n = \frac{1}{2}$$

$$\begin{aligned} \Delta\Sigma &= 0.138 \pm 0.082, \quad (0.150 \pm 0.061) \\ \Delta G &= 1.026 \pm 0.554, \quad (0.931 \pm 0.679) \end{aligned}$$

EMC, 1987: THE NUCLEON SPIN IS NOT THE SUM OF THE LIGHT QUARK SPINS.

MEASURE:

POLARIZED PARTON DENSITIES: $\Delta q_i, \Delta G$

HOW CAN ONE ACCESS THE PARTON ANGULAR MOMENTUM ?

POLARIZED HEAVY FLAVOR CONTRIBUTIONS.

- POLARIZED STRUCTURE FUNCTIONS CONTAIN ALSO TWIST 3 CONTRIBUTIONS.

HOW TO UNFOLD THESE TERMS ?

Polarized Parton Densities:

pioneering work: Dortmund GRSV, 1996, 2001

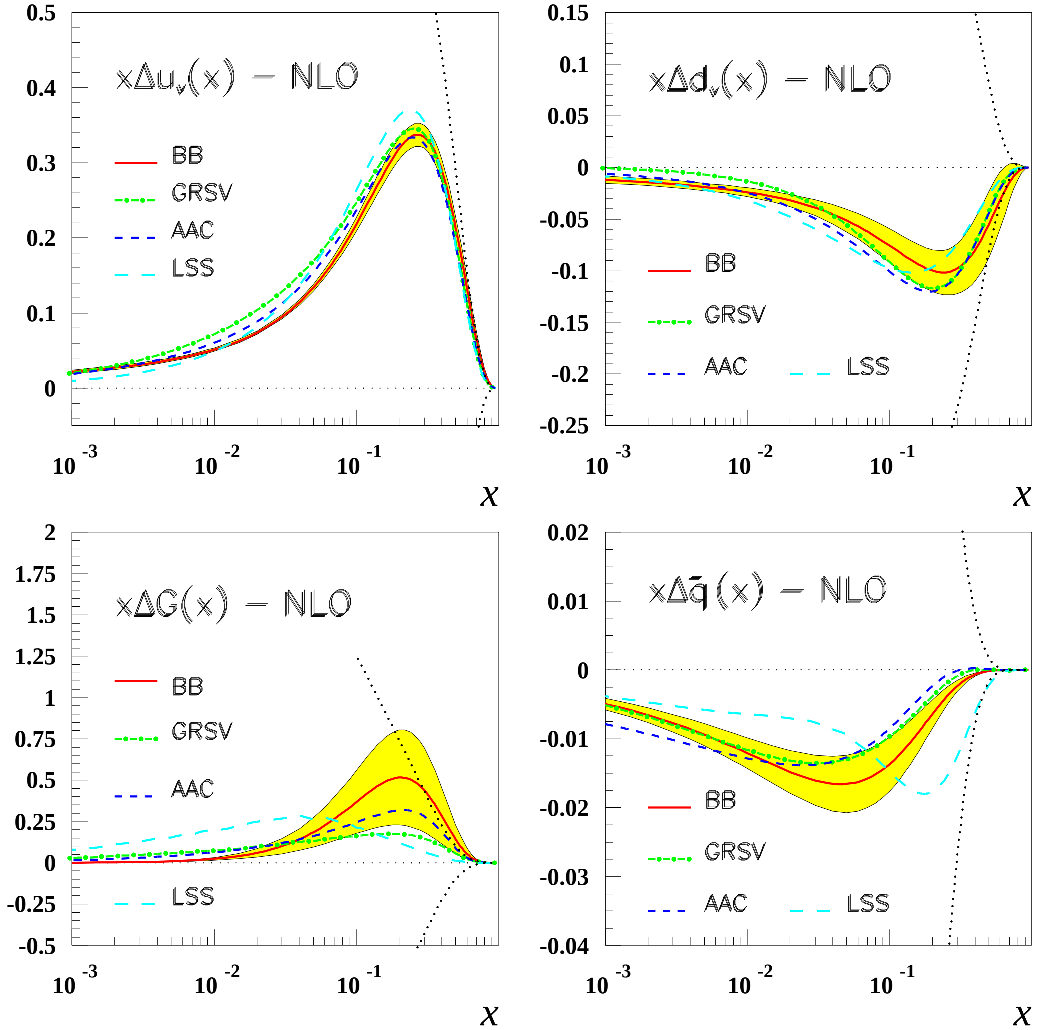
Analysis by other groups:

AAC (Japan), 2000, 2004

J.B., H. Böttcher, 2002

Leader et al., 2002

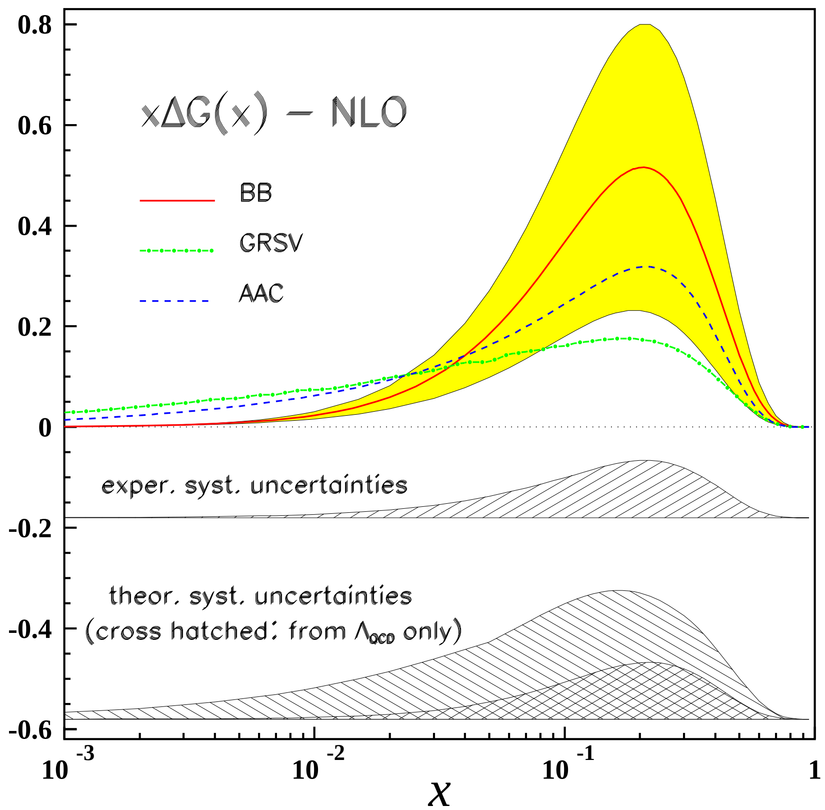
Altarelli et al., 1997



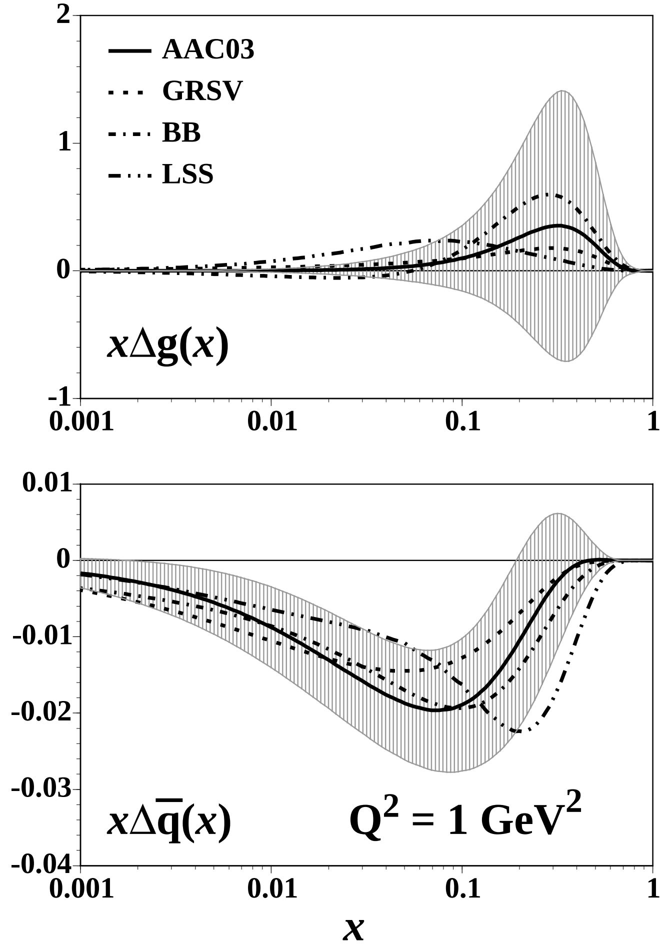
$$\mathrm{NLO}: \quad \alpha_s(M_z^2) = 0.113^{+0.10}_{-0.08}$$

J.B., H. Böttcher, 2002

Polarized Gluon Density



J.B., H. Böttcher, 2002



AAC

$\Longrightarrow$ Currently slight move towards lower values.

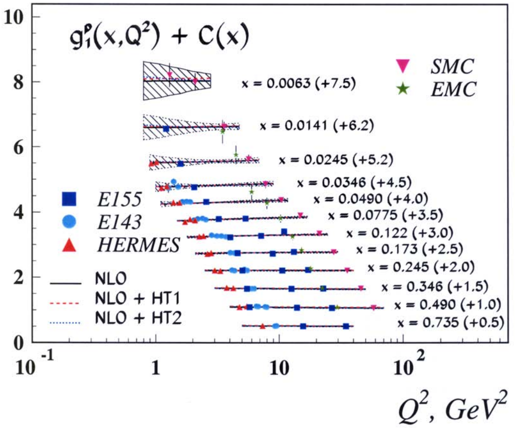


Figure 12: Model fit to potential power corrections in $g_1(x, Q^2)$ as extracted from the world polarization asymmetry data in the present analysis (see text). Dashed line: model I, Eq. (70); dotted line: model II, Eq. (71). The full lines correspond to the parameterization (ISET=4) in the present analysis, to which the corresponding power correction model induces a perturbation. The shaded area corresponds to the 1σ correlated error.

Comparison with Lattice Moments:

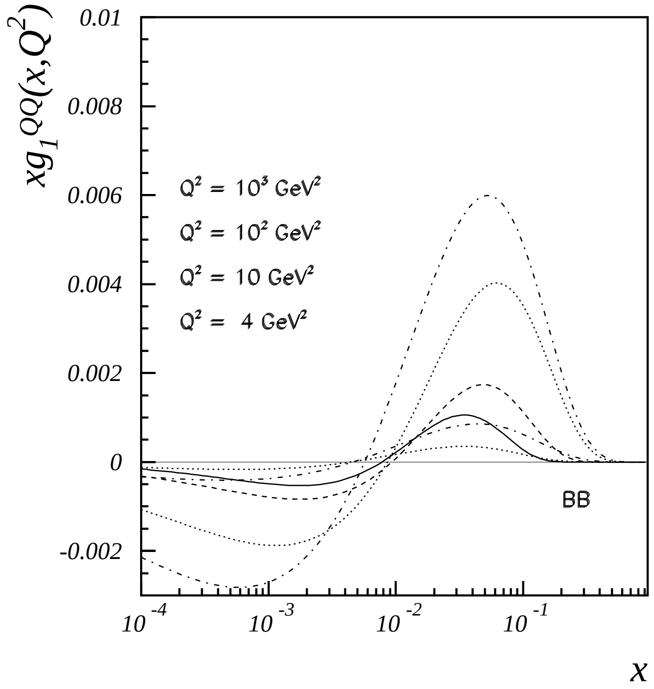
	Moment	BB, NLO	QCDSF	LHPC/SESAM
Δu_v	0	0.926	0.889 ± 0.029	0.860 ± 0.069
	1	0.163 ± 0.014	0.198 ± 0.008	0.242 ± 0.022
	2	0.055 ± 0.006	0.041 ± 0.009	0.116 ± 0.042
Δd_v	0	-0.341	-0.236 ± 0.027	-0.171 ± 0.043
	1	-0.047 ± 0.021	-0.048 ± 0.003	-0.029 ± 0.013
	2	-0.015 ± 0.009	-0.028 ± 0.002	0.001 ± 0.025
$\Delta u_v - \Delta d_v$	0	1.267	1.14 ± 0.03	1.031 ± 0.081
	1	0.210 ± 0.025	0.245 ± 0.009	0.271 ± 0.025
	2	0.070 ± 0.011	0.069 ± 0.009	0.115 ± 0.049

1st moments: Still problematic.

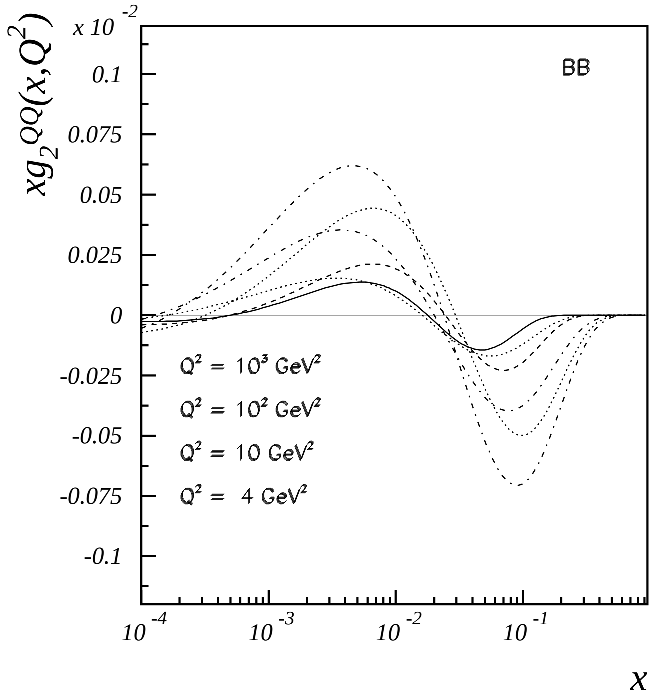
Heavy flavor:

g_1 : Watson, 1982; Vogelsang, 1990

g_2 : J.B., Ravindran, van Neerven, 2003



$g_1^{c\bar{c}}$



$g_2^{c\bar{c}}$

Sum Rules and Integral Relations:

Twist 2:

$$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

Wandzura, Wilczek, 1977;

Piccione, Ridolfi 1998; J.B., A. Tkabladze, 1998 : with TM

$$g_3(x,Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4(y,Q^2)$$

J.B., N. Kochelev, 1996; J.B., A. Tkabladze, 1998 : with TM

Twist 3:

Include nucleon mass effects.

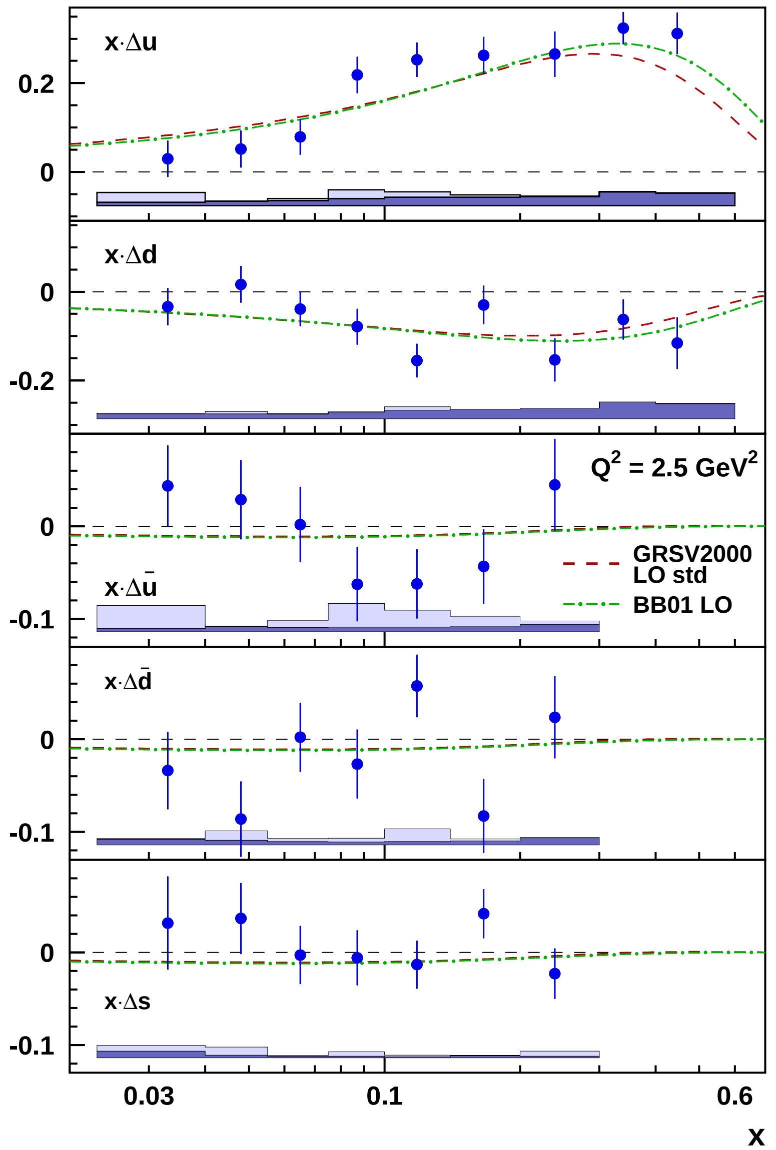
J.B., A. Tkabladze, 1998

$$\begin{aligned} g_1(x,Q^2) &= \frac{4M^2x^2}{Q^2}\left[g_2(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2(y,Q^2)\right] \\ \frac{4M^2x^2}{Q^2} g_3(x,Q^2) &= g_4(x,Q^2)\left(1 + \frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g_4(y,Q^2) \\ 2xg_5(x,Q^2) &= -\int_x^1 \frac{dy}{y} g_4(y,Q^2) \end{aligned}$$

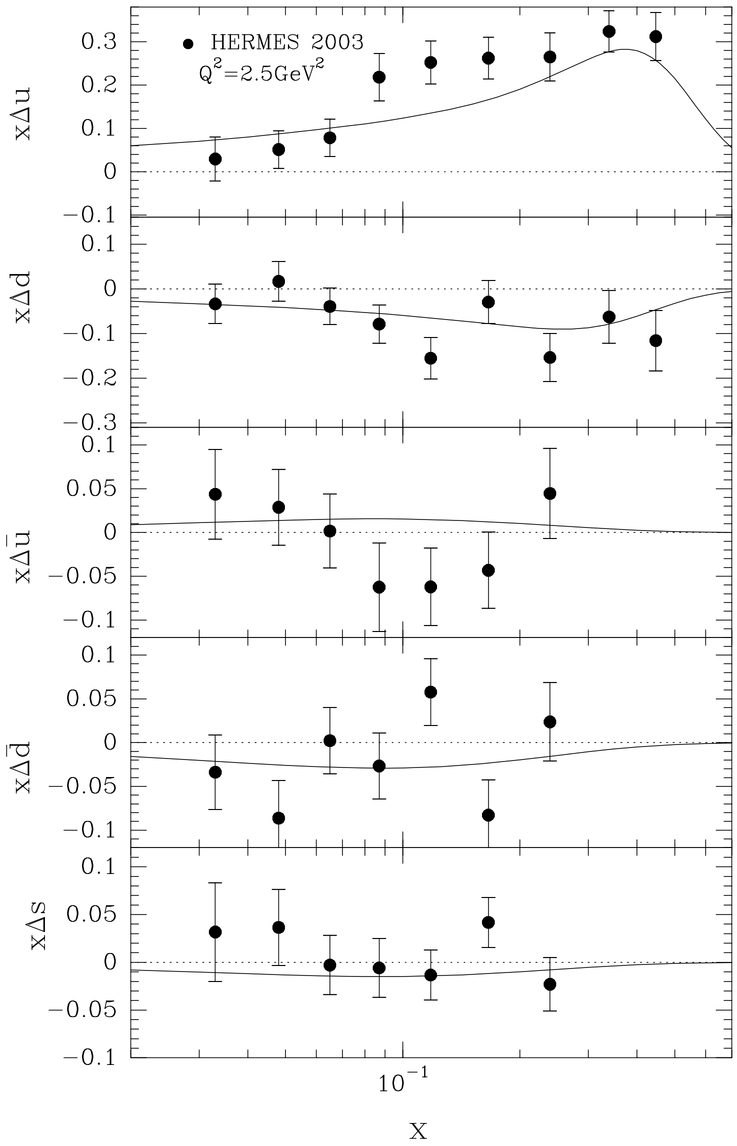
Quark Helicity Distributions

HERMES Experiment, hep-ex/0407032

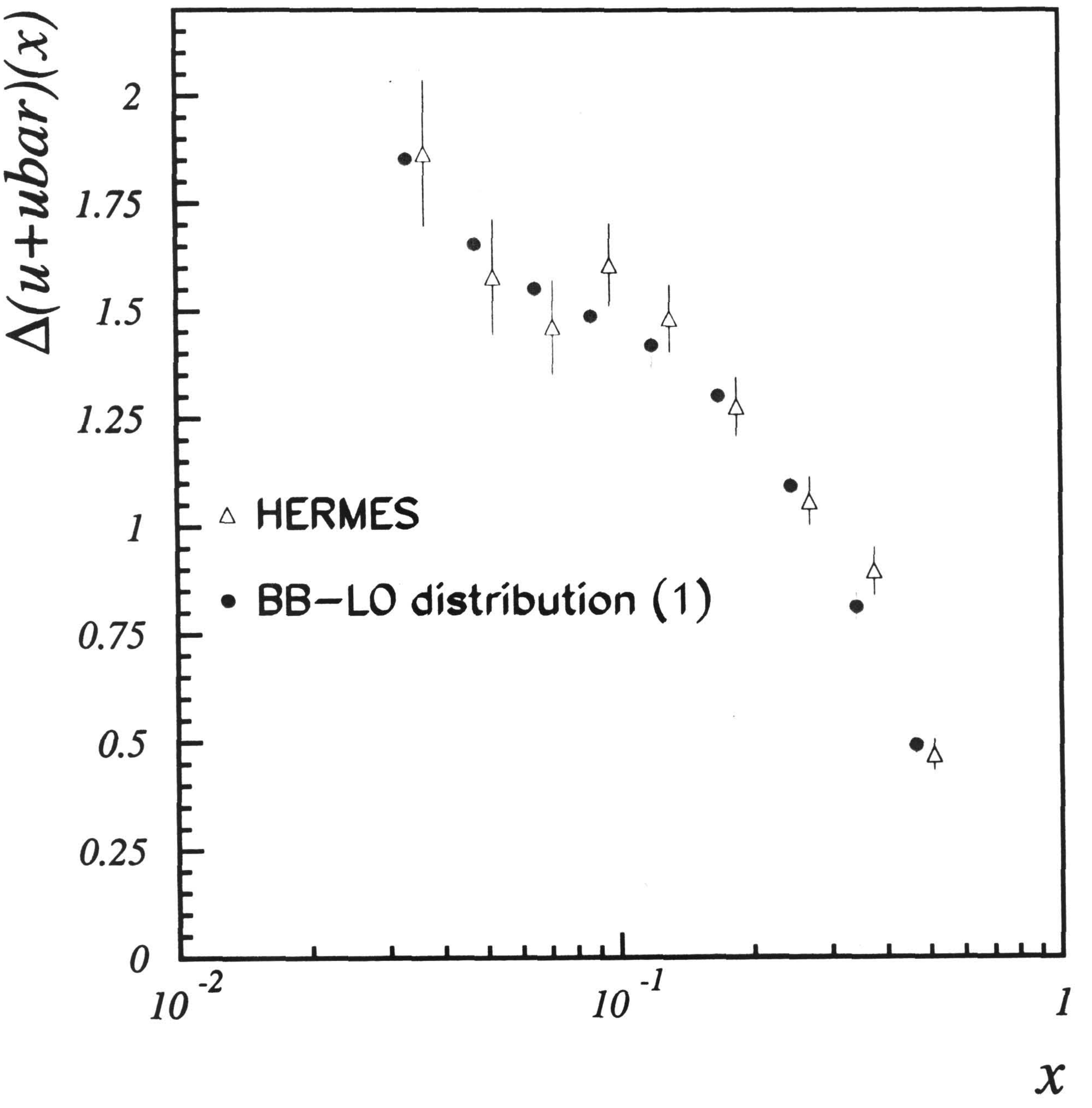
(⟶ A. Miller's talk)



Statistical PDFs: C.Bourrely, J.Soffer and F.Buccella, hep-ph/0109160



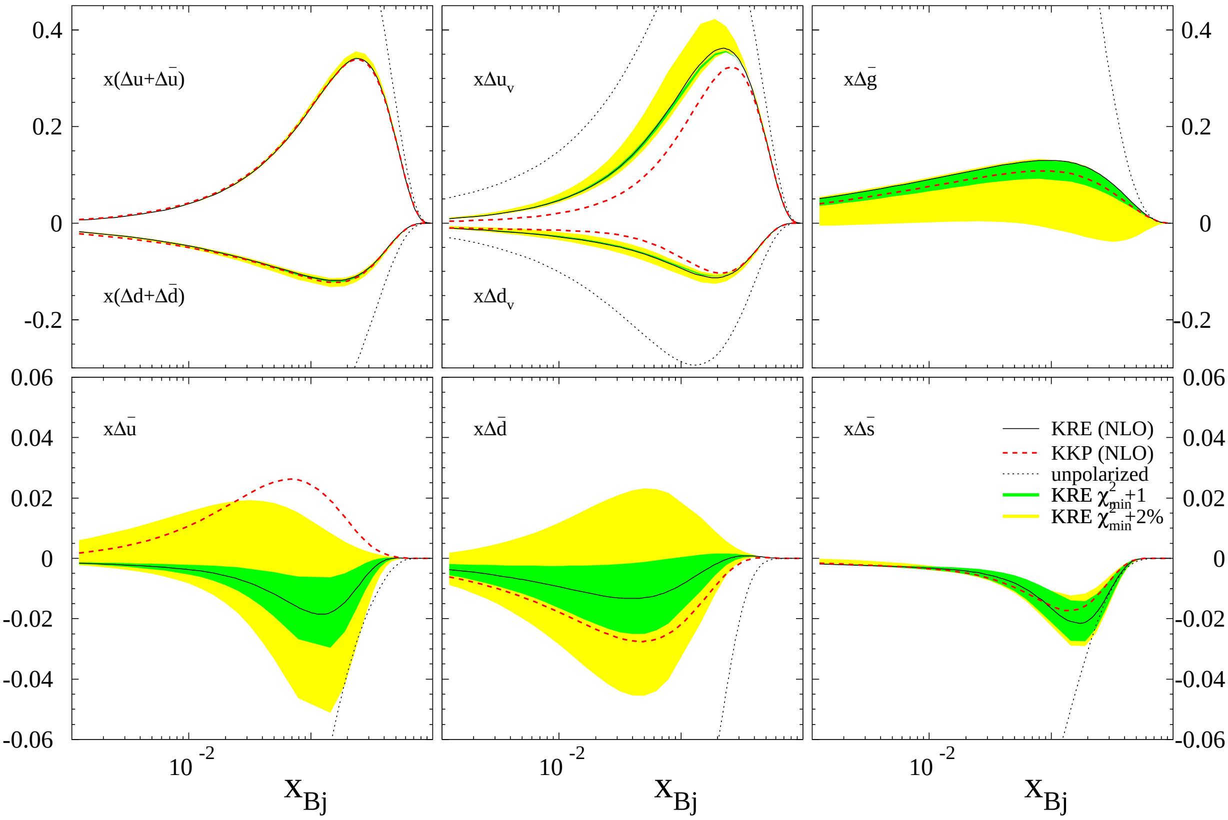
Comparison with Δq from Semi-Incl. Data



$\Longrightarrow$ z–range in the Semi–Incl. Analysis: $0.2 < z < 0.7$

Inclusive + Semi-inclusive Analysis

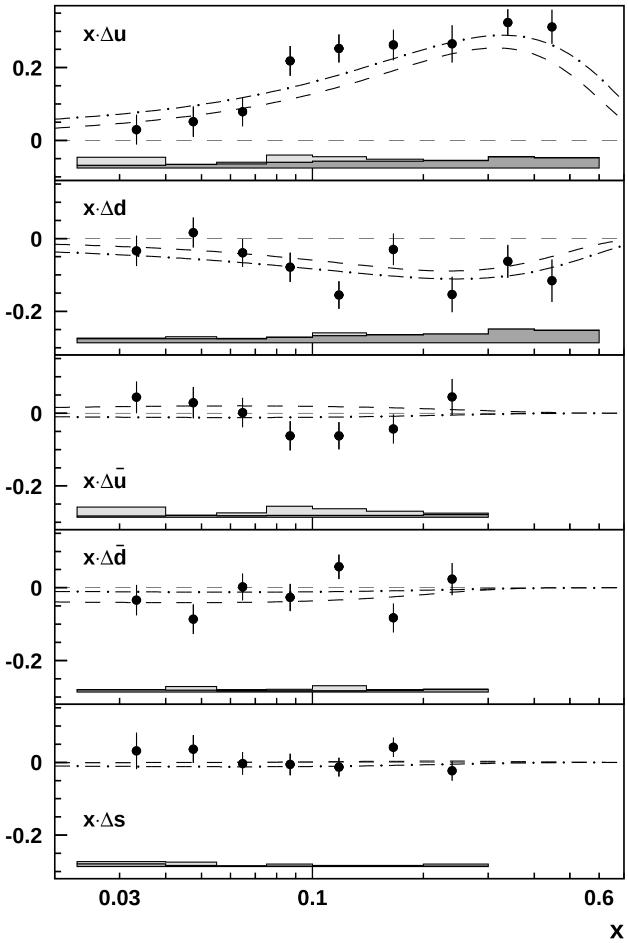
D. de Florian, G. Navarro, R. Sassot, hep-ph/0504155



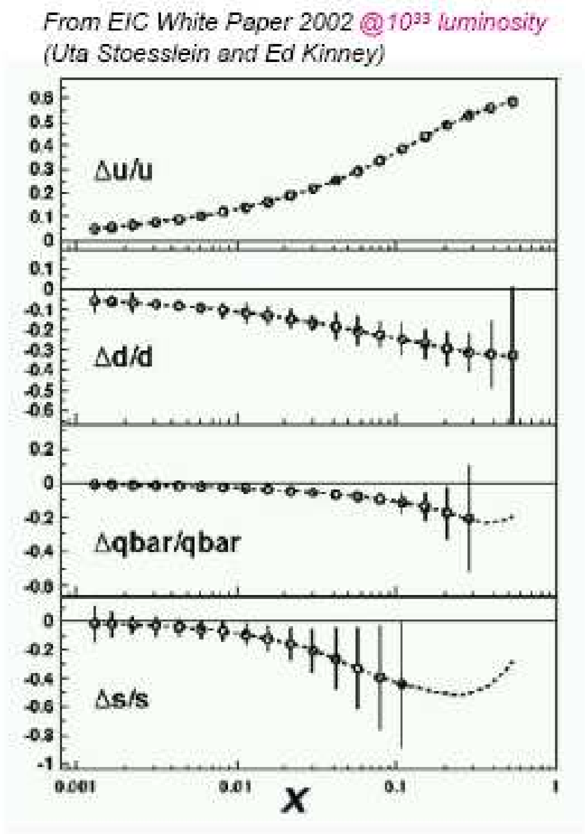
Parton densities at $Q^2 = 10 GeV^2$; error bands: $\Delta\chi^2 = 1; 2\%$.

.... allows very precise measurements

Example : Flavor Separation of polarized PDF's



HERMES



EIC

$\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
BB (pol)	0.113	± 0.004	$^{+0.009}_{-0.006}$	[7]

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
BBG	0.1139	$+0.0026/-0.0028$		[9]

BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^3)$:

$$\Lambda = 233 \pm 30\,\text{MeV}$$

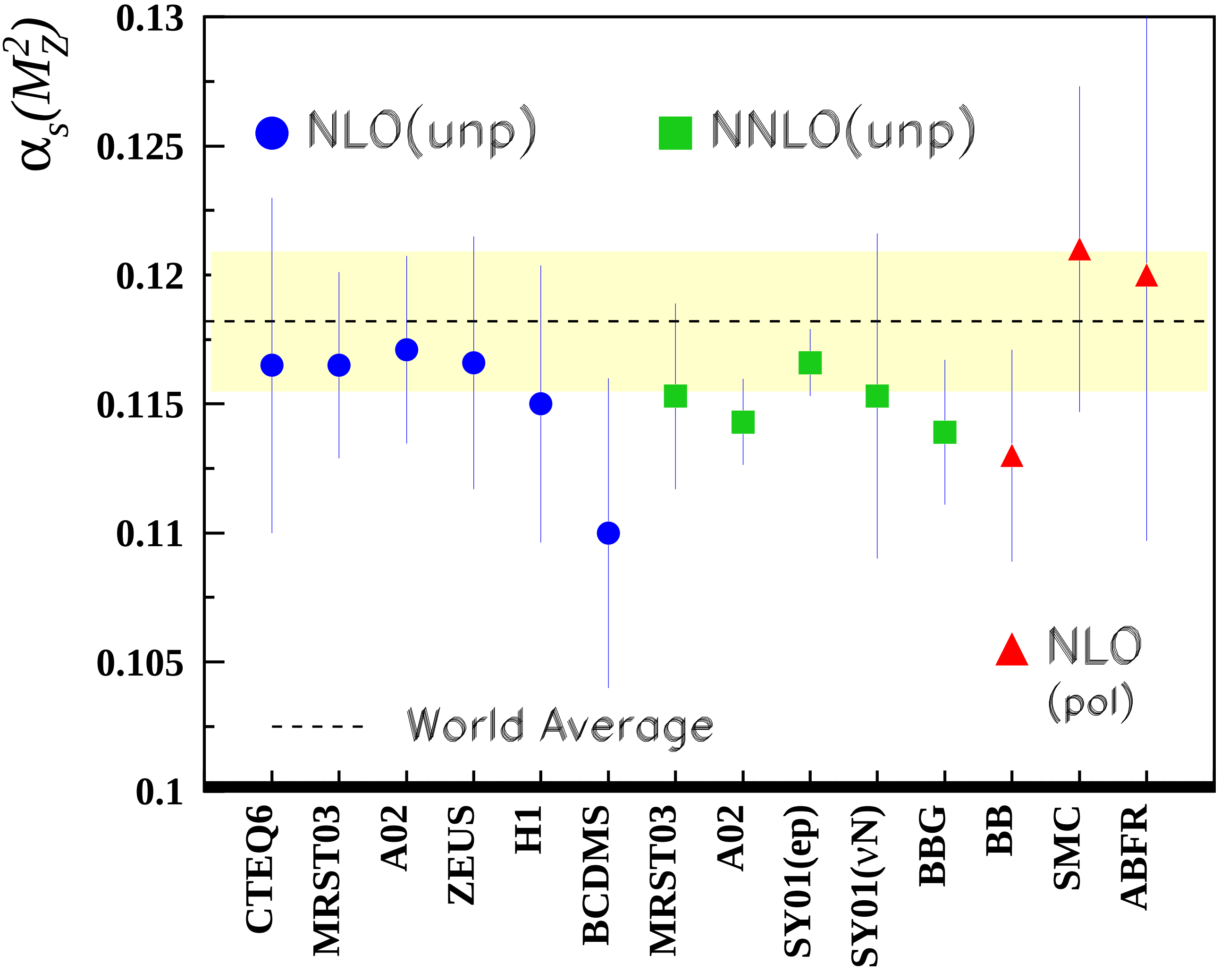
Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization

$$\Lambda = 245 \pm 16 \pm 16\,\text{MeV}$$

QCDSF Collab: $N_f = 2$ Lattice, pert. reno.

$\Lambda = 249 + 13 + 13/-8-17\,\text{MeV}$ also other collab., (cf. PDG).

DIS: $\alpha_s(M_Z^2)$



Future Avenues

HERA:

- Collect high luminosity for $F_2(x, Q^2)$, $F_2^{c\bar{c}}(x, Q^2)$, $g_2^{c\bar{c}}(x, Q^2)$, and measure $h_1(x, Q^2)$.
- Measure : $F_L(x, Q^2)$. This is a key-question for HERA.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized.

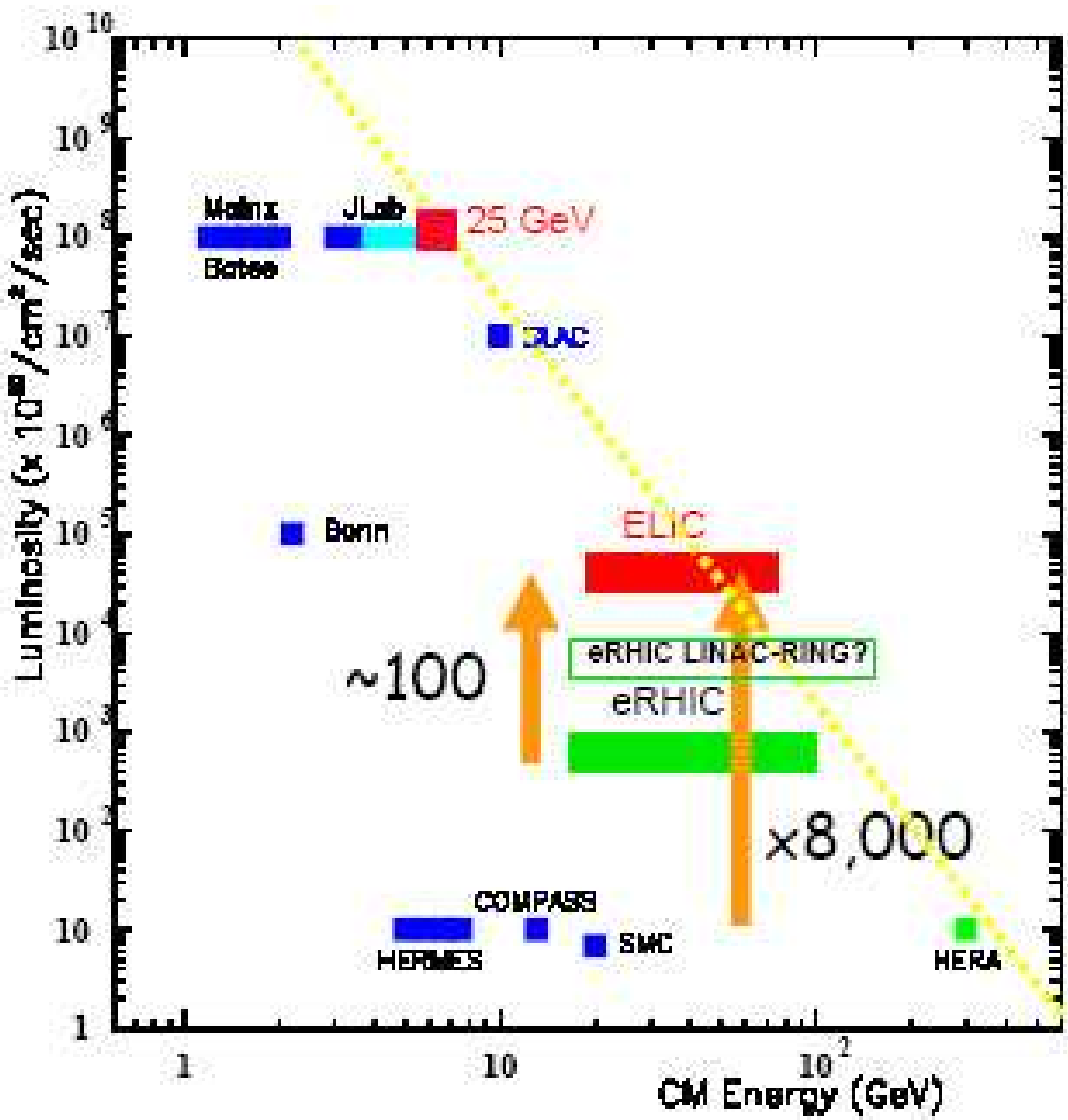
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

ELIC:

- High precision measurements in the medium x domain; both unpolarized and polarized

The quest for large luminosity !



- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\rm MS}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor.[Theory & Experiment]
- Flavor Structure of Sea-Quarks: More studies needed.[All Experiments]
- Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed.[Theory]
- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions [High Precision polarized experiments, JLAB, EIC]
- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Angular Momentum.
- Calculation of more hard scattering reactions at the 3–loop level: ILC, LHC
- Further perfection of the mathematical tools:
  $\Longrightarrow$ Algorithmic simplification of Perturbation theory in higher orders.
- Even higher order corrections needed ?