First Workshop on Quark-Hadron Duality and the Transition to pQCD

Spin Structure of the Nucleon and Aspects of Duality

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Frascati, June 06, 2005

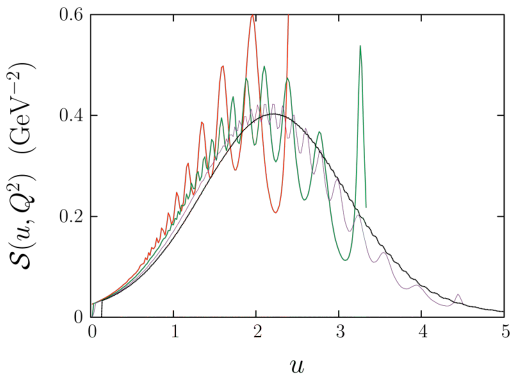
- Moments of spin structure functions and sum rules
- Size of higher twists and duality
- Polarizabilities; spin and color
- Conclusion

Aspects of duality

Duality in spin structure functions?

Local versus Global

- Moments of spin structure functions
  - First Moments:
    - Dispersive Sum rules (Gerasimov-Drell-Hearn, Bjorken, Ellis-Jaffe,Burkhardt-Cottingham)
  - Higher moments:
    - Spin polarizabilities
    - Color polarizabilities
- Q^2 evolution is important to investigate aspects of duality in the response
  - Low Q^2, low W spectrum (resonances) dominate response
  - High Q^2, high W (DIS) dominate the response.
  - Within OPE size of higher twists as a measure of how well duality is working



Response of scalar quark in a harmonic oscillator vs scaling variable u
Melnitchouk et al.

Moments of spin structure functions

GDH
Sum Rule
$I_{GDH}(0)$

Generalized
GDH Integral
$I_{GDH}(Q^2)$

Bjorken
Sum Rule
$\Gamma_1^{p\text{-}n}$

Burkhardt-Cottingham
sum rule
$\Gamma_2(Q^2)$

Spin
Polarizabilities
$\gamma_0(Q^2), \delta_{LT}(Q^2)$

Higher twists & color
Polarizabilities
$d_2(Q^2), f_2(Q^2)$

0 1 10 ∞

Q^2

χPT *Lattice QCD* *OPE* *pQCD*

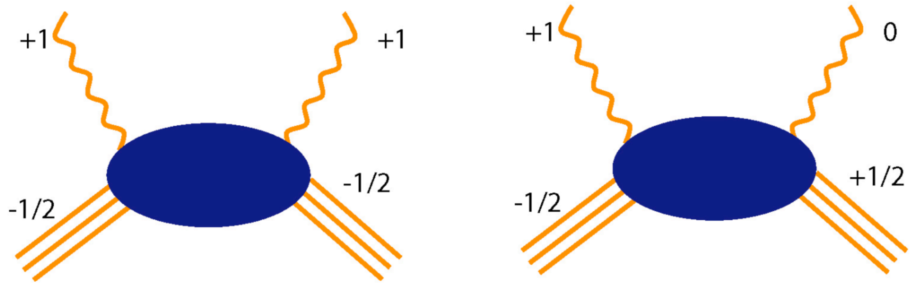
Dispersive sum rules

Forward Virtual Compton Scattering Amplitudes

$$S_1(\nu, Q^2), \quad S_2(\nu, Q^2)$$

The spin structure functions $G_{1,2}(\nu, Q^2)$ are proportional to the virtual Compton amplitudes:

$$\mathrm{Im} S_i(\nu, Q^2) = 2\pi G_i(\nu, Q^2)$$



Dispersion Relations

$$S_1(\nu, Q^2) = 4 \int_{Q^2/2M}^{\infty} \frac{\nu' G_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu' \qquad \text{(a)}$$

$$S_2(\nu, Q^2) = 4 \int_{Q^2/2M}^{\infty} \frac{\nu' G_2(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu' \qquad \text{(b)}$$

(a) leads to the extended GDH sum rule valid at all Q^2.

(b) leads to the Burkhardt-Cottingham sum rule.

⊙Unitarity relates forward scattering amplitudes to physical cross sections

⊙At low energy $S_{1,2}$ can be evaluated using:

- ➥Low-energy theorem
- ➥Chiral perturbation theory

Dispersive Sum rules; Useful for What?

- Check basic assumptions going into the derivation of the sum rule
  - Causality and analyticity
  - Low energy effective theory
  - Asymptotic behavior of the amplitudes
- A way to understand the physical content of low energy constants
- Use of the sum rule to determine the low-energy scattering amplitude

Tools (Hall A)

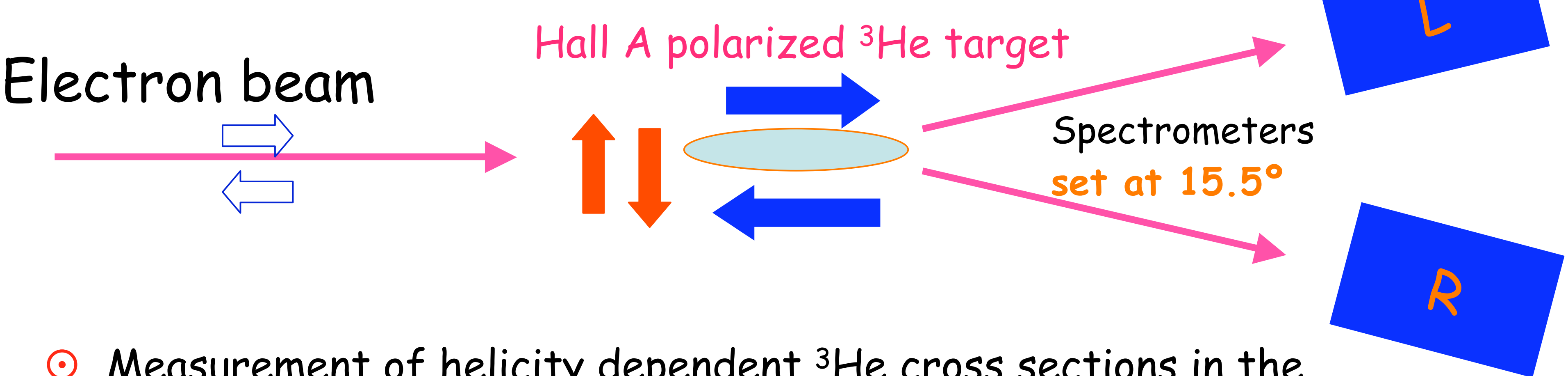
Polarized beam

- Energy: 0.86-5.1 GeV
- Polarization: > 70%
- Average Current: 5 to 15 μA

Hall A polarized ^3He target

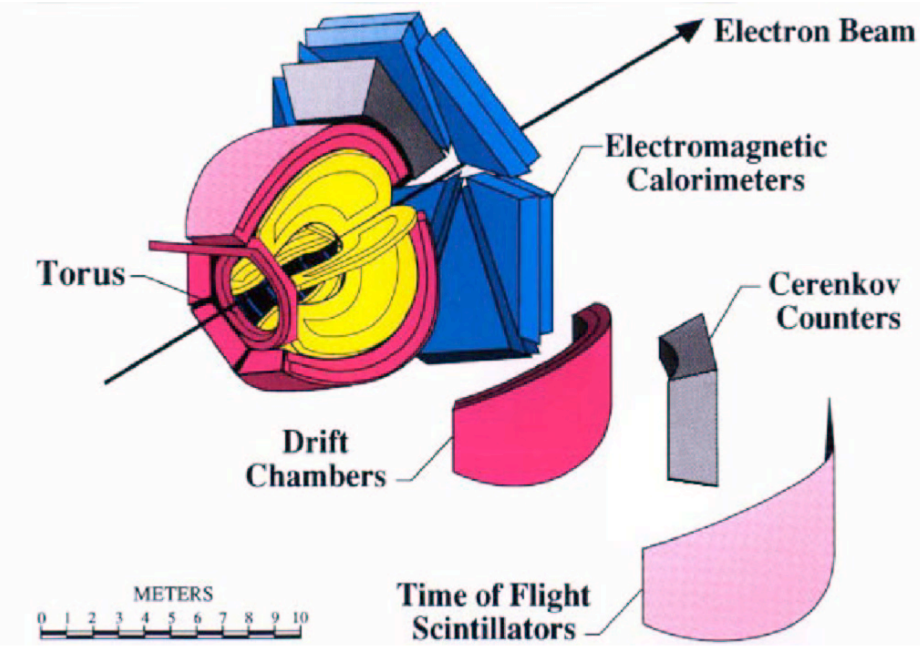
- Pressure ~ 10 atm
- Polarization average: 35%
- Length: 40 cm with 100 μm thickness

Highest polarized luminosity~ $10^{36} cm^{-2} s^{-1}$



- Measurement of helicity dependent ^3He cross sections in the quasielastic and resonance regions
- Extract g_1 and g_2 spin structure functions of ^3He
- Extract moments of spin structure functions of ^3He and Neutron

Tools (Hall B)



Polarized NH_3 &ND_3

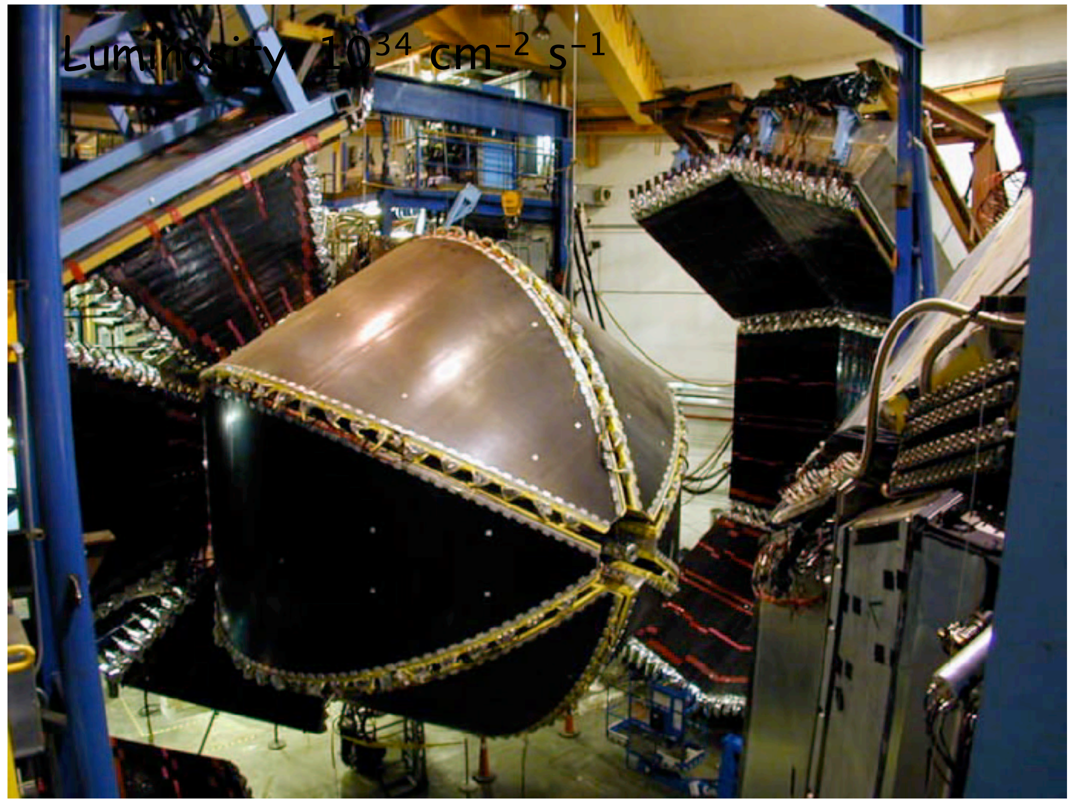
75% (NH_3) or 30% (ND_3)

Longitudial polarization only

Acceptance ~2.5π

Luminosity 10^{34} cm^{-2} s^{-1}

CEBAF
Large
Acceptance
Spectrometer



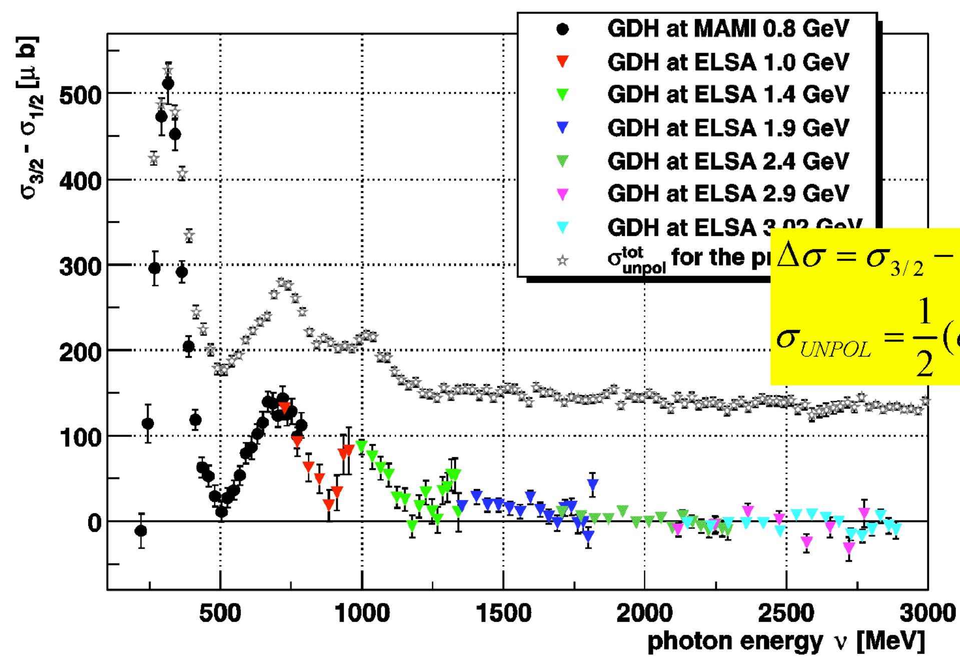
- Large kinematical coverage
- detection of charged and neutral particles
- Multiparticle final state

GDH Sum Rule on the Proton (MAMI+ELSA)

	ν (MeV)	I_{GDH} (μb)
MAMI PRL 87 (2001) 022003	200-800	226±5±12
ELSA PRL 91 (2003) 192001 PRL 93 (2004) 032003	680-1820 800-2900	49.9±2.4±2.2 27.5±2.0±1.2
Low Energy MAID (2003)	140-200	-27.5±3
High Energy PLB 450 (1999)439	>2900	-14
		212±5±12

$$\int_{\nu_{in}}^{\infty} \frac{d\nu}{\nu}\left[\sigma_{3/2}(\nu) - \sigma_{1/2}(\nu)\right] = \frac{2\pi^2\alpha_{em}}{M^2}\kappa^2$$

Total cross sections on the proton



$$\Delta\sigma = \sigma_{3/2} - \sigma_{1/2}$$

$$\sigma_{UNPOL} = \frac{1}{2}(\sigma_{3/2} + \sigma_{1/2})$$

gdh–Collaboration

MAMI data: J. Ahrens et al., Phys. Rev. Lett. 87 (2001) 022003
ELSA data: H. Dutz et al., Phys. Rev. Lett 91 (2003) 192001
H. Dutz et al., Phys. Rev. Lett 93 (2004) 032003

To be compared to 205 (mb)

Sum rules at finite Q^2

Ji and Osborne, Phys. Lett. B472, 1 (2000)

$$S_1(\nu=0,Q^2) = 4\int_{Q^2/2M}^{\infty}\frac{d\nu'}{\nu'}G_1(\nu',Q^2)$$

$Q^2 < 0.1\ \mathrm{GeV}^2$

$Q^2 > 0.5\ \mathrm{GeV}^2$

A mixed region
important to our understanding of
the nucleon structure

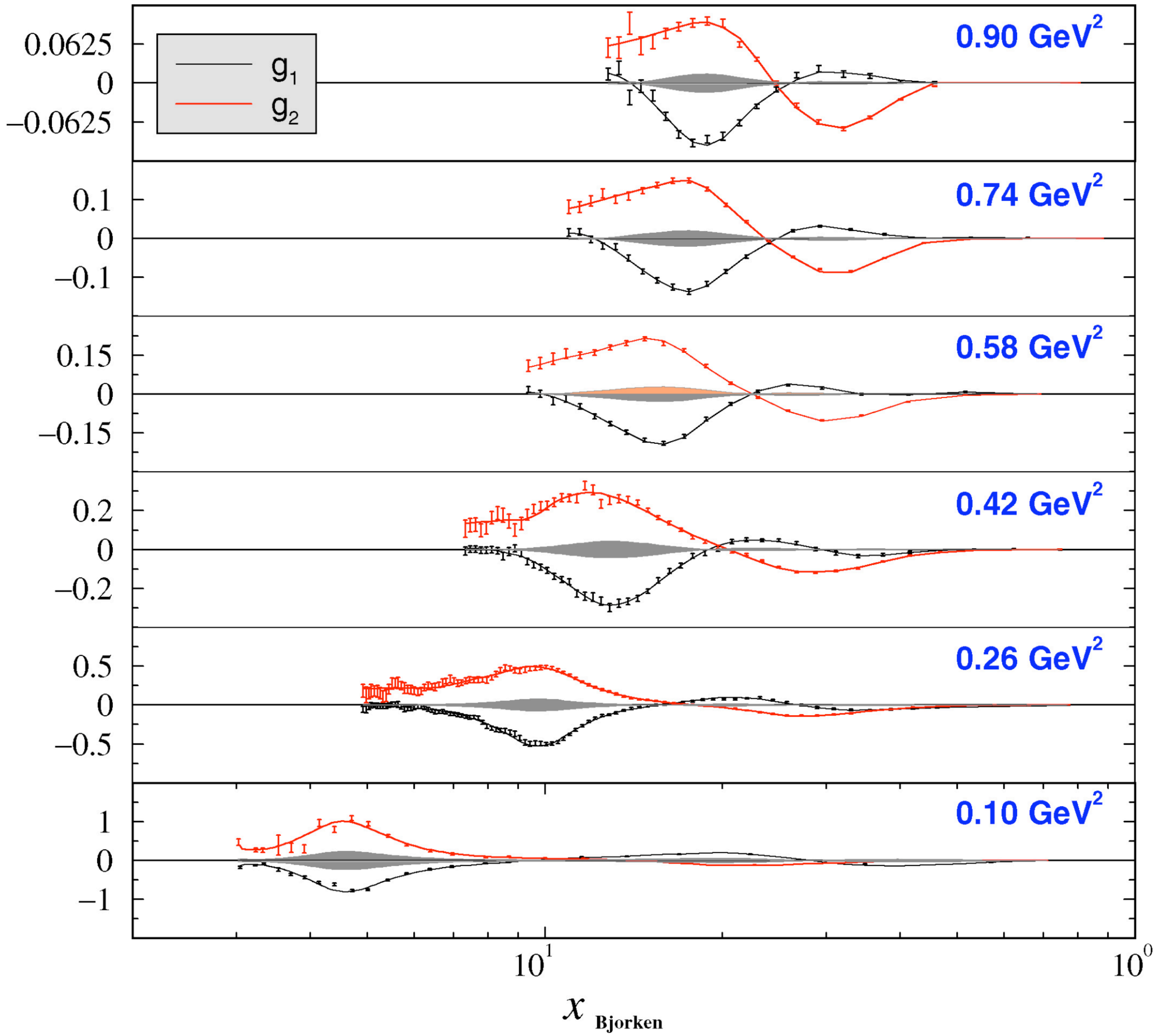
$$-\frac{\kappa^2}{8M^2}Q^2 + cQ^4 + \mathcal{O}(Q^6)$$

$$\sum_{\tau=2,4,\ldots}\frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$

GDH sum rule and Chiral Perturbation
(low resolution and long time exposure picture
of the nucleon)

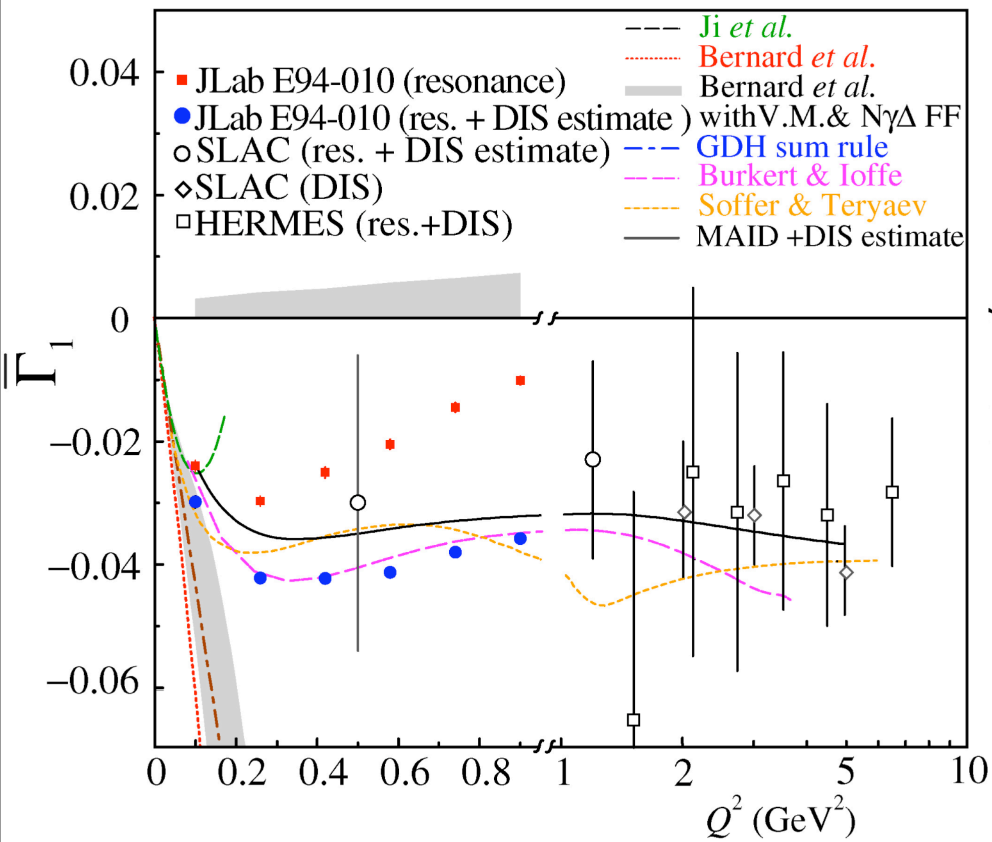
OPE and Bjorken sum rule
(High resolution and short time
Exposure picture of the nucleon

g_1 and g_2 extracted at constant Q^2



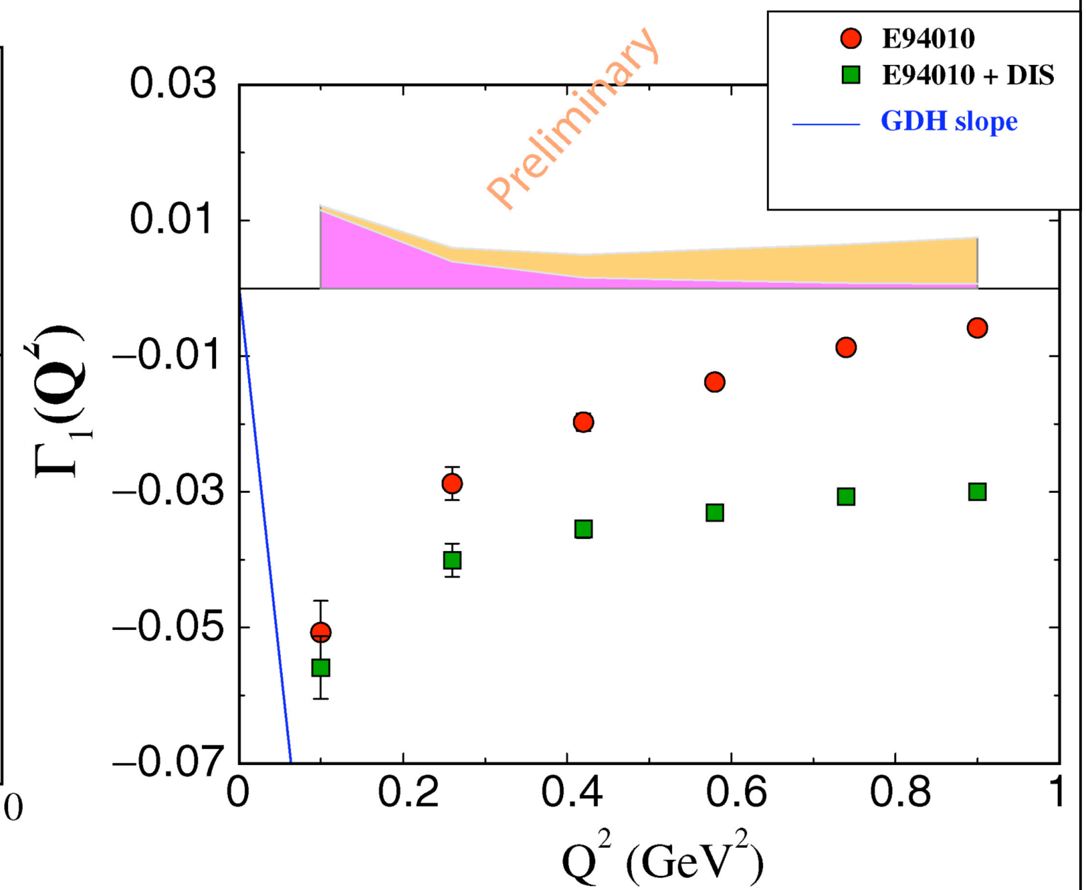
Neutron and ³He

Neutron



M. Amarian et al., Phys. Rev. Lett. 92, 022301 (2004)

³He

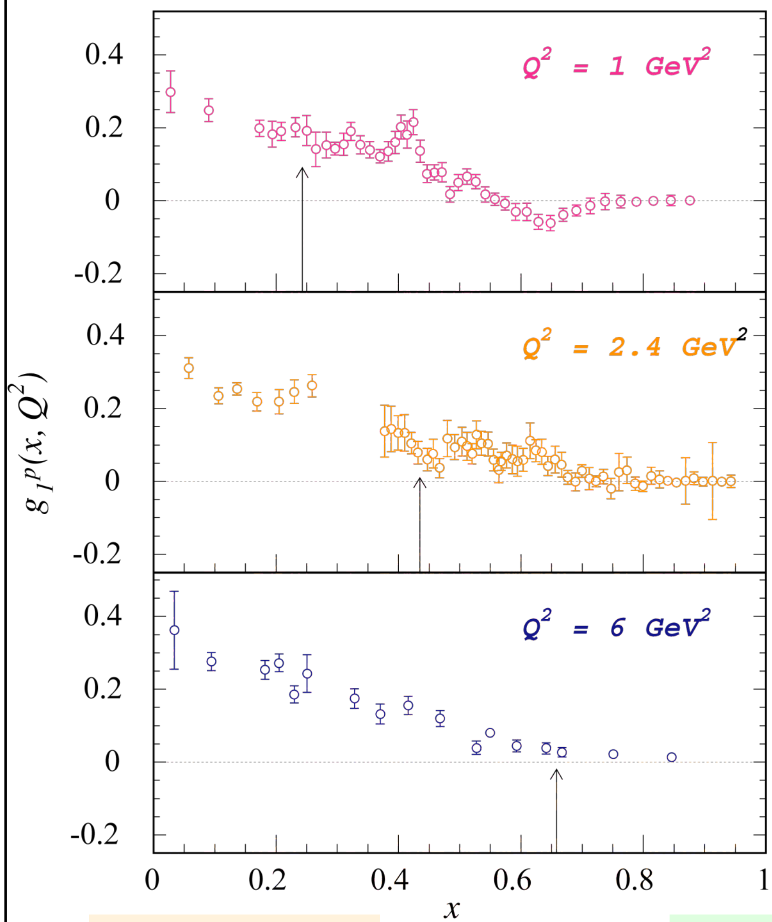


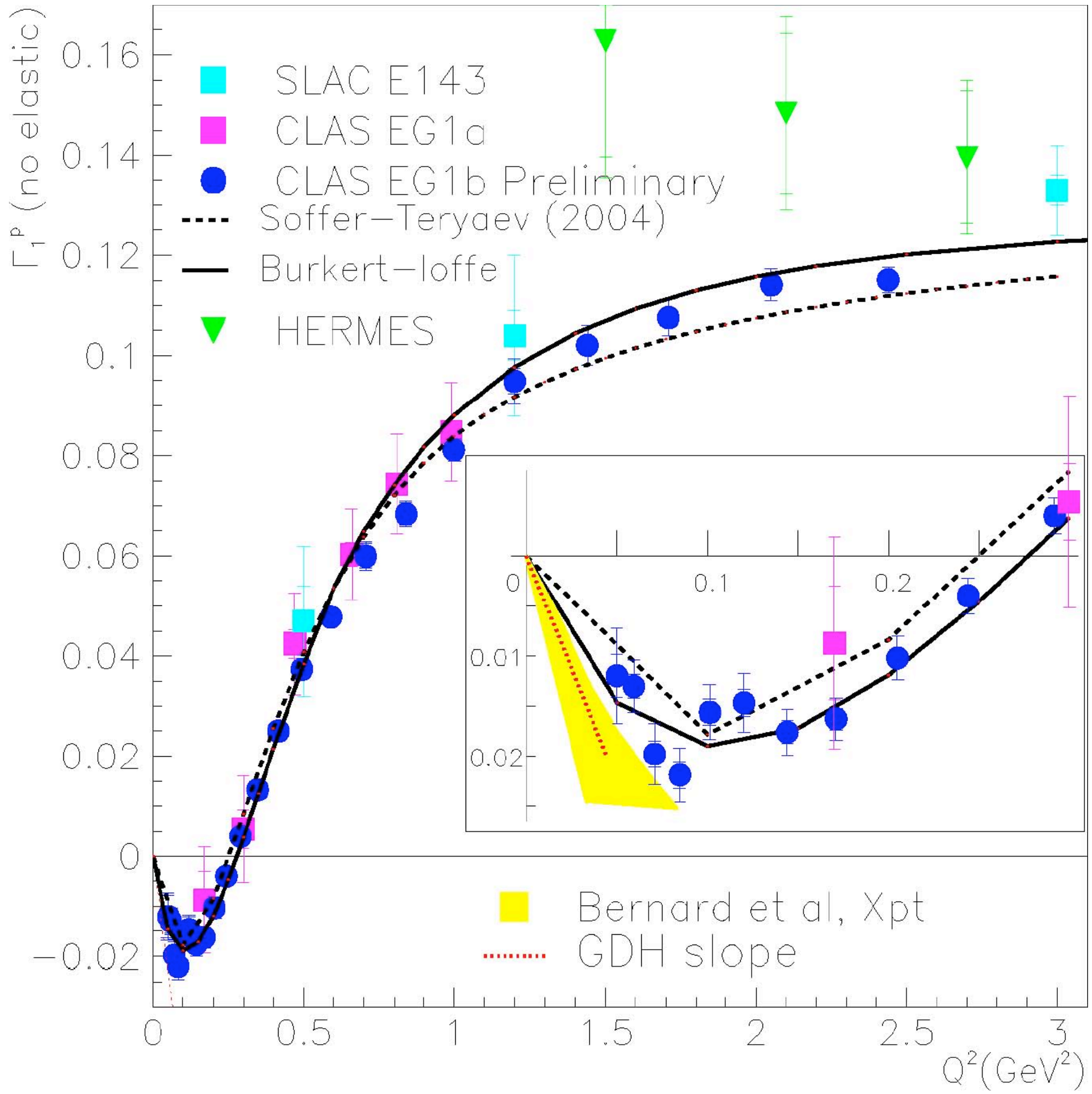
M. Amarian et al., in preparation

Proton

eg1b data: Preliminary
Data analysis: Y. Prok,UVA

eg1a data: R. Fatemi et al.
PRL, 91: 222002 (2003)





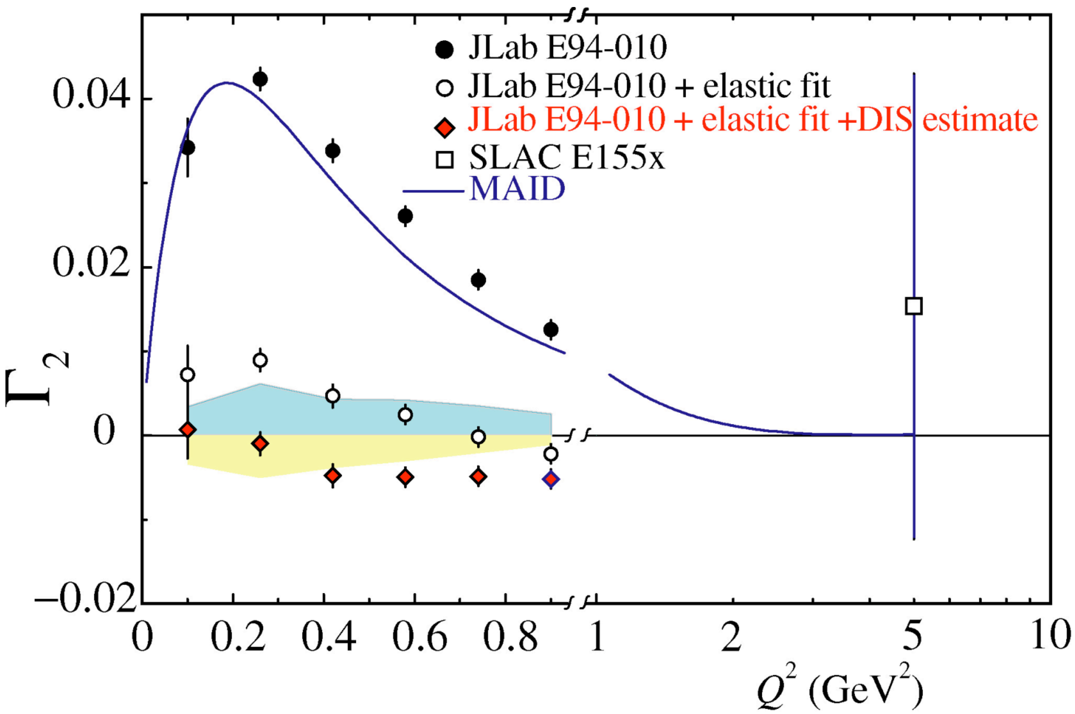
Burkhardt-Cottingham Sum Rule (1965-1966)

$$\Gamma_2(Q^2) = \int_0^1 g_2(x, Q^2)\, dx = 0$$

- Dispersion relation for a spin-flip Compton amplitude
  - *Causality*
  - *Analyticity*
  - *Absence of a J=0 pole with non polynomial residue*
- Doesn't follow from Operator Product Expansion and is valid at all Q^2
- Many scenarios of g_2's low x behavior which would invalidate the sum rule are discussed in the literature.

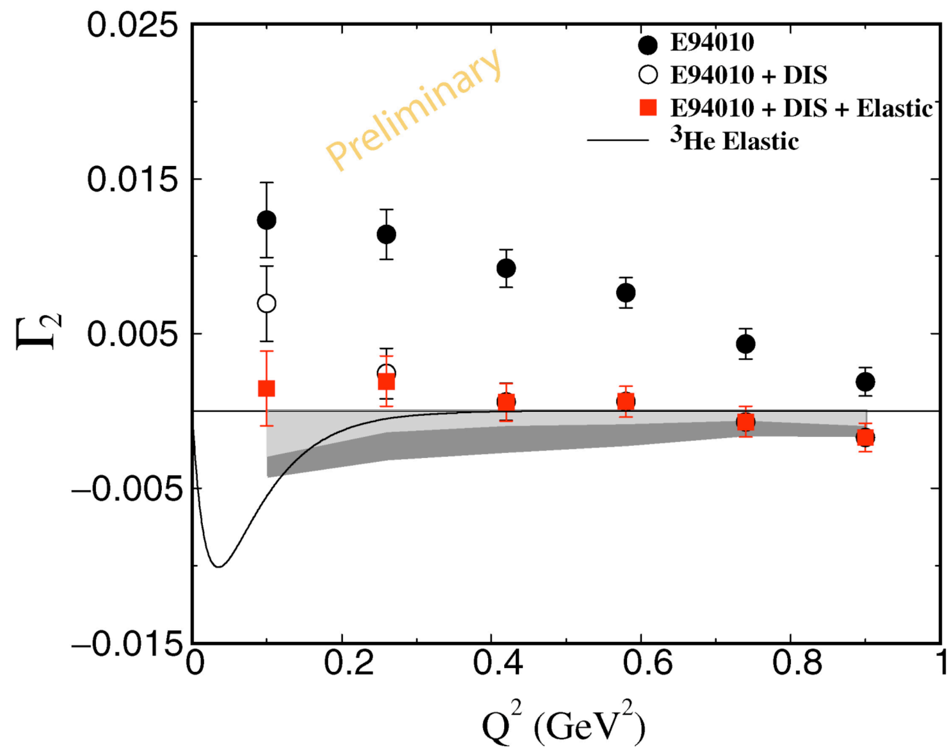
E94-010 B-C sum rule; results

Neutron



M. Amarian et al., Phys. Rev. Lett. 92, 022301 (2004)

^{3}He



M. Amarian et al., in preparation

Moments of Structure Functions

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)\, dx = \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

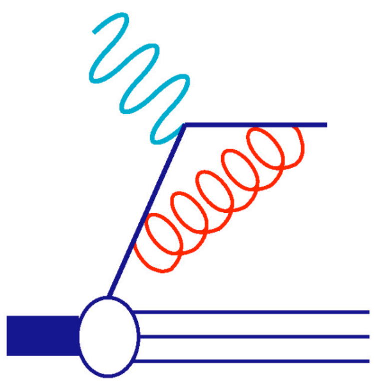
leading twist

higher twist

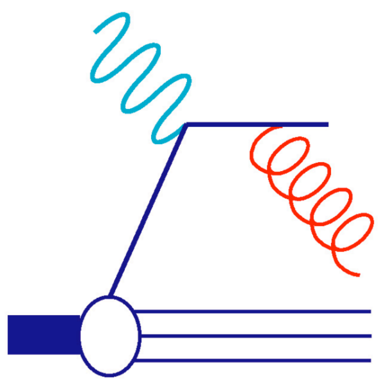
$$\mu_2^{p,n}(Q^2) = \left(\pm \frac{1}{12} g_A + \frac{1}{36} a_8\right) + \frac{1}{9}\Delta\Sigma$$ + pQCD corrections

$g_A = 1.257$ and $a_8 = 0.579$ are the triplet and octet axial charge, respectively

$\Delta\Sigma$ = singlet axial charge



$$\begin{aligned} g_A &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ \Delta\Sigma &= \Delta u + \Delta d + \Delta s \end{aligned}$$



pQCD radiative corrections

Study of Higher Twists

$$\mu_4(Q^2) = \frac{M^2}{9}\left[a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)\right]$$

Twist - 2 (TMC) Twist - 3 Twist -4

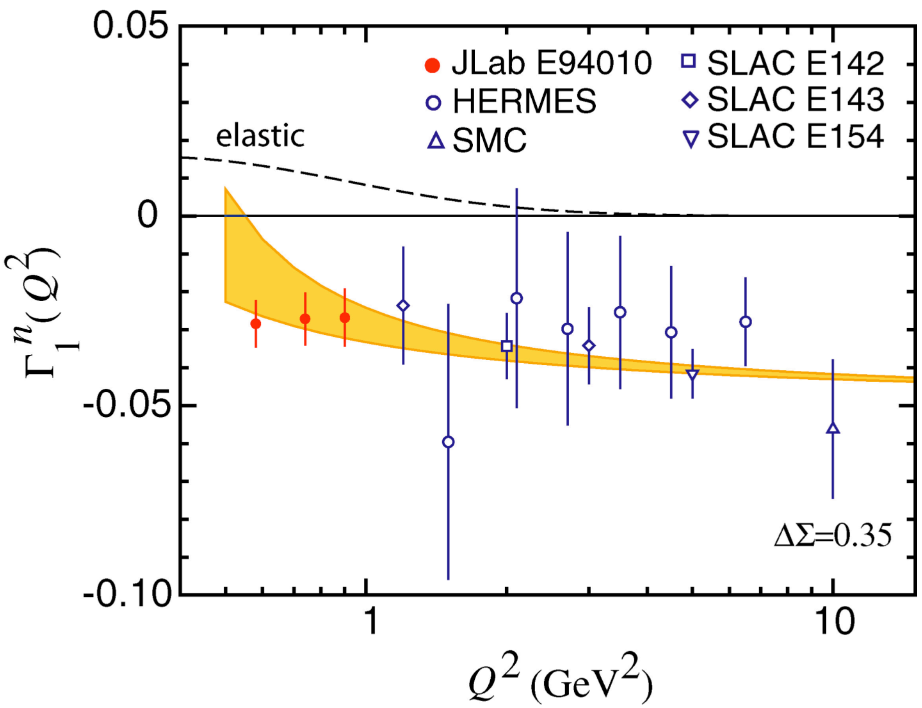
where a_2, d_2 and f_2 are higher moments of g_1 and g_2

e.g. $$d_2(Q^2) = \int_0^1 x^2[2g_1(x,Q^2) + 3g_2(x,Q^2)]dx = \int_0^1 x^2\,\overline{g_2}(x,Q^2)dx$$

$$a_2(Q^2) = \int_0^1 x^2 g_1(x,Q^2)\,dx$$

- To extract f_2, d_2 needs to be determined first.
- Both d_2 and f_2 are required to determine the color polarizabilities

Twist-4 Matrix element f_2

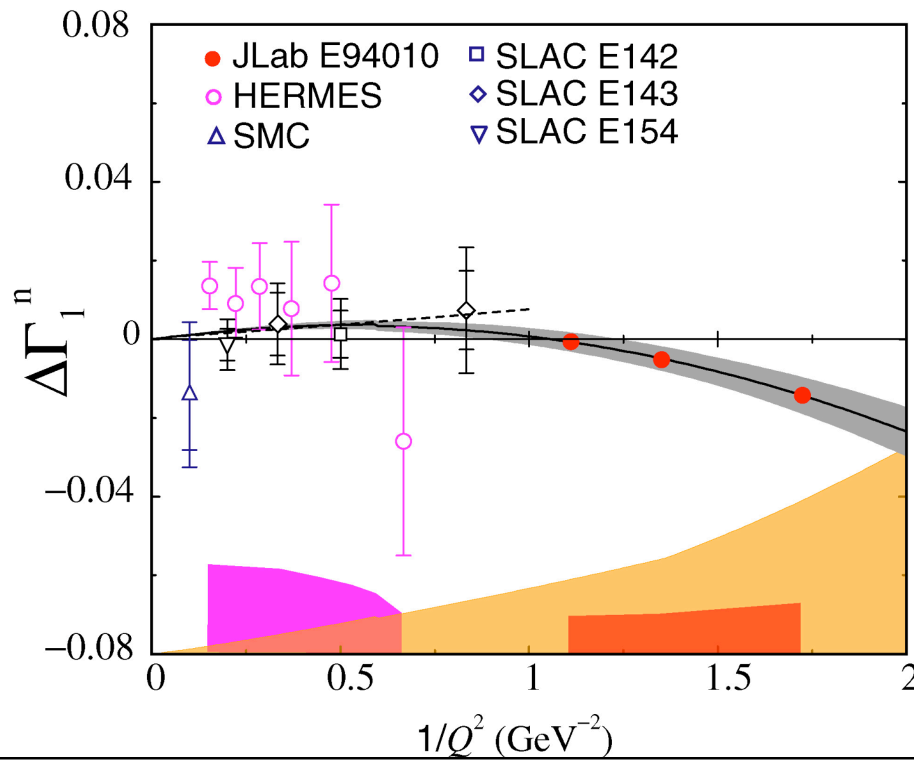


$$\Delta\Gamma_1^n(Q^2) \equiv \Gamma_1^n(Q^2) - \mu_2^n(Q^2) = \frac{\mu_4^n(Q^2)}{Q^2} + \frac{\mu_6^n(Q^2)}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right)$$

$$\mu_4^n = \frac{1}{9}M^2\left(a_2^n + 4d_2^n + 4f_2^n\right)$$

$$f_2^n = 0.033 \pm 0.005\ ,\quad \mu_6^n = (-0.019 \pm 0.002)M^4$$

$$f_2^n = 0.034 \pm 0.043\ ,\quad \mu_6^n = (-0.019 \pm 0.017)M^4$$



Adding $1/Q^6$ term gives the same f_2 and μ_6 with $\mu_8 = (0.00 \pm 0.03)M^2$

If one performs a one parameter fit down to $Q^2 = 0.5$ GeV^2 ; $f_2 = -0.014 \pm 0.010$

If one performs a one parameter fit for $Q^2 > 1$ GeV^2 ; $f_2 = 0.012 \pm 0.029$

Z.-E. M, W. Melnitchouk et al., Phys. Lett. B613,148 (2005)

Proton Analysis

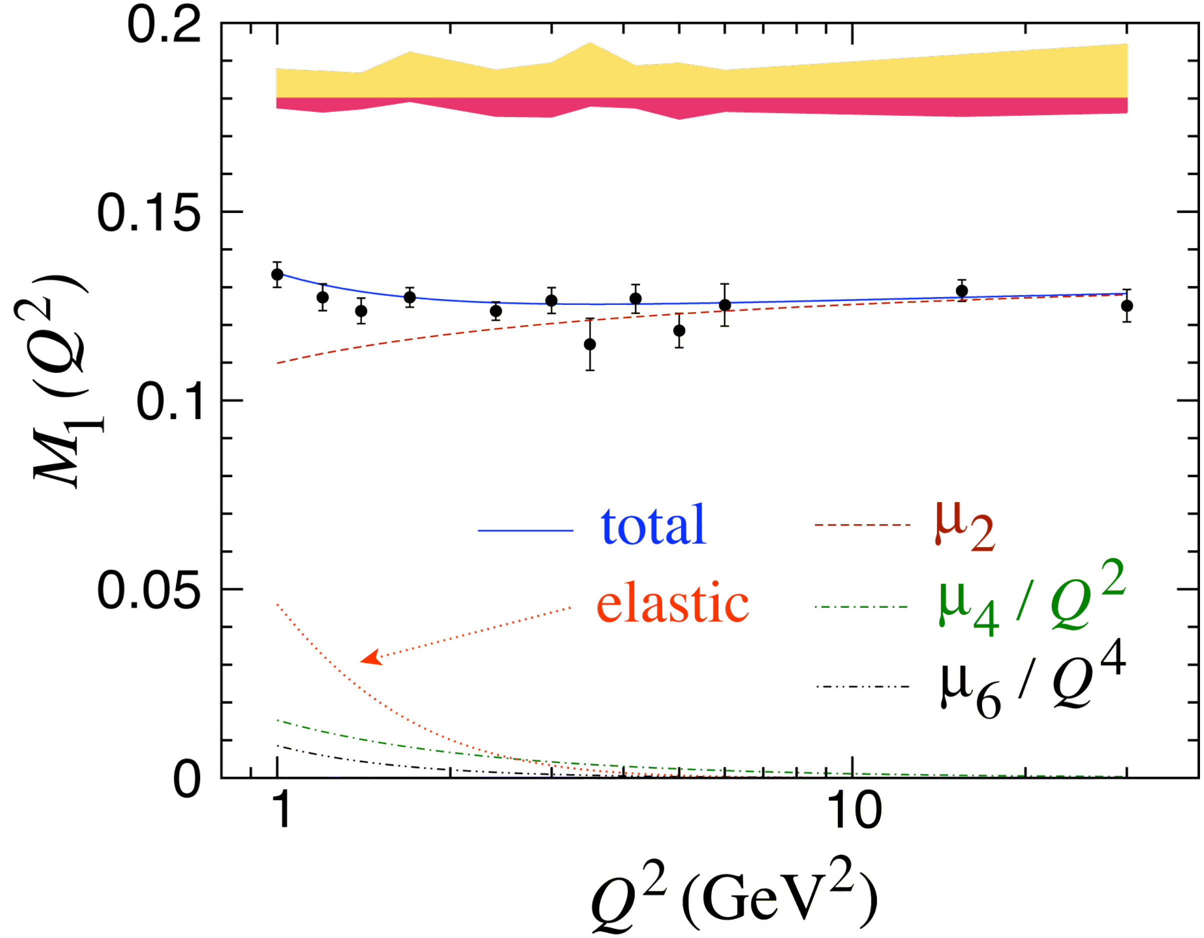
World data +

EG1a data: R. Fatemi et al., PRL, 91 222002 (2003)

Osipenko et al. Phys. Lett. B 609, 258 (2005)

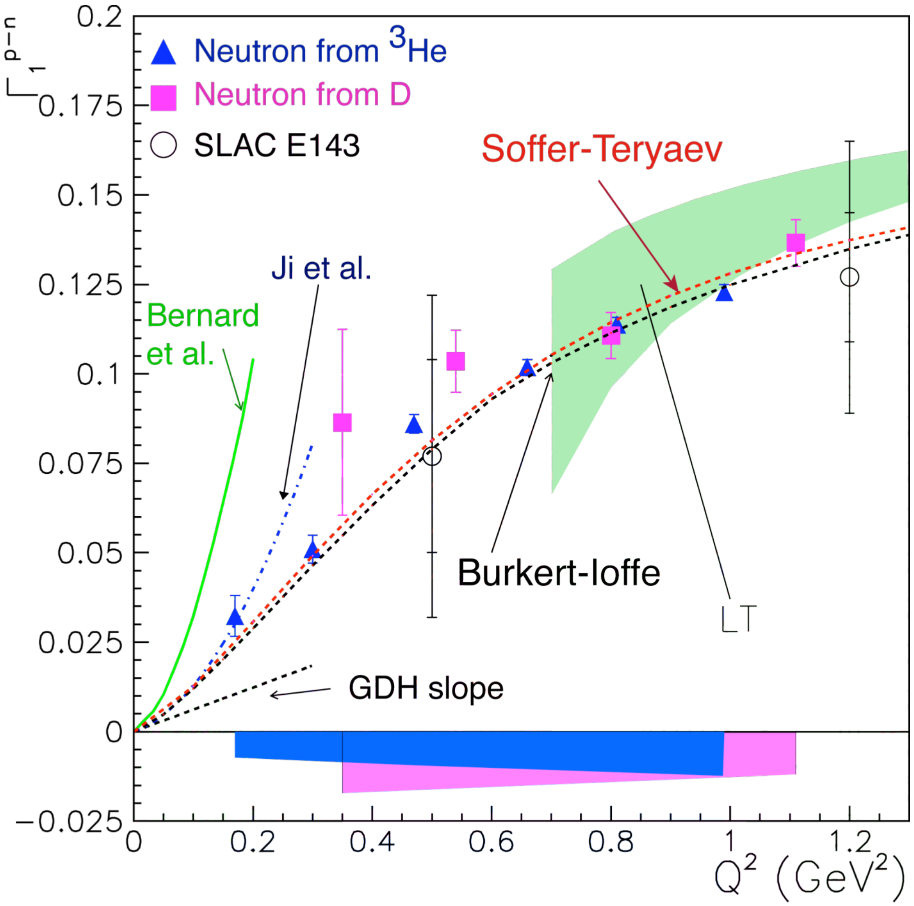
$f_2=0.039^{+0.037}_{-0.043}$

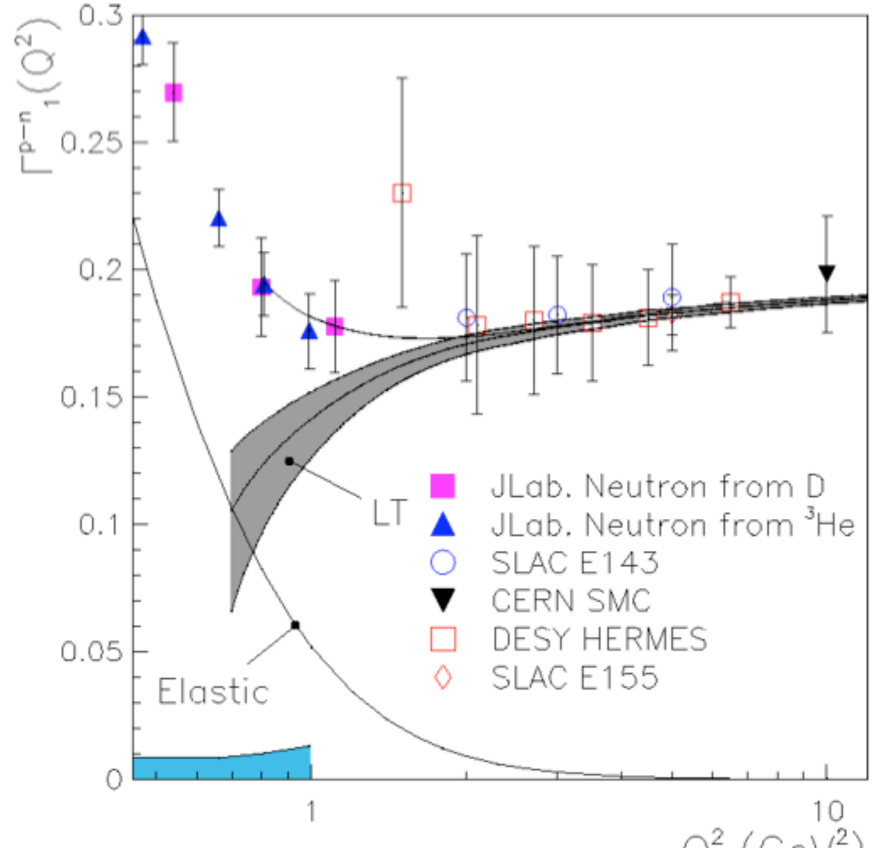
$\mu_6/M^4=0.011^{+0.017}_{-0.013}$



Bjorken Sum Q2 evolution and higher twists

eg1a + E94-010 data
A. Deur et al. PRL 93, 212001 (2004)





- Good quantity to test Chiral P. T. at low Q^2
  - Little or no contribution from the Delta
  - A well defined sum rule at large Q^2

f_2=-0.18±0.10
μ_4/M^2=-0.06±0.02
μ_6/M^4=0.09±0.03

Color polarizabilities

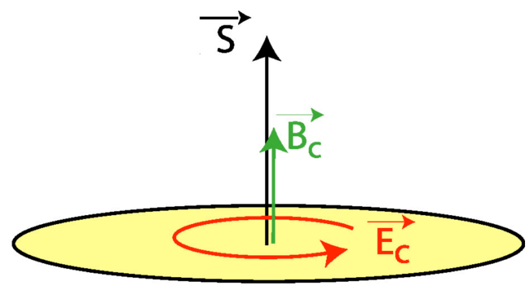
How does the gluon field respond when a nucleon is polarized ?

Define color magnetic and electric polarizabilities (in nucleon rest frame):

$$\chi_{B,E}\, 2M^2 \vec{S} = \langle PS | \vec{O}_{B,E} | PS \rangle$$

where $\vec{O}_B = \psi^\dagger g \vec{\boldsymbol{B}} \psi$

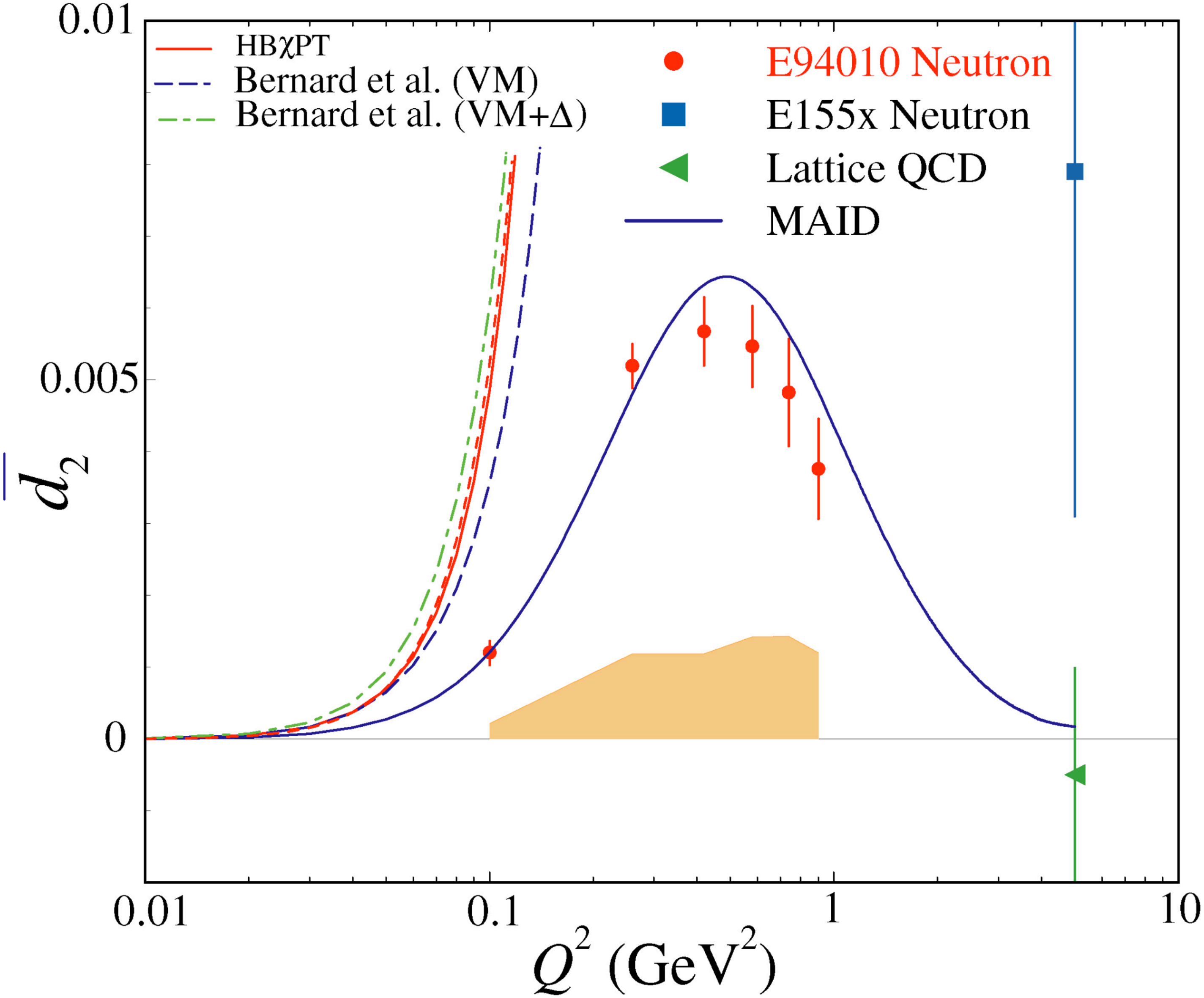
$$\vec{O}_E = \psi^\dagger \vec{\alpha} \times g \vec{\boldsymbol{E}} \psi$$



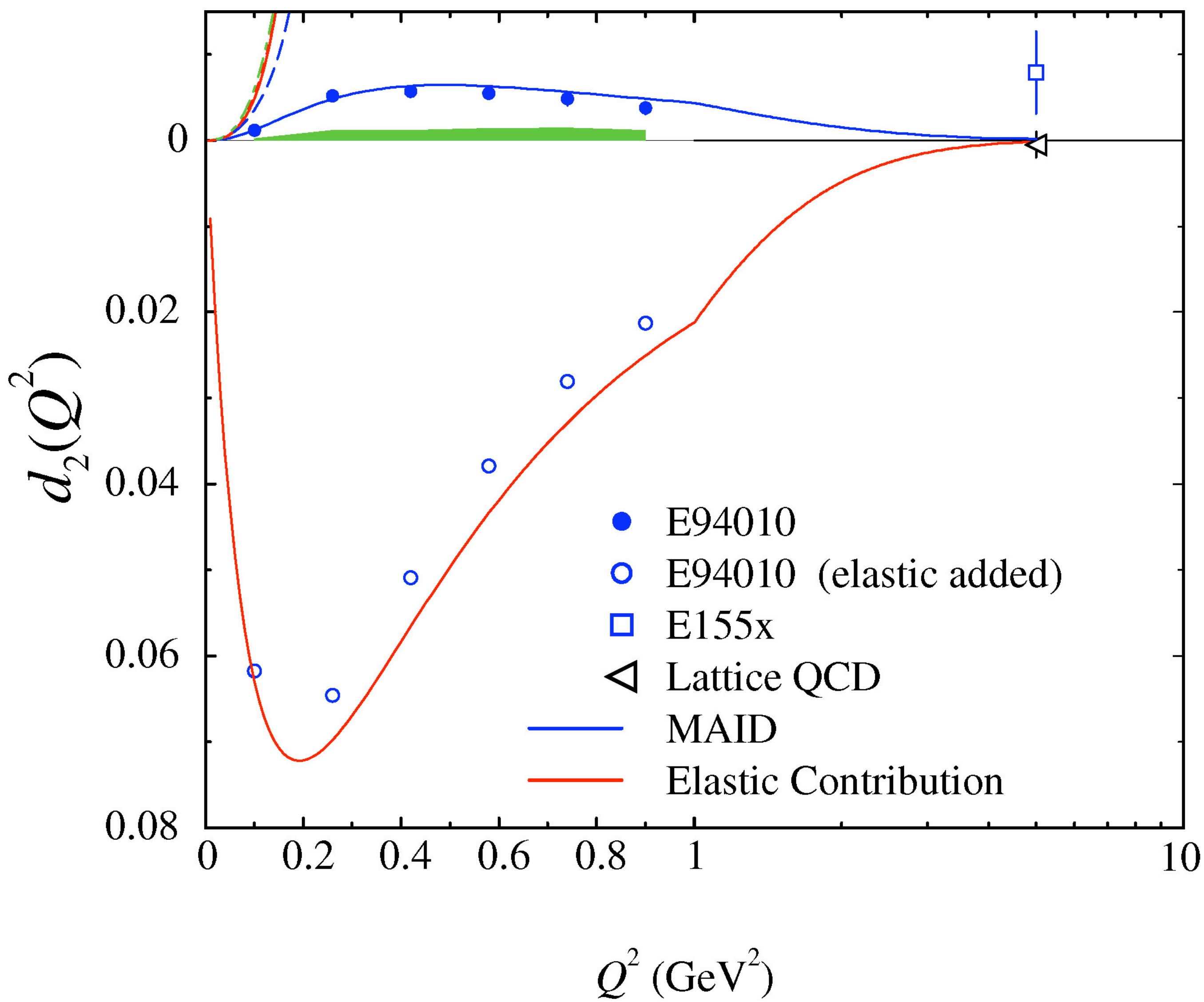
$$\boxed{\begin{aligned} d_2 &= (\chi_E + 2\chi_B)/8 \\ f_2 &= (\chi_E - \chi_B)/2 \end{aligned}}$$

d_2 and f_2 represent the response of the color $\vec{B}$ & $\vec{E}$ fields to the nucleon polarization

Scale dependence of d_2



Adding the elastic contribution



Generalized Spin Polarizabilities of the Neutron

$$\begin{aligned} T(\nu, Q^2) &= \varepsilon'^* \cdot \varepsilon f_T(\nu, Q^2) + f_L(\nu, Q^2) \\ &\quad + i\sigma \cdot (\varepsilon'^* \times \varepsilon) g_{TT}(\nu, Q^2) - i\sigma \cdot [(\varepsilon'^* - \varepsilon) \times \hat{q}] g_{LT}(\nu, Q^2) \end{aligned}$$

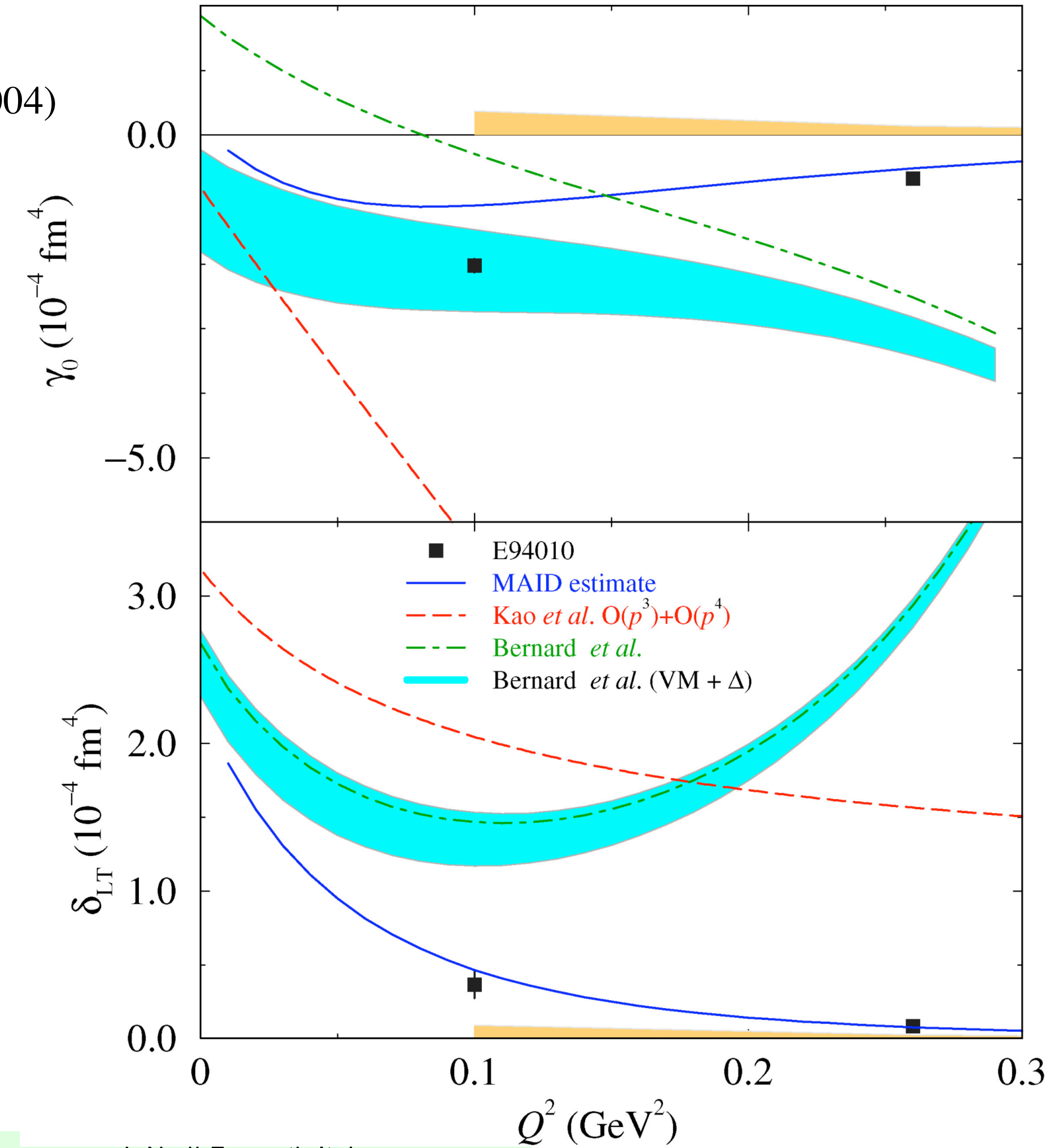
$$\begin{aligned} \mathrm{Re}\, g_{TT}{}^{\mathrm{nonpole}}(\nu, Q^2) &= \frac{2\alpha_{\mathrm{em}}}{M^2} I_A(Q^2)\nu + \gamma_0(Q^2)\nu^3 + \mathcal{O}(\nu^5) \\ \mathrm{Re}\, g_{LT}{}^{\mathrm{nonpole}}(\nu, Q^2) &= \frac{2\alpha_{\mathrm{em}}}{M^2} Q I_3(Q^2) + Q\delta_{LT}(Q^2)\nu^2 + \mathcal{O}(\nu^4) \end{aligned}$$

$$\begin{aligned} \gamma_0(Q^2) &= \frac{16 M^2 \alpha_{\mathrm{em}}}{Q^6} \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) - \frac{Q^2}{\nu^2} g_2(x, Q^2) \right\} dx \\ \delta_{LT}(Q^2) &= \frac{16 M^2 \alpha_{\mathrm{em}}}{Q^6} \int_0^{x_0} x^2 \left\{ g_1(x, Q^2) + g_2(x, Q^2) \right\} dx \end{aligned}$$

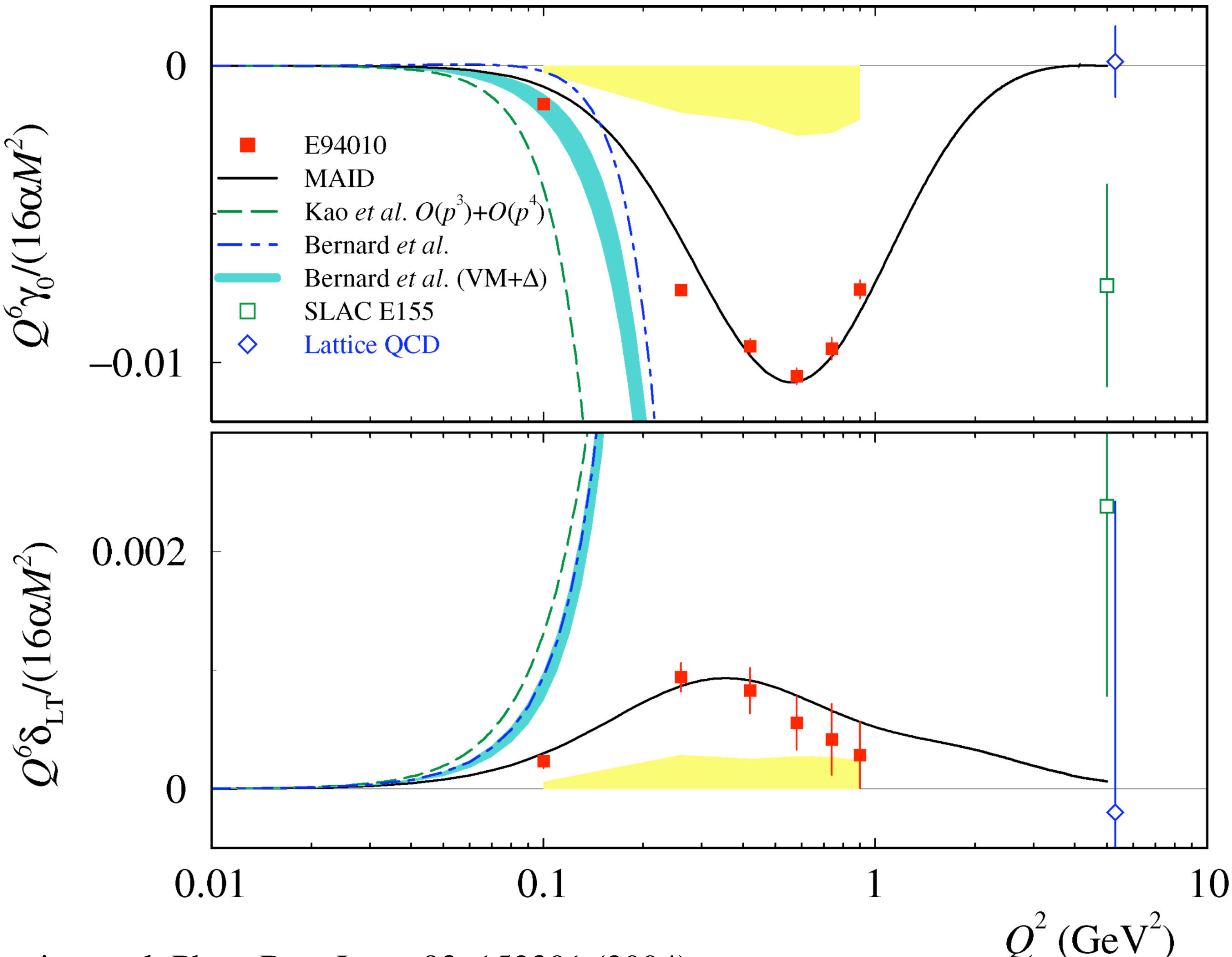
$$\delta_{LT}(Q^2) \to \frac{1}{3}\gamma_0(Q^2), \quad Q^2 \to \infty$$

Spin polarizabilities at low Q^2

M. Amarian et al.,
Phys. Rev. Lett. 93, 152301 (2004)



Q^2 evolution of the Spin Polarizabilities



M. Amarian et al. Phys. Rev. Lett. 93, 152301 (2004)

Conclusion

- The determination of Γ_1 below Q^2 of 1 GeV, dominated by the resonance contributions, is important to gauge the size higher twists and to extract the twist-4 matrix element f2.
- Higher twists are overall small which might indicate that "global duality" holds
- The Burkhardt-Cottingham sum rule seems verified in ^{3}He and the neutron within errors for $Q^2 < 1$ GeV^2; a region dominated by the elastic and Delta resonance contributions which approximately cancel each other.
- Neutron d_2^n resonance contribution is small but finite. d_2^n is dominated by the elastic contribution below 1 GeV^2
- Precision measurements of g_1 and g_2 in the range $1 < Q^2 < 4$ GeV^2 are needed for reducing the error on the extraction of the color polarizabilities. This needs to be pursued at JLab 11 GeV.
- More investigation is needed to understand the discrepancy between chiral perturbation calculations and the data for the spin polarizabilities δ_{LT} at low Q^2