Precise determination $\alpha_{\overline{\rm MS}}(M_Z)$ from realistic lattice QCD

Howard Trottier
Simon Fraser University, Vancouver
HPQCD Collaboration

Duality 05
Frascati
June 2005

(1) Precision *Unquenched* LQCD

- Most LQCD simulations have ignored effects of "sea" quarks (or used unrealiably large m_{sea})
  - so-called quenched "approximation"
- New "staggered" discretization for light quarks
  - much more efficient & accurate unquenched simulations
- Reduce systematic errors to few % many quantities

(2) Determination of $\alpha_{\overline{\text{MS}}}(M_Z)$

- "Blend" of long- and short-distance QCD

(1) Recent *Unquenched* LQCD

$$\langle \mathcal{O} \rangle = \int [dU_\mu(x)][d\bar{\psi} d\psi]\, \mathcal{O}\, e^{-\beta(S_{\mathbf{gluon}} + S^{\mathbf{stagg}}_{\mathbf{quark}})} \Longleftarrow \textbf{!}$$

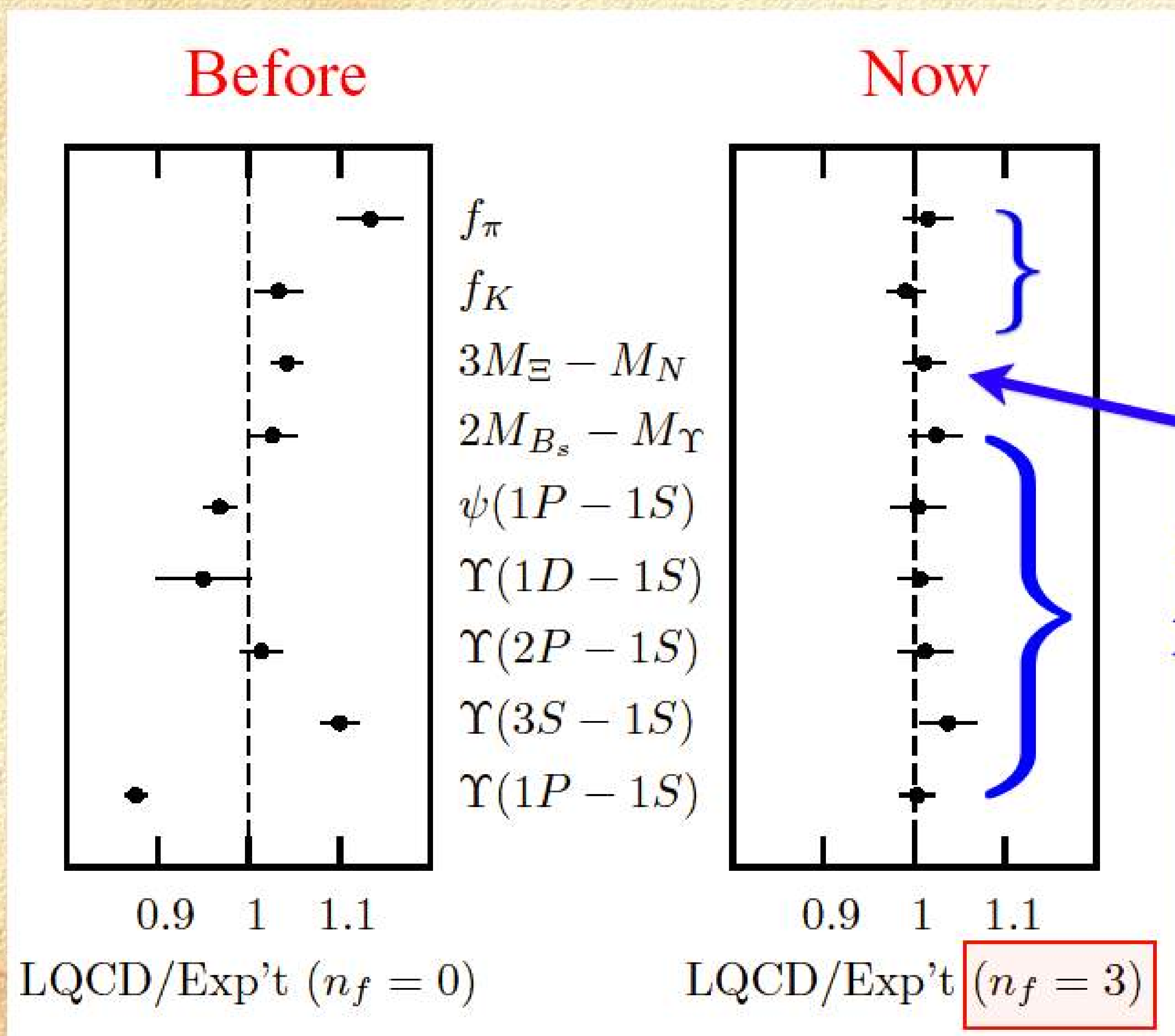
- Only 5 input parameters (same as in continuum QCD)
  - $m_u (= m_d),\ m_s,\ m_c,\ m_b,\ a\ (\Leftrightarrow \alpha_s)$
    - $m_{u/d} \leftarrow m_\pi^2$
    - $m_s \leftarrow 2m_K^2 - m_\pi^2$
    - $m_c \leftarrow m_D$
    - $m_b \leftarrow m_\Upsilon$

    } Each experimental quantity roughly $\propto$ the one m_{quark} and roughly independent of the other masses

    - $a \leftarrow m_{\Upsilon'} - m_\Upsilon$ ⟵ This mass difference roughly independent of all m_{quark}'s
- Same ***a*** using *any* input: $f_\pi, m_K, m_{B_s}, \ldots$

HPQCD, MILC, UKQCD, Fermilab, Collab'ns PRL 92, 2004

LQCD / Experiment



Light-quark physics & form-factors

Baryons

Heavy-quarks

Note "unified" treatment of e.g. *B* & Υ

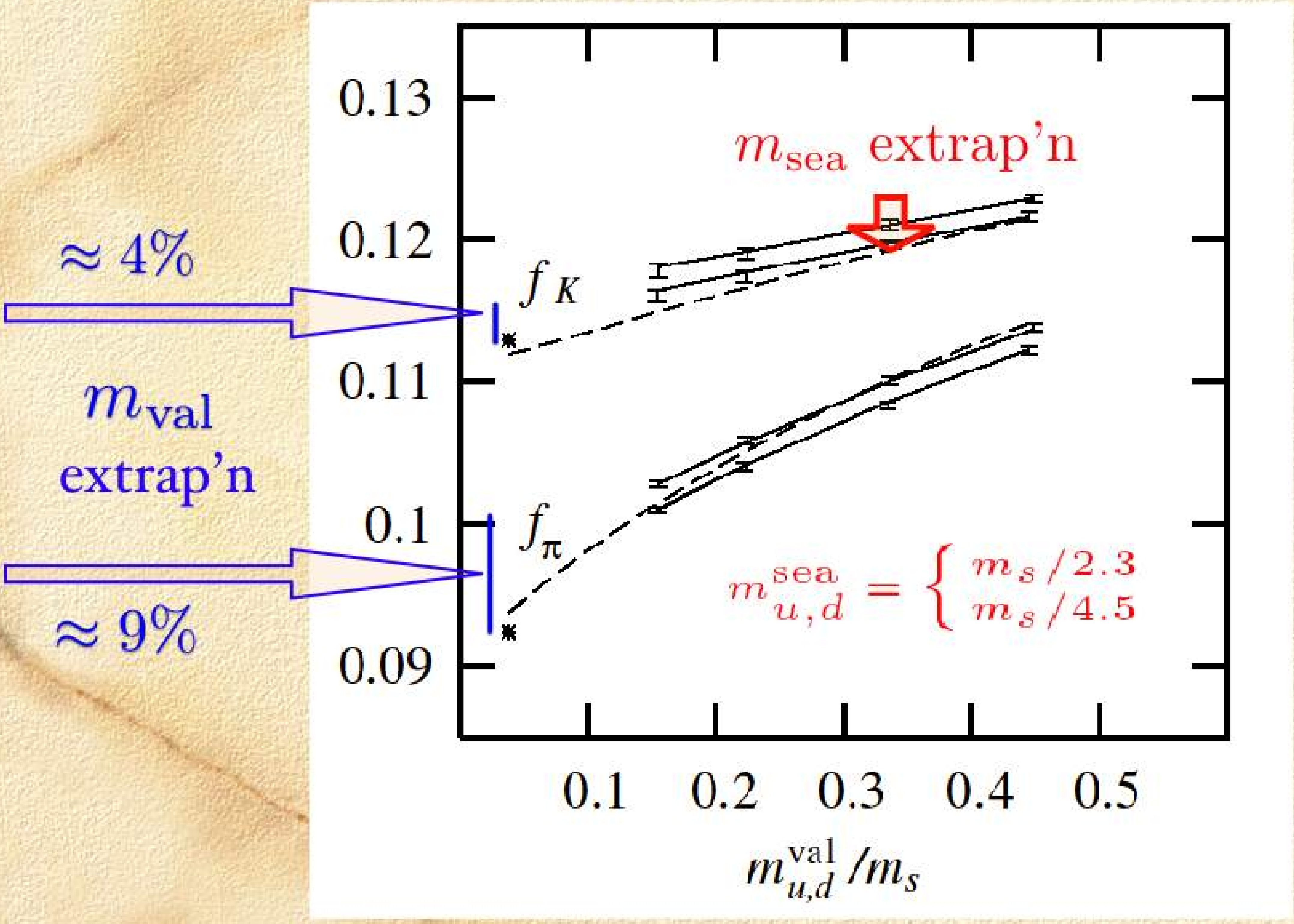
Errors ~ 10-15%

Errors < 3%

Partially Quenched χ PT

Lee & Sharpe
Bernard & Aubin

Run at several $m_{u/d,s}^{\text{valence}}$ not necessarily equal to m_{sea}



Sea quark masses ~3-5 times smaller than in previous unquenched

A potential pit-fall

- Staggered quarks plagued by flavour doubling:

$$\bar{\psi}(x)\,\gamma\cdot D\,\psi(x) \;\Rightarrow\; \sin(p_\mu a)\times\bar{\psi}(p)\gamma_\mu\psi(p)$$

==> low-energy modes at $p_\mu = 0, \pi/a$

$\therefore$ 2^4 copies ("tastes") (reduce to 4 tastes by "staggering")

To get desired (2+1)-flavours instead of 4

$$\det(\gamma\cdot D + m) \rightarrow \det(\gamma\cdot D + m)^{1/4}$$

☂ potentially worrisome non-localities

{ ▶ Staggered quarks cheap to simulate: remnant χ symmetry }

$$\det(\gamma \cdot D + m) \rightarrow \det(\gamma \cdot D + m)^{1/4}$$

Is this a local effective theory ?

☺ Correct to all orders in PT (Batrouni et al 1985)

☺ Chiral anomalies correctly handled (Sharatchandra et al 1981; Smit & Vink 1988)

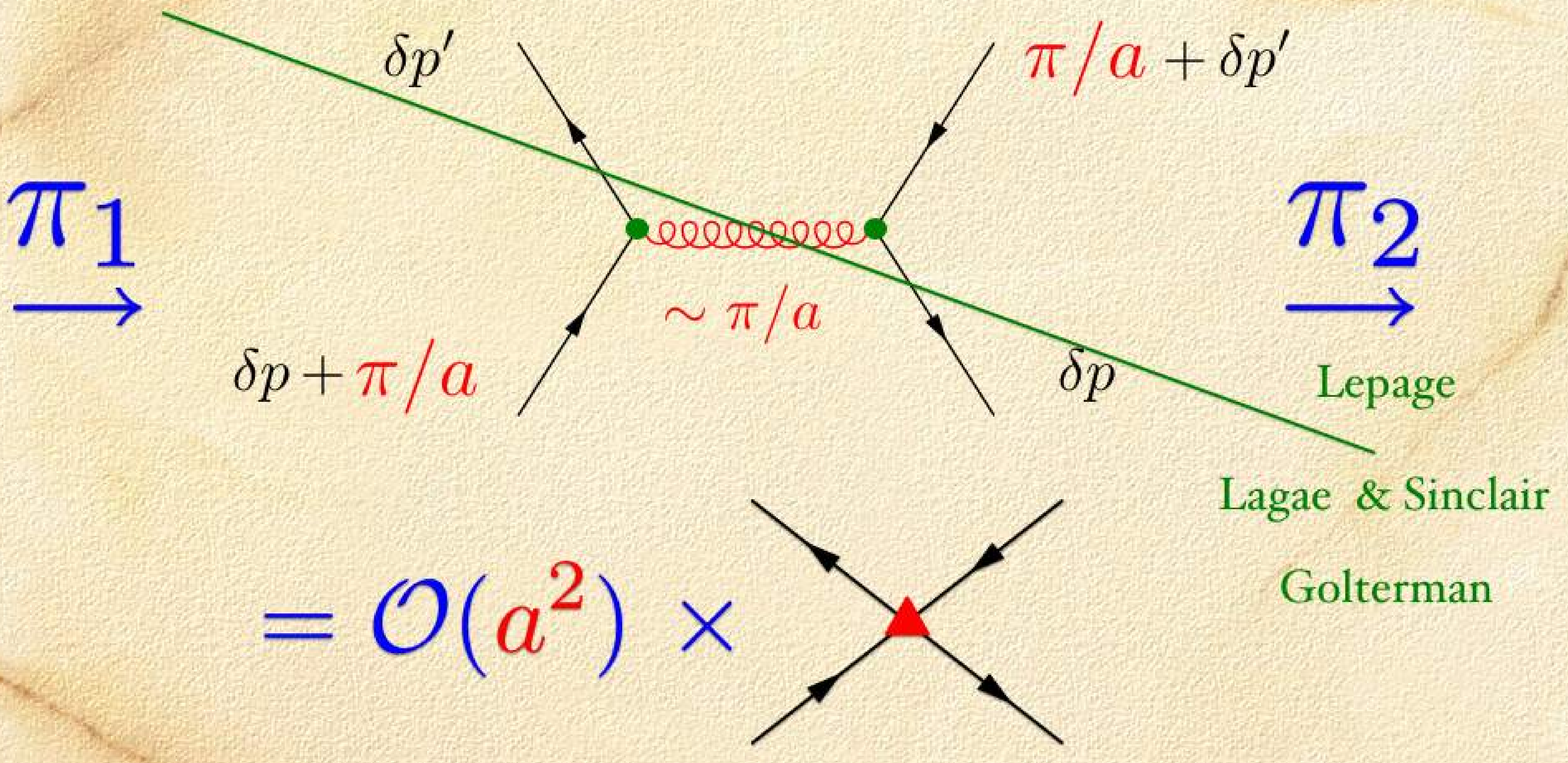
☺ Controlled by short-distance interactions

$$[\det(\gamma \cdot D + m)]^{1/4} = [\Pi_n(\lambda_n + m)]^{1/4}$$

- 1/4 root OK if eigenvalues are ≈ quadruply degenerate
- eigenvalue- or "taste-" splittings controlled by short-distance interactions

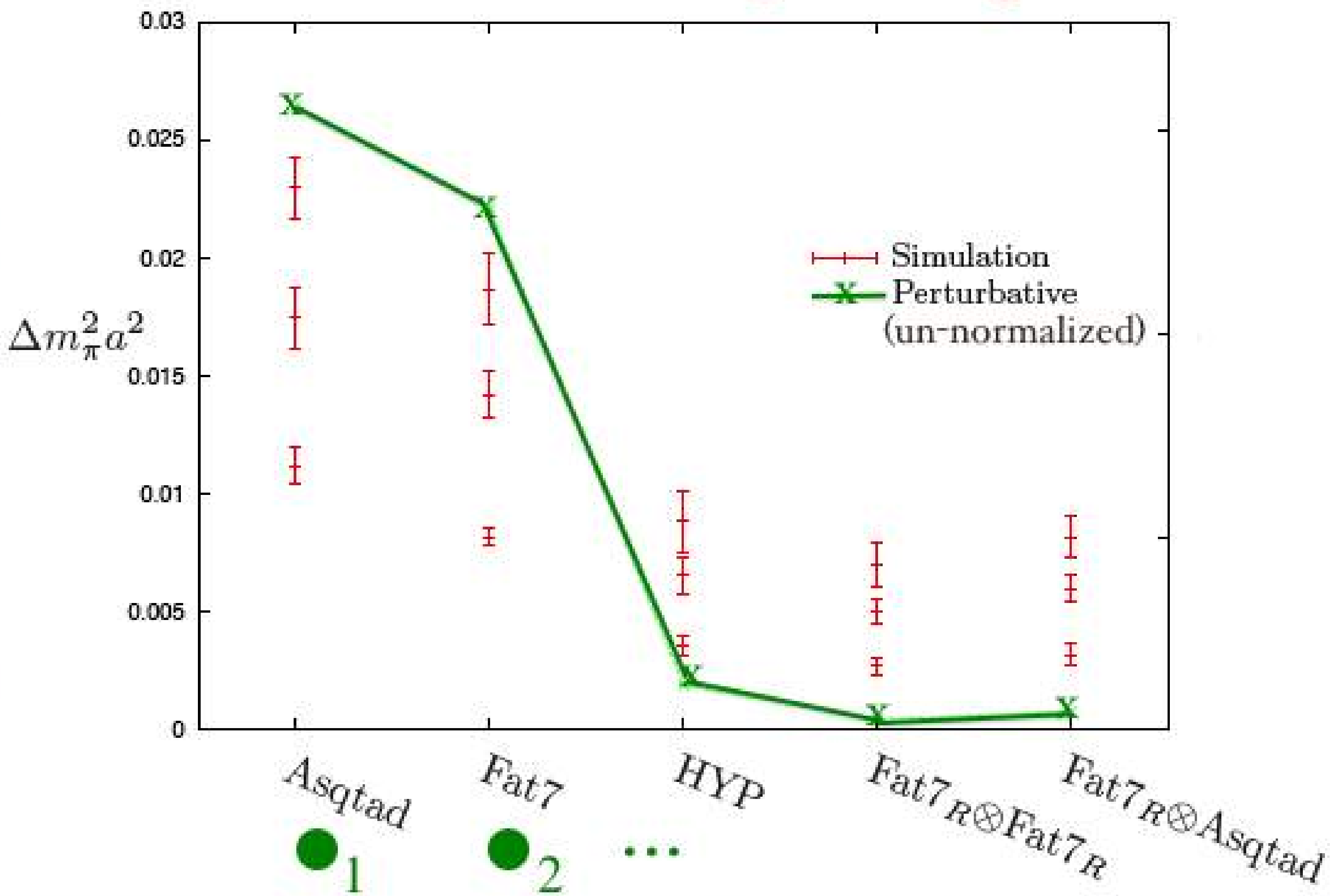
.....

Taste-changing interactions



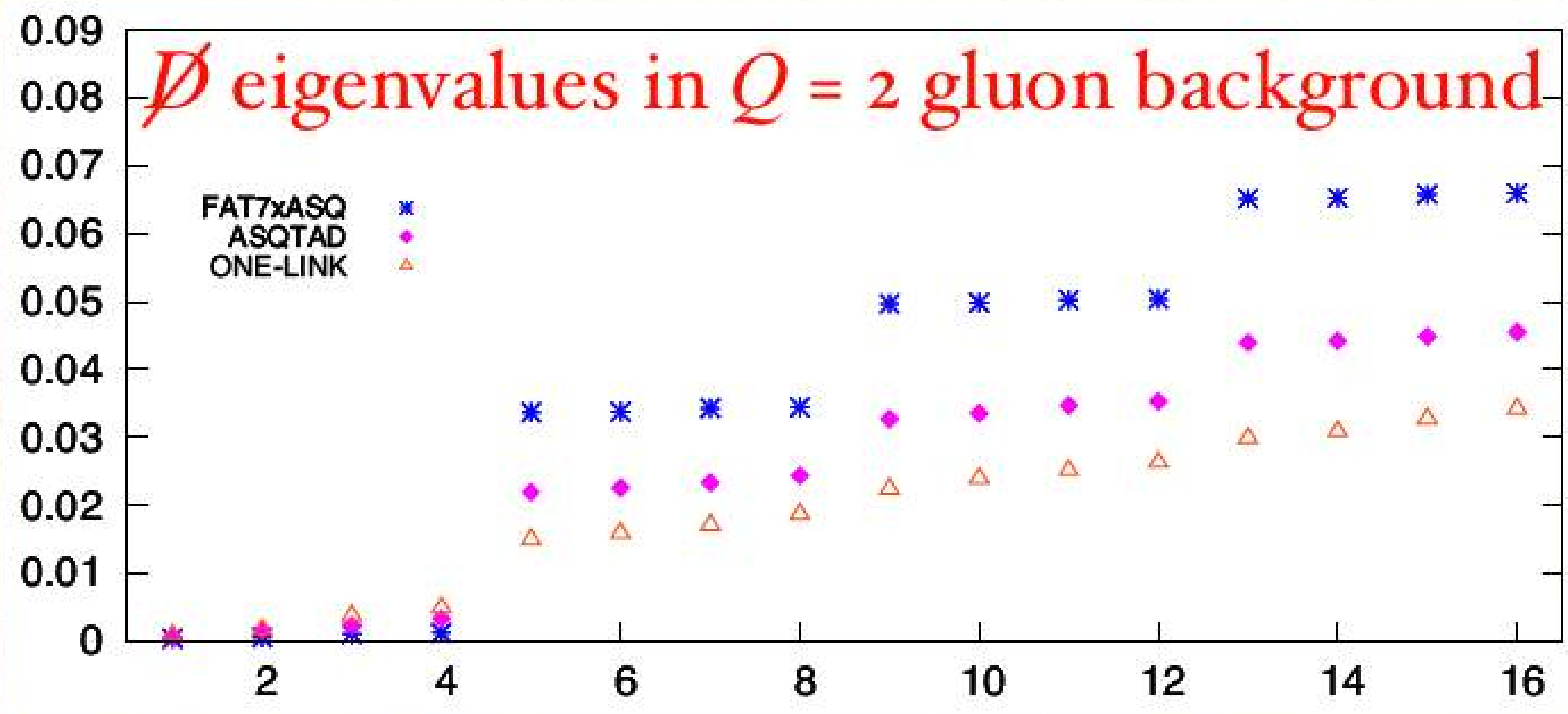
Minimize taste-changing interactions by perturbative analysis of the effective interaction ●

Pion "taste" splittings



1/4 root ☺

Follana, Davies, Hart, Lepage, Mason, *HDT* Lattice 2003

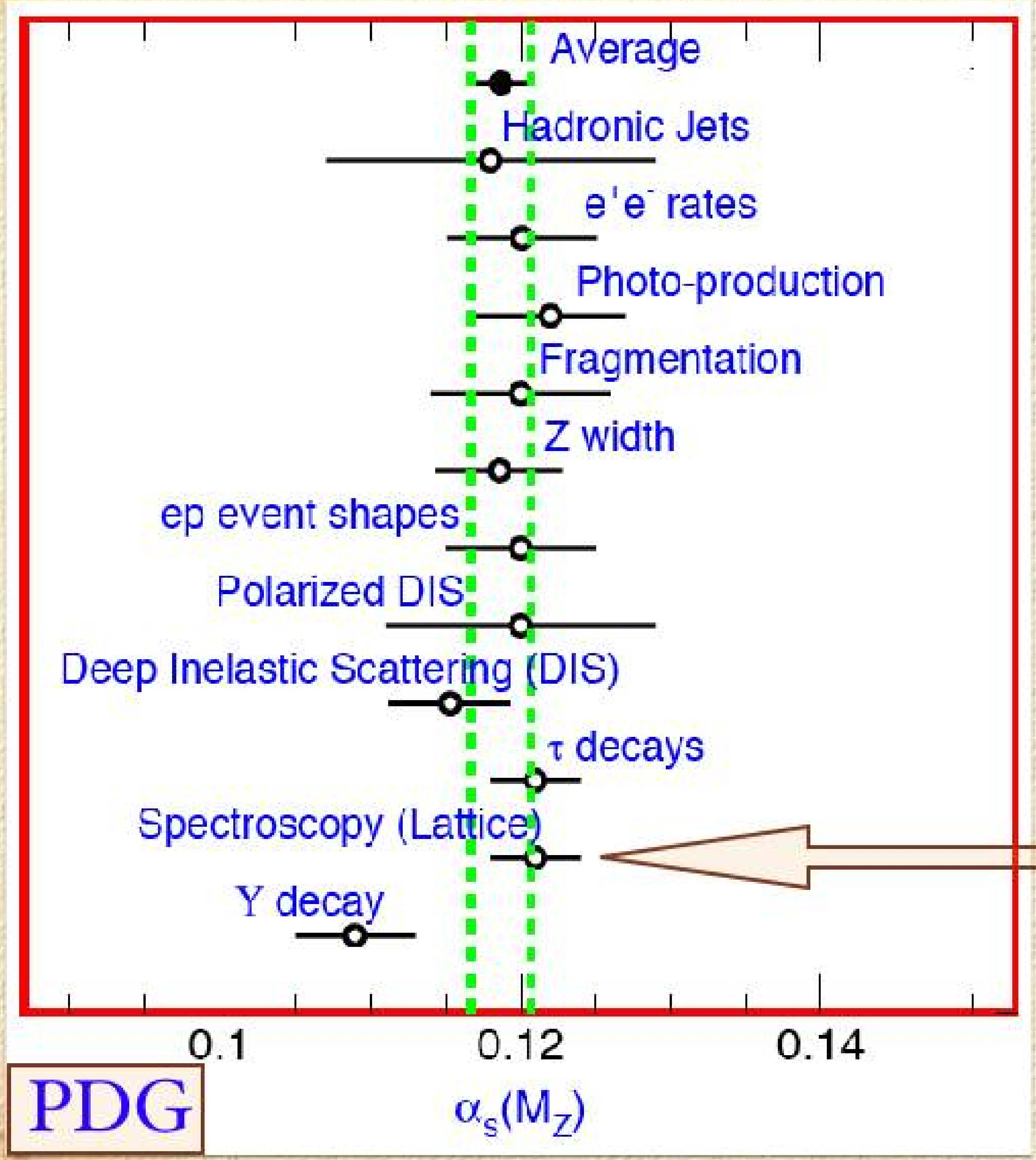


Also: Woloshyn, Wong;

Durr, Hoelbling, Wengner

Follana, Hart, Davies, PRL 2004

(2) Determination of $\alpha_{\overline{\text{MS}}}(M_Z)$



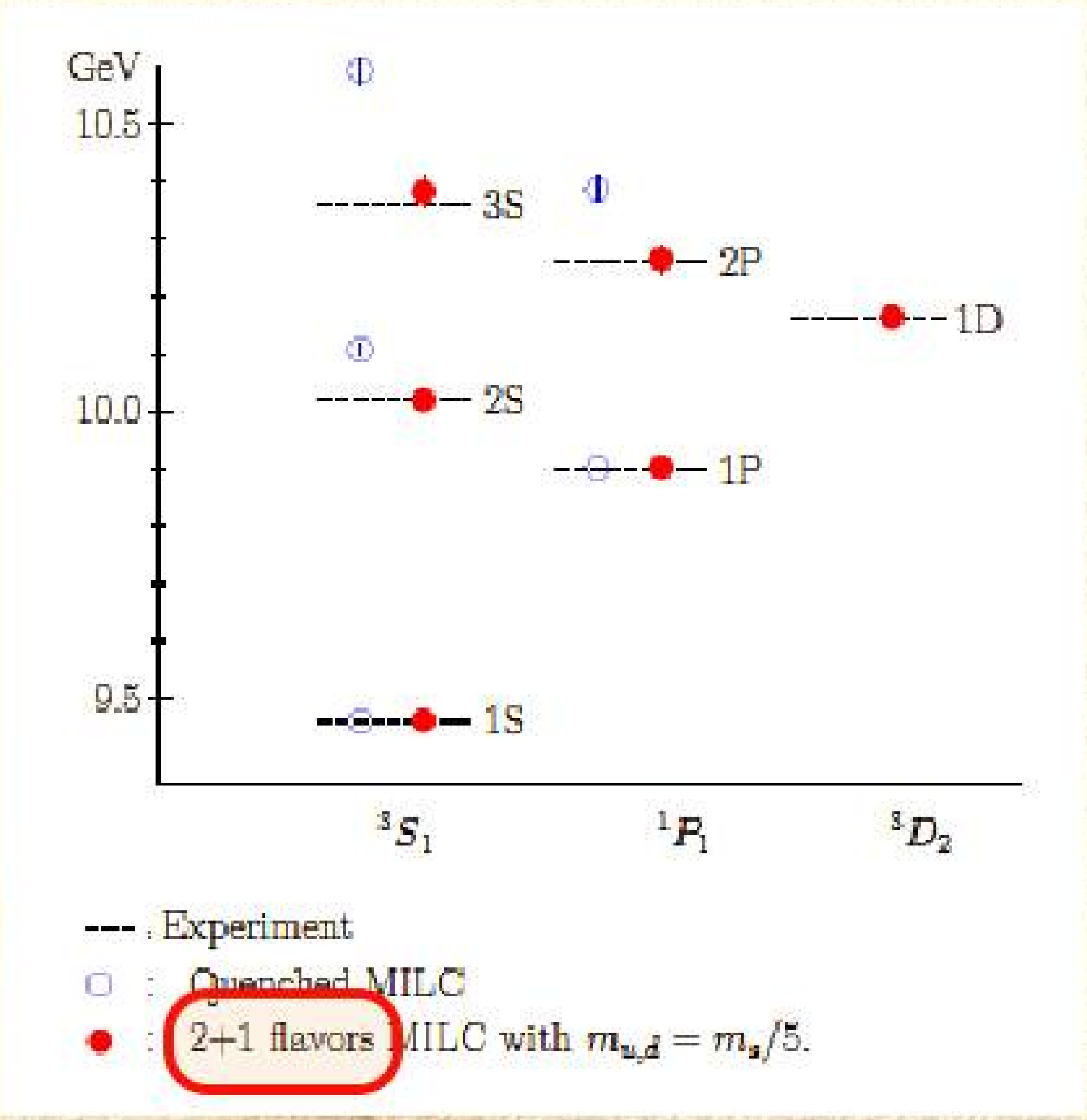
HPQCD Collab'n

Previous lattice NLO analyses

NNLO: reduce uncertainty factor ~ 2

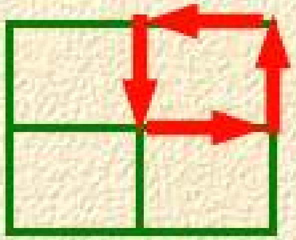
"Novel" use of *both* long- & short-distance QCD

(NRQCD Collab'n 1997)



(i) NPT input e.g. $\Upsilon' - \Upsilon \Rightarrow a$

(ii) Measure short-distance quantity



Wilson loop

▶ Characteristic scale $q^* \propto 1/a$

(iii) Use perturbation theory:

$$\langle \mathcal{O} \rangle = c_1\alpha(q^*) + c_2\alpha^2(q^*) + c_3\alpha^3(q^*) + \ldots$$

(iv) Evolve $\alpha(q^*)$ to $\alpha_{\overline{\rm MS}}(M_Z)$

What's new?

Lattice PT essential, but hard

$$\langle \mathcal{O} \rangle = \int [dU_\mu(x)][d\bar{\psi} d\psi]\, \mathcal{O}\, e^{-\beta(S_{\mathrm{gluon}} + S^{\mathrm{stagg}}_{\mathrm{quark}})}$$

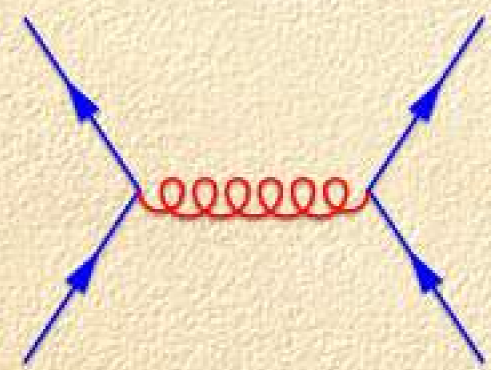
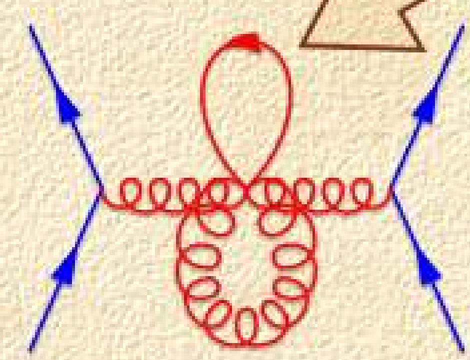
$$\text{e.g.} - \ln \mathrm{W}_{1\times 1} = 3.0684\, \alpha_{\mathrm{lat}} \left[1 + 2.421 \alpha_{\mathrm{lat}} + 8.436(5) \alpha^2_{\mathrm{lat}}\right] + \ldots$$

- Input is lattice-regularized bare coupling α_{lat}

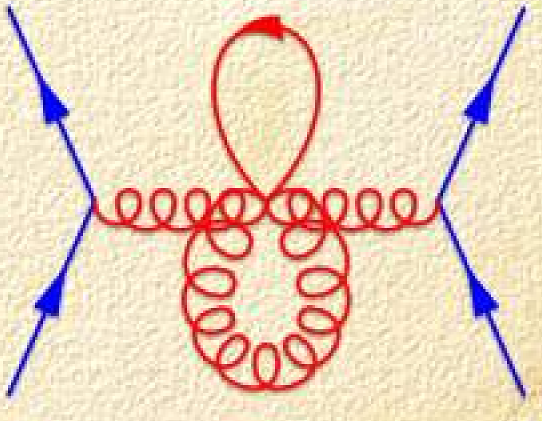
$$= 3.0684\, \alpha_V(3.33/a) \left[1 - 1.068 \alpha_V + 1.69(4) \alpha_V^2\right] + \ldots$$

- We need to reorganize series $\alpha_{\overline{\mathrm{MS}}}$

- Peculiarities lattice regulator

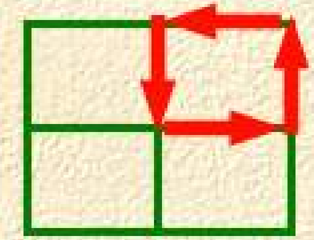
$$-4\pi C_F \frac{\alpha_V(q^2)}{q^2} =$$  $+$  $+ \ldots$

Lattice Regulator Unhappiness

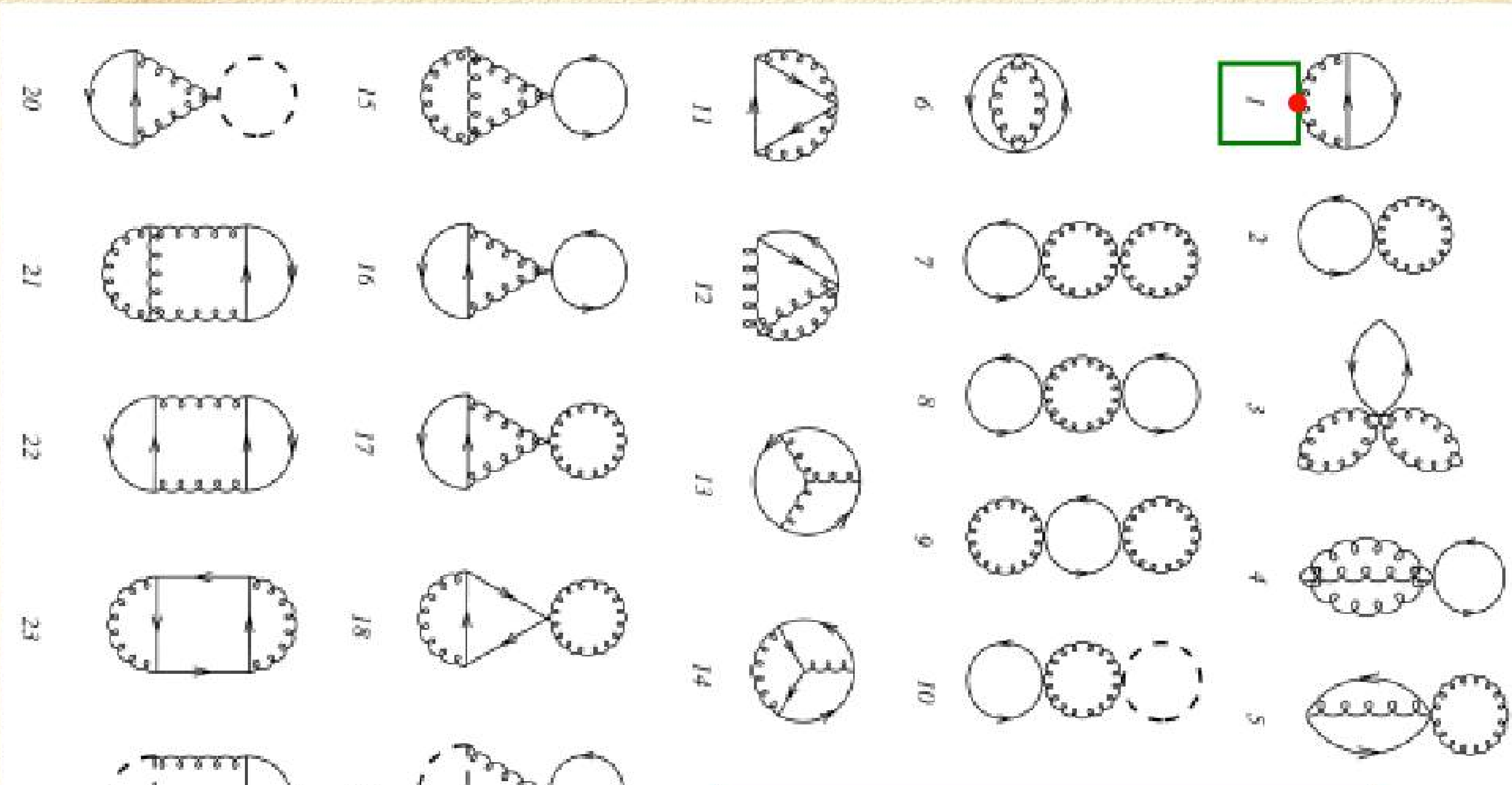


Fortunately, PT algebra can be automated, loop integrals done numerically

NNLO PT Wilson loops



Three-loop diagrams with fermion bubbles



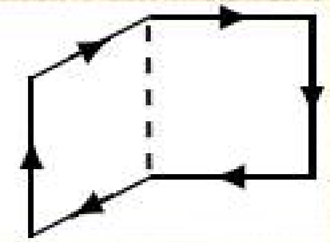
& comparable # gluon loops

$\alpha_{\overline{\text{MS}}}(M_Z)$ analysis

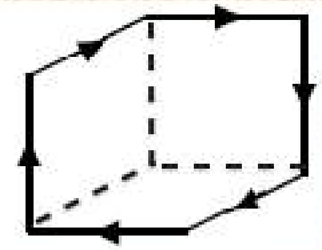
- **NNLO perturbation theory**
  - 8 different Wilson loops:

$$W_{1\times 1}, W_{1\times 2}, \ldots, W_{2\times 3},$$

$W_{\text{BR}} =$



$W_{\text{CC}} =$



  - static potential @ 6 R's

$$\underset{continuum}{V(R)} = -C_F \frac{\alpha_V(0.5614/R)}{R}\left[1 + \frac{\beta_0^2}{48}\alpha_V^2 + \ldots\right]$$

remove perturbative discretizations

- **Simulations** at 3 different lattice spacings:

$$a^{-1} = 1.239(49),\ 1.596(30),\ 2.258(32)\ \text{GeV}$$

  - m_s brackets physical value, $m_{u/d} \downarrow \frac{1}{5} m_s$

} MILC

- Estimate systematic uncertainties, incl.
  - higher-order perturbative corrections
  - non-perturbative condensates

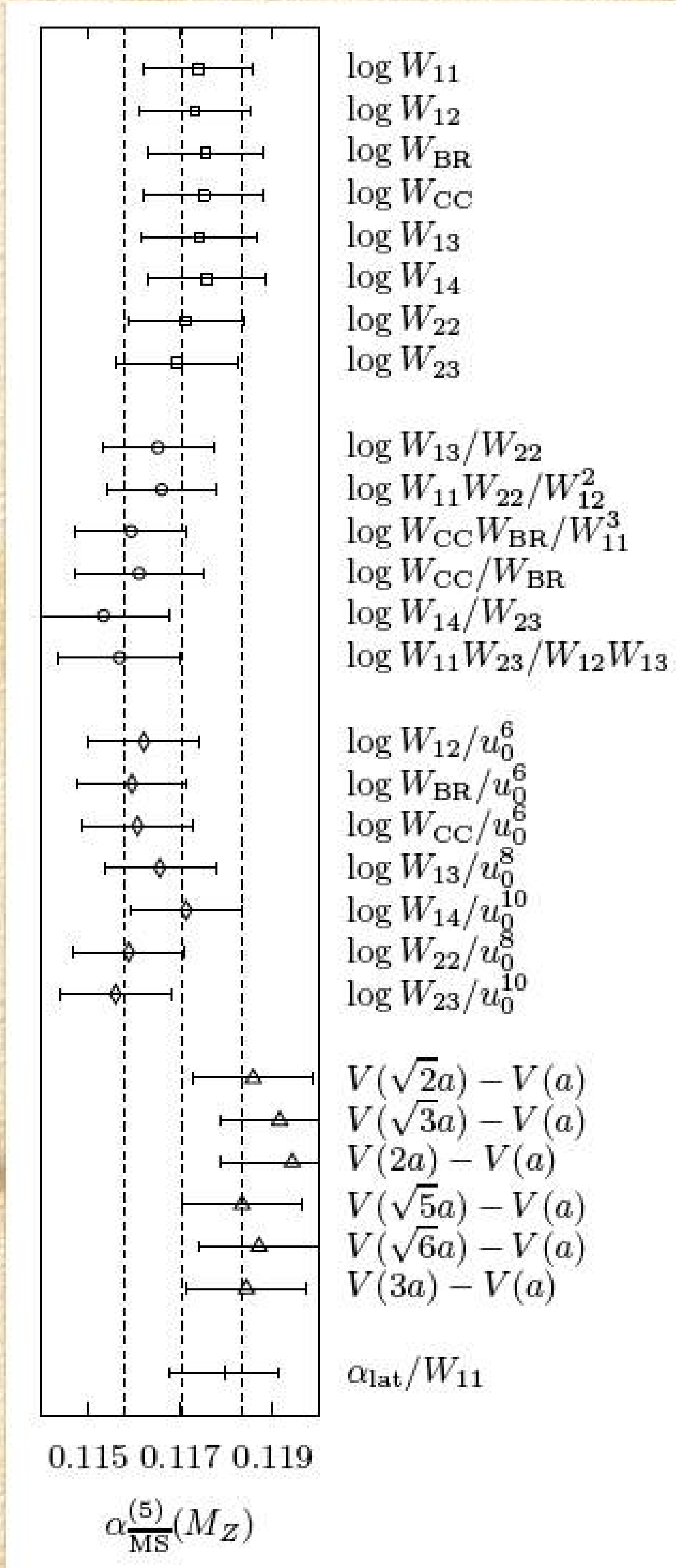
} Thanks to three a's

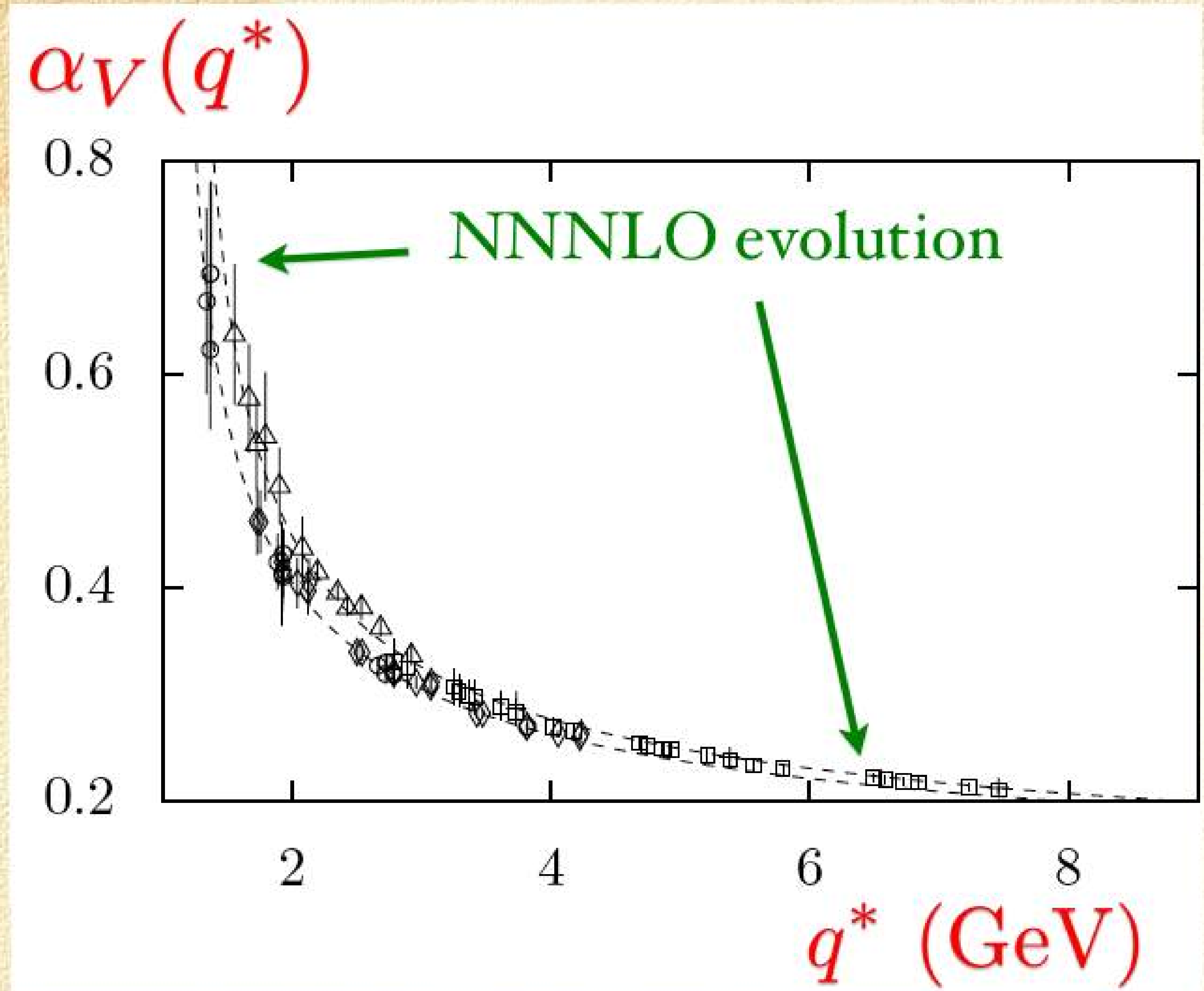
- $$\log W_{RT} = \sum_{n=1}^{3+\ldots} c_n \alpha_V^2(d/a) \; - \frac{\pi}{36} a^4 (RT)^2 \langle \alpha_s F^2 \rangle + \ldots$$

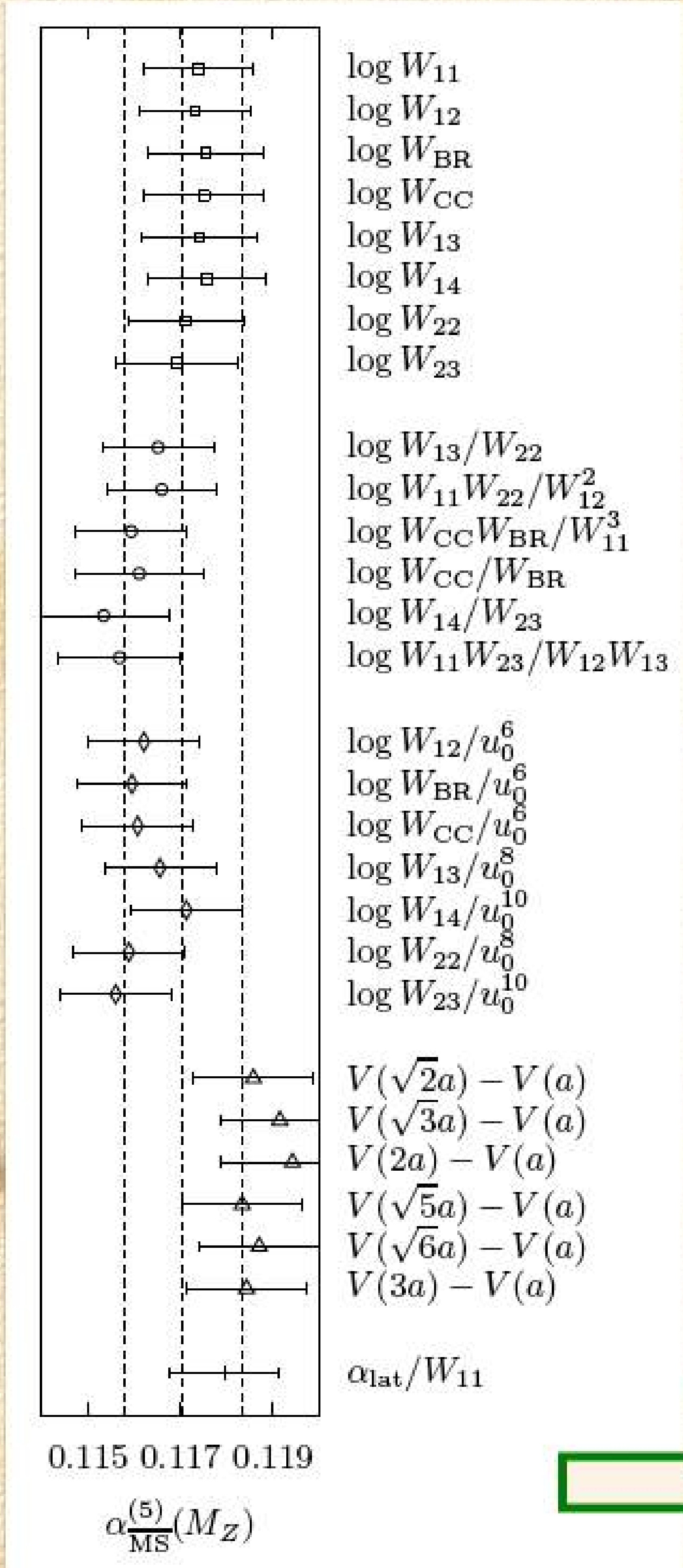
  - use NNNLO $\beta_{\text{“}V\text{”}}$ evolve from one input: $\alpha_V(7.5 \text{ GeV})$ $=$ $0.208(4)$
  - use constrained curve-fitting:

$$c_{n\geq 4} = \mathcal{O}(1)$$

$$\langle \frac{\alpha_s}{\pi} F^2 \rangle = (0.009 \pm 0.007) \text{ GeV}^4$$ (Ioffe & Zyablyuk)


log W_{11}
log W_{12}
log W_{BR}
log W_{CC}
log W_{13}
log W_{14}
log W_{22}
log W_{23}
log W_{13}/W_{22}
log $W_{11}W_{22}/W_{12}^2$
log $W_{CC}W_{BR}/W_{11}^3$
log W_{CC}/W_{BR}
log W_{14}/W_{23}
log $W_{11}W_{23}/W_{12}W_{13}$
log W_{12}/u_0^6
log W_{BR}/u_0^6
log W_{CC}/u_0^6
log W_{13}/u_0^8
log W_{14}/u_0^{10}
log W_{22}/u_0^8
log W_{23}/u_0^{10}
$V(\sqrt{2}a) - V(a)$
$V(\sqrt{3}a) - V(a)$
$V(2a) - V(a)$
$V(\sqrt{5}a) - V(a)$
$V(\sqrt{6}a) - V(a)$
$V(3a) - V(a)$
α_{lat}/W_{11}
0.115 0.117 0.119
$\alpha_{\overline{MS}}^{(5)}(M_Z)$


$\alpha_V(q^*)$
0.8
0.6
0.4
0.2
NNNLO evolution
2
4
6
8
q^* (GeV)



Error Budget

	$\log W_{11}$	$\log W_{13}/W_{22}$	$V(\sqrt{2}a) - V(a)$
a^{-1}	0.0008	0.0010	0.0008
$c_1 \ldots c_3$	0.0001	0.0004	0.0006
c_n for $n \geq 4$	0.0008	0.0005	0.0006
$V \to \overline{\text{MS}} \to M_Z$	0.0001	0.0001	0.0001
condensate	0.0002	0.0001	0.0001
m_u, m_d, m_s	0.0003	0.0001	0.0001
m_c, m_b	0.0002	0.0002	0.0002
simulation errors	0.0000	0.0000	0.0001
total uncertainty	0.0012	0.0012	0.0012

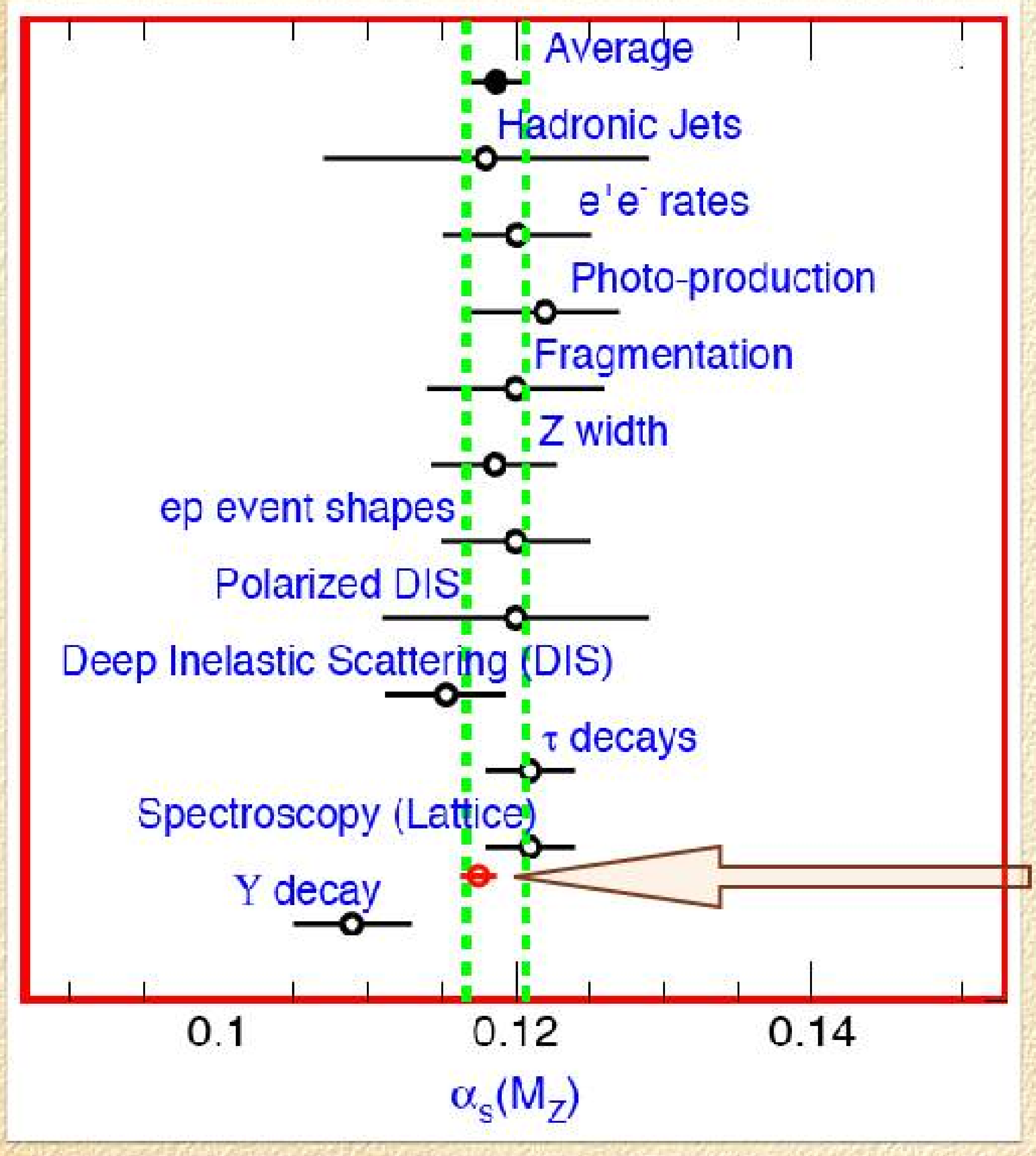
Weighted average

$$\alpha_{\overline{MS}}(M_Z) = 0.1171(13)$$

Result of new NNLO analysis

Mason,
HDT,
Davies,
Foley,
Gray,
Lepage,
Nobes,
Shigemitsu

PRL,
to appear



$\alpha_{\overline{\mathrm{MS}}}(M_Z)$
=0.1171(13)

PDG world avg = 0.1187(20)

Summary

- Unquenched LQCD: few-% precision possible for many quantities
- Perturbation theory key to most phenomenology
- Some important high precision applications done
  - NNLO $\alpha_{\overline{\text{MS}}}(M_Z)$ ✔
  - NNLO m_s, m_c, m_b soon
- Next: leptonic & semileptonic D-decays
  - precision measurements @ CLEO-c