I : A Unified Model for inelasitc e-N and neutrino-N cross sections at all Q^2

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II : Duality and QCD based fits to Nucleon Form Factors

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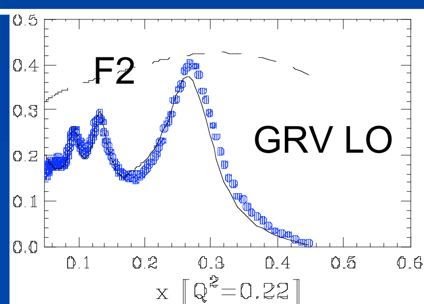
Duality 2005 Frescati , June, 2005

Modeling on neutrino cross sections

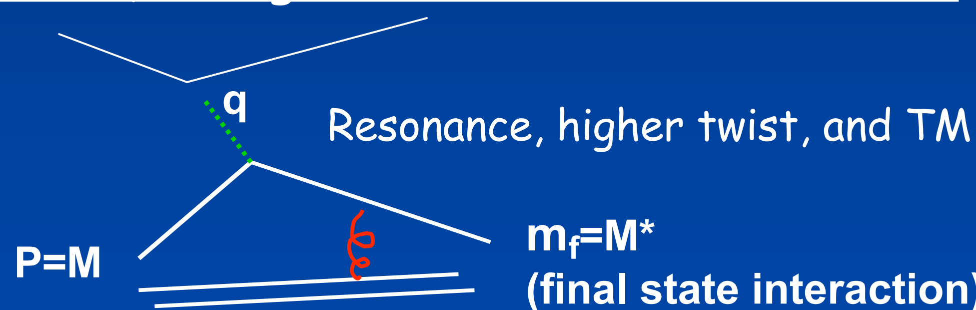
- Describe DIS, resonance, even photo- production (Q^2=0) in terms of quark-parton model. With PDFS, it is straightforward to convert charged-lepton scattering cross sections into neutrino cross section.
- Challenge:
  - Understanding of high x PDFs at very low Q^2?
  - Understanding of resonance scattering in terms of quark-parton model?



- NNLO QCD+TM approach

  explains non-pert. QCD eff ects at Q^2 1-3 GeV^2
- Effective LO approach at lower Q2
- (pseudo NNLO: for MC)

  Use effective LO PDFs with a n ew scaling variable, ξw to absorb target mass, higher twist, missing QCD higher orders



$$\xi_W = \frac{Q^2+m_f^2+O(m_f^2-m_i^2)+A}{M\nu\,(1+(1+Q^2/\nu^2))^{1/2}+B} \longrightarrow X_{bj}= Q^2/2M\nu$$

NEW Scaling Variable - Absorb QCD + Higher Twist into A and B

For Q2 > 1.5 GeV duality works because QCD Moments e.g. momentum sum rules work at a logarithmic level

(I.e. sum of Quark Momenta is about 0.5 with small QCD corrections).

Resonances above the elastic peak and Delta are composed of many final states, so their average value is the same as that of quark model - If one uses a proper scaling variable

For Q2 < 1.5 GeV down to Q2 =0, QCD sum rules (momentum, GLS, Bjorken etc) break down.

However, Valence Number sum rules such as the Adler Sum rule, are exact down all the way to Q2 = 0

But -need to include the Elastic Form Factors and the Delta Form Factor separately, since they are single resonances for which the quark model does not work

Adler Sum rule EXACT all the way down to $Q^2=0$ includes W_2 quasi-elastic

S. Adler, Phys. Rev. 143, 1144 (1966) Exact Sum rules from Current Algebra. Sum Rule for W2 DIS LIMIT is just Uv-Dv =1

$\beta-$ = W_2 (Anti-neutrino -Proton)

'$\beta+$ = W_2 (Neutrino-Proton) $q0=\nu$

$$g_A(q^2)^2 + \int_{M_\pi+(q^2+M_\pi^2)/2M_N}^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1,$$

Elastic $g_A=(\mathbf{-1.267})^2$ Q2=0

Elastic g_A = **0** high Q2

Axial W_2 = non zero at **Q2=0**

Axial W_2 **=1** at high Q2, Inelastic

The vector current part of the original sum rule of Adler for neutrino scattering can be written

$$\int_0^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1. \qquad (18)$$

If we explicitly separate out the nucleon Born term in Eq. (18), we have

Elastic Vector =**1** Q2=0

Elastic Vector = **0** high Q2

$$[F_1^V(q^2)]^2+q^2\left(\frac{\mu^V}{2M_N}\right)[F_2^V(q^2)]^2 + \int_{M_\pi+(q^2+M_\pi^2)/2M_N}^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)]=1,$$

Vector Part of W2, 0 at Q2=0, 1 at high Q2- Inelastic

Adler is a number sum rule at high Q^2

DIS LIMIT is just Uv-Dv.

$$\int_0^{\infty} dq_0[\beta^{(-)}(q_0,q^2)-\beta^{(+)}(q_0,q^2)] \quad =1 \text{ is}$$

$$\int_0^1 \frac{[F_2^-(\xi)-F_2^+(\xi)]}{\xi}d\xi = \int_0^1 [U_v(\xi)-D_v(\xi)]d\xi = 2-1$$

F_2^- = F_2 (Anti-neutrino -Proton) = νW_2

F_2^+ = F_2 (Neutrino-Proton) = νW_2

we use: d (q0) = d (ν) = (ν)d ξ / ξ

at fixed $q^2 = Q^2$

+ Similar sum rules for W1, W3, and strangeness changing structure functions

Note: Elastic Form Factors separated out

(Here V not equal A)

Also need to separate out form factors for Delta (First Resonance) Production (e.g. use Model of Paschos or others - for Delta form factors)

(Here V not equal A)

for High Q2 scattering from Quarks PDFs

Expect V=A

So Adler sum rule gives relationships between Elastic, resonance and DIS, as well as between V and A form factors.

Effective LO model - 2003

1. Start with GRV98 LO (Q^2_{min}=0.80 GeV^2)

   - dashed line- describe F2 data at high Q^2

2. Replace the Xbj with a new scaling, ξ_w
3. Multiply all PDFs by K factors for photo prod. limit and higher twist

   $[\ \sigma(\gamma) = 4\pi\alpha/Q^2 * F_2(x, Q^2)\ ]$

   Ksea = $Q^2/[Q^2+Csea]$

   Kval = $[1 - G_D^{\ 2}(Q^2)]$ $*[Q^2+C2V] / [Q^2+C1V]$ motivated by Adler Sum rule

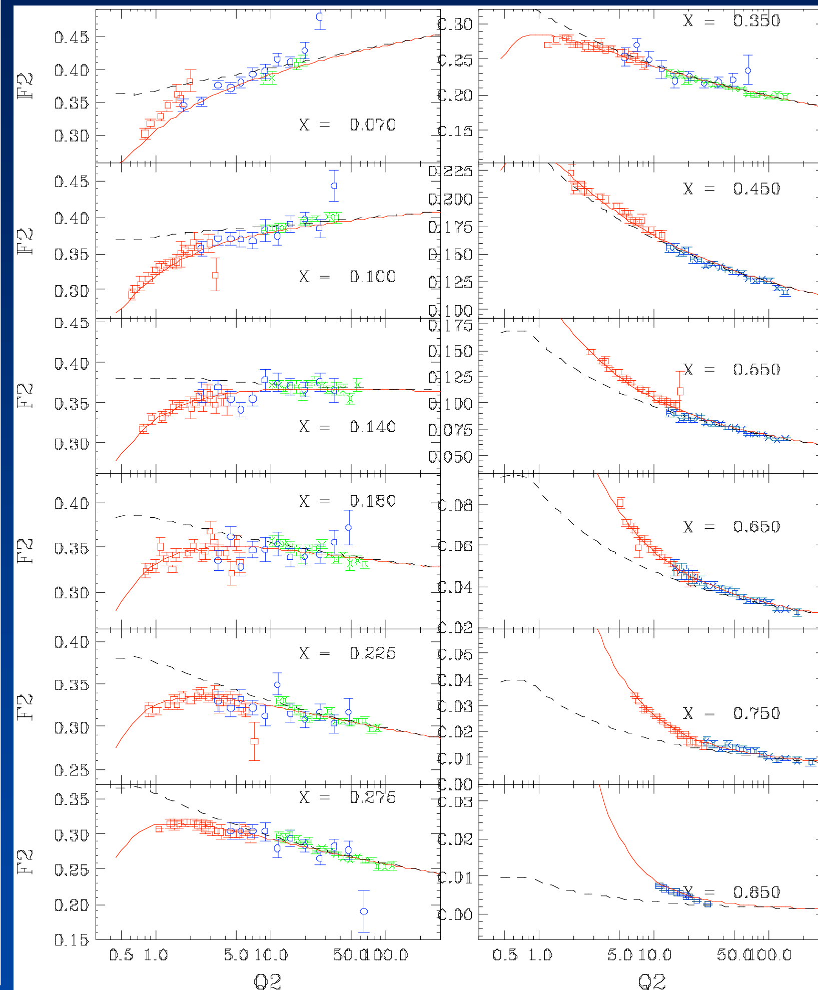
   where $G_D^{\ 2}(Q^2) = 1/[\ 1+Q^2/0.71\ ]^4$
4. Freeze the evolution at $Q^2 = Q^2_{min}$

   - $F_2(x, Q^2 < 0.8) = K(Q^2)*F_2(Xw, Q^2=0.8)$

➢ Fit to all DIS F2 P/D (with low x HERA data) A=0.418, B=0.222

Csea = 0.381, C1V = 0.604, C2V= 0.485

χ^2/DOF = 1268 / 1200 Solid Line



A : initial binding/TM effect+ higher order

B : final state mass m_f^2, Δm^2,

K Factor: Photo-prod limit (Q^2 =0), Adler sum rule

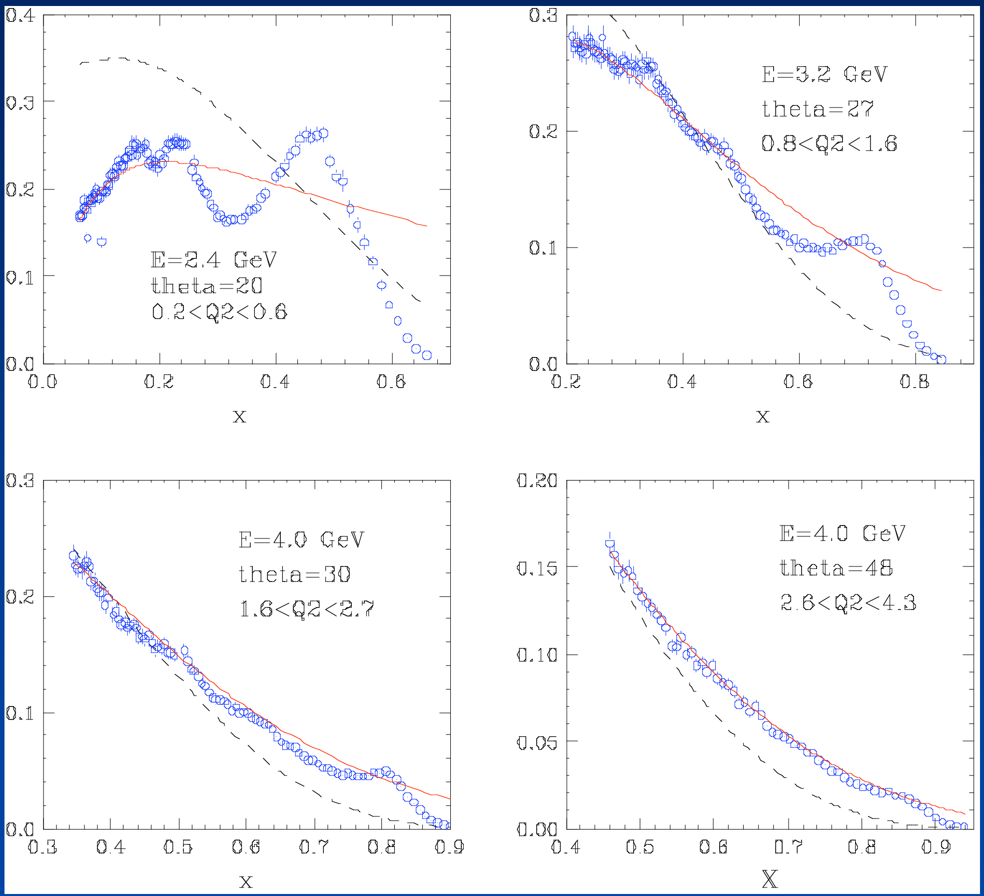
F2 e-Proton

Solid- GRB98 PDFs

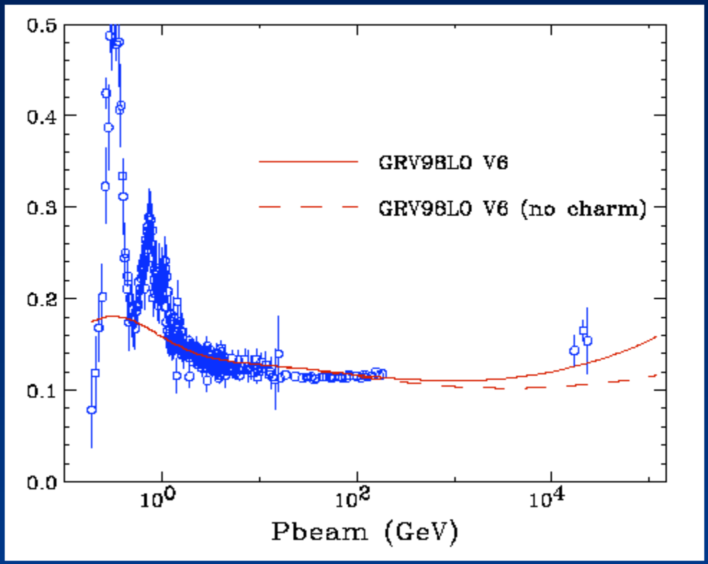
Dashed -Modified GRB98 PDFs

Comparison with effective LO model

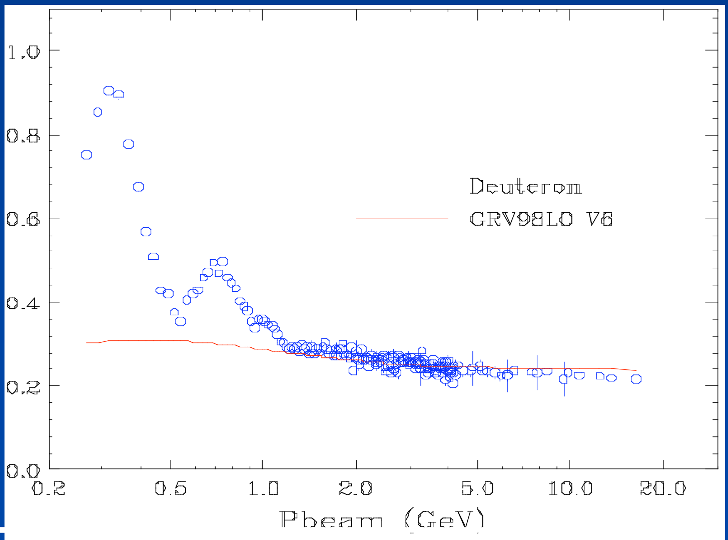
Elastic peak and Delta need to be treated separately



F_2(d) resonance low Q2



Q2=0 Photo-production (P)



Q2=0 Photo-production (d)

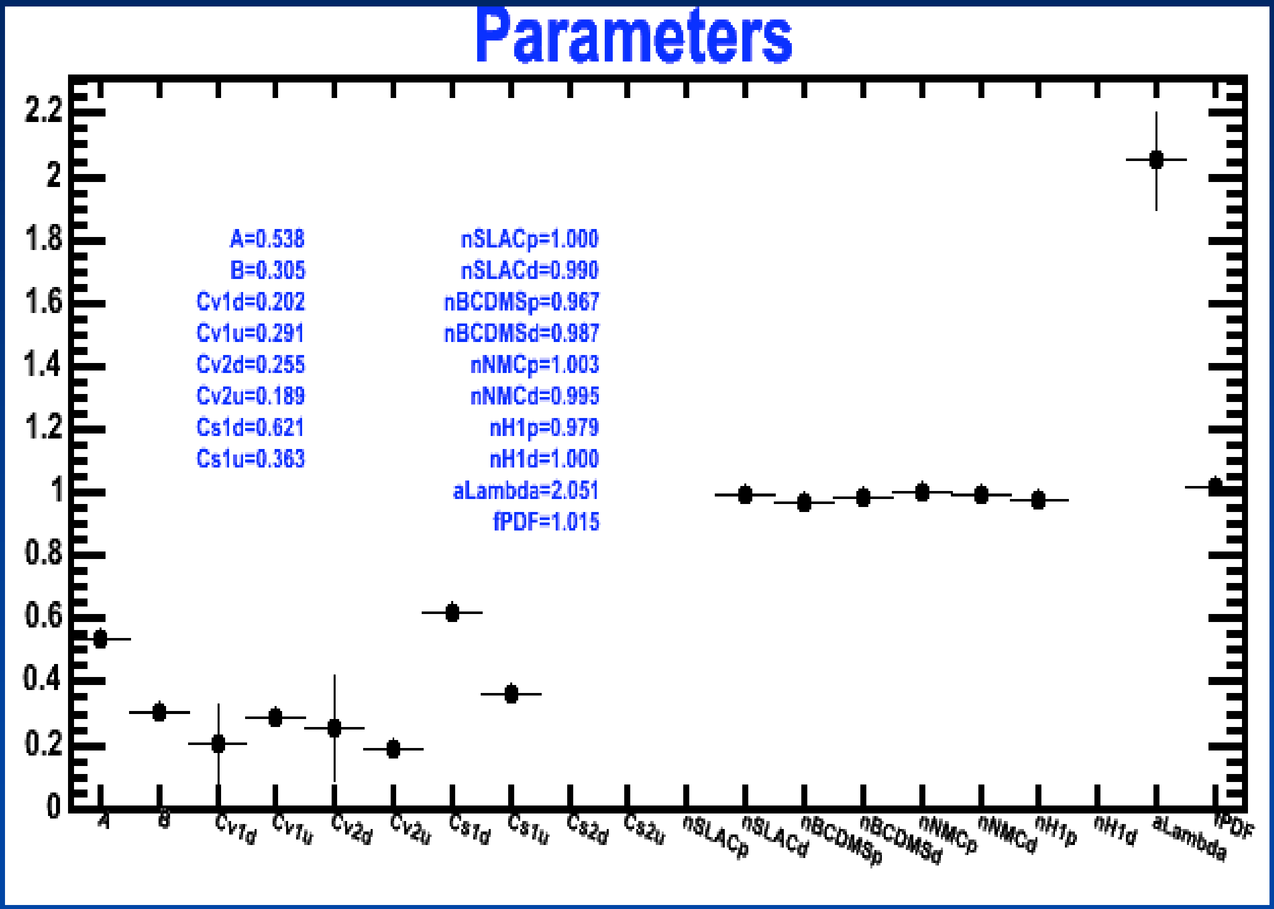
2004 Updates on effective LO model

- Improvements in our model
  - Separate low Q^2 corrections to d and u valence quarks, and sea quarks
  - Include all inelastic F2 proton/deuterium (SLAC/NMC/BCDMC /HERA), photo-production on proton/deuterium in the fits (the c-cbar photon-gluon fusion contribution is included, important at high energy)
- Toward axial PDFs (vector PDFs vs axial PDFs)
  - Compare to neutrino data (assume V=A)

    CCCFR-Fe, CDHS-Fe, CHORUS-Pb differential cross section (without c-cbar boson-fusion in yet - to be added next since it is high energy data)
  - We have a model for axial low Q2 PDFs, but need to compare to low energy neutrino data to get exact parameters - next.

    Kvec = $Q^2/[Q^2+C1]$ -> Kax = $/[Q^2+C2]/[Q^2+C1]$

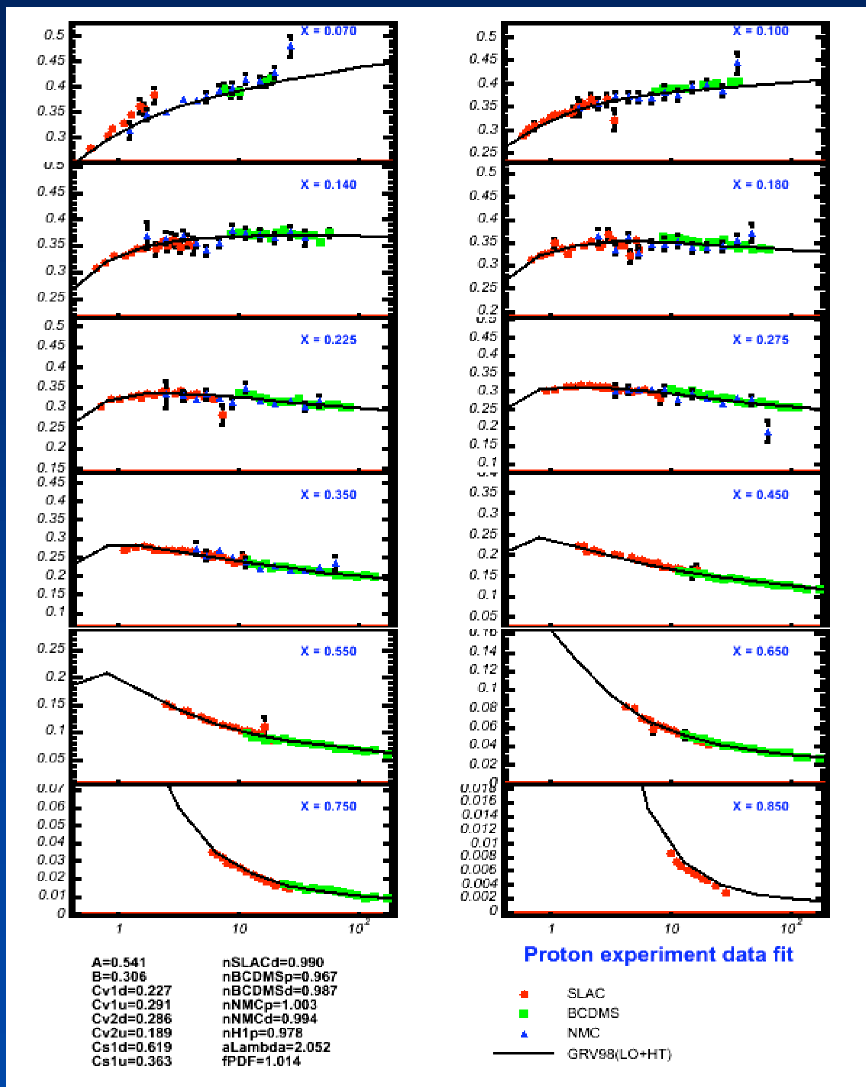
Fit results using the updated model



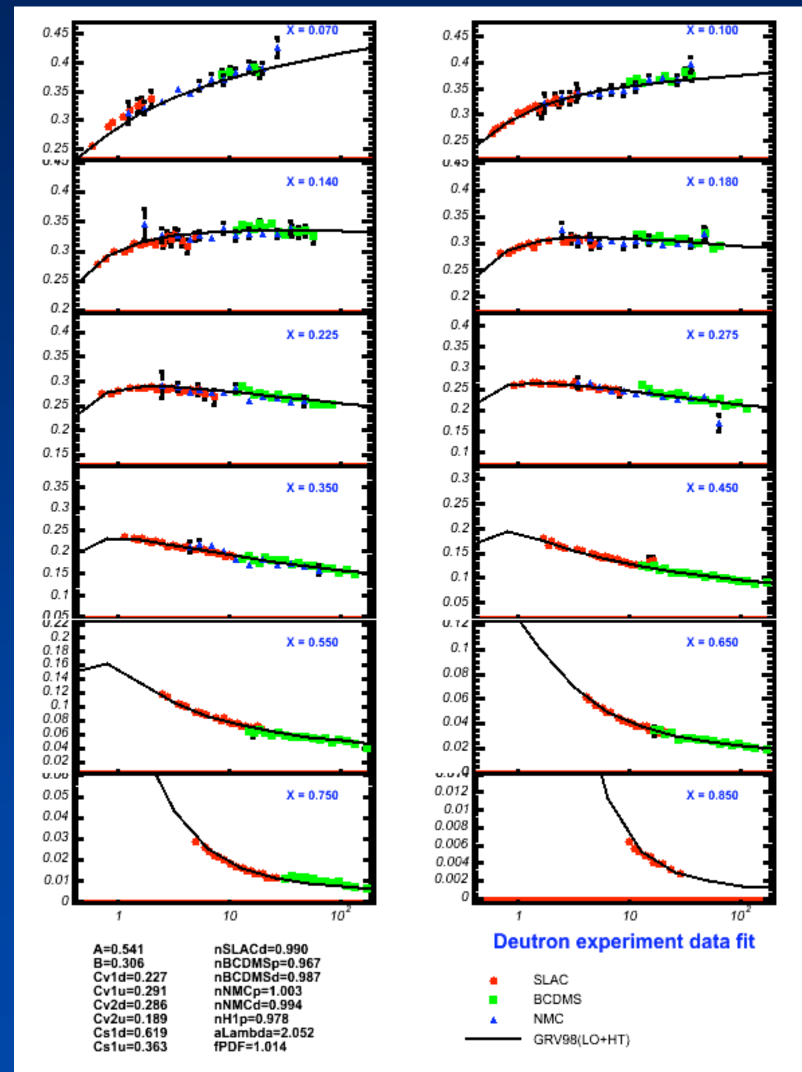
Separate K factors for uv, dv,us,ds

http://web.pas.rochester.edu/~icpark/MINERvA/

Fit results



F2 proton



F2 deuterium

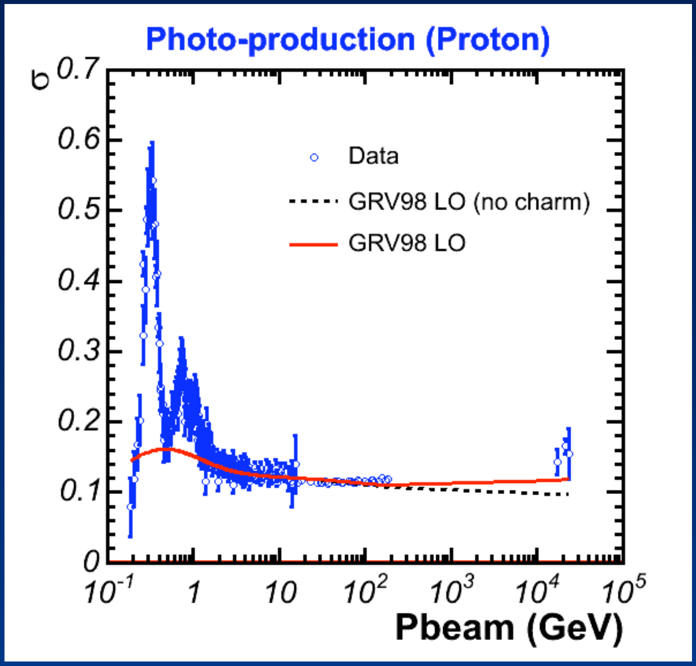


Photo-production (Proton)
σ
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Data
GRV98 LO (no charm)
GRV98 LO
10^{-1} 1 10 10^2 10^3 10^4 10^5
Pbeam (GeV)

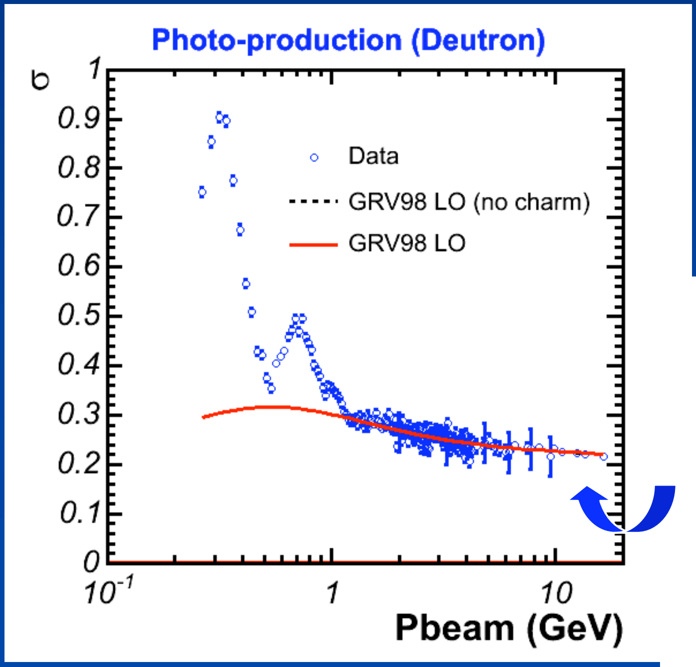
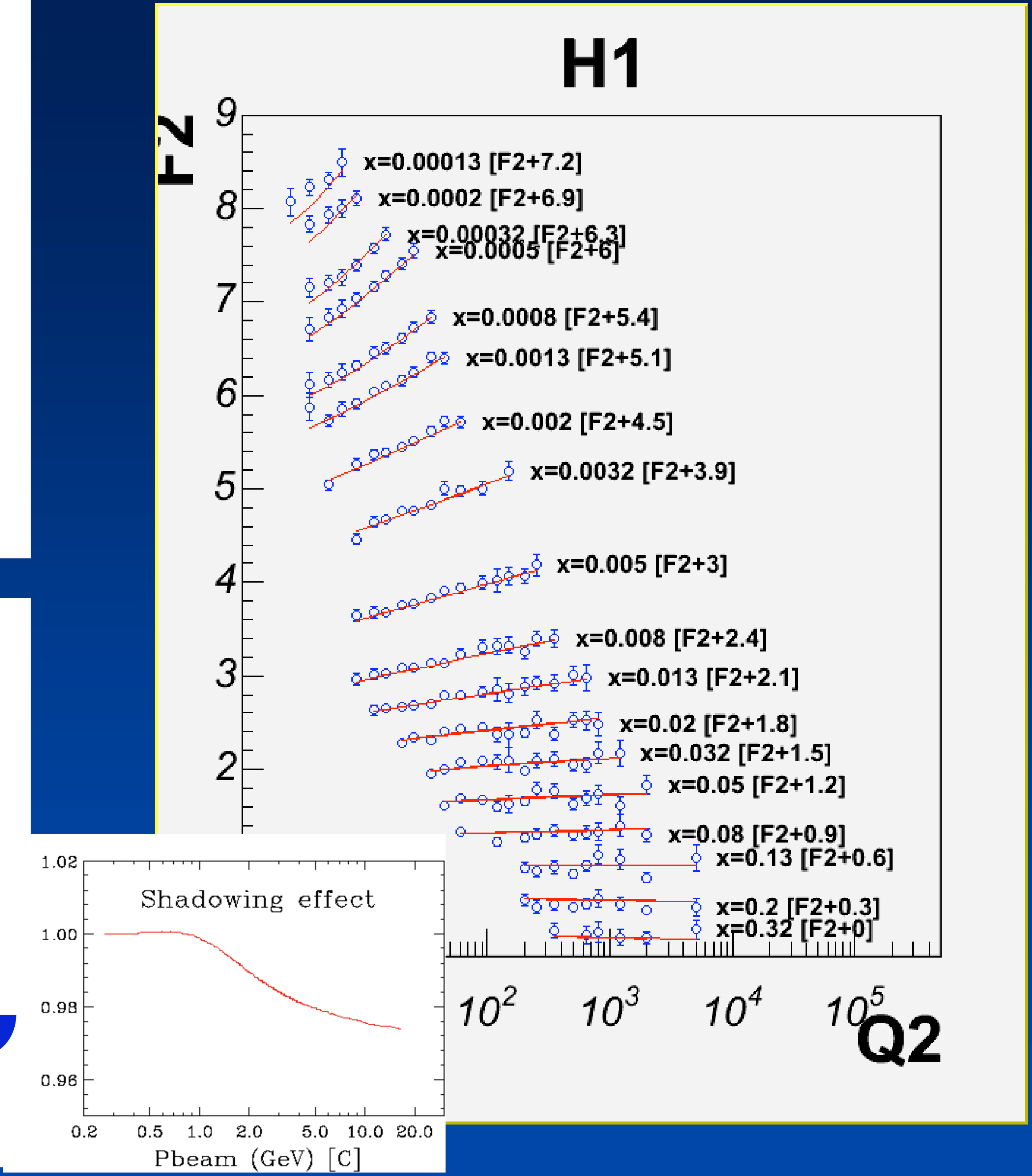
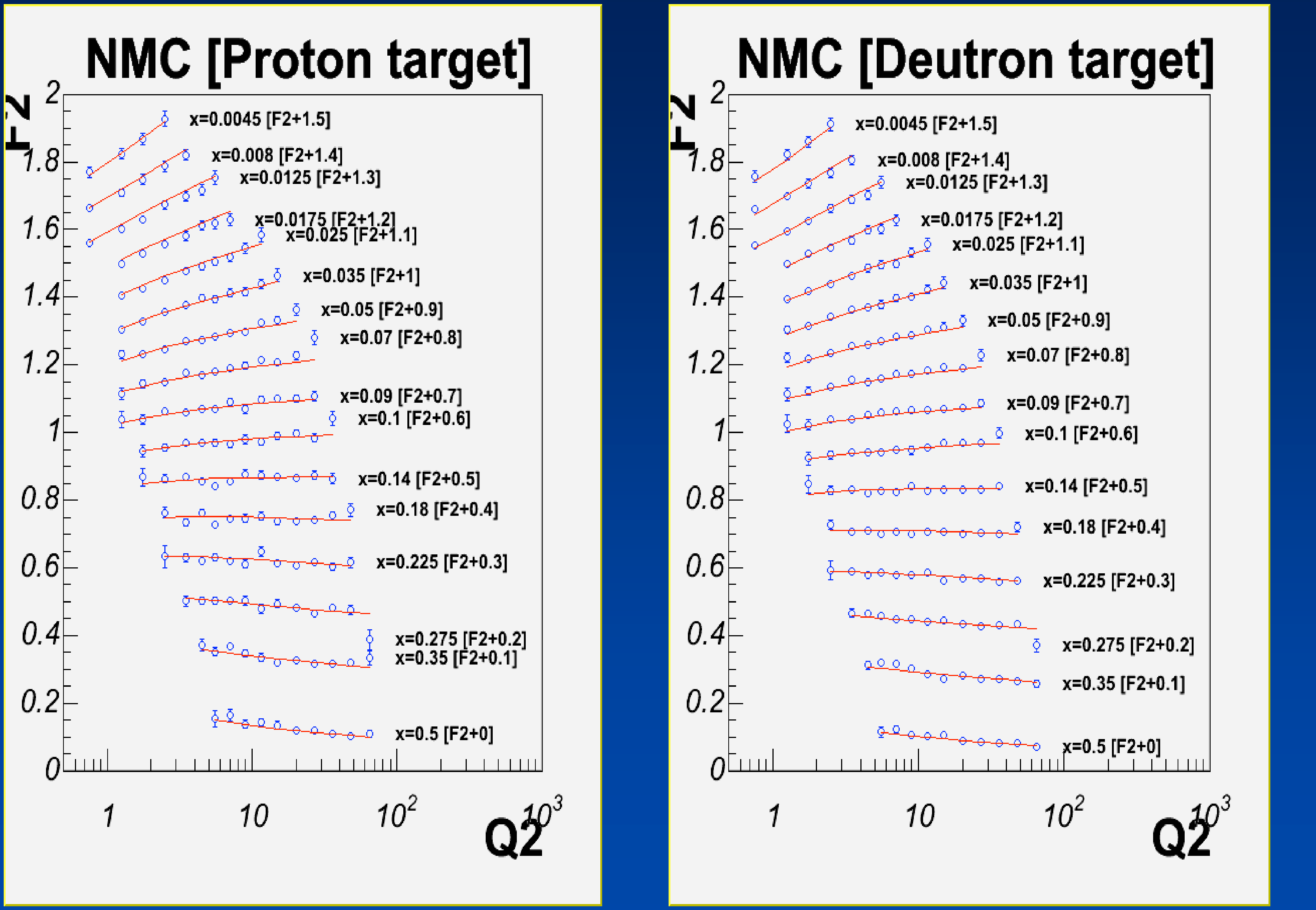


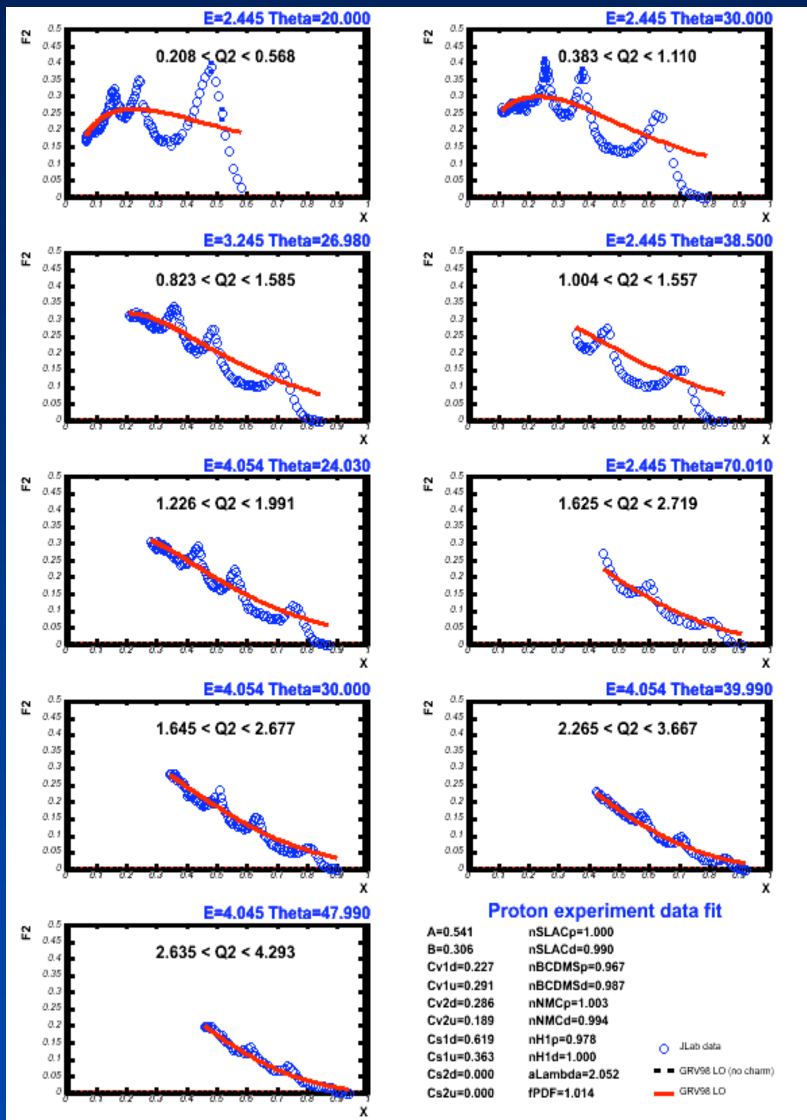
Photo-production (Deutron)
σ
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Data
GRV98 LO (no charm)
GRV98 LO
10^{-1} 1 10
Pbeam (GeV)



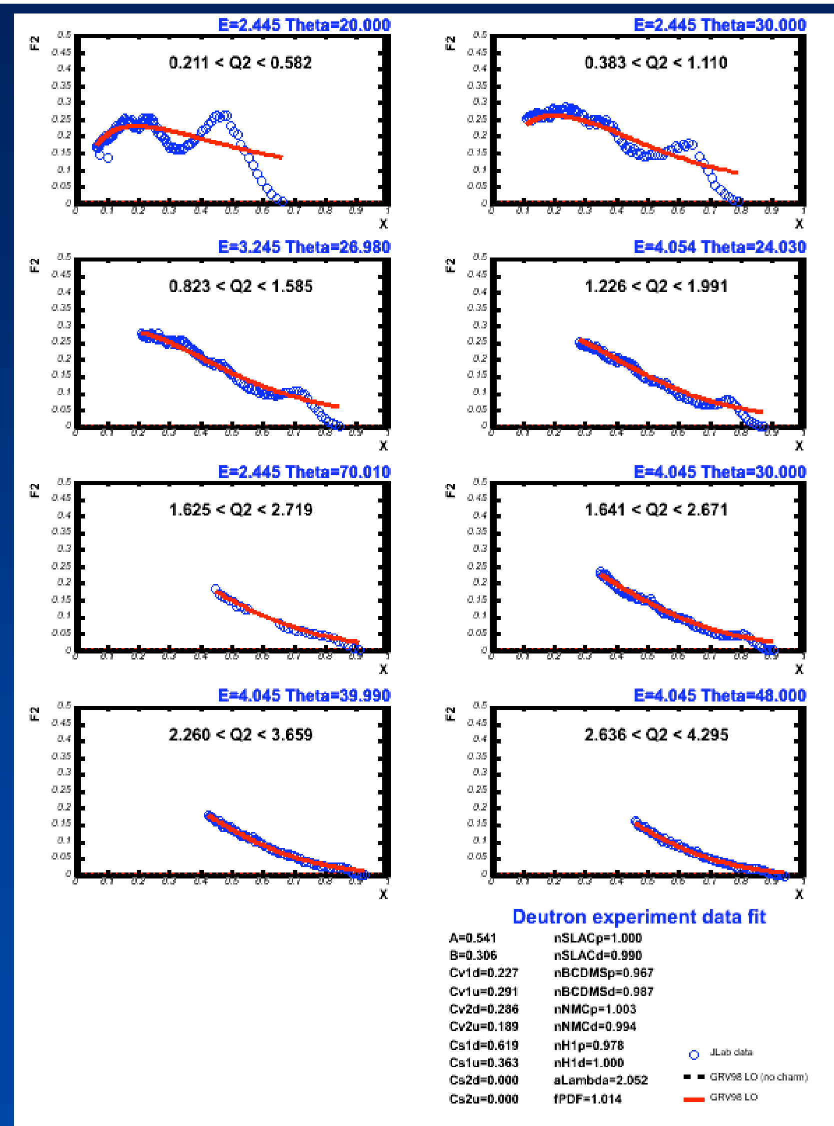
H1
F2
9
8
7
6
5
4
3
2
x=0.00013 [F2+7.2]
x=0.0002 [F2+6.9]
x=0.00032 [F2+6.3]
x=0.0005 [F2+6]
x=0.0008 [F2+5.4]
x=0.0013 [F2+5.1]
x=0.002 [F2+4.5]
x=0.0032 [F2+3.9]
x=0.005 [F2+3]
x=0.008 [F2+2.4]
x=0.013 [F2+2.1]
x=0.02 [F2+1.8]
x=0.032 [F2+1.5]
x=0.05 [F2+1.2]
x=0.08 [F2+0.9]
x=0.13 [F2+0.6]
x=0.2 [F2+0.3]
x=0.32 [F2+0]
10^2 10^3 10^4 10^5
Q2
Shadowing effect
1.02
1.00
0.98
0.96
0.2 0.5 1.0 2.0 5.0 10.0 20.0
Pbeam (GeV) [C]


NMC [Proton target]
F2
Q2
x=0.0045 [F2+1.5]
x=0.008 [F2+1.4]
x=0.0125 [F2+1.3]
x=0.0175 [F2+1.2]
x=0.025 [F2+1.1]
x=0.035 [F2+1]
x=0.05 [F2+0.9]
x=0.07 [F2+0.8]
x=0.09 [F2+0.7]
x=0.1 [F2+0.6]
x=0.14 [F2+0.5]
x=0.18 [F2+0.4]
x=0.225 [F2+0.3]
x=0.275 [F2+0.2]
x=0.35 [F2+0.1]
x=0.5 [F2+0]
NMC [Deutron target]
F2
Q2
x=0.0045 [F2+1.5]
x=0.008 [F2+1.4]
x=0.0125 [F2+1.3]
x=0.0175 [F2+1.2]
x=0.025 [F2+1.1]
x=0.035 [F2+1]
x=0.05 [F2+0.9]
x=0.07 [F2+0.8]
x=0.09 [F2+0.7]
x=0.1 [F2+0.6]
x=0.14 [F2+0.5]
x=0.18 [F2+0.4]
x=0.225 [F2+0.3]
x=0.275 [F2+0.2]
x=0.35 [F2+0.1]
x=0.5 [F2+0]

Resonance F2 proton

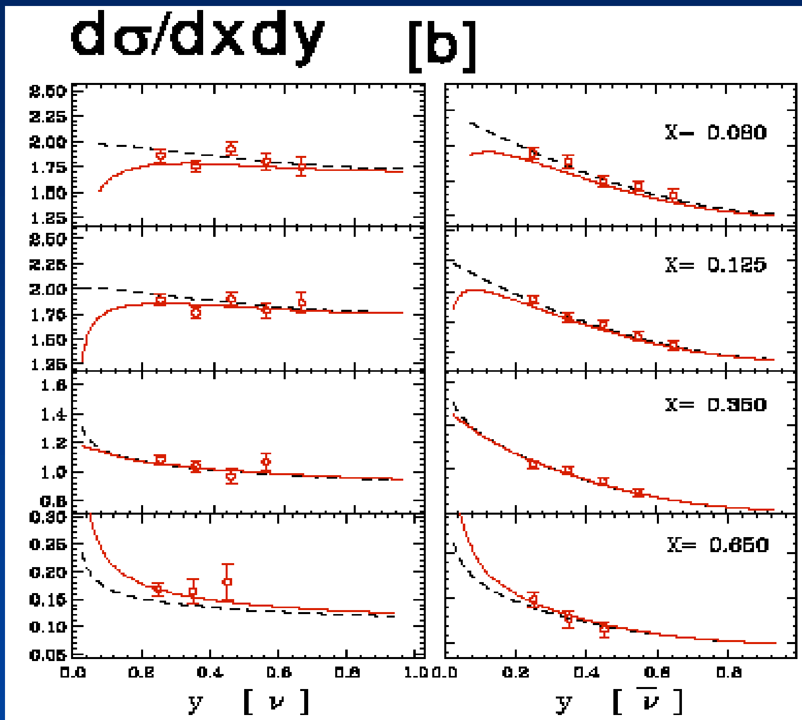


Resonance F2 deuterium



Resonance data are not included in the fit!!!

Comparison with neutrino data (assume V=A)



- Apply nuclear corrections using e/m scattering data.
- Calculate F_2 and xF_3 from the modified PDFs with ξw
- Use R=Rworld fit to get $2xF_1$ from F_2
- Implement charm mass effect through ξw slow rescaling algorithm, for F_2 $2xF_1$, and XF_3

—ξw PDFs GRV98 modified

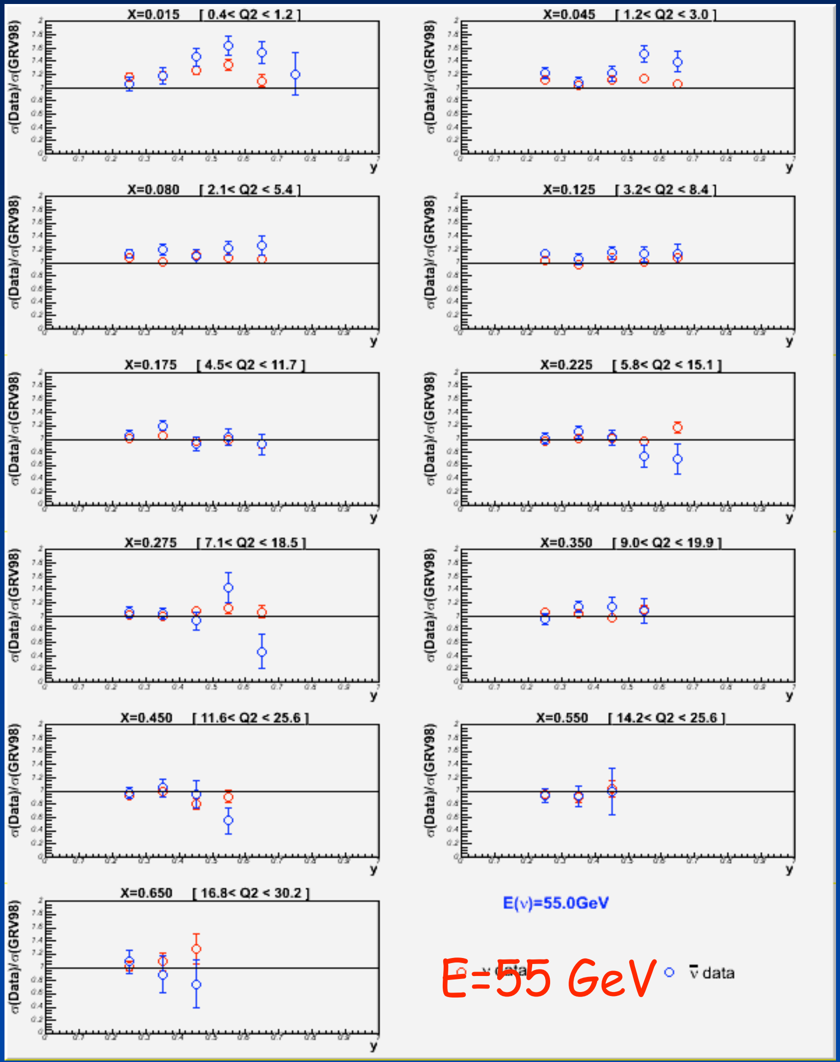
---- GRV98 (x,Q²) unmodified

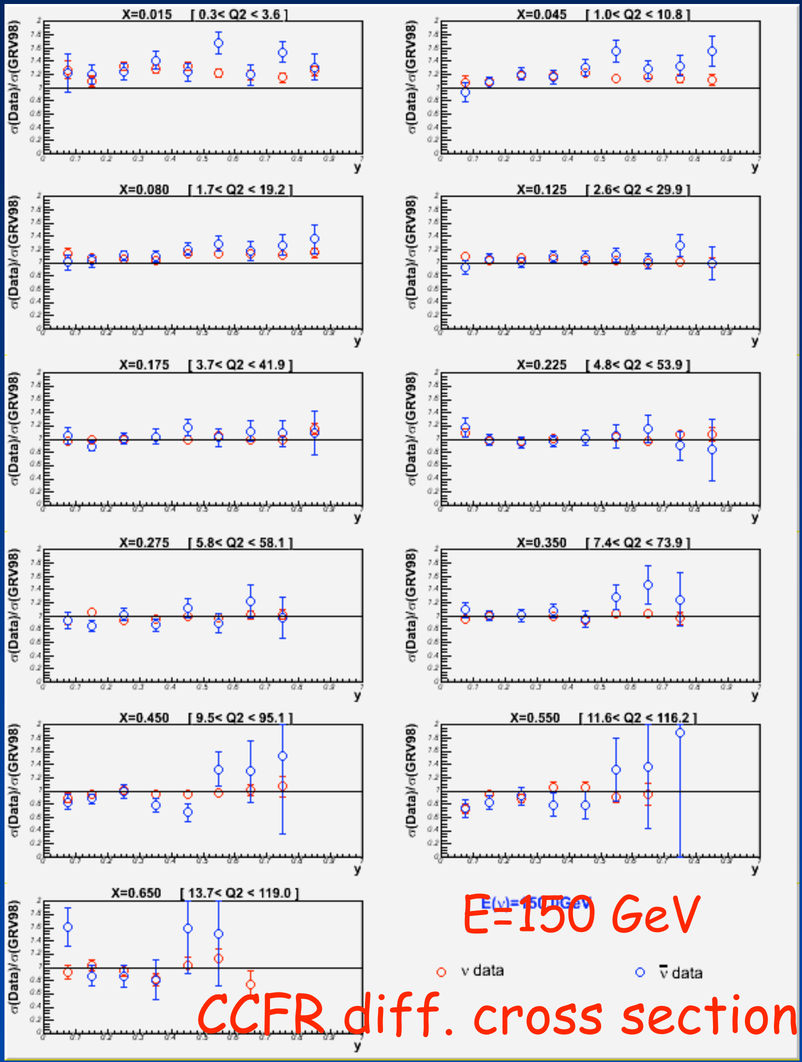
Left: (neutrino), right anti-neu

(NuFact03 version)

Our model describe CCFR diff. cross sect. (En=30-300 GeV) well (except at the lowest x)

Comparison with updated model (assume V=A)



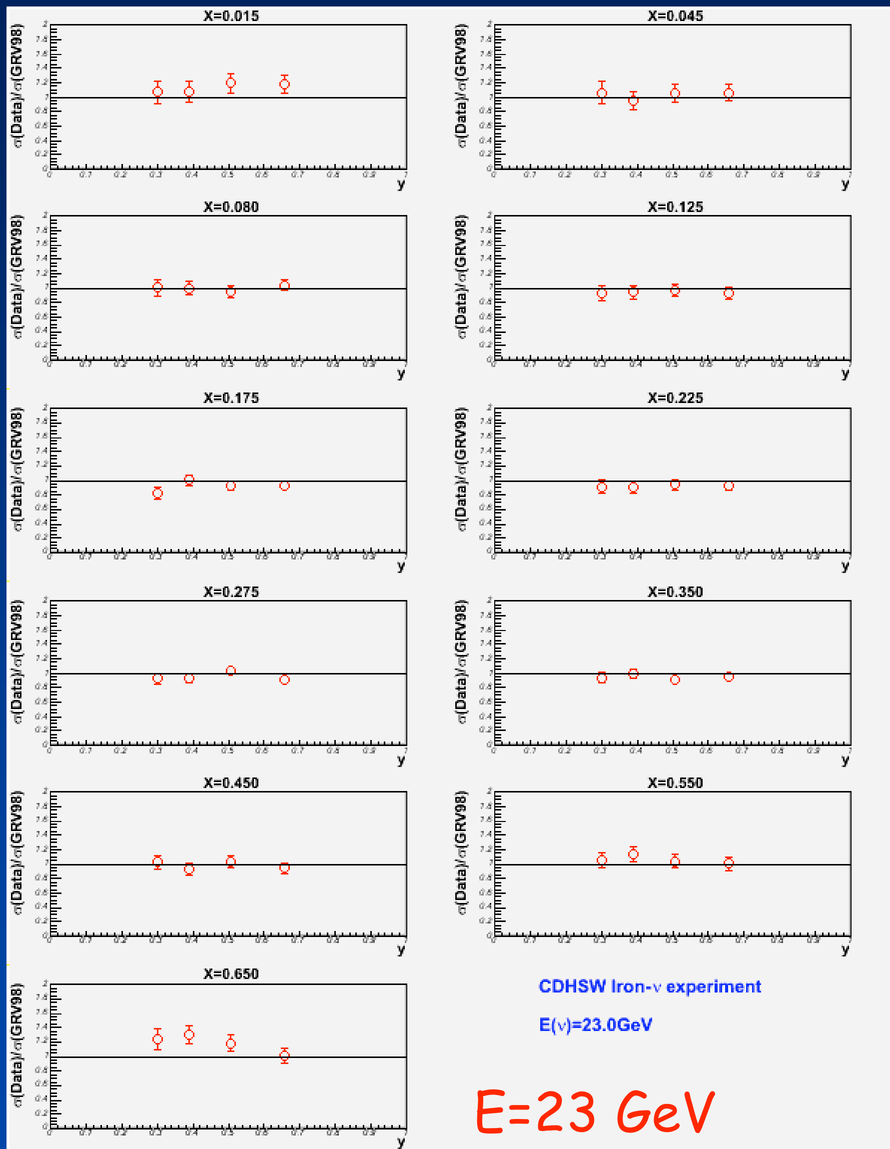


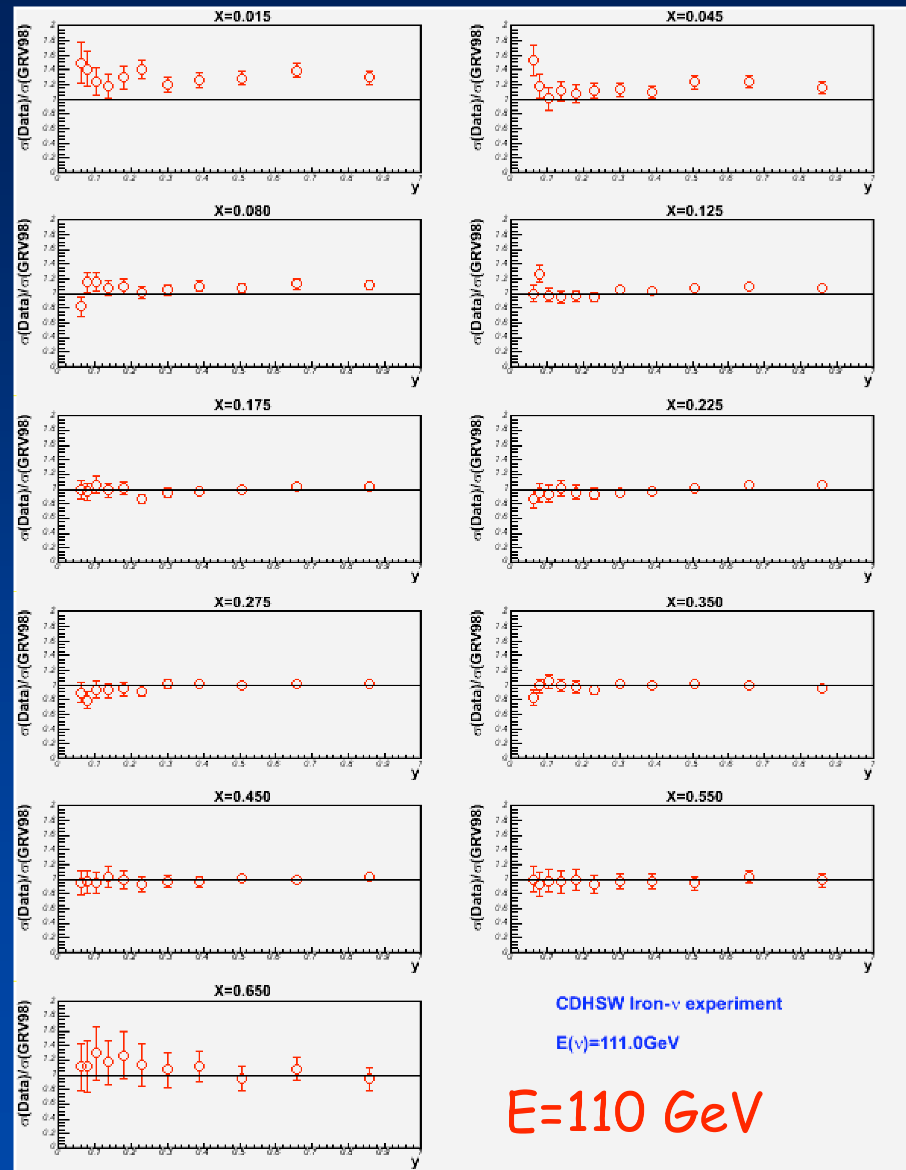
CCFR diff. cross section data

Plots for all energy regions:

http://web.pas.rochester.edu/~icpark/MINERvA/

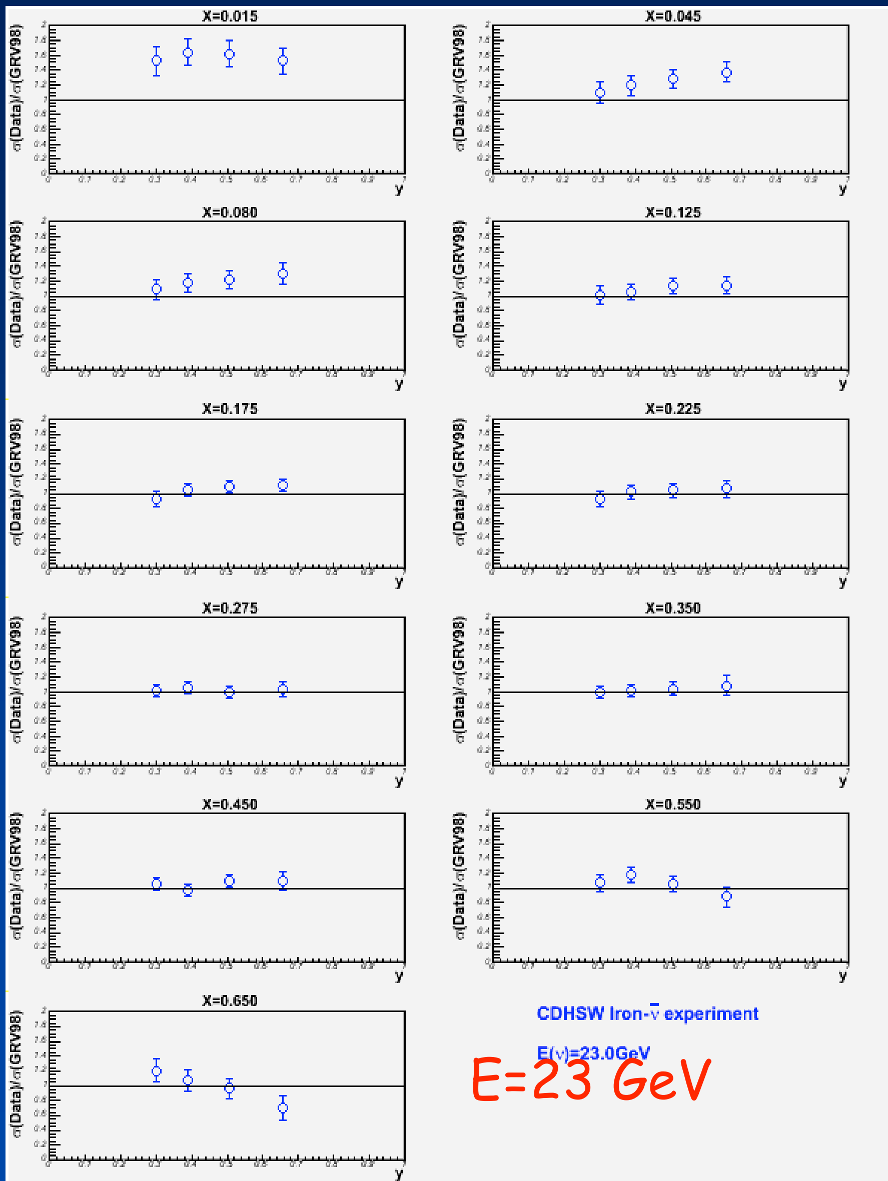
Comparison with CDHSW neutrino data

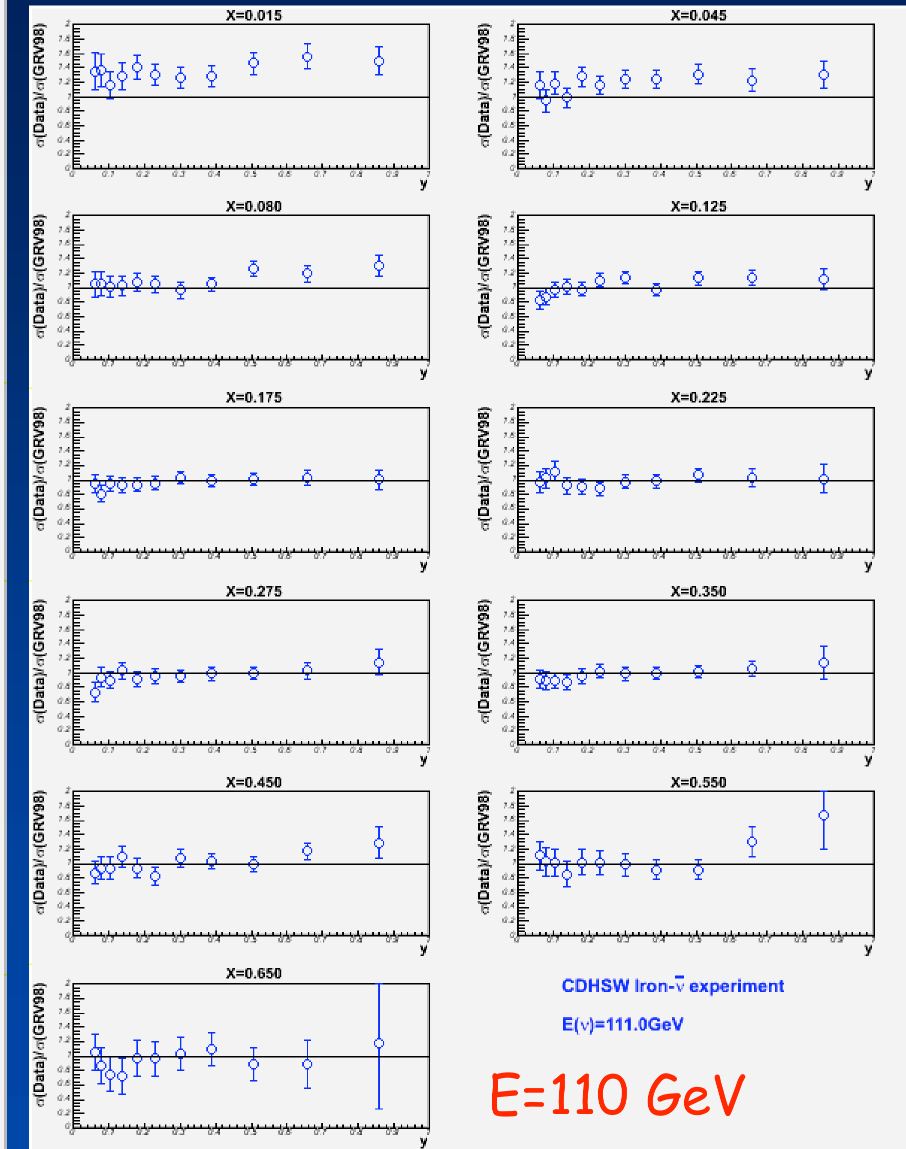




Radiative correction, ccbar contribution at low x

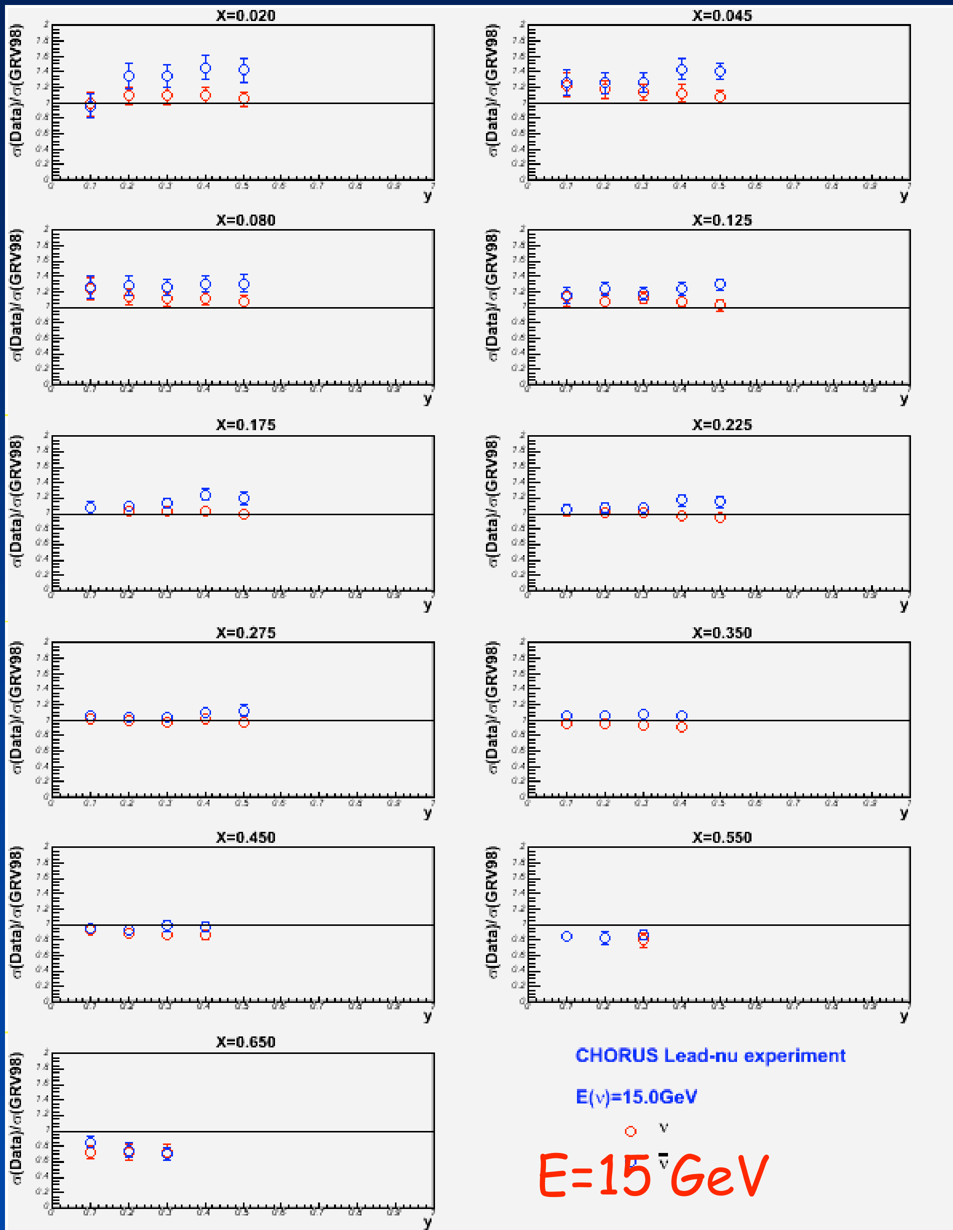
Comparison with CDHSW anti-neutrino data

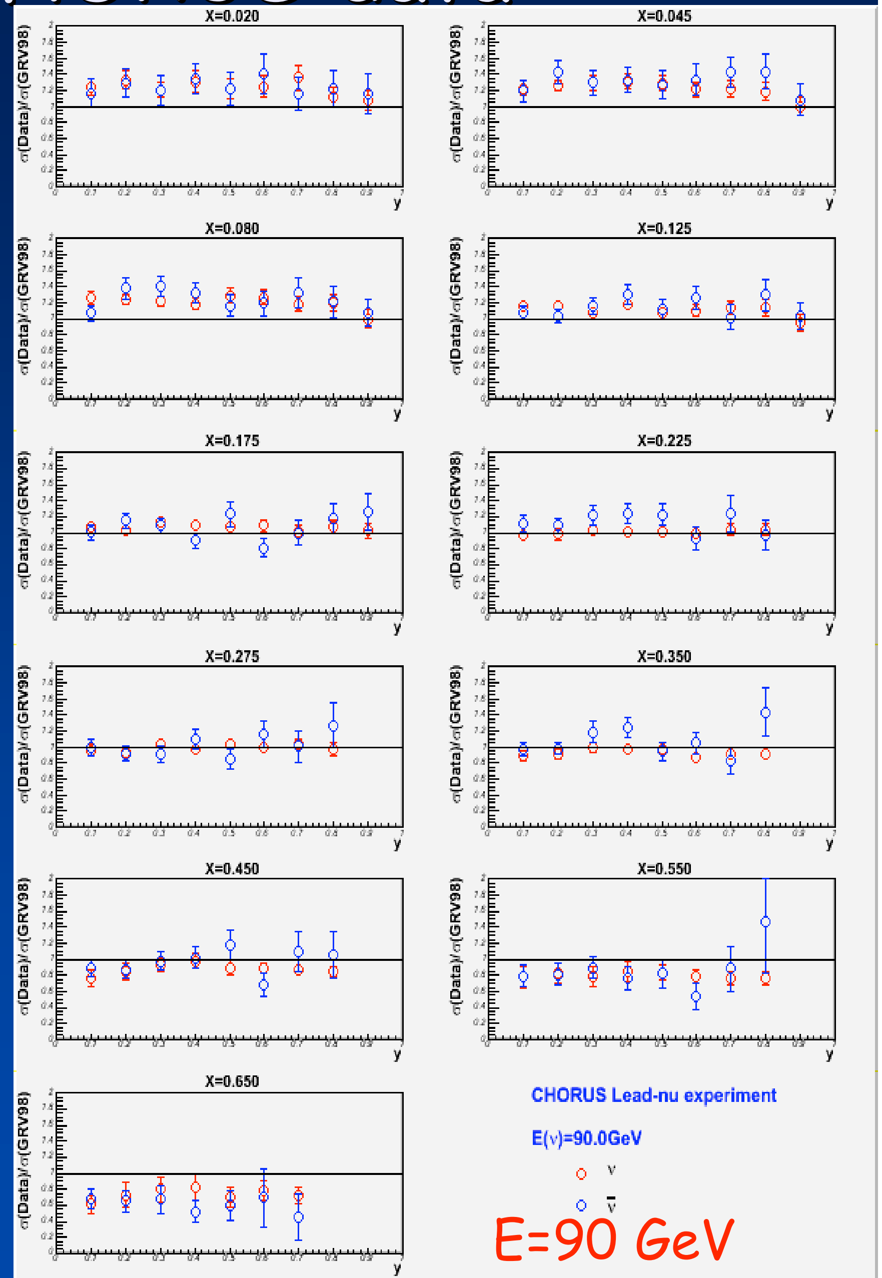




Radiative correction, ccbar contribution at low x

Comparison with CHORUS data





Correct for Nuclear Effects measured in e/muon expt.

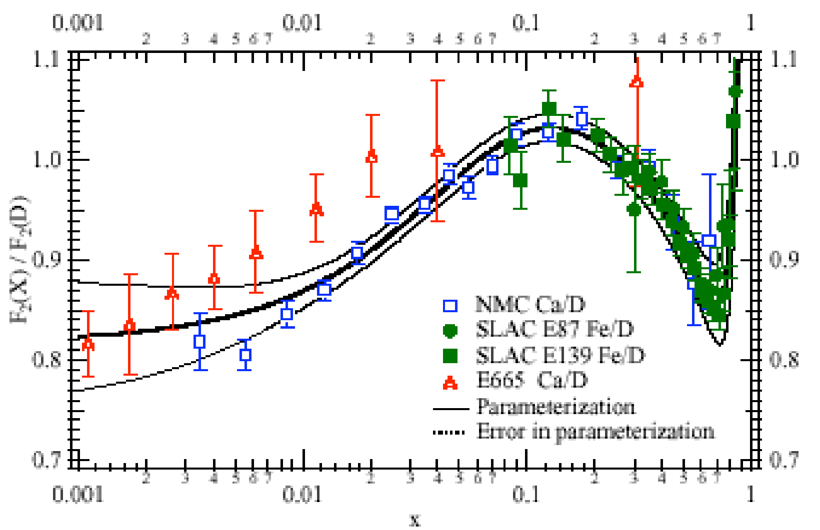
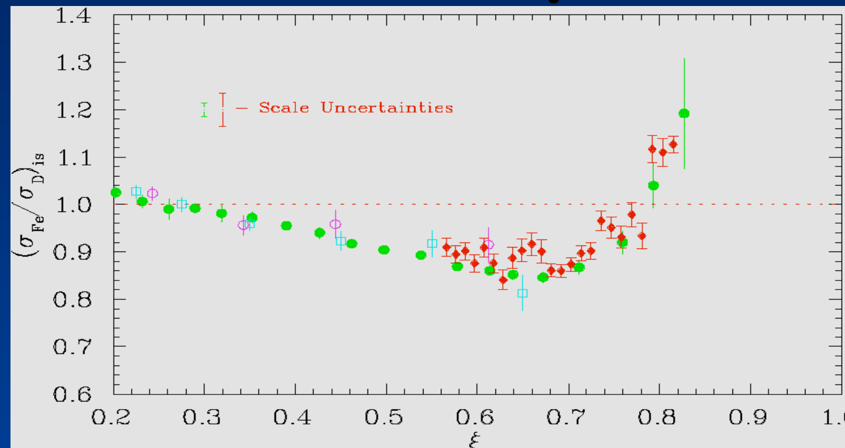


Figure 5. The ratio of F_2 data for heavy nuclear targets and deuterium as measured in charged lepton scattering experiments(SLAC,NMC, E665). The band show the uncertainty of the parametrized curve from the statistical and systematic errors in the experimental data [16].



Comparison of Fe/D F2 data

In resonance region (JLAB)

Versus DIS SLAC/NMC data

In $_{TM}$ (C. Keppel 2002).

Summary of Unified LO Approach works from Q2=0 to high Q2

For applications to Neutrino Oscillations at Low Energy (down to Q2=0) the best approach is to use a LO PDF analysis (including a more sophisticated target mass - scaling variable) and modify to include the missing QCD higher order terms via Empirical Higher Twist Corrections.

Reason:

For Q2>5 both Current Algebra exact sum rules (e.g. Adler sum rule) and QCD sum rules (e.g. momentum sum rule) are satisfied. This is why duality works in the resonance region (Here we can also use NNLO QCD analysis or a modified leading order analysis): Use duality + Adler to constrain elastic vector and axial form factors.

For Q2<1, QCD corrections diverge, and all QCD sum rules (e.g momentum sum rule) break down, and duality breaks down in the resonance region. In contrast, Current Algebra Sum rules e,g, Adler sum rule which is related to the Number of (U minus D) Valence quarks) are valid.

Our unified approach uses sum-rules combines both inelastic and elastic.

I. Summary and Plans

- Our effective LO model describe all F2 DIS, resonance, and photo-production data well.
- This model provide a good description on the neutrino cross section data (except axial vector contribution).
- Now working on the axial structure functions and next plan to work on resonance fits.
- JUPITER at Jlab (Bodek, Keppel) taken January 05 - provides electron-Carbon (also e-H and e-D and other nuclei such as e-Fe) in resonance region (summer 05)

- Future: MINERvA at FNAL (McFarland, Morfin) will provide Neutrino-Carbon data at low energies.

II : Duality and QCD based fits to Nucleon Form Factors - to study Adler Sum rule and Axial form factor

Arie Bodek, R. Bradford, H. Budd -U. of Rochester

John Arrington- Argonne National Lab

$$2xF_1^{inel}(x,Q^2) = x^2 G_M^2(Q^2)\delta(x-1)$$
$$F_2^{inel}(x,Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\delta(x-1)$$

$$R(x=1,Q^2) = \frac{4M^2}{Q^2}\left(\frac{G_E^2}{G_M^2}\right)$$

Duality: Rp=Rn (inelastic) at high Q2 near x=1.

$$G_D(Q^2) \equiv \frac{1}{(1+Q^2 r_0^2)^2}$$

$$r_0^2 = (0.24\ \text{fm})^2 = 1/0.71\ (\text{GeV})^{-2}$$

QCD and Duality: At High Q2 near x=1 F1n/F1p = $(Gmn/Gmp)^2$

Use F1n/F1p predicted with d/u = 0.2 at x=1 (From QCD)

Use a form proposed by J. J. Kelly Phys. Rev. C 70, 068202 (2004). This form satisfies QCD constraints at High Q2 with 4 parameters for Gep, Gmp, Gmn.

$$G(Q^2) \propto \frac{\sum_{k=0}^{n} a_k \tau^k}{1 + \sum_{k=1}^{n+2} b_k \tau^k}, \qquad (1)$$

where both numerator and denominator are polynomials in $\tau = Q^2/4m_p^2$ and where the degree of the denominator is larger than that of the numerator to ensure that $G \propto Q^{-4}$ for large Q^2. For magnetic form factors we include a factor of μ on the right-hand side, such that $a_0 \approx 1$ if the data for low Q^2 are normalized accurately. With $n=1$ and $a_0=1$, this parametrization provides excellent fits to G_{Ep}, G_{Mp}/μ_p, and G_{Mn}/μ_n using only four parameters each. However, this approach is less successful for G_{En} because the existing data are still too limited. Therefore, for G_{En} I continue to use the Galster parametrization [8],

$$G_{En}(Q^2) = \frac{A\tau}{1 + B\tau} G_D(Q^2),$$

where $G_D = (1 + Q^2/\Lambda^2)^{-2}$ with $\Lambda^2 = 0.71\ (\mathrm{GeV}/c)^2$

For Gen Kelly uses the Galster Parametrization

PHYSICAL REVIEW C **70**, 068202 (2004)

TABLE I. Parameters fitted to data for nucleon electromagnetic form factors. The normalization parameter $a_0 = 1$ was held constant. The second column lists chi-square per datum.

Quantity	χ^2/N	a_1	b_1	b_2	b_3	$r_{\rm rms}$ (fm)	A	B	$\langle r_n^2 \rangle$ (fm^2)
G_{Ep}	0.78	−0.24±0.12	10.98±0.19	12.82±1.1	21.97±6.8	0.863±0.004			
G_{Mp}/μ_p	1.06	0.12±0.04	10.97±0.11	18.86±0.28	6.55±1.2	0.848±0.003			
G_{Mn}/μ_n	0.51	2.33±1.4	14.72±1.7	24.20±9.8	84.1±41	0.907±0.016			
G_{En}	0.80						1.70±0.04	3.30±0.32	−0.112±0.003

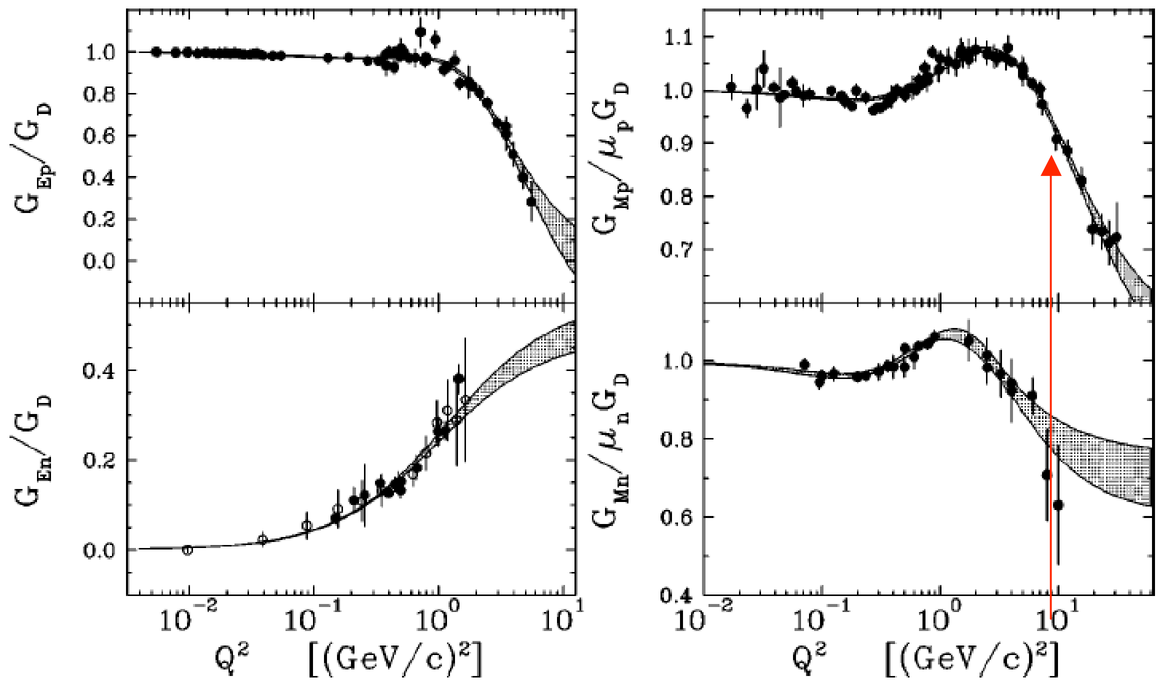


FIG. 1. Fits to nucleon electromagnetic form factors. For G_{En}, data using recoil or target polarization [16–22] are shown as filled circles while data obtained from the deuteron quadrupole form factor [23] are shown as open circles.

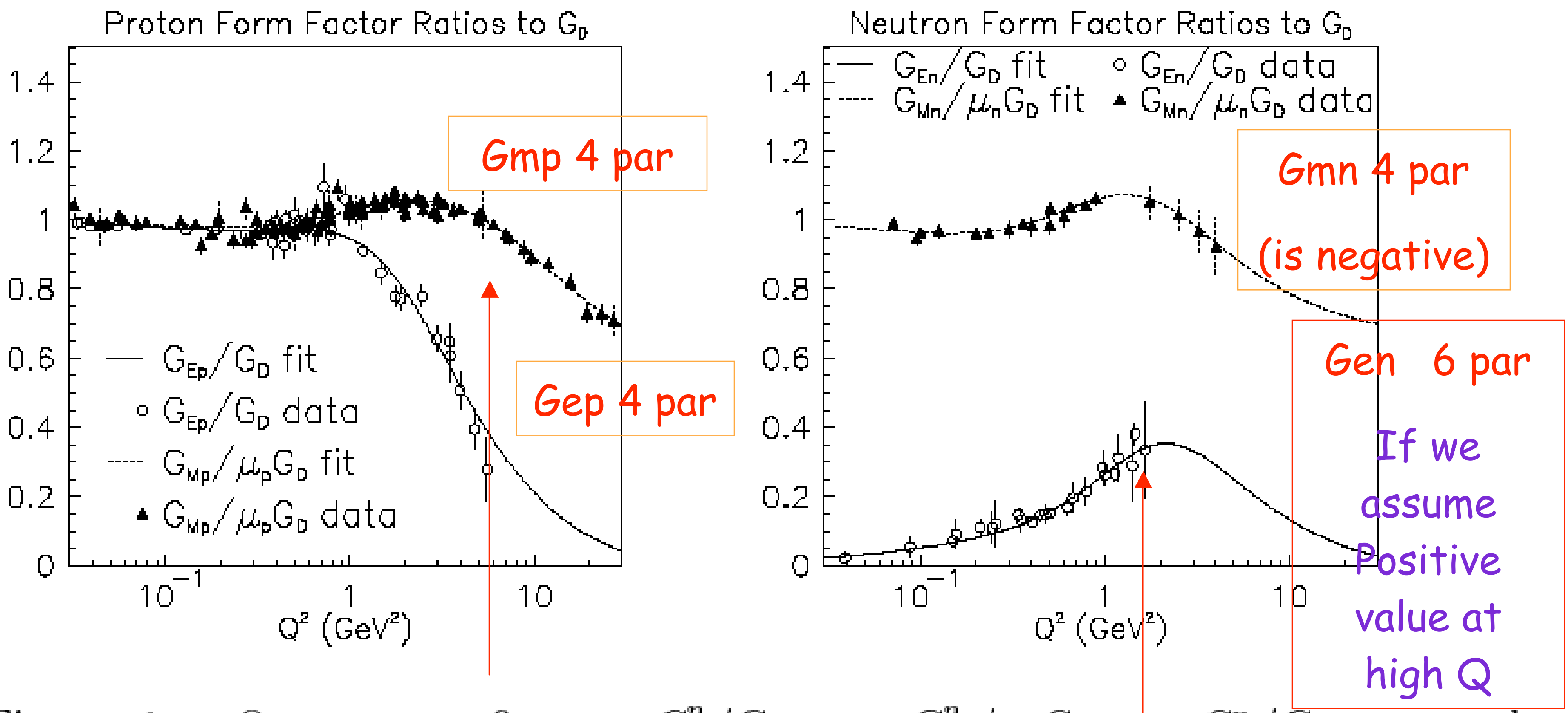
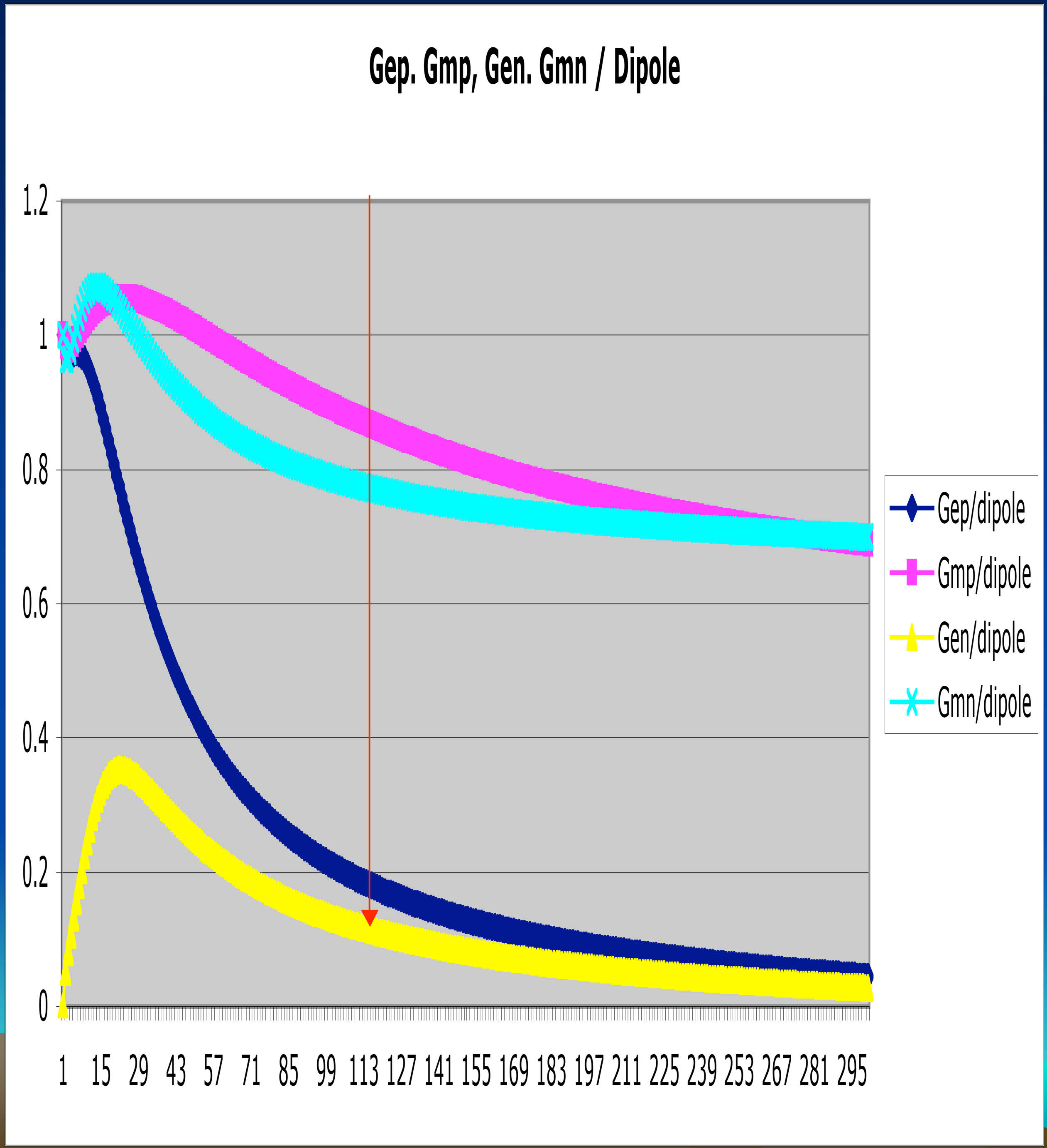
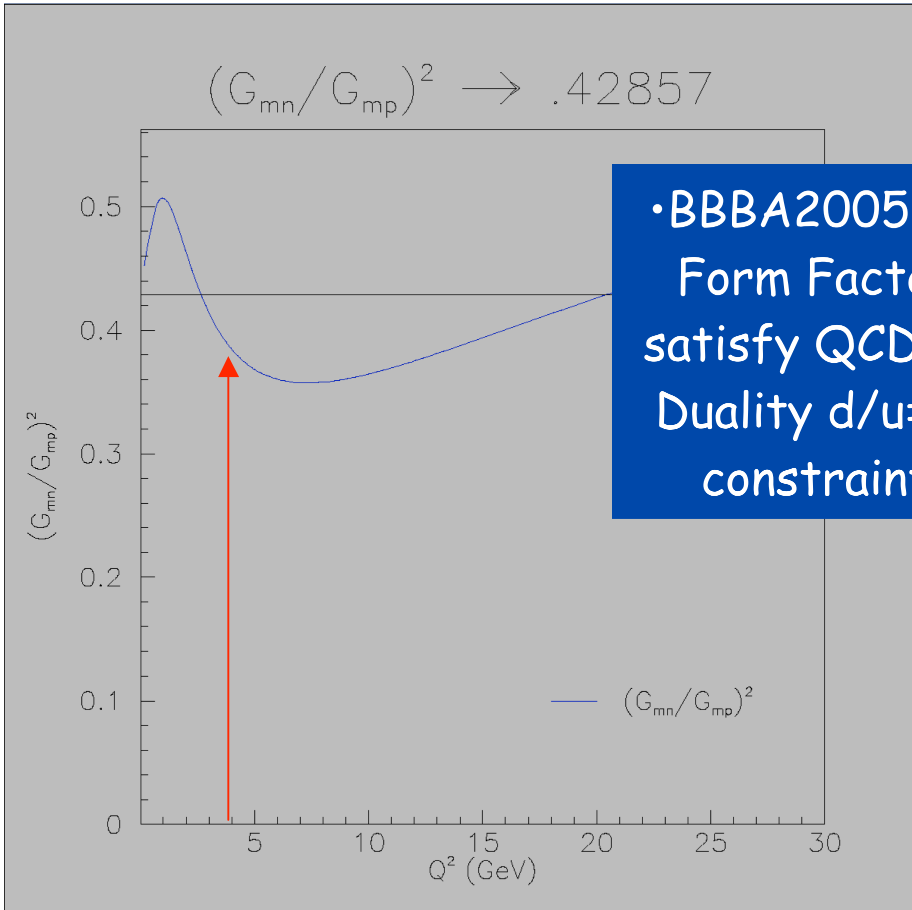


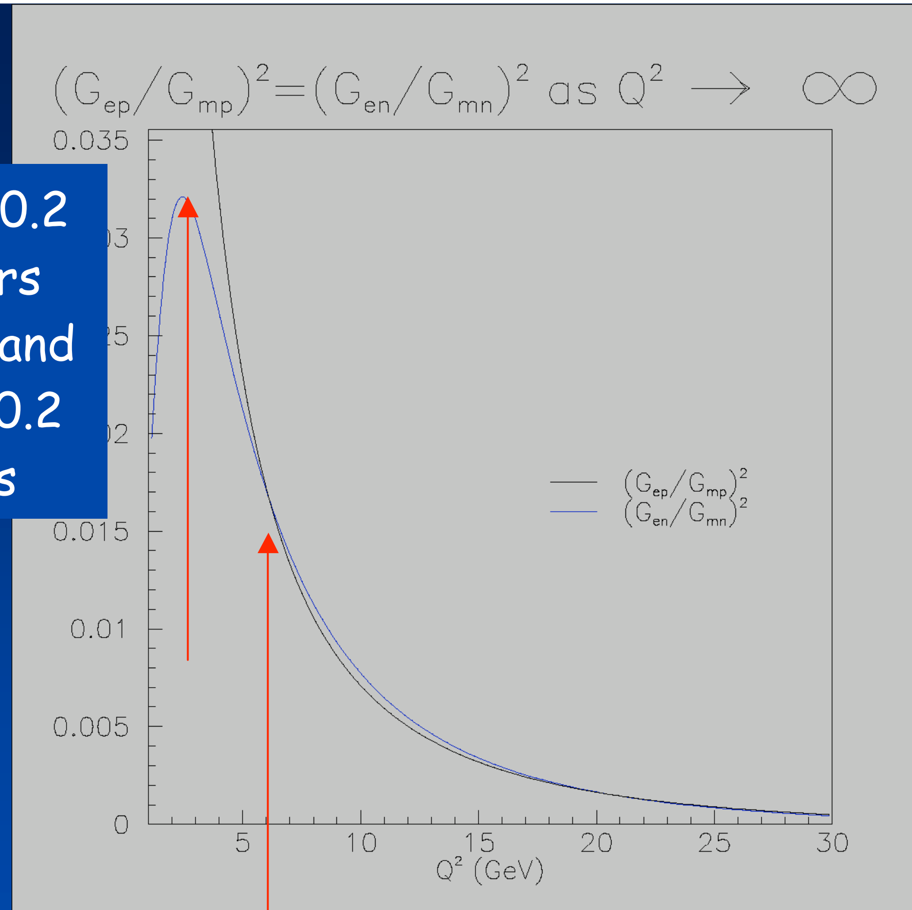
Figure 1: Our recent fits to G_E^p/G_{Dipole}, $G_M^p/\mu_p G_{Dipole}$ G_E^n/G_{Dipole}, and $G_M^n/\mu_p G_{Dipole}$. The data used to extract these fits are shown. These fits use constraints from QCD and duality at high Q^2, where there is no data.

BBBA- 2005 - Bodek, Bradford, Budd, Arrington QCD-Duality Constraint Form Factors-----We refit the Form Factors using the Kelly Pramaterization for Gep, Gmp, Gmn, In addition, we use this parametrization (with 6 parameters) to also fit Gen. All parameters are varied such as to satisfy ACD duality constraints at high Q2.


Gep. Gmp, Gen. Gmn / Dipole
1.2
1
0.8
0.6
0.4
0.2
0
1 15 29 43 57 71 85 99 113 127 141 155 169 183 197 211 225 239 253 267 281 295
Gep/dipole
Gmp/dipole
Gen/dipole
Gmn/dipole



•BBBA2005p0.2 Form Factors satisfy QCD and Duality d/u=0.2 constraints



QCD and Duality: At High Q2 near x=1

F1n/F1p =(Gmn/Gmp)^2 = 0.43

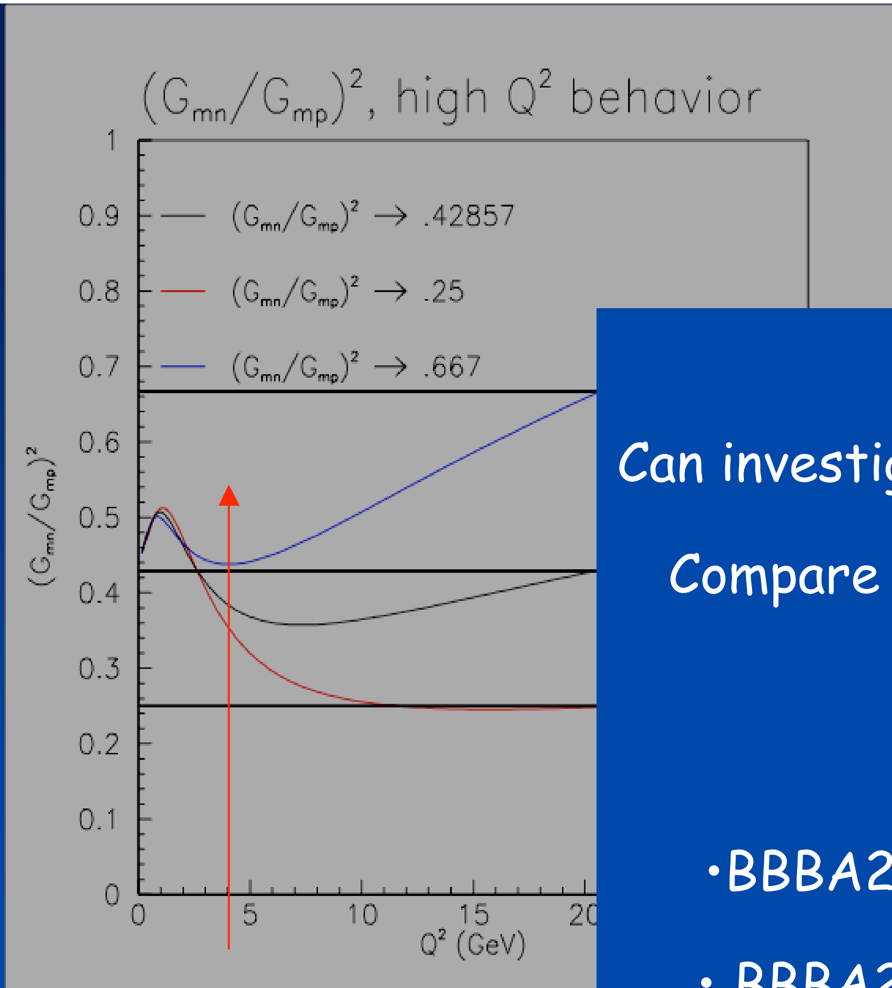
From F1n/F1p DIS prediction with

d/u = 0.2 at x=1 (From QCD)

Duality: Rp=Rn (inelastic) at high Q2 near x=1.

Implies

|Gen/Gmn|=|Gep/Gmp|

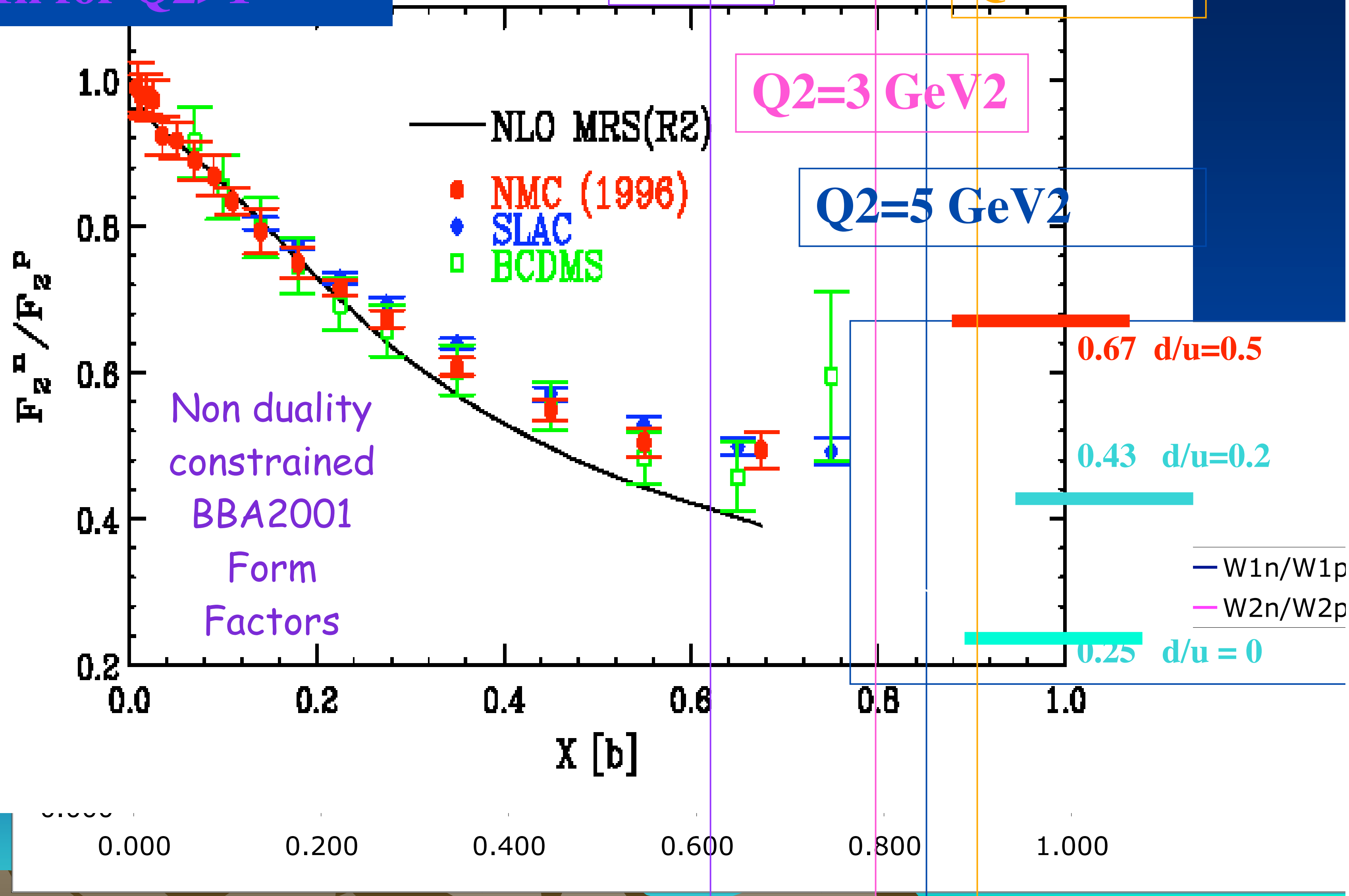


Can investigate different limits for d/u

Compare (Gmn/Gmp)2 and F1n/F2p DIS

- BBBA2005pdu0.5 Form Factors
- BBBA2005pdu0.2 Form Factors
- BBBA2005pdu0.0 Form Factors

Duality works for W1n for Q2>1



BBBApdu0.2- 2005 - Bodek, Bradford, Budd, Arrington QCD-Duality Constraint Form Factors - should work both at low and High Q2

a1	b1	b2	b3	GEP
-.5972961E-01	11.17692	13.62536	32.96109	parameter
0.1852694	0.2370240	1.435371	9.888144	error

a1	b1	b2	b3	GMP
0.1500081	11.05341	19.60742	7.536569	parameter
0.3059846E-01	0.1020370	0.2822719	0.9482885	error

a1	a2	b1	b2	b3	b4	GEN
3.488283	-.2175027	51.54248	16.33405	146.7034	159.0527	param.
0.5096608	0.4067491E-01	9.908267	31.33876	78.36866	25.36869	error

a1	b1	b2	b3	GMN
1.815929	14.09349	20.69266	68.58896	parameter
0.4214175	0.6181164	2.643576	14.75946	error

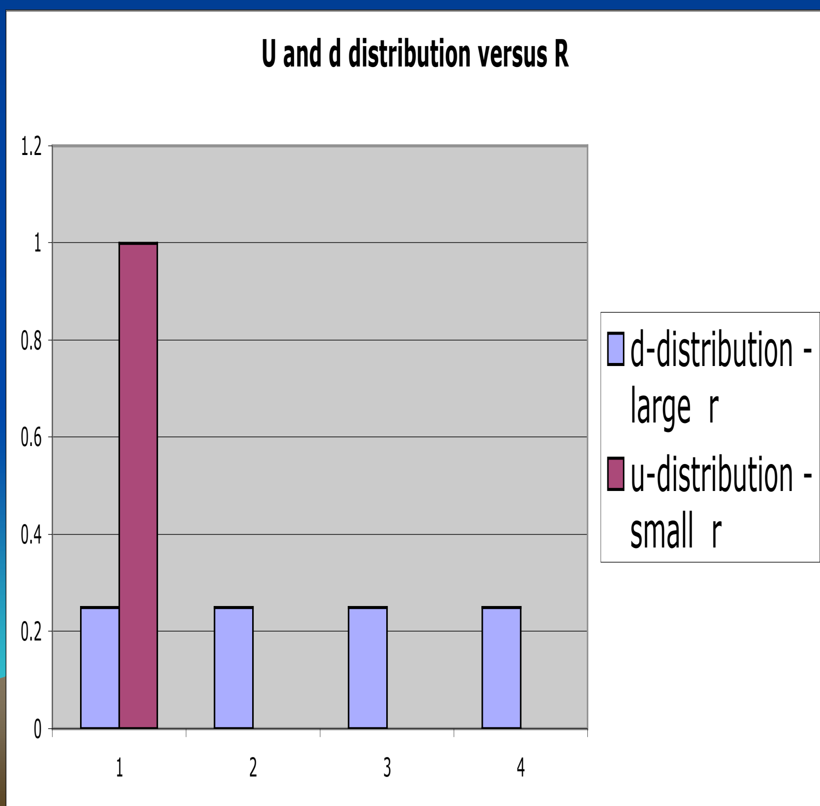
Next (summer 2004) (1) Fit individual quark form factors separately
(1) Working on getting better description of axial form factor Fa(Q2) using High Q2 QCD Duality Constraints for both vector and axial form factors, and the Adler Sum rule.
(2) Compare to absolute value predictions from Duality using standard PDFs at high Q2.

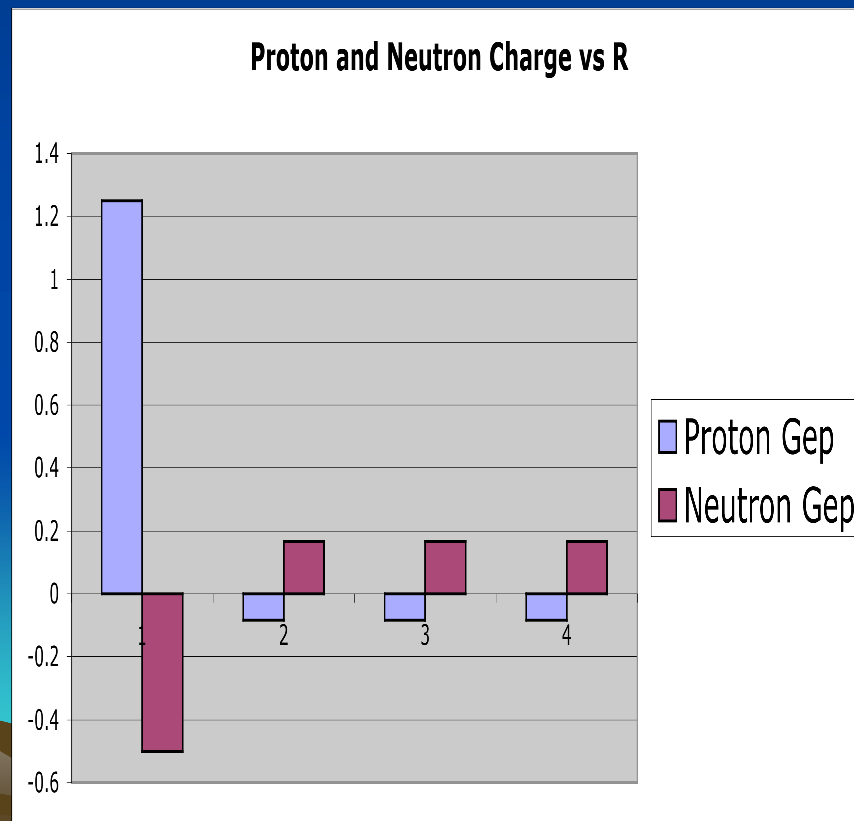
Next step: Duality Constrained fits for Ge(u) Ge(d) Combine QCD, Duality, Lattice QCD, and Q2=0 constraints

Fit Ge(u) and Ge(d) separately - expect Gen to change sign at high Q2 e.g.

Gep = 4/3 Ge(u) -1/3 Ge(d)

Gen = -2/3 Ge(u) + 1/3 Ge(d)





Duality Constraints to be tested with elastic and inelastic form factors

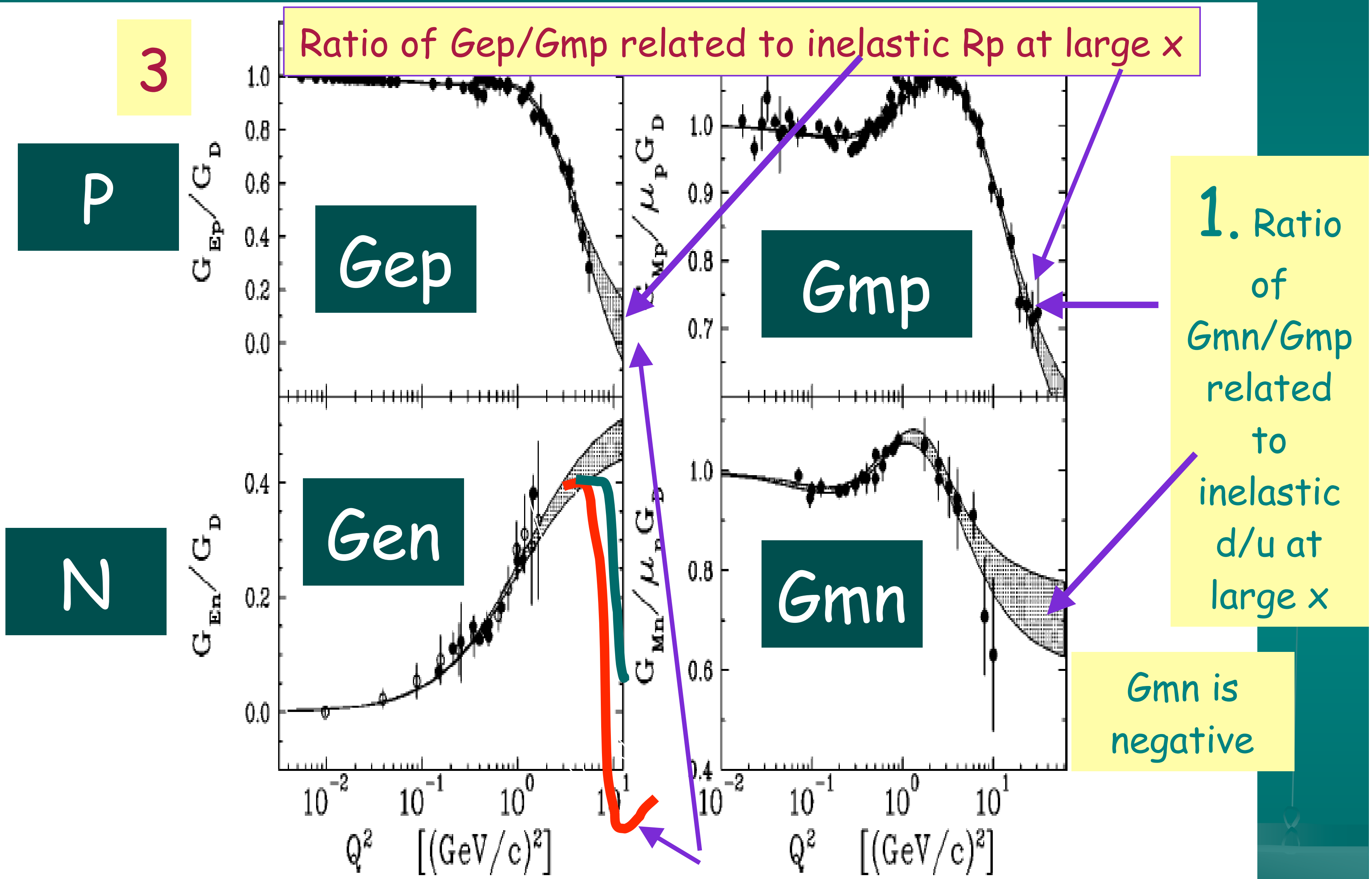
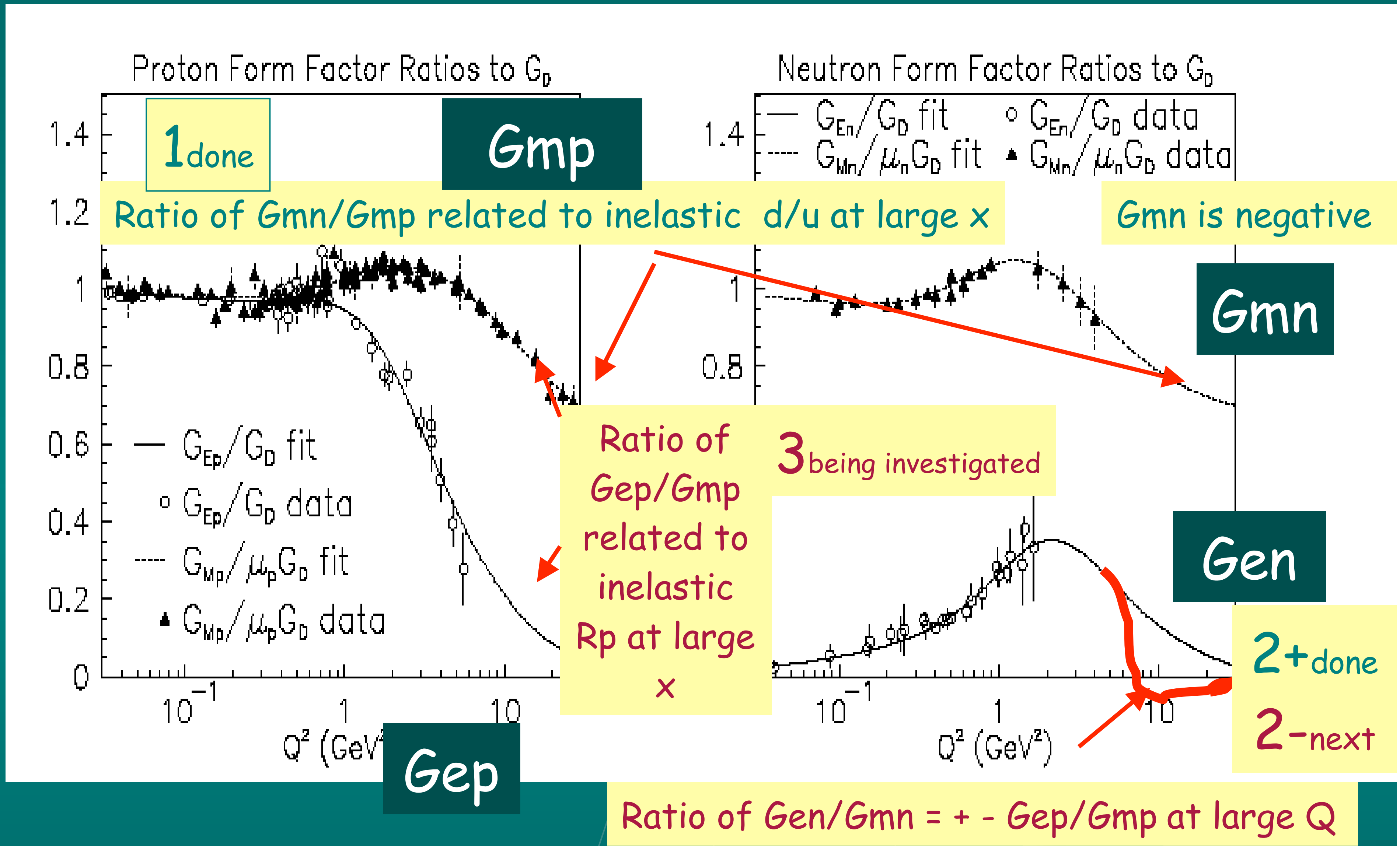


FIG. 1. Fits to nucleon electromagnetic form f...
while data obtained from the deuteron quadrupol...

2. Ratio of Gen/Gmn = + - Gep/Gmp at large Q



Duality Constraints to be tested with elastic and inelastic form factors

Constraints on axial form factor - Next