Effects of Littlest Higgs Model in rare D meson decays

Svjetlana Fajfer and Sasa Prelovsek

Department of Physics, University of Ljubljana and J. Stefan Institute, Ljubljana, Slovenia

(based on hep-ph/0511048)
Outline

• Motivation

• Present status of searches for new physics in charm meson decays

• Littlest Higgs Model : FCNC at tree level

• Effects of LH in charm decays: inclusive and exclusive modes

• Results

• Conclusions
Why do we expect new physics?

Within Standard Model there are quadratic divergences which can be cancelled only by having new physics at very high energy scale.

Solutions:

- **Supersymmetry**
- **Little Higgs Models**
Searches for new physics in rare decays at low energies

we expect new physics in

\[ \Delta F = 1 \]

\[ \Delta F = 2 \]

GIM mechanism very important!
Is there any possibility to search for new physics in the up-like quark sector?

The effects of new physics in the hadronic phenomena are most likely to be seen in the down-like quark sector. Within Standard Model (SM) one expects that the effects of new physics might contribute in the processes which are not possible at tree level \( b \to s, \ s \to d, \ b \to dZ \) or

\[
\begin{align*}
&b\bar{q} \leftrightarrow \bar{b}q \ (q = s, d) \ \text{and} \ s\bar{d} \leftrightarrow \bar{s}d \\
\end{align*}
\]

• \( c \to u \gamma \) transition

\[
M(c \to u) = \sum_{q=d,s,b} V_{uq}^* V_{cq} M_q \sim \begin{cases} 
O(\lambda^5 m_b^2) & : \ b - \text{quark}, \\
O(\lambda m_s^2) & : \ s - \text{quark}, \\
O(\lambda \Lambda_{QCD}^2) & : \ d - \text{quark},
\end{cases}
\]

(at one loop level in SM)
• $c \rightarrow u \gamma$ transition in SM

\[ \mathcal{L} = -A C_{\gamma\gamma}^{\text{eff}} \frac{e}{4\pi^2} F_{\mu\nu} \left[ \bar{u} \sigma^{\mu\nu} \frac{1}{2} (1 + \gamma_5) c \right] \]

GIM cancellation at one loop level and QCD enhancement

\[ V_{cb}^* V_{ub} \hat{C}_7^{\text{eff}} = V_{cs}^* V_{us} (0.007 \pm 0.020i) (1 \pm 0.2) \]

MSSM in $c \rightarrow u \gamma$

The gluino exchange diagrams give largest enhancement

\[ |\delta_{12}^{u}|_{LL} \leq 0.03 \quad \text{for} \quad M_{sq} = M_{gl} = 250 \text{ GeV} \]

\[ |\delta_{12}^{u}|_{LR} , |\delta_{12}^{u}|_{RL} \leq 0.0046 \]

\[ \frac{\text{SM}}{\text{SD}} \frac{\Gamma(c \rightarrow u \gamma)}{\Gamma(D^{0})} \sim 2.5 \times 10^{-8} \]

\[ \frac{\text{BR}(c \rightarrow u \gamma)_{\text{MSSM}}}{\text{BR}(c \rightarrow u \gamma)_{\text{SM}}} \simeq 10^{2} \]

S. Prelovsek, D. Wyler
Standard Model prediction

\[ A(D \rightarrow V + \gamma) = \frac{eG_F}{\sqrt{2}} V_{uq_j} \cdot V_{cq_j} \left\{ \epsilon_{\mu\nu\alpha\beta} k^\mu \epsilon^*_\gamma(\gamma) p^\alpha \epsilon^*_V(\nu) A_{PC} \right. \]

\[ + i \left[ (\epsilon^*_V(\gamma) \cdot k)(\epsilon^*_\gamma(\gamma) \cdot p(\gamma)) - (p(\gamma) \cdot k)(\epsilon^*_V(\gamma) \cdot \epsilon^*_\gamma(\gamma)) \right] \} A_{PV} \]

dominance of LD contributions!

<table>
<thead>
<tr>
<th>( D \rightarrow V \gamma ) Transition</th>
<th>Br Ratio ( \times 10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^0 \rightarrow K^{*0} )</td>
<td>6-36</td>
</tr>
<tr>
<td>( D_s^+ \rightarrow \rho^+ )</td>
<td>20-80</td>
</tr>
<tr>
<td>( D^0 \rightarrow \rho^0 )</td>
<td>0.1-1</td>
</tr>
<tr>
<td>( D^0 \rightarrow \omega )</td>
<td>0.1-0.9</td>
</tr>
<tr>
<td>( D^0 \rightarrow \varphi )</td>
<td>0.4-1.9</td>
</tr>
<tr>
<td>( D^+ \rightarrow \rho^+ )</td>
<td>0.4-6.3</td>
</tr>
<tr>
<td>( D_s^+ \rightarrow K^{*+} )</td>
<td>1.2-5.1</td>
</tr>
<tr>
<td>( D^+ \rightarrow K^{*+} )</td>
<td>0.03-0.44</td>
</tr>
<tr>
<td>( D^0 \rightarrow K^{*0} )</td>
<td>0.03-0.2</td>
</tr>
</tbody>
</table>

Exp. limits

<table>
<thead>
<tr>
<th></th>
<th>(&lt; 7.6)</th>
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</thead>
<tbody>
<tr>
<td>( &lt; 2.4 \times 10^{-4} )</td>
<td></td>
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<tr>
<td>( &lt; 2.4 \times 10^{-4} )</td>
<td></td>
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<tr>
<td>( 2.6_{-0.6}^{+0.7} \times 10^{-5} )</td>
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</tbody>
</table>

Belle collaboration

The very good candidate to search for FCNC in $c \to u \gamma$ is the difference of the decay rates (S.F., S.Prelovsek, P. Singer, D. Wyler) $D^0 \to \rho^0 \gamma$ and $D^0 \to \omega \gamma$

$$R = \frac{Br[D^0 \to \omega \gamma] - Br[D^0 \to \rho^0 \gamma]}{Br[D^0 \to \omega \gamma]} \propto Re \frac{A[D^0 \to u\bar{u}\gamma]}{A[D^0 \to d\bar{d}\gamma]}$$

LD contributions cancel

MSSM: $R$ can amount up to $R = 6 \pm 15\%$

The best candidate to test

$c \to u \gamma$ is $B_c \to B_u^* \gamma$ (SD dominates LD contributions in SM)

The MSSM enhancement of the $c \to u \gamma$
can be probed at LHC expected to produce $2 \times 10^8 B_c$
Inclusive $c \rightarrow ul^+l^-$ decay

\[
M = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} \left[ \hat{C}_7^{\text{eff}} \langle Q_7 \rangle^0 + \hat{C}_9^{\text{eff}} \langle Q_9 \rangle^0 + \hat{C}_{10}^{\text{eff}} \langle Q_{10} \rangle^0 \right]
\]

the dominant contribution comes from

\[
V_{cb}^* V_{ub} \hat{C}_7^{\text{eff}} = V_{cs}^* V_{us} (0.007 \pm 0.020i)(1 \pm 0.2)
\]

with $\langle Q_{7,9,10} \rangle^0$ the tree level matrix elements of the operators. The coefficient $\hat{C}_{10}^{\text{eff}} \simeq 0$ in the Standard Model.

\[
\frac{\text{Br}(c \to u l^+ l^-)}{d\hat{s}} = \frac{G_F^2 \alpha_{\text{QED}}^2 m_c^5}{768\pi^5 \Gamma(D^0)} |V_{cb}^* V_{ub}|^2 (1 - \hat{s})^2 \left[ 4 \left(1 + \frac{2}{\hat{s}}\right) |\hat{C}_{7}^\text{eff}|^2 + \frac{1 + 2\hat{s}}{16} \left( |\hat{C}_{9}^\text{eff}|^2 + |\hat{C}_{10}^\text{eff}|^2 \right) + 3\Re \left( \hat{C}_{7}^\text{eff*} \hat{C}_{9}^\text{eff} \right) \right],
\]

the branching ratio distribution

\[c \to u e^- e^+\]

invariant dilepton mass square

R- parity
MSSM
SM
Exclusive rare leptonic D meson meson searches

Intensive searches of CLEO and FERMILAB; CLEO’s results Phys. Rev. Lett. 95 (2005) 221802

\[ \mathcal{B}(D^+ \rightarrow \pi^+ e^+ e^-) < 7.4 \times 10^{-6}, \quad \mathcal{B}(D^+ \rightarrow \pi^- e^+ e^+) < 3.6 \times 10^{-6}, \]
\[ \mathcal{B}(D^+ \rightarrow K^+ e^+ e^-) < 6.2 \times 10^{-6}, \quad \mathcal{B}(D^+ \rightarrow K^- e^+ e^+) < 4.5 \times 10^{-6} \]

\[ Br^{exp}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) < 8.8 \times 10^{-6} \]

FOCUS collaboration

Littlest Higgs model

• rather simple solution to the gauge hierarchy problem;

• new massive gauge bosons and new heavy quark states;

the quadratic divergences
• the spin ½ contribution of the top quark is cancelled by the new fermion;
• the spin 1 contributions are cancelled by new gauge bosons;
• the spin 0 quadratically divergent contributions are vanishing due to the fact that all scalars in the model are Goldstone bosons at tree level.

  (there is an extra vector –like heavy quark, which is a single of SU(2) group and which mixes with the third generation);

• this mixing induces new tree level flavor changing charged and neutral currents.
• The Littlest Higgs model begins with SU(5) global symmetry, with two locally gauged subgroups SU(2) x U(1). The global SU(5) is spontaneously broken to its subgroup SO(5) at scale $f$ close to 1 TeV.

• The global symmetry breaking results in fourteen massless Goldstone bosons. Among them four massless Goldstone bosons are eaten by the gauge bosons, so that the two SU(2)xU(1) are broken down to its diagonal subgroup SU(2)xU(1).

• The remaining 10 Goldstone bosons give complex doublet (SM Higgs) and complex triplet.

• Higgs doublet is a Goldstone boson at the scale $f$ so there is no mass term for the Higgs doublet at the tree level. The Higgs potential arises from the Coleman – Weinberg potential. Additional heavy gauge bosons and the Higgs triplet give the Higgs doublet a logarithmically enhanced positive mass squared.

• In order to cancel out these positive contributions and get a negative Higgs mass squared an additional quark is introduced in vector - like representation of the SM:

$$\tilde{t}(3, 1)_{+2/3} + \tilde{t}^c(\bar{3}, 1)_{-2/3}$$

it is a singlet under the $SU(2)_L$ gauge group.
Weak currents:

In SM the neutral current weak interaction is given by

\[ \mathcal{L}_{NC} = \frac{g}{\cos \theta_W} Z_\mu (J_{W3}^\mu - \sin^2 \theta_W J_{EM}^\mu) \]

Within LH - Lee model

\[ U_L^m = (u_L, C_L, t_L, T_L)^T, \quad D_L^m = (d_L, S_L, b_L)^T \]

and \( L = \frac{1}{2}(1 - \gamma_5) \)

\[
\Omega = \begin{pmatrix}
1 - |\Theta_u|^2 & -\Theta_u \Theta_c^* & -\Theta_u \Theta_t^* & -\Theta_u \Theta_T^* \\
-\Theta_c \Theta_u^* & 1 - |\Theta_c|^2 & -\Theta_c \Theta_t^* & -\Theta_c \Theta_T^* \\
-\Theta_t \Theta_u^* & -\Theta_u \Theta_t^* & 1 - |\Theta_t|^2 & -\Theta_u \Theta_T^* \\
-\Theta_T \Theta_u^* & -\Theta_T \Theta_c^* & -\Theta_T \Theta_t^* & 1 - |\Theta_T|^2
\end{pmatrix}
\]

\[ J_{W3}^\mu = \frac{1}{2} \bar{U}_L^m \gamma^\mu \Omega U_L^m - \frac{1}{2} \bar{D}_L^m \gamma^\mu D_L^m \]

CKM matrix elements

\[ |V_{id}|^2 + |V_{is}|^2 + |V_{id}|^2 + |\Theta_i|^2 = 1 , \]

\[ V_{id} V_{jd}^* + V_{is} V_{js}^* + V_{ib} V_{jd}^* + \Theta_i \Theta_j^* = 0 \]

There is a tree level FCNC coupling coming from

\[ i g \Omega_{uc} / (2 \cos \theta_W) \]
In our study we use:

- a) the lowest reasonable value \( f = 500 \text{ GeV} \), leading to \( \langle v \rangle = 0.25 \text{ GeV} \)

\[
|\Theta_{\text{u}}| \sim |V_{ub}| \frac{v}{f} \sim 0.001 \\
|\Theta_{\text{c}}| \sim |V_{cb}| \frac{v}{f} \sim 0.01 \\
|\Theta_{\text{t}}| \sim |V_{tb}| \frac{v}{f} \sim 0.25 \\
|\Theta_{\text{T}}| \sim 0.93
\]

Lee’s results hep-ph/0408362

- b) using the unitarity limit

\[
|V_{id}|^2 + |V_{is}|^2 + |V_{id}|^2 + |\Theta_i|^2 = 1, \\
V_{id}V_{jd}^* + V_{is}V_{js}^* + V_{ib}V_{jd}^* + \Theta_i\Theta_j^* = 0
\]

one gets

\[
\Omega_{uc} = |\Theta_{\text{u}}| |\Theta_{\text{c}}| < 0.0028
\]

(we use the lowest value of CKM as given in PDG to get maximal effects)
On the other hand, $D^0 - \bar{D}^0$ mixing limits the effects coming from

$$x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}$$

$$(\Delta m_D)^Z \approx 2 \times 10^{-7} \times |\Omega_{uc}|^2 \text{ GeV}$$

Exp. Result (PDG):

$$|m_{D_1^0} - m_{D_2^0}| < 7 \times 10^{10} \text{ } \hbar \text{ s}^{-1}$$

Maximal value is $|\Omega_{uc}| = 0.0004$

This corresponds to the maximal effect that any new physics possibly have on rare D decays given by model independent constraints on Z mediated flavor changing currents.
SM results

\[ V_{cb}^* V_{ub} \hat{C}^{\text{eff}}_7 = V_{cs}^* V_{us} (0.007 \pm 0.020i)(1 \pm 0.2) \]

\[ \hat{C}^{\text{eff}}_9 \] is significantly suppressed by QCD

\[ C_{10} \simeq 0 \]

Littlest Higgs model modifies Wilson coefficients:

\[ V_{cb}^* V_{ub} \delta C_{9}^{LH} = \frac{8\pi}{\alpha} \Omega_{uc} g_V^l, \quad V_{cb}^* V_{ub} \delta C_{10}^{LH} = \frac{-8\pi}{\alpha} \Omega_{uc} g_A^l \]

\[ g_V^l = -1/2 + 2\sin^2\theta_W \text{ and } g_A = -1/2 \]

(S. Fajfer and S. Prelovsek)
Inclusive decays

\[ c \rightarrow u \, e^+ \, e^- \]

\[ d\Gamma/ds \times \frac{1}{\Gamma_{D^+}} \]

\[ s = m_{ee}^2 / m_c^2 \]

\[ |\Omega_{uc}| \approx |V_{ub}| |V_{cb}| \frac{v^2}{f^2} \approx 10^{-5} \left( \frac{1 \text{ TeV}}{f} \right)^2 \]
\[ \left| \Omega_{\mu\tau} \right| = 0.0004 \]

\[ c \rightarrow u e^+ e^- \]

Graph showing the decay rate \( d\Gamma/ds \) normalized by \( 1/\Gamma_{D^+} \) as a function of \( s=m_{ee}^2/m_c^2 \). The graph compares two scenarios: SM and SM+LH (\( \Omega_{\mu\tau} = 0.0004 \)).
Effects on $D^+ \rightarrow \pi^+ l^+ l^-$ decay

CLEO’s and FOCUS upper upper bounds

$$Br^{exp}(D^+ \rightarrow \pi^+ e^+ e^-) < 7.4 \times 10^{-6}, \quad Br^{exp}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) < 8.8 \times 10^{-6}$$

$$Br^{exp}(D^+ \rightarrow \pi^+ \phi \rightarrow \pi^+ e^+ e^-) = (2.8 \pm 1.9 \pm 0.2) \times 10^{-6}$$

This is consistent with

$$Br(D^+ \rightarrow \phi \pi^+ \rightarrow \pi^+ e^+ e^-) =$$

$$Br(D^+ \rightarrow \phi \pi^+) \times Br(\phi \rightarrow e^+ e^-) = (1.9 \pm 0.2) \times 10^{-6}$$

It already indicates that resonant decay channels $D^+ \rightarrow \pi^+ V_0 \rightarrow \pi^+ l^+ l^-$ with intermediate resonances $V_0 = \rho^0, \omega, \phi$ constitute an important long-distance contribution to the charmed meson decay, which may shadow interesting long distance contribution induced by $c \rightarrow ul^+ l^-$.
The SM short distance contribution is dominated by the

\[ A^{SD}[D(p) \rightarrow \pi(p - q)l^+l^-] = i \frac{G_F}{\sqrt{2}} e^2 V_{cb}^* V_{ub} \left[ \frac{C_{10}}{16\pi^2} f_+(q^2) \bar{u}(p_-) \gamma_5 v(p_+) \right. \]
\[ \left. + \left\{ \frac{C_7}{2\pi^2} m_c s(q^2) + \frac{C_9}{16\pi^2} f_+(q^2) \right\} \bar{u}(p_-) \gamma_5 v(p_+) \right] \]

where \( q^2 = m_{\pi}^2 \) and form factors \( f_+(q^2) \) and \( s(q^2) \) are defined by

\[ \langle \pi(p_\pi) | \bar{u} \gamma^\mu (1 - \gamma_5) c | D(p) \rangle = (p + p_\pi)^\mu f_+(q^2) + (p - p_\pi)^\mu f_-(q^2), \]
\[ \langle \pi(p_\pi) | \bar{u} \sigma^{\mu\nu} (1 \pm \gamma_5) c | D(p) \rangle = i s(q^2) [(p + p_\pi)^\mu q^\nu - q^\mu (p + p_\pi)^\nu \pm i \epsilon^{\mu\nu\lambda\sigma} (p + p_\pi)_\lambda q_\sigma] \]

We use experimental information on \( D \rightarrow \pi \)

\( f_+(0) = 0.73 \pm 0.14 \pm 0.06 \) and D* pole dominance of the form factor

\( s(q^2) = f_+(q^2)/m_D \) (this holds strictly in the heavy quark limit!)
Instead of using the theoretical model we take full advantage of the experimental input that is available to determine long distance contribution. The dominant contribution is:

\[
\frac{d\Gamma_{D\to\pi V_0 \to \pi l^+ l^-}}{dq^2} = \Gamma_{D\to\pi V_0}(q^2) \frac{1}{\pi} \frac{\sqrt{q^2}}{(m_{V_0}^2 - q^2)^2 + m_{V_0}^2 \Gamma_{V_0}^2} \Gamma_{V_0\to l^+ l^-}(q^2)
\]

The success of narrow width approximation allows to write:

\[
Br(D \to \pi V_0 \to \pi l^+ l^-) = Br(D \to \pi V_0) Br(V_0 \to l^+ l^-)
\]

That indicates that the amplitude for a cascade via resonances can be written as:

\[
A^{LD}[D(p) \to \pi V_0 \to \pi(p-q)l^+ l^-] = e^{i\varphi_{V_0}} a_{V_0} \frac{1}{q^2 - m_{V_0}^2 + i m_{V_0} \Gamma_{V_0}} \bar{u}(p_-) \gamma_v(p_+)
\]

Overall phase is unknown comes from the experimental data:

\[a_\rho = 2.9 \times 10^{-9} \text{ GeV}^{-2} \quad \text{and} \quad a_\phi = 4.2 \times 10^{-9} \text{ GeV}^{-2}\]
We argue that the relative sign of $\omega/\rho^0$ amplitudes can be determined by considering the cascade decays (using vector meson dominance of the electromagnetic current).

The remaining part of the difference is due to the weak transition $D^+ \to \pi^+ V^0$ which is induced by the operators $Q_{1,2}^{d,s}$ and can proceed via three ways (detail in appendix)

$$A(\omega)/A(\rho^0) = -1/3$$

$$\left| A(\phi)/A(\rho^0) \right| = a_\phi/a_\rho$$

For the most of remaining amplitudes $D \to Pl^+l^-$ this procedure cannot be used due to the lack of experimental data.
\[ D^+ \to \pi^+ e^+ e^- \]

- **LD only**
- **SD only: SM**
- **SD only: SM+LH (f=0.5 TeV)**
- **SD only: SM+LH (f=1 TeV)**
- **SD only: SM+LH (f=2 TeV)**

![Graph showing the reaction](image-url)
\[ \Omega_{uc} = 0.0004 \]

\[ D^+ \rightarrow \pi^+ e^+ e^- \]

![Graph showing the decay rate in terms of \( m_{ee}^2 \) [GeV^2].](chart.png)

- **SM+LH (\( \Omega_{uc} = 0.0004 \))**
- **LD only**
- **SD only: SM**
Effects on $D^0 \rightarrow \rho^0 l^+ l^-$ decay

Due to the lack of experimental data we are forced to use a model:

- we use factorization of the amplitude: it reduces calculation on the products of the matrix elements of the two currents;

- we use Lorentz decomposition of the matrix elements: form factors, decay constants;

- we use heavy quark symmetries for D and D* and chiral symmetry for light pseudoscalar and vector mesons

References + new experimental input parameters

The long distance contributions in $D \rightarrow P (V) \gamma^*$
$D^0 \rightarrow \rho^0 \, e^+ \, e^-$

- **SD only: LH model ($f=0.5$ TeV)**
- **LD**

![Graph showing $d\Gamma/dm_{ee}$ vs. $m_{ee}^2$ [GeV$^2$]](image)
$D^0 \rightarrow \rho^0 \ e^+ \ e^-$

$SD \text{ only: } LH \ (\Omega_{uc} = 0.0004)$

$LD$

$dB/r/dm_{ee}^2$

$m_{ee}^2 \ [GeV^2]$
Branching ratios for the decays, which are most suitable to probe $c \rightarrow ul^+l^-$ transition experimentally.
For LH model the scale $f = 0.5$ TeV.
The dilepton mass distribution and the forward-backward asymmetry in SM it is 0!

New physics produces nonzero forward/backward asymmetry!
Conclusions

- the parameters of the little Higgs model are rather restricted;

- the effects of LH model in FCNC charm quark inclusive decay into u quark and the lepton pair are small;

- in the exclusive decays of $D^+ \rightarrow \pi^+ l^+ l^-$ and $D^0 \rightarrow \rho^0 l^+ l^-$ the LH model can not sizably modify the total rates and dilepton mass;

- the forward backward asymmetry in $D^0 \rightarrow \rho^0 l^+ l^-$ vanishes in SM while is of the order $10^{-3}$.  


Theoretical framework

\[ \mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \left[ V_{cd}^* V_{ud} \sum_{i=1,2} C_i Q_i^d + V_{cs}^* V_{us} \sum_{i=1,2} C_i Q_i^s - V_{cb}^* V_{ub} \sum_{i=3,\ldots,10} C_i Q_i \right] \]

\[ Q_1^q = (\bar{u}^\alpha q^\beta)_{V-A} (\bar{q}^\beta c^\alpha)_{V-A}, \]

\[ Q_3 = (\bar{u}c)_{V-A} \sum_q (\bar{q}q)_{V-A}, \]

\[ Q_5 = (\bar{u}c)_{V-A} \sum_q (\bar{q}q)_{V+A}, \]

\[ Q_7 = \frac{e}{4\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} P_R c, \]

\[ Q_9 = \frac{e^2}{16\pi^2} (\bar{u}_L \gamma^\mu c_L)(\bar{l}_\mu \gamma_5 l), \]

\[ Q_2^q = (\bar{u}q)_{V-A} (\bar{q}c)_{V-A}, \]

\[ Q_4 = (\bar{u}^\alpha c^\beta)_{V-A} \sum_q (\bar{q}^\beta q^\alpha)_{V-A}, \]

\[ Q_6 = (\bar{u}^\alpha c^\beta)_{V-A} \sum_q (\bar{q}^\beta q^\alpha)_{V+A}, \]

\[ Q_8 = \frac{g_s}{4\pi^2} m_c G^{\alpha}_{\mu\nu} \bar{u} \sigma^{\mu\nu} T^a P_R c, \]

\[ Q_{10} = \frac{e^2}{16\pi^2} (\bar{u}_L \gamma^\mu c_L)(\bar{l}_\mu \gamma_5 l), \]

| - | \( \mu \) (GeV) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) | \( C_9 \) |
|---|---|---|---|---|---|---|---|
| LO | 1.0 | -0.64 | 1.34 | 0.016 | -0.036 | 0.010 | -0.046 | -0.07 |
| NLO | 1.0 | -0.49 | 1.26 | 0.024 | -0.060 | 0.015 | -0.060 | -0.60 |
| NLO | 1.5 | -0.37 | 1.18 | 0.013 | -0.036 | 0.012 | -0.033 | -0.13 |
| NLO | 2.0 | -0.30 | 1.14 | 0.009 | -0.025 | 0.009 | -0.021 | -0.13 |

Values of Wilson coefficients at scales \( \mu = 1.0 \text{ GeV}, 1.5 \text{ GeV}, 2.0 \text{ GeV} \), calculated at next-to-leading order
Inami Lim result recovered!

\[ V_{cb}^* V_{ub} \hat{C}_9^{\text{eff}} = 2 V_{cs}^* V_{us} \left( h(z_s, \hat{s}) - h(z_d, \hat{s}) \right) \left( 3 C_1(m_c) + C_2(m_c) \right) \]

Dominated by the 1-loop insertion of the \[ Q_{1,2}^q \] \[ V_{cb}^* V_{ub} C_9(\mu) \sim 10^{-4}, \]

with \( z_q = m_q / m_c, \hat{s} = (m_{l+/-} / m_c)^2 \) and \( m_{l+/-} \) the mass of the lepton pair, while

\[ h(z, s) = -\frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} x \]

\[ -\frac{2}{9} (2 + x) \sqrt{|1 - x|} \left\{ \begin{array}{ll} \ln \left| \frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1} \right| - i\pi, & \text{for } x < 1, \\ 2 \arctan \left( \frac{1}{\sqrt{x - 1}} \right), & \text{for } x \geq 1, \end{array} \right. \]

where \( x = 4z^2 / s \)

\[ \lim_{\hat{s} \to 0} (h(z_s, \hat{s}) - h(z_d, \hat{s})) \to -\frac{8}{9} \ln \left( \frac{m_s}{m_d} \right) \]

Inami Lim result recovered!
Explaining the signs in:

\[ A^{LD}[D(p) \to \pi(p - q)l^+l^-] = e^{i\varphi} \left[ a_\rho \left( \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} - \frac{1}{3} \frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) - a_\phi \frac{1}{q^2 - m_\phi^2 + im_\phi\Gamma_\phi} \right] \bar{u}(p_-)\gamma_\mu\gamma_5\phi_\nu(p_+) \]

1. The first possibility is due to the operator \( V_{cd}^* V_{ud} Q_1^d + V_{cs}^* V_{us} Q_1^s \approx V_{cd}^* V_{ud} \bar{u}_L \gamma_\mu c_L \times (d_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu s_L) \), which induces the \( D^+ \to \pi^+ \) transition via the \( \bar{u}_L \gamma_\mu c_L \) current and produces \( \rho^0, \omega, \phi \) due to the acting of the \( d_L \gamma_\mu d - \bar{s}_L \gamma_\mu s \) current. The \( \bar{d}d \sim \frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega \) current renders \( \rho^0 \) and \( \omega \) with the opposite phase, while their amplitudes for the EM transition differ by factor 1/3, so \( A_1(\omega)/A_1(\rho^0) = -1/3 \) for this mechanism in the limit of \( SU(3) \) flavor symmetry. Along the same lines \( A_1(\phi)/A_1(\rho^0) = -2/3 \).

2. The operator \( Q_2^d \) can induce \( D^+ \to \rho^0 \) or \( D^+ \to \omega \) transition via the \( \bar{d}_L \gamma_\mu c_L \) current and produce \( \pi^+ \) via \( \bar{u}_L \gamma_\mu d_L \). Since \( \rho^0 \) and \( \omega \) arise from \( \bar{d}d \) again, this mechanism gives the same ratio \( A_2(\omega)/A_2(\rho^0) = -1/3 \), while there is no intermediate \( \phi \) in this

3. The third possibility arises from \( D^+ \) which is annihilated by the \( \bar{d}_L \gamma_\mu c_L \) operator and \( V^0 \pi^+ \) created by the \( \bar{u}_L \gamma_\mu d_L \) operator. It was shown within a model of [9] that this gives rise only to bremsstrahlung diagrams and that the total bremsstrahlung amplitude is equal to zero for \( D \to Pl^+l^- \) decays. So the model of [9] indicates that the contribution from this mechanism is small and will be neglected.

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The branching ratios in SM

<table>
<thead>
<tr>
<th>( D \rightarrow P l^+l^- )</th>
<th>( Br_{SM}^{SD} )</th>
<th>( Br_{SM} \simeq Br^{LD} )</th>
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</thead>
<tbody>
<tr>
<td>( D^0 \rightarrow K^0 l^+l^- )</td>
<td>0</td>
<td>( 4.3 \times 10^{-7} )</td>
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<tr>
<td>( D_s^+ \rightarrow \pi^+ l^+l^- )</td>
<td>0</td>
<td>( 6.1 \times 10^{-6} )</td>
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<td>( D^0 \rightarrow \pi^0 l^+l^- )</td>
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<td>( 2.1 \times 10^{-7} )</td>
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<tr>
<td>( D^0 \rightarrow \eta l^+l^- )</td>
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<td>( D^0 \rightarrow \eta' l^+l^- )</td>
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<td>( 2.4 \times 10^{-10} )</td>
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<tr>
<td>( D^+ \rightarrow \pi^+ l^+l^- )</td>
<td>( 9.4 \times 10^{-9} )</td>
<td>( 1.0 \times 10^{-6} )</td>
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<tr>
<td>( D_s^+ \rightarrow K^+ l^+l^- )</td>
<td>( 9.0 \times 10^{-10} )</td>
<td>( 4.3 \times 10^{-8} )</td>
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<tr>
<td>( D^+ \rightarrow K^+ l^+l^- )</td>
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<td>( 7.1 \times 10^{-9} )</td>
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<tr>
<td>( D^0 \rightarrow K^0 l^+l^- )</td>
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</table>

<table>
<thead>
<tr>
<th>( D \rightarrow V \mu^+\mu^- )</th>
<th>( BR_{SD} )</th>
<th>( BR_{LD} )</th>
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<tbody>
<tr>
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<tr>
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<tr>
<td>( D^+ \rightarrow \rho^+ \mu^+\mu^- )</td>
<td>( 4.8 \times 10^{-9} )</td>
<td>( [1.5 - 1.8] \times 10^{-6} )</td>
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<td>( D_s^+ \rightarrow K^{*+} \mu^+\mu^- )</td>
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<td>( [5.0 - 7.0] \times 10^{-7} )</td>
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<tr>
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<tr>
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<td>( [4.4 - 5.1] \times 10^{-9} )</td>
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The R parity violating MSSM

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<th>Decay Mode</th>
<th>SM</th>
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<tbody>
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<td>$D^+ \rightarrow \pi^+ e^+ e^-$</td>
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<td>$2.3 \times 10^{-6}$</td>
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<tr>
<td>$D^0 \rightarrow \rho^0 e^+ e^-$</td>
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<td>$5.1 \times 10^{-6}$</td>
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<tr>
<td>$D^+ \rightarrow \pi^+ \mu^+ \mu^-$</td>
<td>$1.9 \times 10^{-6}$</td>
<td>$1.5 \times 10^{-5}$</td>
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<tr>
<td>$D^0 \rightarrow \rho^0 \mu^+ \mu^-$</td>
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<td>$8.7 \times 10^{-6}$</td>
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