The strong coupling constant at low Q^2

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First Workshop on Quark-Hadron Duality and the Transition to pQCD

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Experimental determination of α_s

Moments of structure functions.

OPE: $$M = \sum_{twist} \frac{\mu_t(\alpha_s)}{Q^{2-t}}$$

Ex: generalized Bjorken sum rule:

$$\int g^p_1 - g^n_1 \, dx = \Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{M^2}{9Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon axial charge (Bjorken limit)

pQCD radiative corrections

Higher Twists

If Q^2 is large enough, Higher twists are negligible

$\Rightarrow \alpha_s$ can be extracted.

But if we are interested by low Q^2, higher twists cannot be neglected and those are not well known...

Effective coupling constant

A way out is to fold the higher twists and pQCD radiations into the definition of the coupling constant (Grunberg, Brodsky *et al.*).

$\Rightarrow$effective coupling constant

$$\Rightarrow \Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_s^{eff}}{\pi}\right)$$

By doing so we obtain a coupling constant that is:

- Extractable at any Q^2
- Free of divergence
- Not renormalization scheme dependent
- Analytic when crossing quark threshold

but that is:

- Process dependent

$\Rightarrow$There is a priori a different α_s^{eff} for each different process.

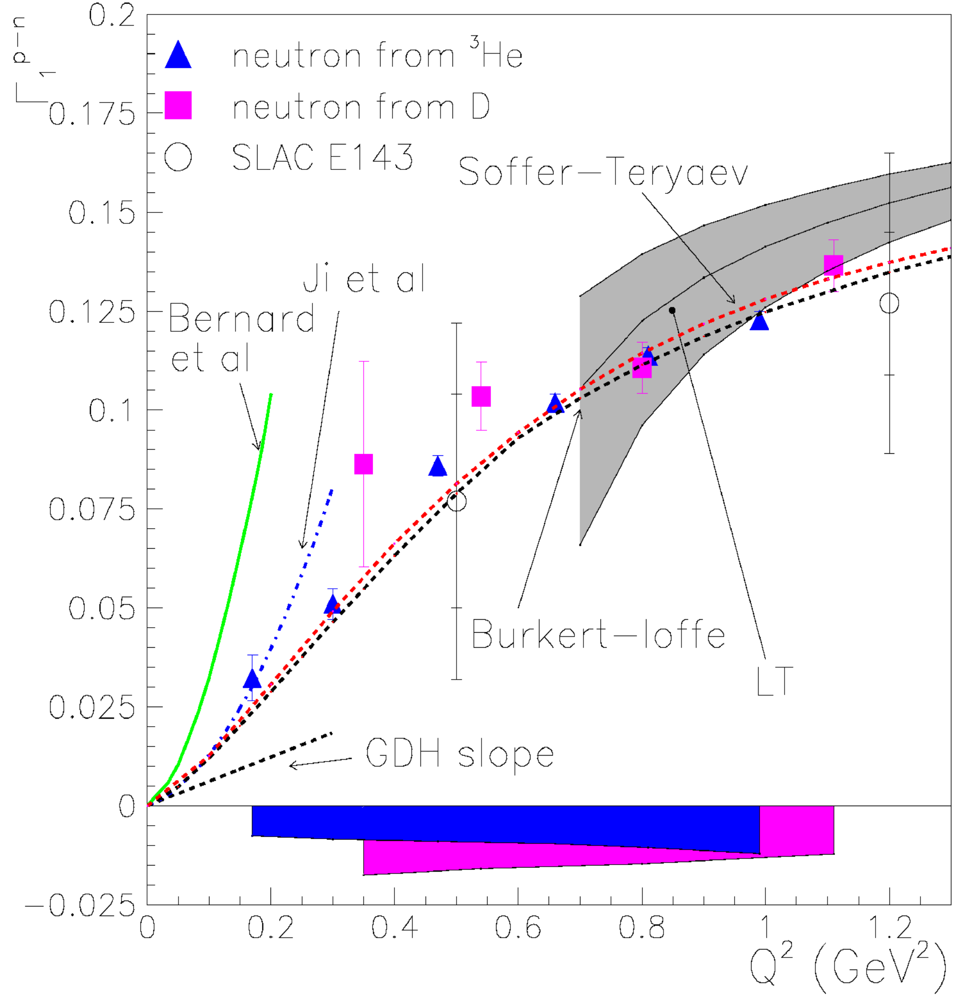
<u>However</u> these α_s^{eff} can be related, so they are not useless quantities.

Measurement of the Bjorken sum at intermediate Q^2

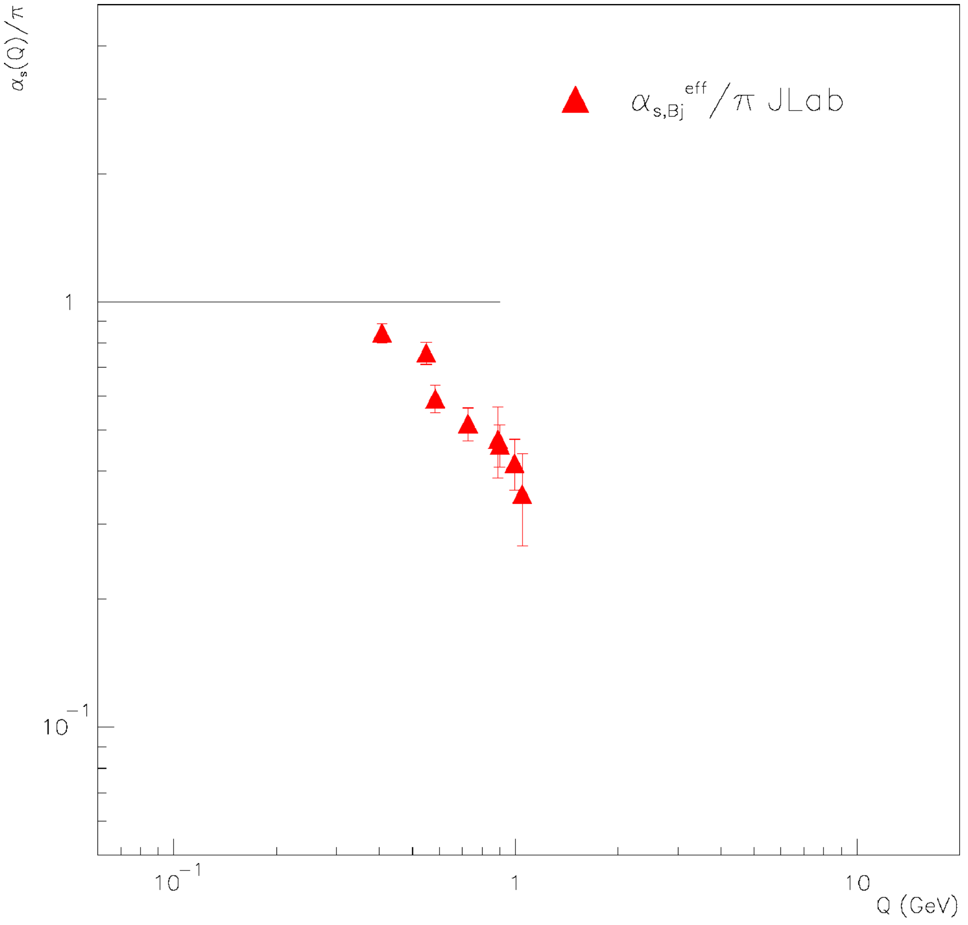
Proton data: Jlab CLAS EG1a

Neutron data: Jlab Hall A E94010 (^3He) and Hall B (ND_3)

(A. Deur *et al.* PRL 93 212001 (2004))



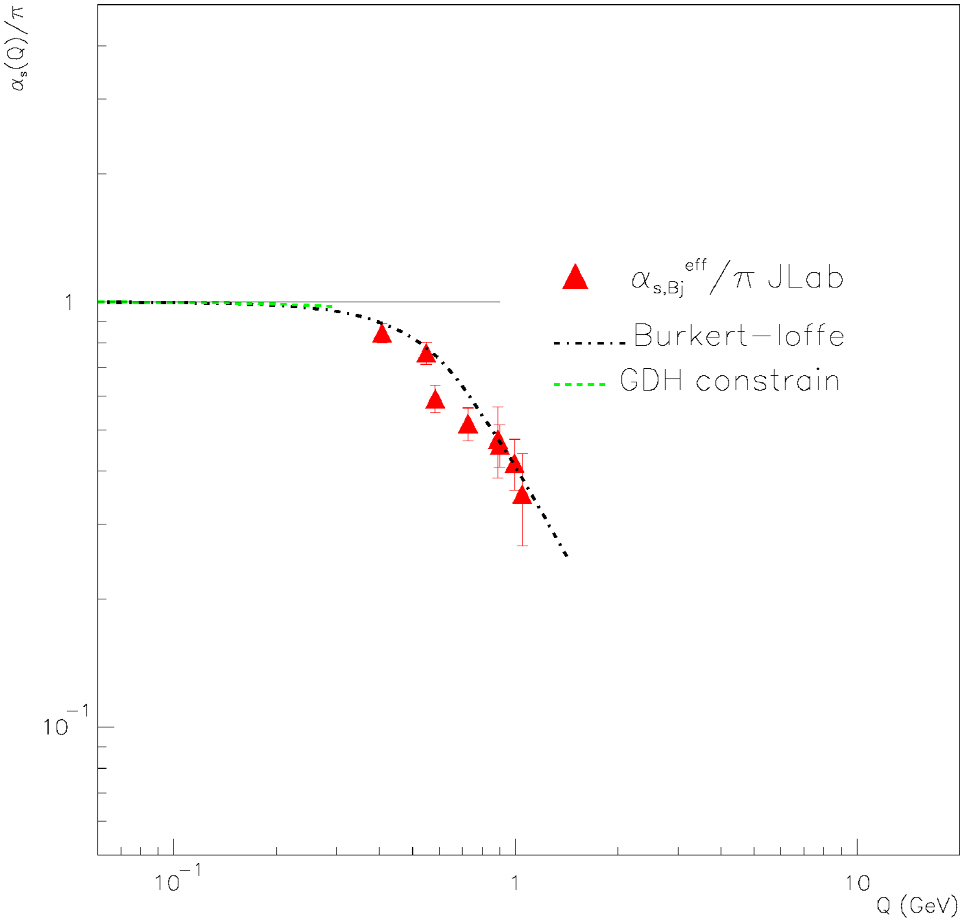
Effective coupling constant at intermediate Q^2 from Bjorken sum



⇒Determination of α_s^{eff} in the parton-hadron transition region.

But we can do more...

Low Q^2



Bjorken and Gerasimov-Drell-Hearn sum are related:

At $Q^2 = 0$, GDH sum rule:

$$\int_{\nu_{thr}}^{\infty} \sigma^{TT} \frac{d\nu}{\nu} = \frac{-2\alpha\pi^2\kappa^2}{M^2} = \frac{16\alpha\pi^2}{Q^2}\Gamma_1$$

κ: anomalous magnetic moment

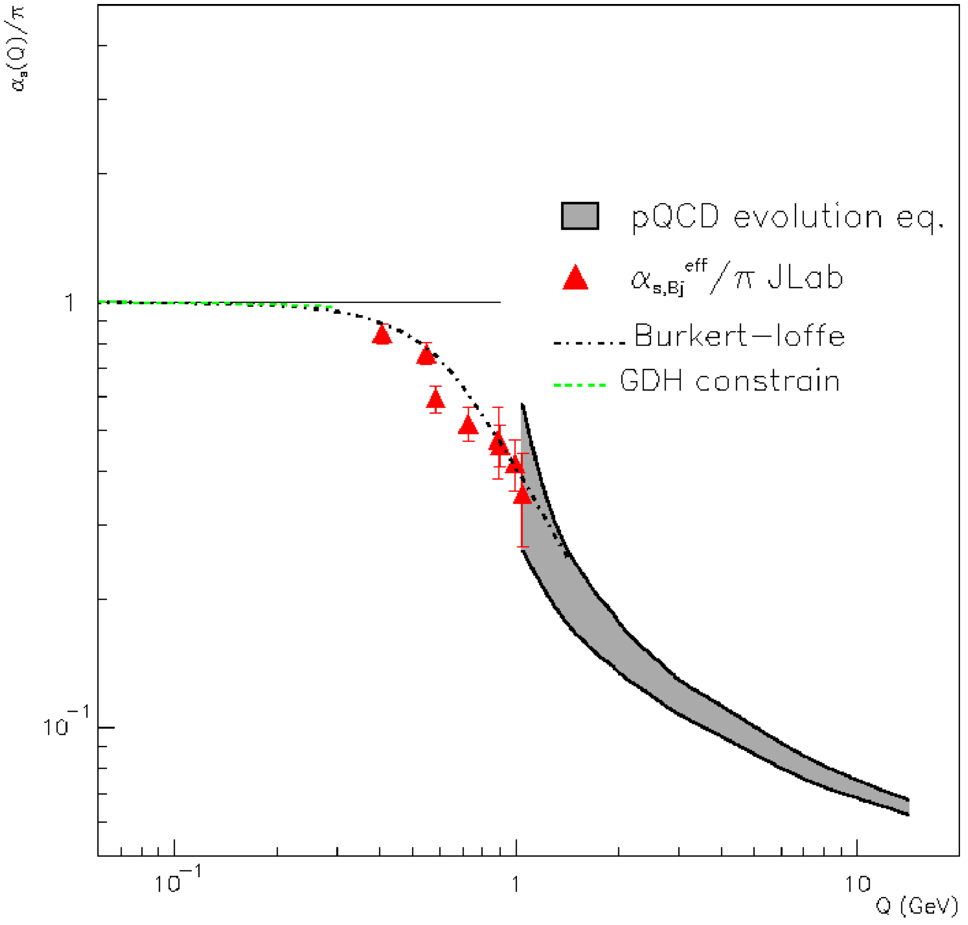
$\Rightarrow$ $Q^2 = 0$ constraints:

$$\Gamma_1^{p\text{-}n} = \frac{Q^2}{16\alpha\pi^2}\left(GDH^p\text{-}GDH^n\right)$$

$$\Rightarrow \begin{cases} \alpha_s^{eff} = \pi \\ \dfrac{d\alpha_s^{eff}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^{\ 2}}{M_n^{\ 2}} - \dfrac{\kappa_p^{\ 2}}{M_p^{\ 2}}\right) \end{cases}$$

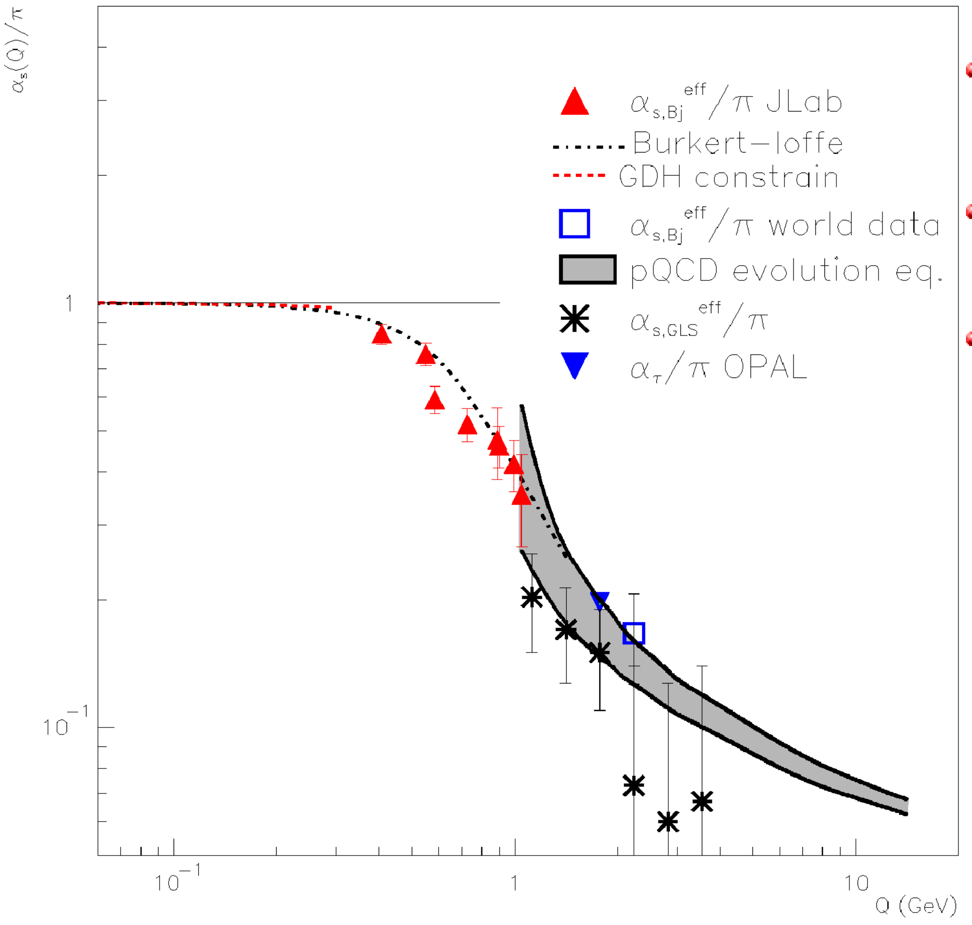
We can also determine α_s^{eff} at large Q^2.

Large Q^2



$$\Gamma_1^{p-n} = \frac{g_A}{6}\left(1-\frac{\alpha_s^{pQCD}}{\pi} - 3.58\left(\frac{\alpha_s^{pQCD}}{\pi}\right)^2 - \ldots\right) = \frac{g_A}{6}\left(1-\frac{\alpha_s^{eff}}{\pi}\right)$$

Other extractions of α_s^{eff}



- World data on Γ_1^{p-n} (CERN, HERMES, SLAC). $\langle Q^2 \rangle = 5$ GeV^2
- α_τ^{eff} using data on hadronic decays of τ leptons. (Brodsky *et al.*)
- $\alpha_{s\ GLS}^{eff}$ using the GLS sum rule:

$$\int F_3 dx = n_v \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) = n_v\left(1 - \frac{\alpha_s^{eff}}{\pi}\right)$$

Number of valence quarks in the nucleon

<u>All in all: we have a parametrization of the strong force at any scale.</u>

Connection with theory

Some warnings:

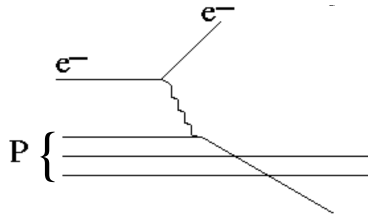
- Many theoretical or phenomenological predictions exist.
- As for the α_s^{eff} experimentally defined, there exist many definitions for α_s^{theory}.
- The connection between these definitions is not fully known.
- The calculations should be viewed as indications rather than firm predictions.

It is still interesting to compare our extraction to calculations to see if they share common features. However we need to make sure we compare similar objects.

Connection with theory

To study the connection between α_s^{eff} and theories, we should recall how α_s comes into Γ_1

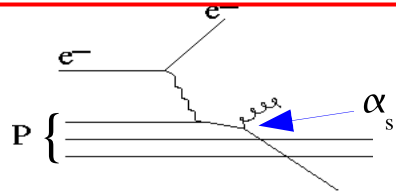
- Bjorken limit:



$$\Gamma_1^{p-n} = \frac{g_A}{6}$$

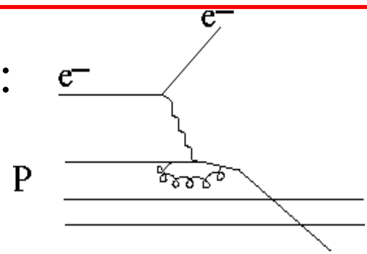
No α_s (free quarks)

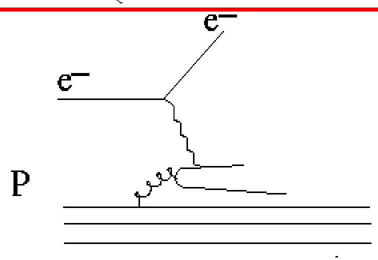
- Finite but very large Q^2:

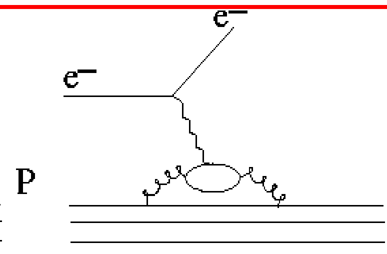


$$\Gamma_1^{p-n} = \frac{g_A}{6}\left(1 - \frac{\alpha_s}{\pi}\right)$$

- Large Q^2:

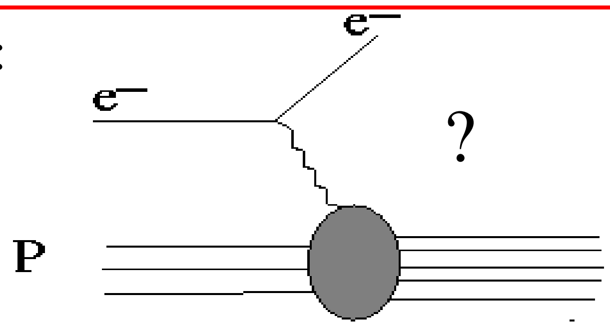






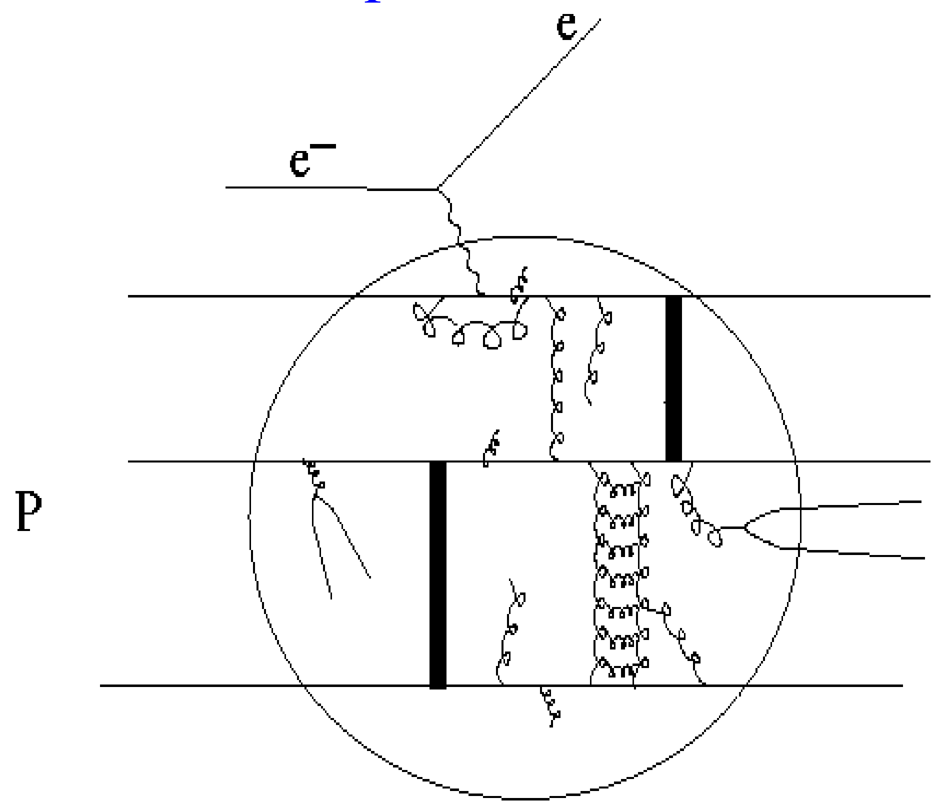
$$\Gamma_1^{p-n} = \frac{g_A}{6}\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 \ldots\right)$$

But for α_s^{eff} we have:



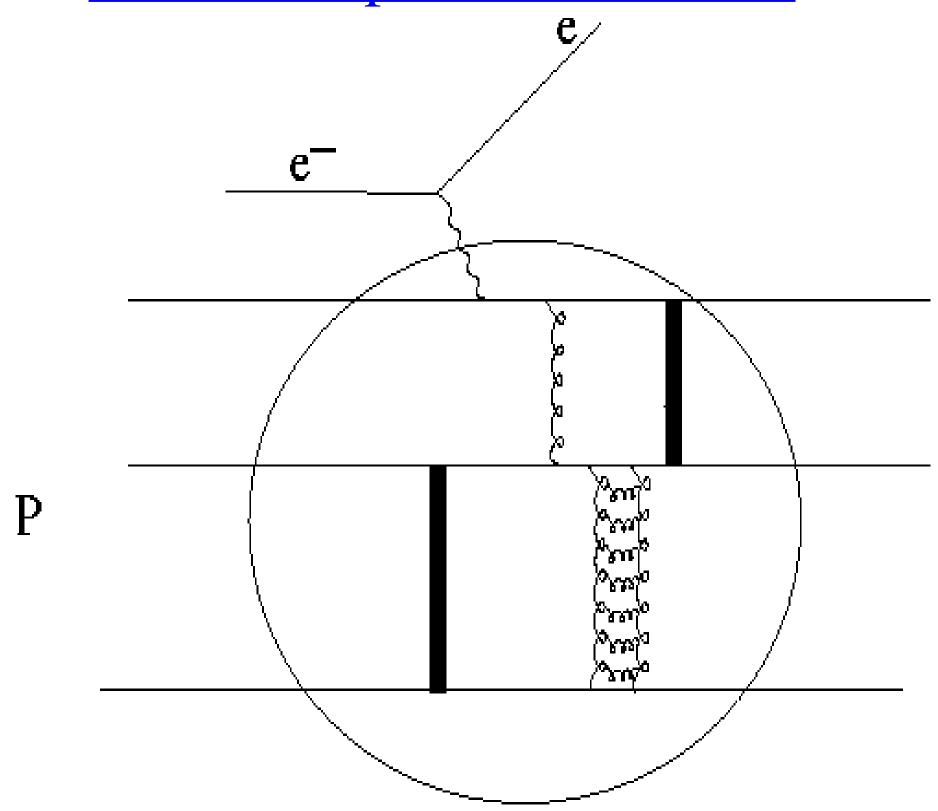
What is the connection between α_s^{eff} and α_s^{theory} ?

In the non-perturbative blob



If we remove the effects of short-distance physics using pQCD evol. eq. we get:

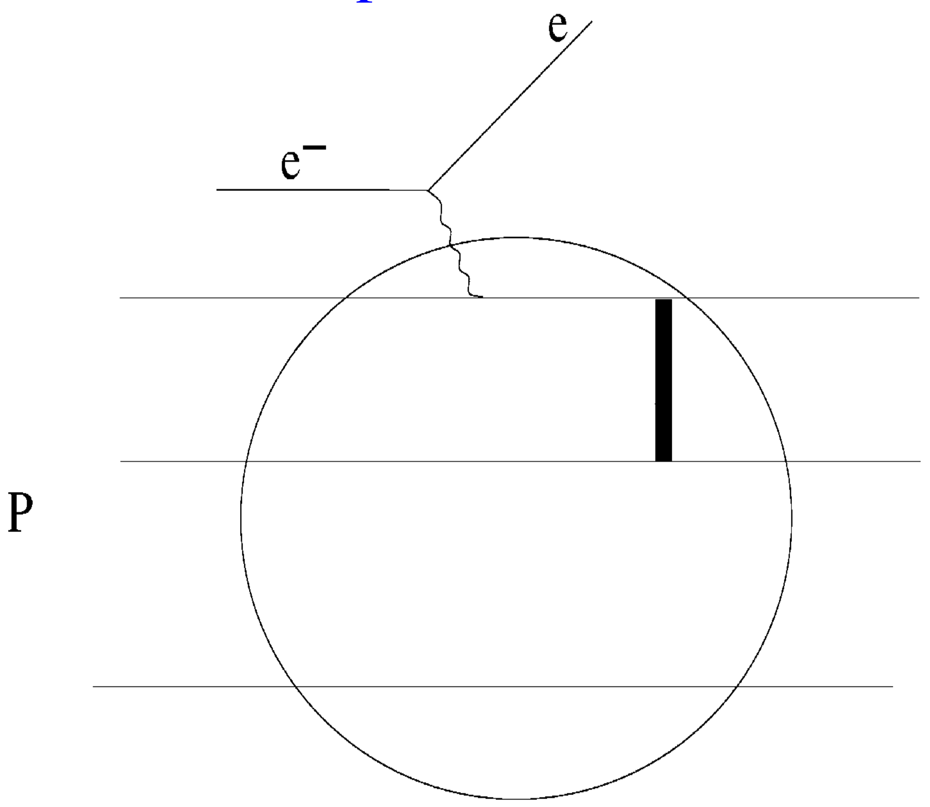
In the non-perturbative blob



Now, if we require no elastic reaction (x=1 excluded from $\Gamma_1^{p\text{-}n}$) and if we use a moment for which the resonant contribution is minimized, we get:

Bjorken sum: the Δ_{1232} contribution, which drives Γ_1^{p} and Γ_1^{n} Q^2-dependence cancels. The rest of the resonances account for ~15% (MAID)

In the non-perturbative blob

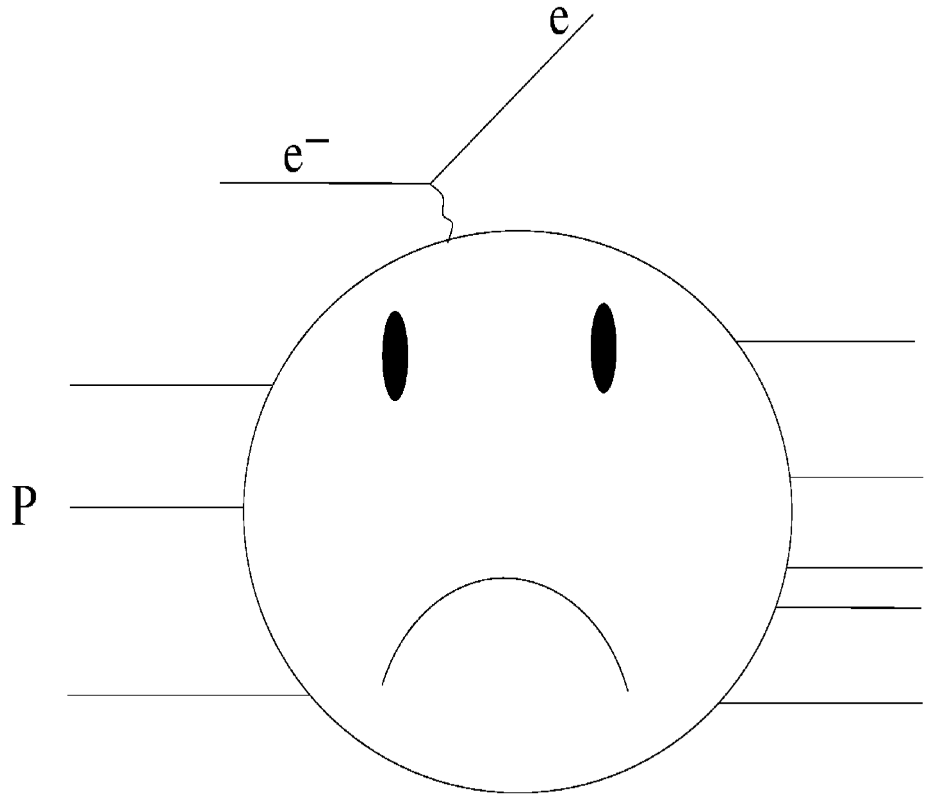


Now, we can use a LO eq. to extract an α_s^{eff} that we can compare to theoretical calculations.

⇒Prescription:

- Use the Bjorken sum rule
- Exclude elastic
- Account for QCD radiative corrections

But trying to see what is in the blob may be too naive

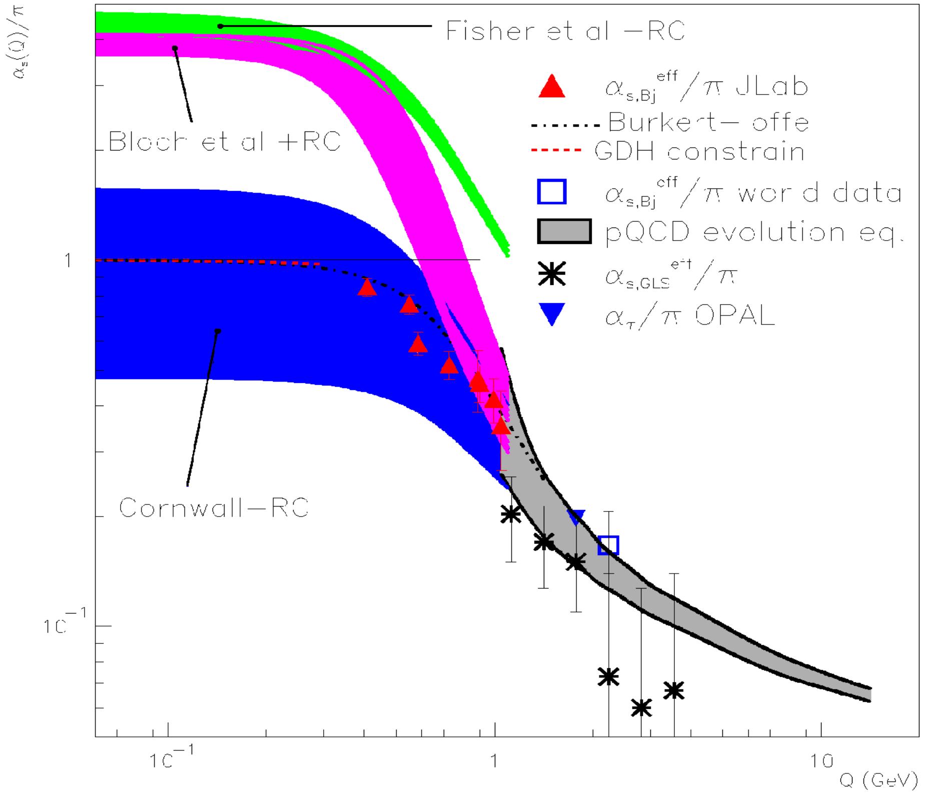


We can use a alternate explanation:

By using a quantity where non-resonant background is largely dominant, we are back to the DIS situation where the connection between the α_s extracted and the calculated coupling constant is direct.

Comparison with theory

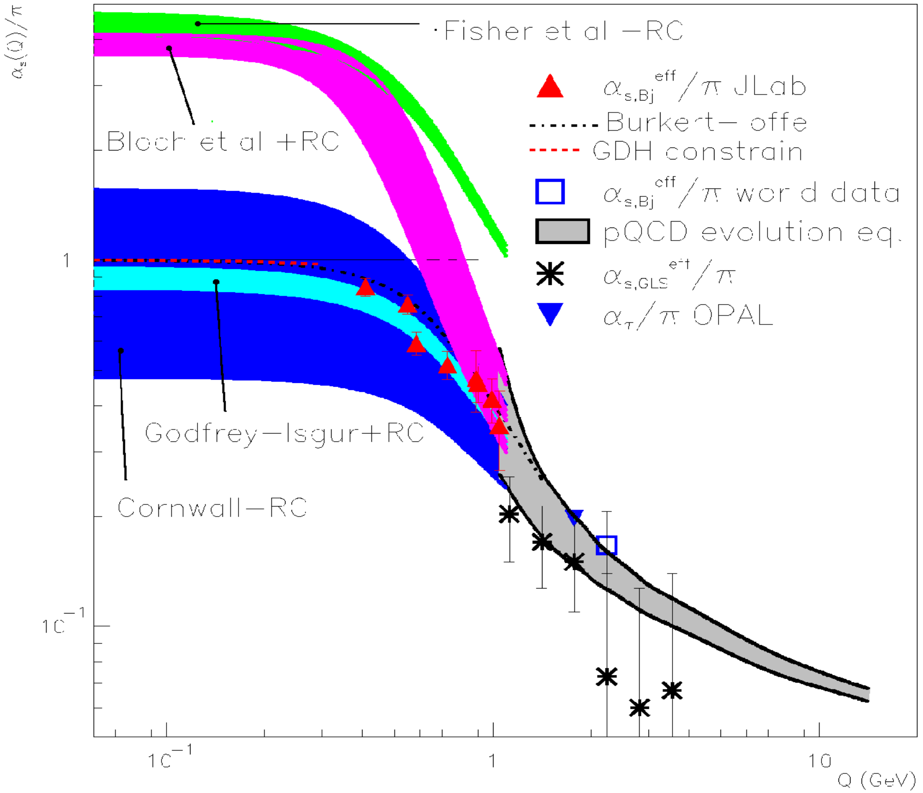
Schwinger-Dyson equations:



Note: The pQCD radiations were applied on the theory results.

Comparison with theory

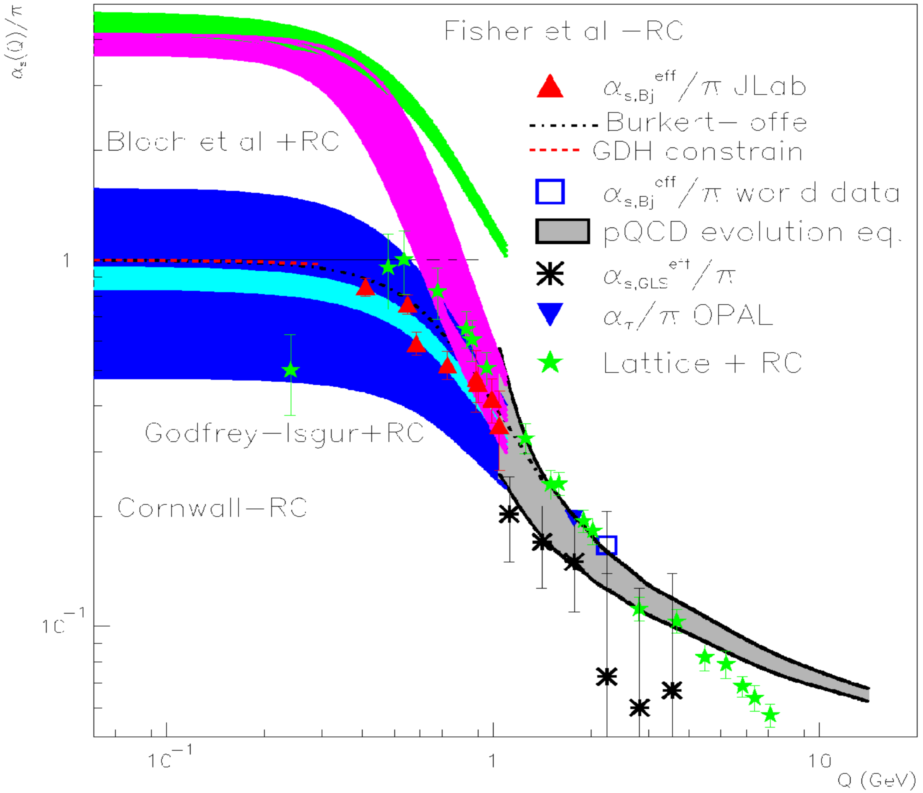
α_s from Godfrey-Isgur quark model:



Note: The pQCD radiations were applied on the model.

Comparison with theory

Lattice QCD:



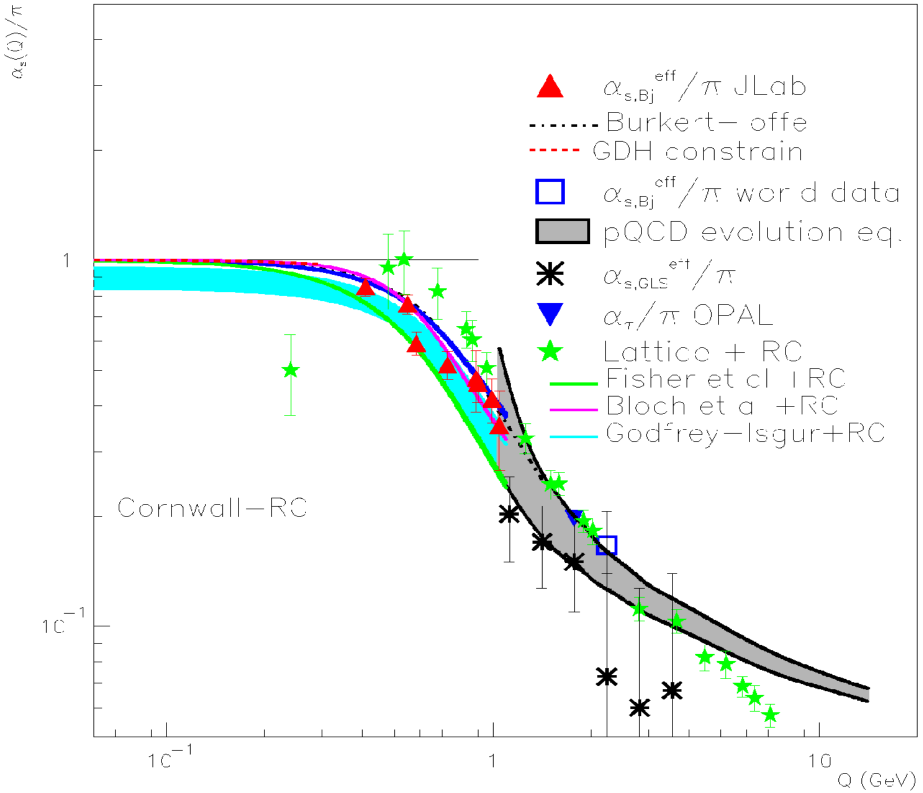
Note: Results are normalized to world data on Bjorken sum.

(<Q^2>=5 GeV^2)

Comparison with theory

Theory results are only indicative. Comparison of Q^2-dependence may be more relevant.

If the SDE are normalized using $Q^2 = 0$ constraints:



⇒ QCD radiative corrections uncertainty is gone.

Remarkable agreement between different estimates of α_s from different QCD sectors.

Summary

- Extraction of an effective coupling constant at any Q^2 using JLab data and sum rules.

- Gives the strength of strong interaction at any scale. Freezing of $\alpha_{s,\ bjorken}^{eff}$ at low Q^2.

- Many ways to define α_s^{eff}. All can be related. However, $\alpha_{s,\ bjorken}^{eff}$ has advantages:
  - Low Q^2 data available
  - Near-real photon data taken and available soon.
  - Sum rules constrain $\alpha_{s,\ bjorken}^{eff}$ at $Q^2 \rightarrow \infty$ and $Q^2 \simeq 0$
  - Quantity comparable to theory ?

- Comparison with theories.

- Need to clarify connection between various theories and between theories and extracted $\alpha_{s,\ bjorken}^{eff}$

- Calculated Q^2-evolutions are similar and agree with data when an extraction of α_s à la DIS is used.

  $\Rightarrow$ Some duality at work ?

- Up-coming Jlab data: Proton+neutron: CLAS EG1b (0.07-2.4 GeV^2)
  Proton: CLAS E03-006, Neutron: Hall A E97-110 (0.02-0.5 GeV^2)