Introductory Remarks on Duality

Carl E. Carlson

Particle Theory Group

College of William and Mary

Williamsburg, VA, USA

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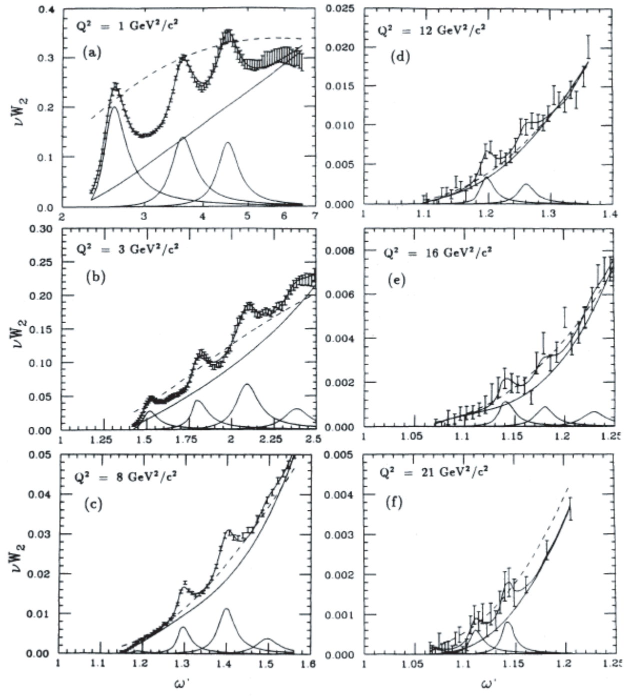
Theoretical remarks on duality in electron scattering

- What duality is—mainly talk about electron scattering.
- Duality when pQCD is applicable
- The disappearing Delta(1232)
- Mass dependences
- Longitudinal and polarized structure functions
- QM models for duality
- Application in a related field: atomic HFS calculations
- What to look for with high $k_\perp$ semi-exclusive data

Duality in electron scattering has been known since 1970 (Bloom-Gilman); here is figure from Stoler (1991). [$\omega' \approx 1/x$]



Remarks

In Bloom-Gilman duality, look, initially, at resonance bumps in F_2 and average them over some region.

Let $\langle \ldots \rangle$ denote the average over a region including the chosen resonance.

Compare $\langle F_2(x, Q^2) \rangle$ for real data at low Q^2, in the resonance region, to the same quantity, same x, but for the scaling curve evolved to the same Q^2. They are pretty much the same.

Won't discuss: reduced F_2 evolution in the resonance region? try BHL, Banff lecture, 1981

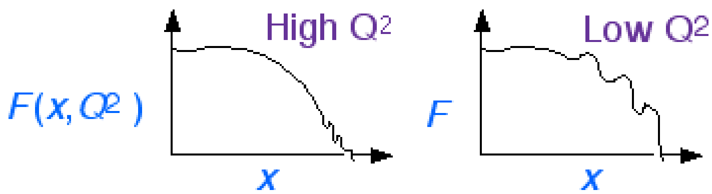
Broadly, the single quark reaction rate determines accurately the reaction rate for the entire process (including FSI)—on the average.

"Proof", or at least "Demystification," of duality offered by DeRújula, Georgi, and Politzer in 1977. I would like a more explicit understanding.

May note that duality can be gotten in (at least) two ways:

- The peak always remains visible, and the signal to continuum S/C remains constant

- The peak gets washed out with rising Q^2, but the average over the region that includes the resonance satisfies duality



There always is a resonance region. As Q^2 increases it slides closer to the endpoint $x = 1$.

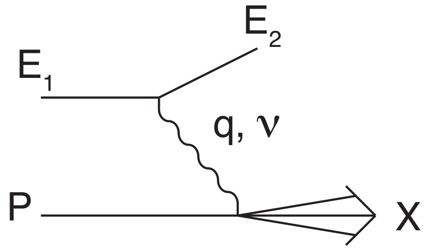
Wish, in physics professor style, to show that the constancy of the S/C at all Q^2 is "predicted" by QCD given

- $(1-x)^3$ for the F_2 scaling curve as $x \to 1$
- pQCD scaling (in Q^2) of leading (helicity conserving) resonance form factor

Works for most known resonances.

explicitly $\longrightarrow \ldots$

Deep inelastic scattering (DIS)



From general definitions of structure functions,

$$\frac{d\sigma}{d\Omega_{lab}\,dx} = \frac{\sigma_{NS}}{x}\Big(F_2(\nu,Q^2) + 2xF_1(\nu,Q^2)\cdot 2\tau\tan^2(\theta/2)\Big). \tag{1}$$

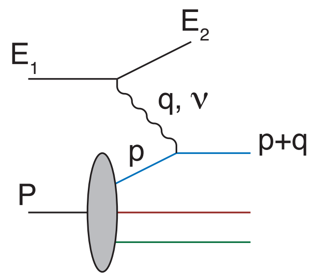
where

$$x = x_{Bj} \equiv \frac{Q^2}{2M\nu}\,, \qquad \tau \equiv \nu^2/Q^2 \tag{2}$$

and the “no structure” cross section is

$$\sigma_{NS} \equiv \frac{4\alpha^2\cos^2(\theta/2)}{Q^4}\,\frac{E_2^3}{E_1} \tag{3}$$

In the parton model,



$$\frac{d\sigma}{d\Omega_{lab}\,dx} = \sum_a e_a^2 \frac{d\sigma}{d\Omega_{lab}} \frac{dN_a}{dx} = \sum_a e_a^2 \frac{d\sigma}{d\Omega_{lab}} f_a(x) \tag{4}$$

Here: $x \equiv \xi =$ percentage of proton's momentum carried by struck quark.

Need: proof that this $x = \xi$ is same as the previous $x = x_{Bj}$. Will show!

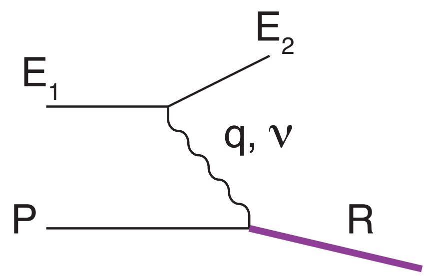
For pointlike spin-1/2 partons,

$$\frac{d\sigma}{d\Omega_{lab}} = \sigma_{NS}\left\{1 + 2\tau \tan^2(\theta/2)\right\} \tag{5}$$

Hence,

$$F_2(\nu, Q^2) = x \sum_a e_a^2 f_a(x) = F_2(x) \qquad and \qquad 2xF_1 = F_2\,. \tag{6}$$

Next: resonance production



Helicity matrix elements (good for unified treatment with elastic scatt.)

$$G_m = \left\langle R, \lambda' = m - \frac{1}{2} \middle| \epsilon_m \cdot J \middle| N, \lambda = \frac{1}{2} \right\rangle \Big/ (2m_N) \qquad (7)$$

Cross section for stable resonances,

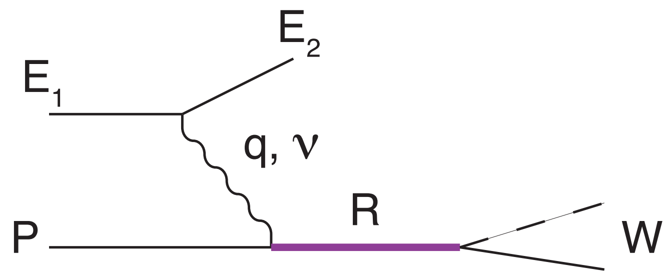
$$\frac{d\sigma}{d\Omega_{lab}} = \frac{\sigma_{NS}}{1+\tau} \left(1 - \frac{m_R^2 - m_N^2}{2m_N E_1}\right)^{-1} \left(G_0^2 + \frac{1}{2\epsilon}\left(G_+^2 + G_-^2\right)\right) \qquad (8)$$

with $$1/\epsilon \equiv 1 + 2(1+\tau)\tan^2(\theta/2)\,.$$

(For the elastic case,

$$G_E = G_0\,, \qquad G_M = \sqrt{2\tau}G_+\,, \quad \text{and} \quad G_- = 0. \qquad) \qquad (9)$$

Of course, resonances decay,



With Breit-Wigner form for propagator,

$$\frac{d\sigma_R}{d\Omega_{lab}\,dx} = \frac{\sigma_{NS}}{1+\tau}\,\frac{\tau}{\pi}\,\frac{4m_N^2 m_R \Gamma_R}{\left(W^2 - m_R^2\right)^2 + m_R^2\Gamma_R^2}\left(G_0^2 + \frac{1}{2\epsilon}\left(G_+^2 + G_-^2\right)\right) \tag{10}$$

At peak ($W = m_R$)

$$\frac{d\sigma_R}{d\Omega_{lab}\,dx} = \frac{\tau\,\sigma_{NS}}{1+\tau}\,\frac{4m_N^2}{\pi m_R \Gamma_R}\left(G_0^2 + \frac{1}{2\epsilon}\left(G_+^2 + G_-^2\right)\right) \tag{11}$$

Compare this to the DIS cross section, after working it into the form

$$\frac{d\sigma_{DIS}}{d\Omega_{lab}\,dx} = \frac{\tau\,\sigma_{NS}}{1+\tau}\,\frac{1}{x}\left(F_L + \frac{1}{\epsilon}F_T\right) \tag{12}$$

For latter, replaced the $\tan^2(\theta/2)$ using $1/\epsilon$ and defined

$$\begin{aligned} F_T &= 2xF_1 \\ F_L &= \left(1 + \frac{1}{\tau}\right) F_2 - 2xF_1 \end{aligned} \tag{13}$$

Hence for $x \to 1$

$$\begin{aligned} F_T &\propto G_+^2 + G_-^2 \\ F_L &\propto G_0^2 \end{aligned} \tag{14}$$

The LHS depends on x only; the RHS depends on Q^2 only. They are correlated because we fix $W = m_R$,

$$W^2 = (P+q)^2 = m_N^2 + 2m_N\nu - Q^2 \qquad \text{or} \qquad (1-x) = \frac{m_R^2 - m_N^2}{Q^2} \tag{15}$$

the latter for $x \to 1$.

Finish:

The counting rules, which come from QCD and the knowledge that baryons are made from 3 quarks, tell us that

$$G_+^2 \propto Q^{-6} \qquad G_0^2 \propto Q^{-8} \qquad G_-^2 \propto Q^{-10} \tag{16}$$

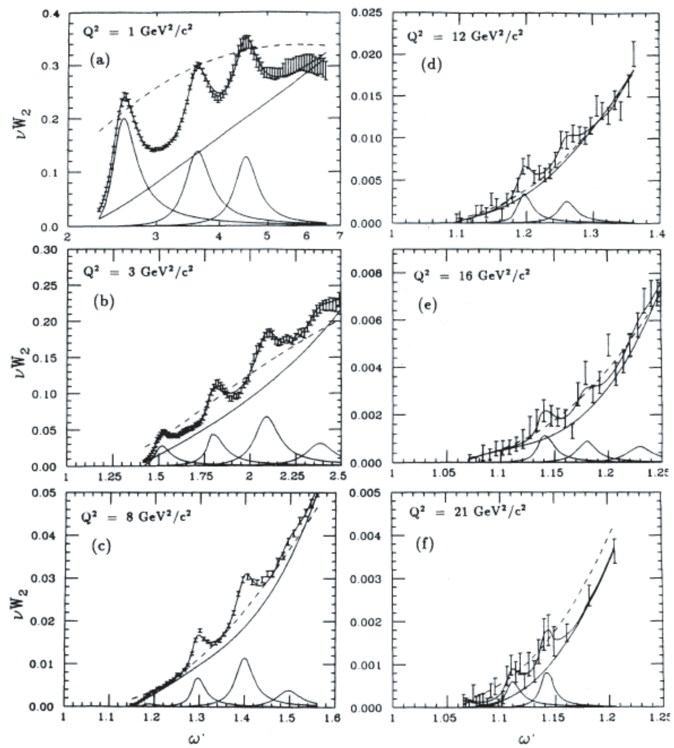
Which says that as the resonances slide down the curves describing $F_{T,L}$, they slide along curves

$$F_T \propto (1-x)^3 \qquad F_L \propto (1-x)^4 \tag{17}$$

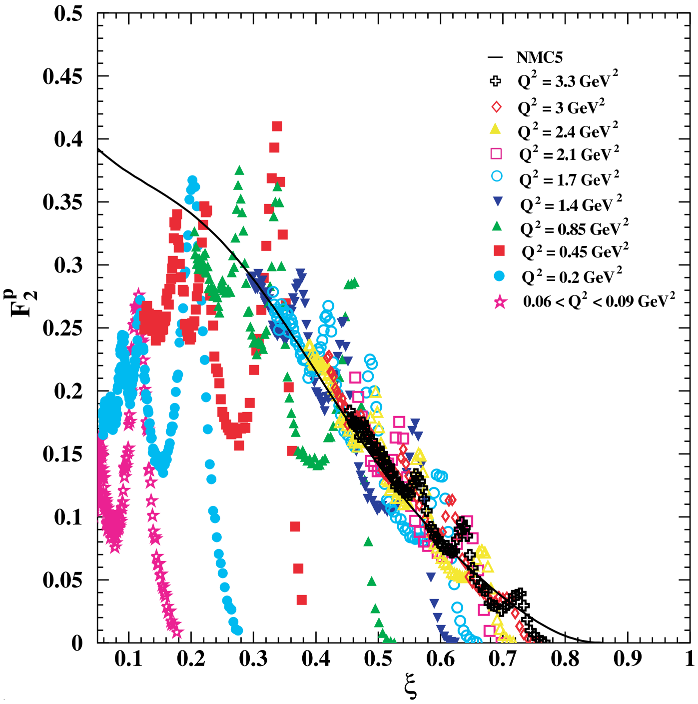
But this is what we know they do anyway, from their own counting rules (and to some decent approximation from data).

Similar analysis works for the polarized structure function g_1.

Stoler (1991) plot again.



JLab data for F_2 (essentially F_T).



The Delta(1232) disappears with increasing Q^2.

First note: Duality is stronger than the disappearing Δ resonance.

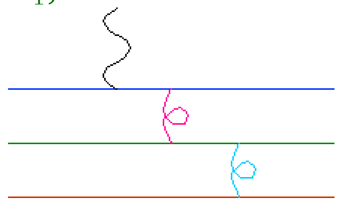
Duality here means average over resonance region matches scaling curve.

It does even for $\Delta(1232)$. As peak falls, background rises, and average/continuum = const. CM93

Conclude: Background knows about the Δ.

Co-conclude: Don't use just simple π-Nucleon Born terms for background.

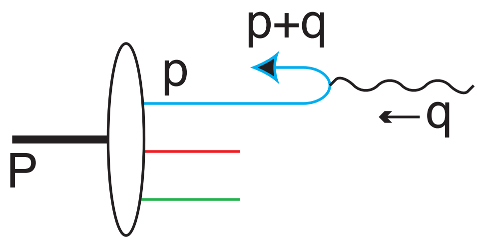
Why Δ disappears: the leading helicity form factor is anomalously small. This is result of calculation, equivalent to pQCD calculation of high Q^2 nucleon elastic form factor F_1, based on



integrated with distributions amplitudes ($\approx$ wave functions) for incoming and outgoing baryons.

What we mainly see in even in spin-summed $N \to \Delta$ are asymptotically subleading amplitudes. Lousy circumstance for pQCD.

Nachtmann: relate momentum fraction ξ to x_{Bj}, w/ proton mass effects.



Define ξ using light front variables ($p^{\pm} \equiv p^0 \pm p^3$),

$$\xi \equiv p^+/P^+$$

$$\text{and} \qquad x_{Bj} \equiv Q^2/(2m_N\nu) \tag{18}$$

Work in Breit frame:

$$\begin{aligned} q &= (q^+, q^-, q_\perp) = (-Q, Q, 0_\perp) \\ p &= (p^+, 0, 0_\perp) = (\xi P^+, 0, 0_\perp) \\ P &= (P^+, M^2/P^+, 0_\perp) \end{aligned} \tag{19}$$

Neglected quark mass and $p_\perp$, but not nucleon mass.

Quark is on-shell:

$$(p+q)^2 = p^2 \qquad \Longrightarrow \qquad 2p \cdot q = \xi P^+ Q = Q^2 \tag{20}$$

Standard definition: $2P \cdot q = 2M\nu = P^+ Q - QM^2/P^+$

Solve for P^+:

$$P^+ = \left(\frac{1}{2}\right) \frac{2M\nu}{Q} \left(1 + \sqrt{1 + Q^2/\nu^2}\right)$$

Solve for ξ:

$$\xi = \frac{2}{1 + \sqrt{1 + Q^2/\nu^2}}\, x = \frac{2}{1 + \sqrt{1 + 4M^2 x^2/Q^2}}\, x \tag{21}$$

- internal variable measured by external variable!
- modified by mass correction; significant at low Q^2
- quark mass corrections considered [Greenberg-Bhaumik (1971); Barbieri et al (1974)].

Re: Longitudinal structure function

Still expect duality to work. Already noted predicted by QCD, given

- $(1-x)^4$ for scaling curve as $x \to 1$
- pQCD scaling (in Q^2) of longitudinal resonance form factor CM90

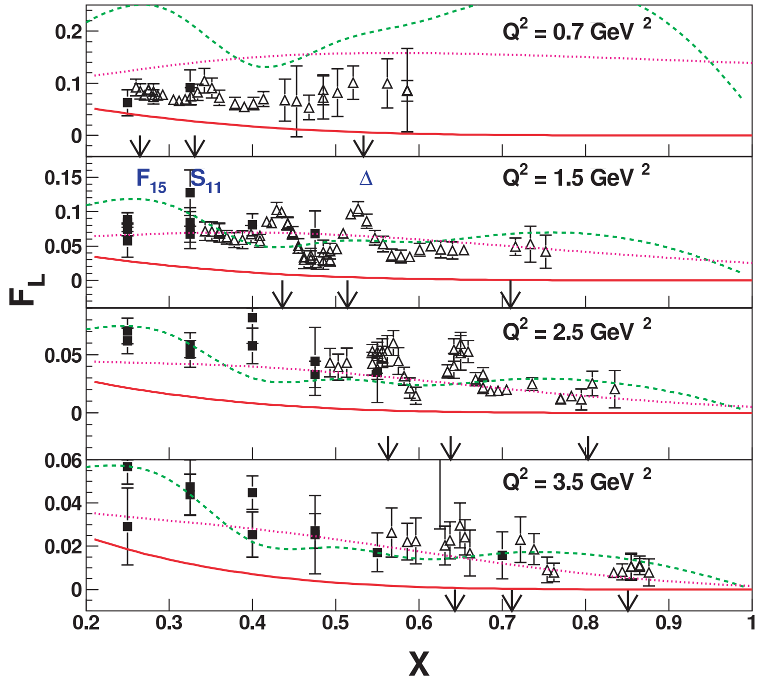
But there may be some differences. E.g.,

- S/C = constant even for Δ?

  Depends on next-to-leading helicity ampl.

- Maybe the Roper, $N^*(1440)$, will appear. Interesting: **If** Roper is hybrid baryon (qqqg), its leading electroproduction amplitude is asymptotically $1/Q^2$ smaller than qqq, but its longitudinal amplitude has normal falloff. CM91

JLab [△] and SLAC [■] data for F_L.



dashed: Alekhin model
solid: MRST at NNLO w/o target mass effects
dotted: MRST at NNLO w/ target mass effects

Re: g_1, expect scaling to work also—at high enough Q^2

Anticipate failure at low Q^2 because of $\Delta(1232)$:

for a resonance contribution

$$g_1 \propto |G_+|^2 - |G_-|^2 + (another\ term)$$

The scaling curve comes from higher Q^2, where G_+ dominates. Hence the g_1 scaling curve is a positive function.

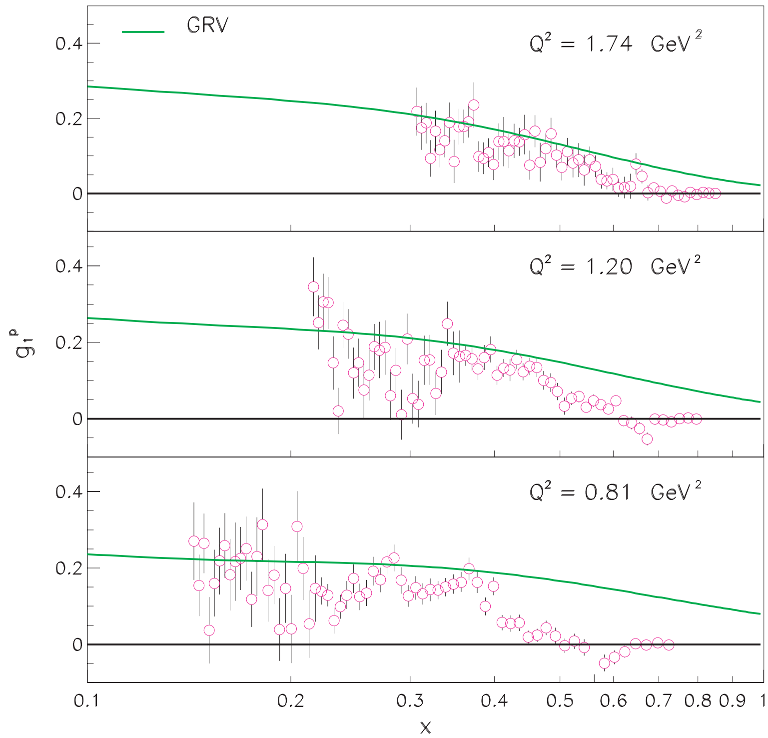
But, at low Q^2, the $N \to \Delta$ transition is known to be an M1 transition, with the E2 amplitude nearly zero, so that

$$E2 \propto -\sqrt{3}\, G_+ + G_- \approx 0.$$

Hence $|G_-|$ is larger than $|G_+|$, and g_1 is negative.

That the Delta disappears could be useful for scaling.

JLab data for g_1.



Low in region of $\Delta(1232)$ resonance.

Modeling scaling from QM and bound state models

Ingredients: bound states in QM, pointlike particles, confining potential.

E.g., Paris & Pandharipande; Isgur, Jeschonnek, Melnitchouk, & Van Orden; Gurvitz & Rinat; Greenberg; Pace *et al.*

A bound state starts in the ground state, gets hit, goes to an excited state, with some transition form factor. One can do a exactly solvable relativistic harmonic oscillator to get definite results, like

$$F_{0\to N} = \frac{i^N}{\sqrt{N!}} \left(\frac{|\vec{q}\,|}{\beta\sqrt{2}} \right)^N e^{-\vec{q}^{\,2}/4\beta^2} \tag{22}$$

The transition form factor is small at low and high $\vec{q}^{\,2}$, and peaks at some $\vec{q}^{\,2}$ that happens to be at the same x_{Bj} for each transition form factor regardless of the final state.

(The practitioners prefer to define the scaling variable as

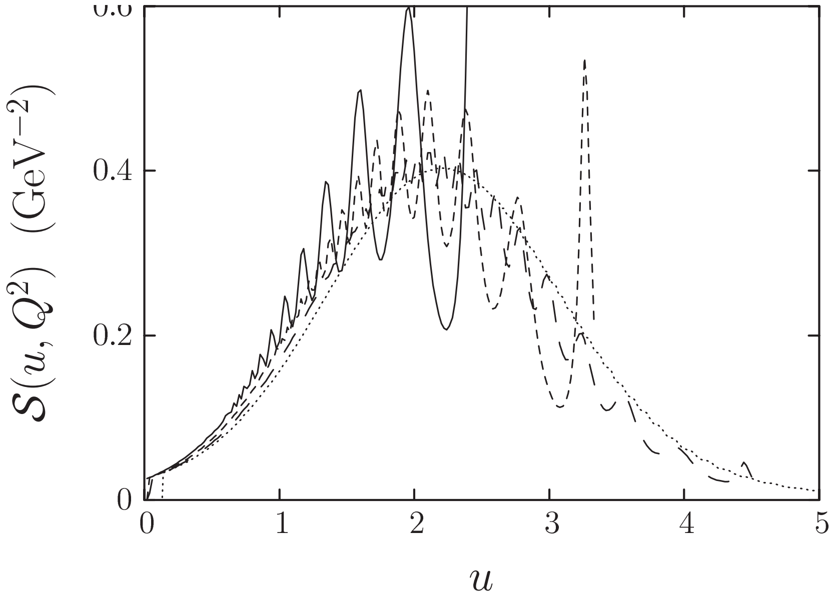
$$u \equiv \frac{m}{M} x_{Bj} \tag{23}$$

where m is the mass of the light struck quark and M is the mass of the heavy quark it is bound to.)

The response is

$$\frac{d\sigma}{dE_f\, d\Omega_f} \propto \mathcal{S} \sim \sum |F_{0\to N}|^2\, \delta(E_N - E_0 - \nu) \tag{24}$$

which is a collection of delta-functions. They can be given some artificial width for visual purposes, and then one gets a set of curves that looks like the figure.



$Q^2 = 0.5$ (solid), 1 (short-dashed), 2 (long-dashed), and 5 GeV^2 (dotted). [from IJMV]

Amazing: the limiting curve is the same as one would get from the initial state wave function, treating the final "quarks" as free.

Another use of experimental g_1 information: “Zemach radius,” or

Proton structure and atomic hyperfine splittings

HFS numbers:

$$E_{\rm hfs}(ep) = 1\,420.405\,751\,766\,7(9)\ \mathrm{MHz},$$
$$E_{\rm hfs}(e\mu) = 4\,463.302\,78(5)\ \mathrm{MHz}.$$

Former is 14 figures, and accuracy on latter is 11 ppb.

Leading order calculation due to Fermi,

$$E_F = \frac{8}{3\pi}\,\alpha^3 \mu_B \mu_N \frac{m_e^3 m_N^3}{(m_N + m_e)^3}$$

where “N” stands for either p or μ^+ and the μ_i’s are magnetic moments.

There are corrections, as

$$E_{\rm hfs}(ep) = E_F(ep) \times (1 + \Delta_{\rm QED} + \Delta_Z + \Delta_{\rm pol} + \Delta_R)$$

- $\Delta_{\rm QED}$ same for hydrogen and muonium.
- Δ_R is recoil and radiative recoil correction. Also really QED.
- $\Delta_Z + \Delta_{\rm pol}$ together are proton structure corrections.
- Δ_Z is purely elastic part of correction, worked out by Zemach,

$$\Delta_Z = -2\alpha m_e \langle r \rangle_Z \times \left(1 + \delta^{\rm radiative}\right)$$

where $\langle r \rangle_Z$ is the "Zemach radius,"

$$\langle r \rangle_Z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[G_E(Q^2) \frac{G_M(Q^2)}{\mu_p} - 1 \right] ,$$

for $G_M(0) = \mu_p$. [$\delta^{\rm radiative}$ is a known $\mathcal{O}(\alpha)$ correction ($\approx 1.53\%$).]

• The polarizability corrections come mainly from inelastic intermediate states:

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2),$$

$$\begin{aligned}
\Delta_1 &= \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + 4m_p\int_{\nu_{\text{th}}}^\infty \frac{d\nu}{\nu^2}\beta\left(\frac{\nu^2}{Q^2}\right)g_1(\nu, Q^2)\right\}, \\
\Delta_2 &= -12m_p\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{\text{th}}}^\infty \frac{d\nu}{\nu^2}\beta_2\left(\frac{\nu^2}{Q^2}\right)g_2(\nu, Q^2),
\end{aligned}$$

F_2 is Pauli form factor, g_1 and g_2 are spin-dependent structure functions,

$$\begin{aligned}
\beta(\tau) &= \frac{4}{9}\left(-3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}\right), \\
\beta_2(\tau) &= 1 + 2\tau - 2\sqrt{\tau(\tau+1)}\,.
\end{aligned}$$

• Faustov and Martynenko (2002) evaluated $\Delta_{\text{pol}} = 1.4 \pm 0.6$ ppm.

How to use:

- Can use proton HFS alone and calculated QED and infer $\Delta_Z + \Delta_{\rm pol}$.
- Can use difference of proton and muonium HFS to eliminate (big) calculated QED correction, and infer $\Delta_Z + \Delta_{\rm pol}$
- Either way, can use calculated $\Delta_{\rm pol}$ to infer Δ_Z or $\langle r \rangle_Z$, getting

$$\langle r \rangle_Z = 1.043(16)\ {\rm fm}$$

which then becomes a constraint on any form factor parameterization.

- Or one can take "best" form factor knowledge to calculate $\langle r \rangle_Z = 1.086(12)\ {\rm fm}$, and infer $\Delta_{\rm pol} = 3.05(49)$.
- Either way $\Delta_{\rm pol}$ is weak point. Its evaluation is sensitive to $g_{1,2}$ at low Q^2. The newer, better data proves important. (Side note: if GDH sum rule fails, then proton HFS is infinite.)

• Re-evaluation of $\Delta_{\rm pol}$ underway (Griffioen *et al.*). Preliminary look indicates central value about half of old result, and uncertainty limit can be about half of what F&S found.

• Read more about all but very latest: Brodsky, Carlson, Hiller, Hwang, PRL (2005)

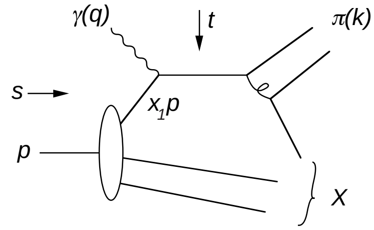
• Cf.: Volotka *et al.* (physics/0405118); Friar & Sick (PL B 2004); Dupays *et al.* (PR A, 2003).

Possible duality with semi-exclusive data?

Consider pions produced at high transverse momentum in

$$\gamma(q) + p \to \pi(k) + X.$$

sometimes the pion is produced directly (at short range), as in



.

There will be a scaling function, $F(x_1, t, q^2)$, dependent mainly on x_1,

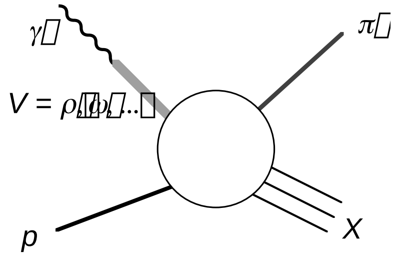
$$x_1 \equiv \frac{-t}{s + u - 2m_N^2 - q^2 - m_\pi^2},$$

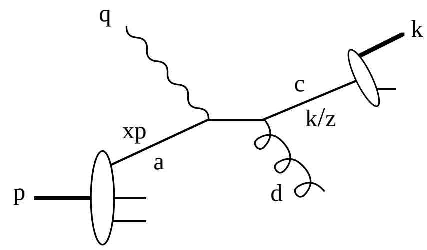
where s, t, and u are Mandelstam variables, and—for the direct process—x_1 is the momentum fraction of the struck quark. ACW,BDHP

For scaling, need s, t, u, and m_X large.

Go into the resonance region with fixed q^2 and diminishing t. Will we see an inclusive-exclusive connection as in the DIS case?

To see scaling, in addition to large s, t, u, and m_X; also competing processes, such as the soft or VMD process, and fragmentation, must be small.

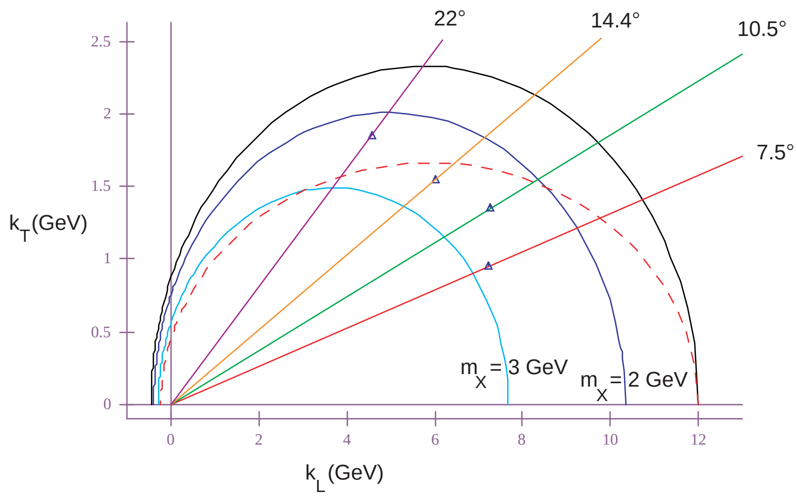




VMD serious at 12 GeV. Use isolation cut to emphasize direct process. Decrease size of VMD process by using spacelike off-shell photon, rather than real photon, $1/m_\rho^2 \to 1/\left(Q^2 + m_\rho^2\right)$. Consider latter.

Have means to calculate direct process and estimate VMD process.

Consider $\gamma(q) + p \to \pi^+(k) + X$ with $E_\gamma = 12$ GeV, $Q^2 = 1$ GeV^2.



Solid ellipses show $m_X = m_N$, 2 GeV, 3 GeV.

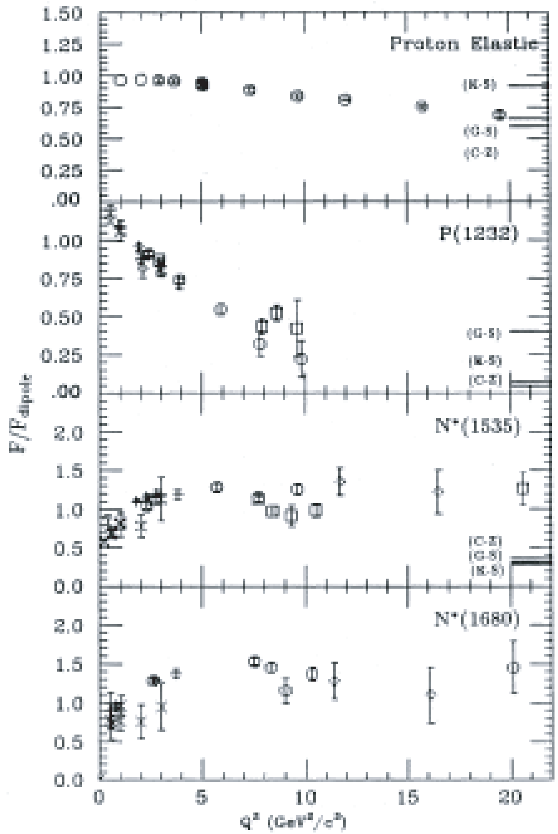
Dashed ellipse is for fixed $x = 0.5$.

Direct processes dominate above a to right of small triangles.

Final remarks:

- Have reason to think duality works when pQCD is applicable.
- Duality seems to appear also in QM models with confinement.
- Still want more general understanding, particularly since we want to use duality in applications like studying the structure functions for $x_{Bj} \to 1$ using data in the resonance region.
- Have useful applications of data in related area, as in calculating proton HFS splitting.
- Can expect duality in other hadronic physics processes, for example in semiexclusive processes.

The End


Proton Elastic
P(1232)
N*(1535)
N*(1680)
F/F_{dipole}
Q^2 (GeV^2/c^2)