Recent Measurements of V_{us}

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DESY

V_{us} and Unitarity check

CKM matrix describes the quark mixing:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (1)$$

V_{us} is the oldest known mixing element (Cabibbo angle). Yet many exciting developments have happened in the last two years !

Unitarity of CKM matrix requires:

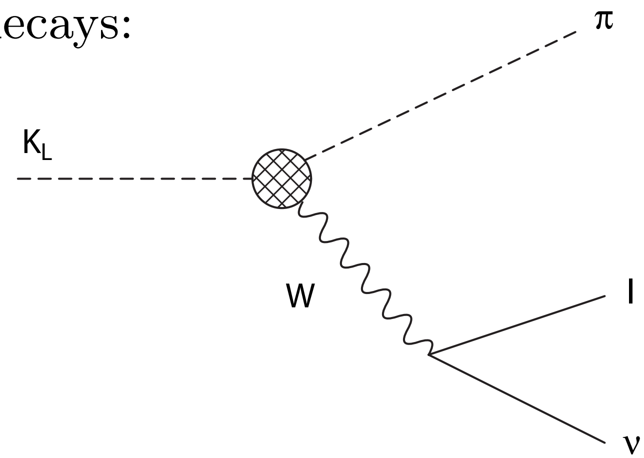
$$1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = \delta = 0 \qquad (2)$$

Largest contribution comes from $|V_{ud}|$, next from $|V_{us}|$, negligible from $|V_{ub}|$.

According PDG-02, $\delta = 0.0043 \pm 0.0019$, about 2.2σ deviation from unitarity, with uncertainty from V_{us} of 0.0010.

Methods to extract V_{us}

The most accurate approach to extract V_{us} is to use rate of semileptonic kaon decays:



$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1+\delta_K^\ell) C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell, \qquad (3)$$

Here:

- S_{EW}, δ_K^ℓ – universal short- and mode dependent long-distance radiative corrections.
- $C = 1$ for K_L and $C = 1/2$ for $K^\pm$.
- $f_+^2(0)$ is calculated in theory form factor value for $t = 0$
- I_K^ℓ are mode and form factor ($f+(t)$ for $Ke3$ and $f_+(t)$, $f_0(t)$ for $K\mu 3$) dependent decay phase space integrals.

Situation before 2004

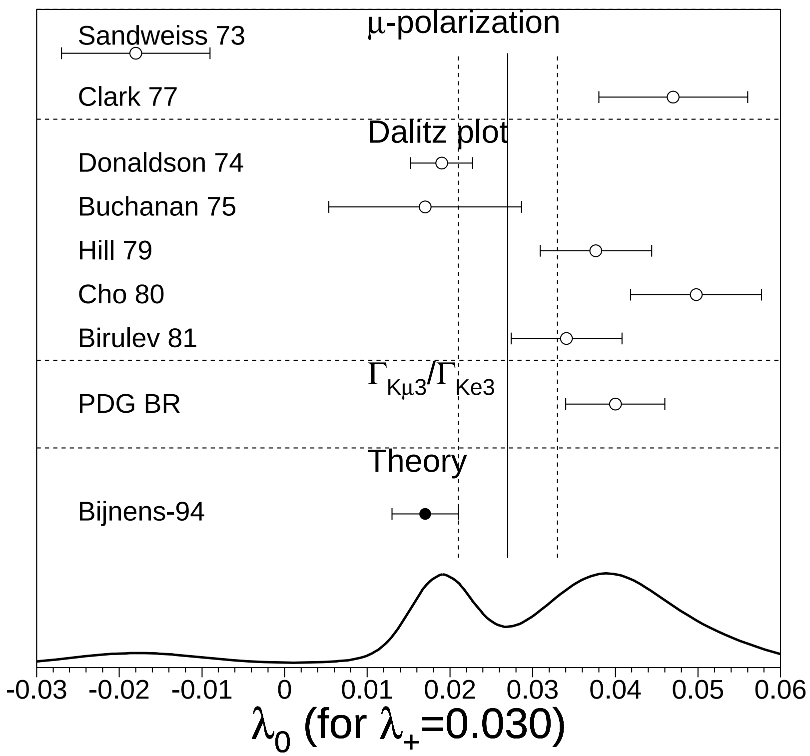
Apart from unitarity problem, V_{us} seemed to be well understood before the new data has arrived:

- Measured with $K_L e3$ ($0.2182 \pm 0.0012_{\mathrm{exp}}$), $K^{\pm}e3$ ($0.2208 \pm 0.0016_{\mathrm{exp}}$) and Hyperon decays ($0.2176 \pm 0.0026$). The most precise measurement came from $K_L e3$ decays.
- $K_L e3$ branching fraction is extracted from various measurements of 36 different experiments performed between 1967-1995, they show good internal agreement
- $f_+(t)$ form factor is measured by ~ 10 experiments, well described by linear λ^+ term. The value of λ^+ is consistent between $K^{\pm}$ (0.028 ± 0.003) and K_L (0.030 ± 0.002) as well as with theory (chiral QCD) expectations (~ 0.028).
- $f_+(0)$ is calculated by Leutweyler and Roos in 1984, their analysis shows that $K^{\pm}e3$ and $K_L e3$ data are consistent.

The only problem in this picture was BNL E865 determination of V_{us} based on $K^{\pm}e3$ data (PRL **91** 261802, published on 31 Dec 2003) which triggered a lot of new experimental activity.

Consistency check: $Ke3$ vs $K\mu3$

V_{us} measured with $Ke3$ should be equal to V_{us} measured with $K\mu3$ ("lepton universality"). Also, $f_+^{Ke3}(t) = f_+^{K\mu3}(t)$. For a linear parameterization of $f_0(t)$ this allows to extract λ_0 from $Br(K\mu3)/Br(Ke3)$:.



- unsatisfactory experimental situation.
- theory (which is used for $f_+(0)$) largely disagree with BR result

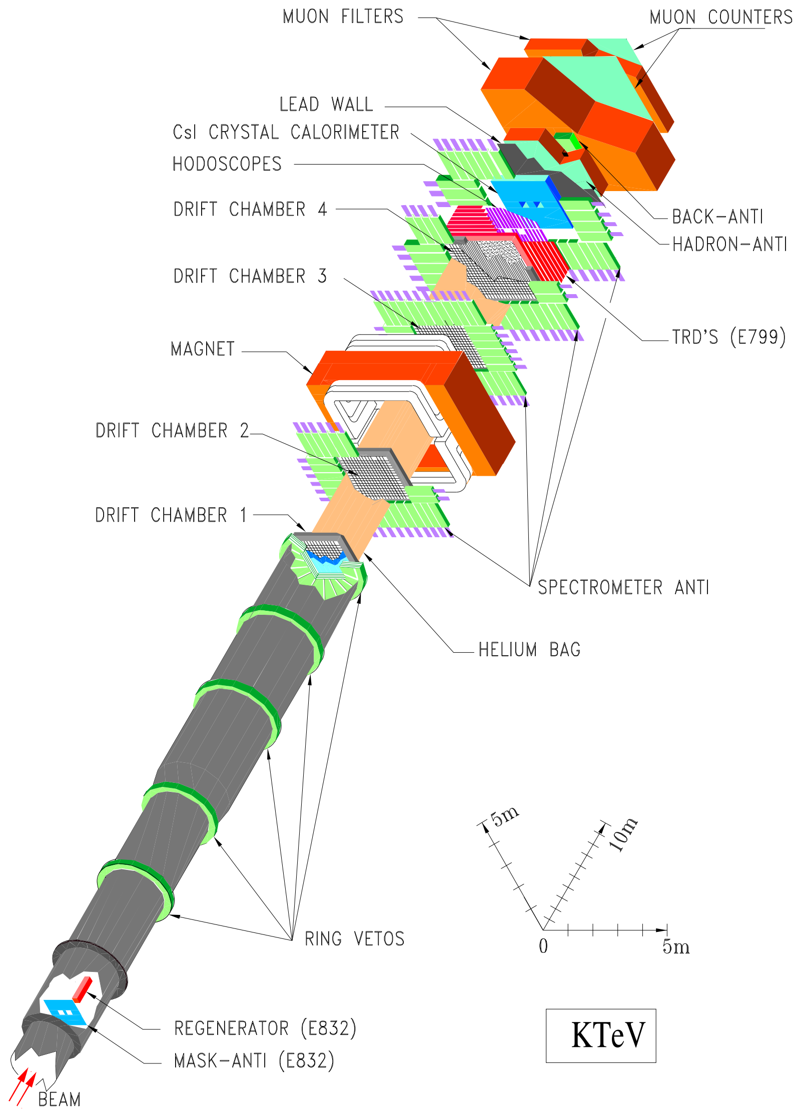
$Ke3$ vs $K\mu 3$ – long standing problem

"For λ_0 measurements, $\chi^2/DF = 88/16$ ... In view of large χ^2/DF, the fit results should be taken with a grain of salt."
T. Tripp, PDG82

"Concerning λ_0 experimental situation is not clear. The value $\lambda_0 = 0.019 \pm 0.004$ obtained in a high statistics experiment in 1974 (Donaldson *et al*) confirmed the theoretical expectations. ... More recent measurements of this parameter however disagree with the above value. The muonic phase space integrals are quite sensituve to λ_0 (If λ_0 is increased from 0.019 to 0.046 (Cho *et al*, 1980) the phase space integrals for $K^+_{\mu 3}$ and $K^0_{\mu 3}$ increase by 6%)."
H. Leutwyler, M. Roos, Z. Phys. **C25**, 91 (1984)

Key measurements to resolve this issue would be λ_0 and $Br(K\mu 3)/Br(Ke3)$, the latter is rather easy for $Re(\epsilon'/\epsilon)$ hadron beam experiment (0.4% error for 1 day low intensity KTeV run), could have been done in late 80s – early 90s.

KTeV detector



KTeV measurement of K_L branching fractions

Since there is no way to tag the kaon, measure all six largest decay modes in terms of five branching fraction ratios and use the constraint that the remaining width is just 0.03%. Use external τ_L to convert branching fractions into partial widths.

The five measured ratios are:

$$\Gamma_{K\mu 3}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^\pm \mu^\mp \nu)/\Gamma(K_L \to \pi^\pm e^\mp \nu) \quad (4)$$

$$\Gamma_{+-0}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^+ \pi^- \pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu) \quad (5)$$

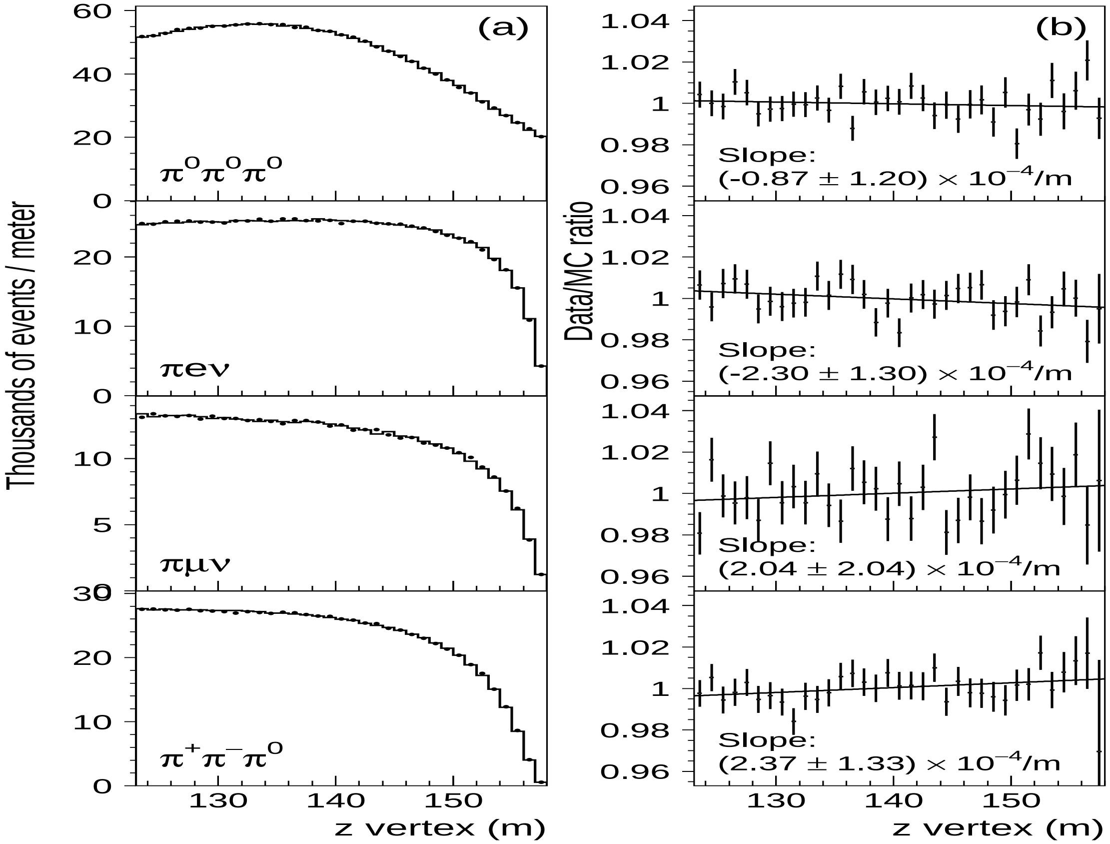
$$\Gamma_{000}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^0 \pi^0 \pi^0)/\Gamma(K_L \to \pi^\pm e^\mp \nu) \quad (6)$$

$$\Gamma_{+-}/\Gamma_{Ke3} \equiv \Gamma(K_L \to \pi^+ \pi^-)/\Gamma(K_L \to \pi^\pm e^\mp \nu) \quad (7)$$

$$\Gamma_{00}/\Gamma_{000} \equiv \Gamma(K_L \to \pi^0 \pi^0)/\Gamma(K_L \to \pi^0 \pi^0 \pi^0), \quad (8)$$

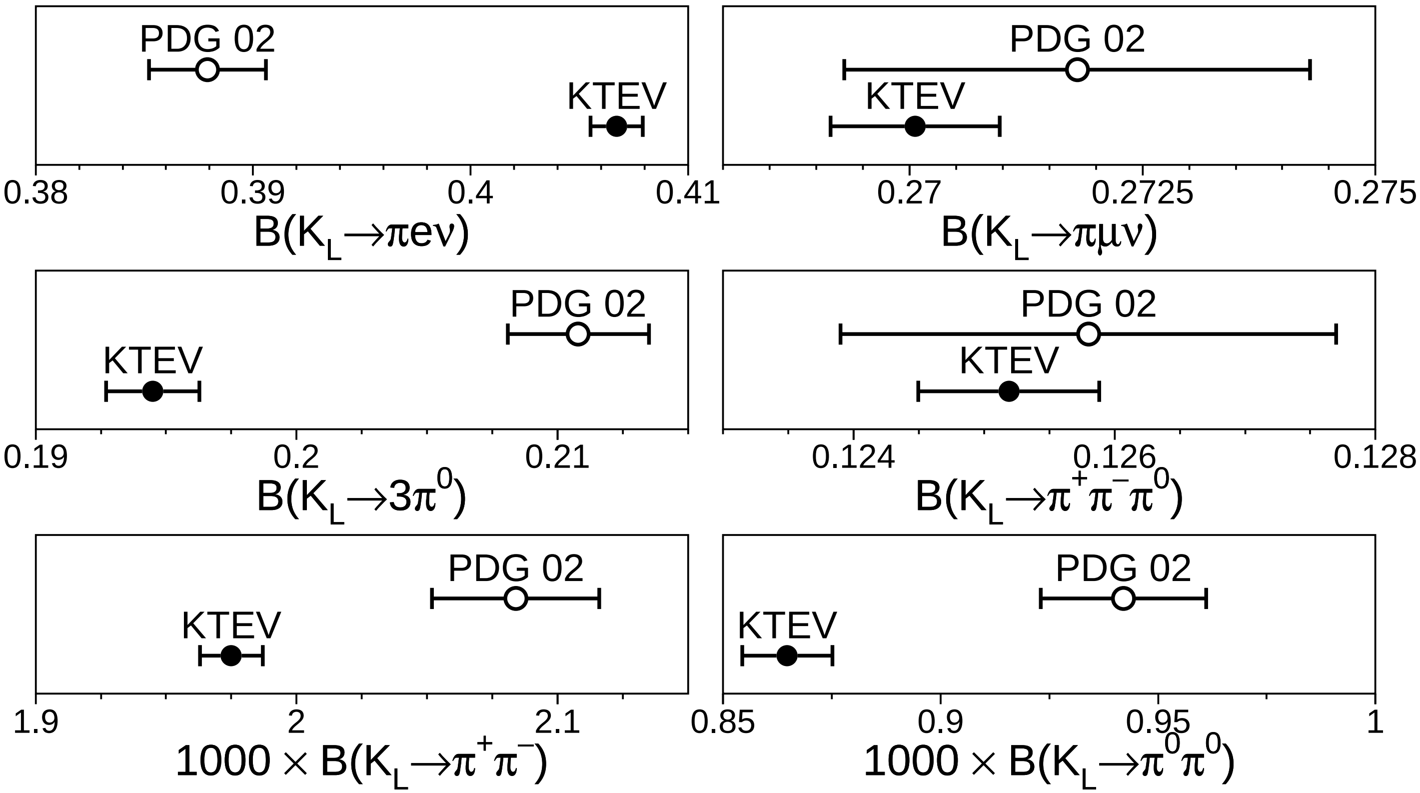
The ratios are formed between **charged** (2-track), **neutral** (0-tracks) decay modes to cancel systematic uncertainties. The "mixed" ratio $\Gamma_{000}/\Gamma_{Ke3}$ is selected to have a common trigger.

KTeV: Acceptance vs Z



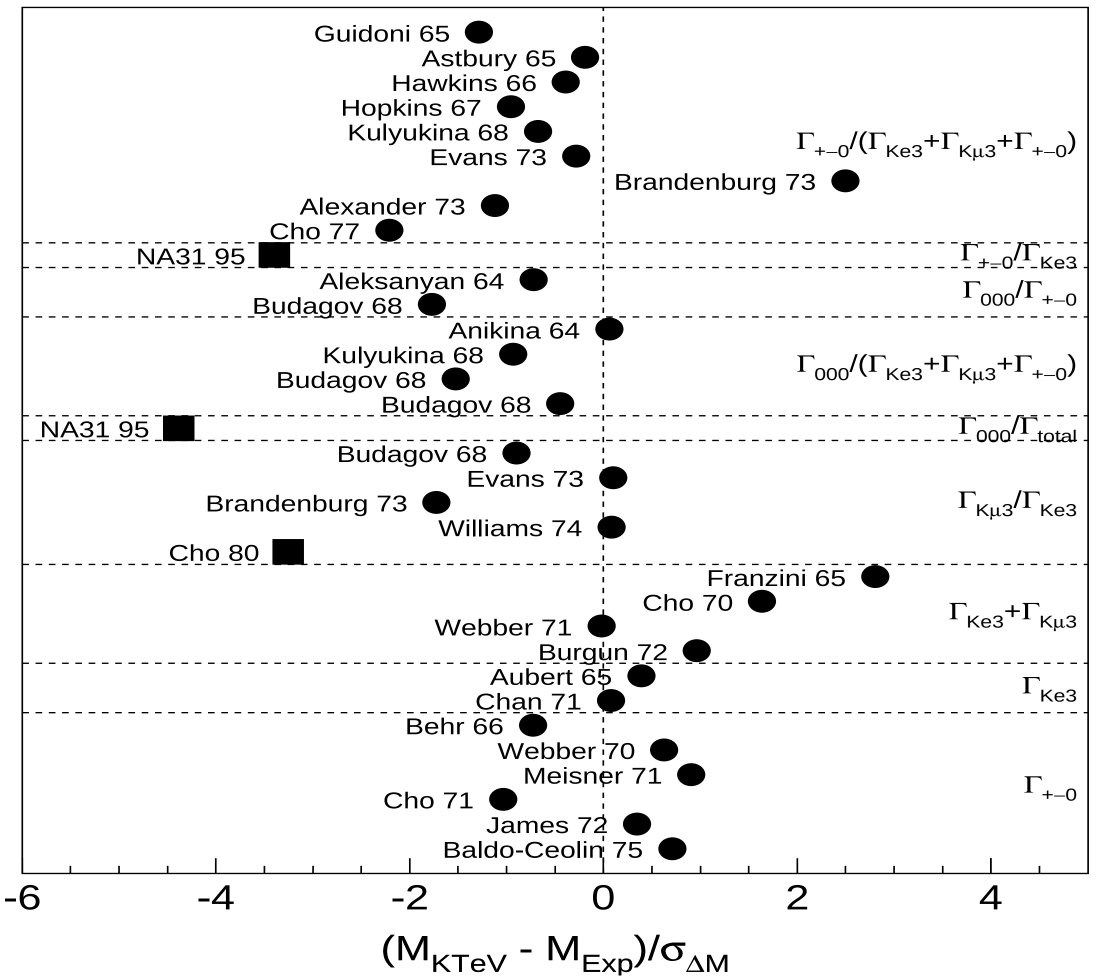
- Acceptance is different for different modes but well described by MC
- Special effort to minimize effects from different particle types (e.g. μ vs π). For example, μ system is not used in the main $K\mu3$ analysis and π^0 decay products are ignored for $\pi^+\pi^-\pi^0$.

KTeV results for K_L Branching Fractions



Large change compared to PDG for 4 out of 6 decay modes. In particular, $Ke3$ is about 5% higher. But $K\mu3$ is consistent with older values.

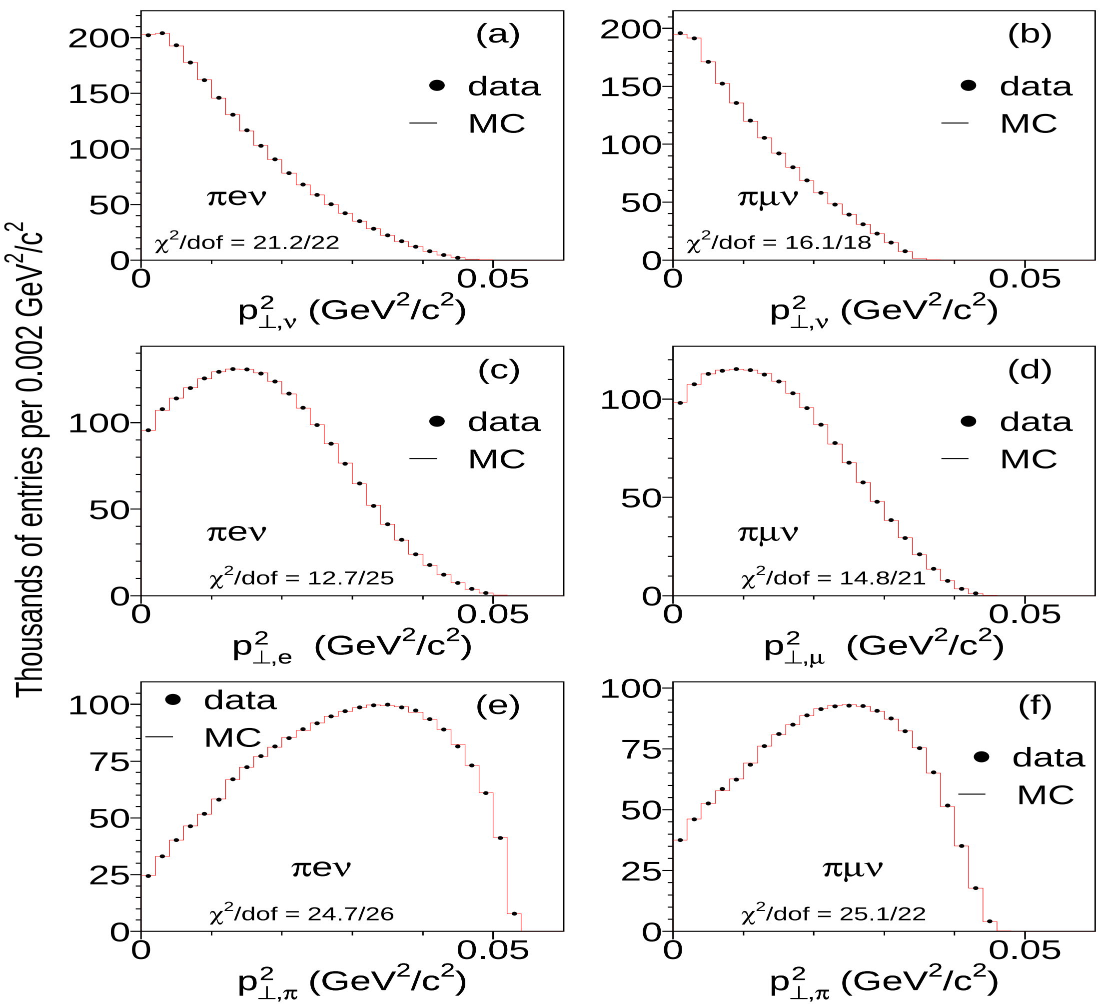
KTeV vs old experiments



- For all experiments: $\chi^2/dof = 83/34$
- Excluding Cho80, NA31: $\chi^2/dof = 42/31$

KTeV measurement of semileptonic form factors

Since kaon energy is unknown (2-fold ambiguity) use boost invariant transverse-t determined using $p_\perp$ of the particles.

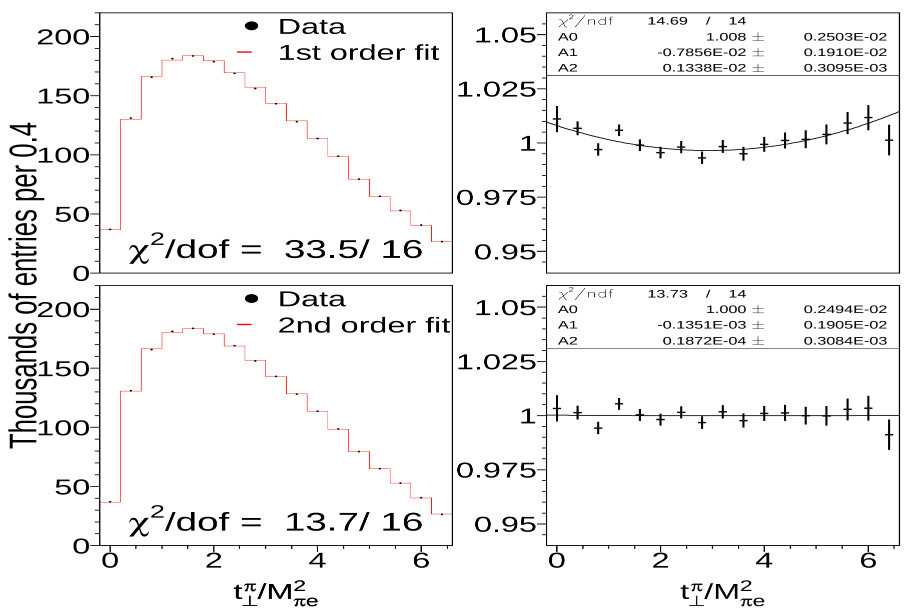


Form factors: non-linear term

Parameterization of the form factors:

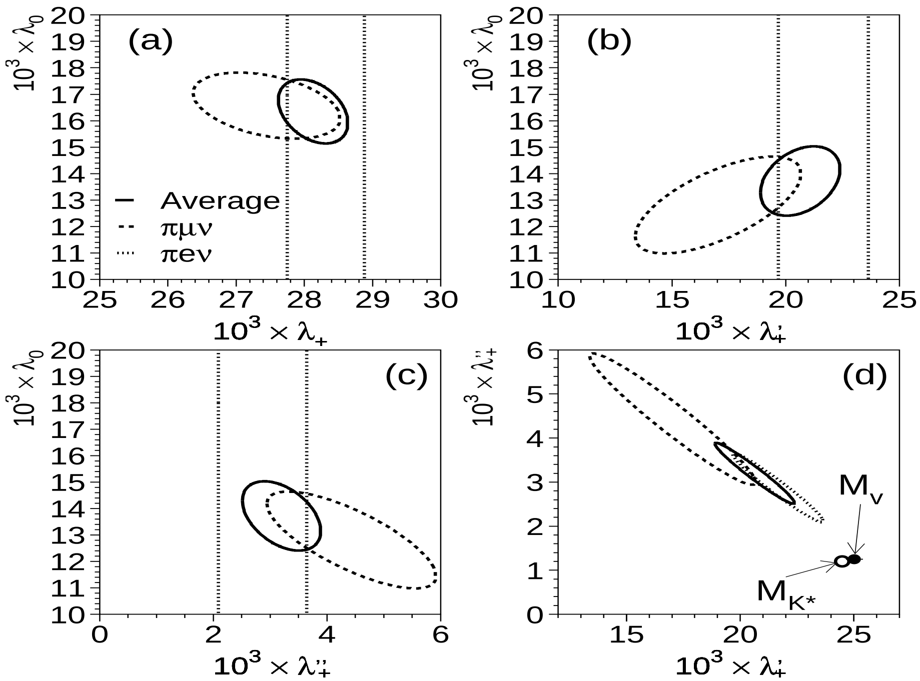
$$\begin{aligned} f_+(t) &= f_+(0) \times \left[1 + \lambda'_+ \frac{t}{M_\pi^2} + \frac{1}{2}\lambda''_+ \frac{t^2}{M_\pi^4}\right] \\ f_0(t) &= f_+(0) \times \left[1 + \lambda'_0 \frac{t}{M_\pi^2}\right] \end{aligned} \quad (9)$$

KTeV sees improvement in the fit to $t_\perp$ distribution using the quadratic parameterization for $f_+(t)$:.



→ the second order fit changes I_K integrals by about -1%

Form factor results



	λ'_+	λ''_+	λ_0 (for $\lambda_+ = 0.0277$)
		$\times 10^{-3}$	
KTeV	20.64 ± 1.75	3.20 ± 0.69	16.5 ± 1.1
ISTRA+	23.24 ± 1.55	1.68 ± 0.82	18.3 ± 1.1
NA48	28.0 ± 2.4	0.2 ± 0.4	
KLOE	25.5 ± 1.5	1.4 ± 0.7	

KTeV check: lepton universality

V_{us} measured with $Ke3$ and $K\mu3$ should be the same – lepton universality. More directly, the ratio of the Fermi coupling constants for electrons and muons must be the same:

$$\left(\frac{G_F^\mu}{G_F^e}\right)^2 = \left[\frac{\Gamma(K_L \to \pi^\pm \mu^\mp \nu)}{\Gamma(K_L \to \pi^\pm e^\mp \nu)}\right] \Big/ \left(\frac{1+\delta_K^\mu}{1+\delta_K^e} \cdot \frac{I_K^\mu}{I_K^e}\right) \qquad (10)$$

- Theoretical uncertainties in $f_+(0)$ cancel for this ratio
- "Matching scale" uncertainties for δ_K^ℓ are reduced: $(1+\delta_K^\mu)/(1+\delta_K^e) = 1.0058 \pm 0.0010$
- Uncertainties for the "rate" measurement of $\Gamma(K_L \to \pi^\pm \mu^\mp \nu)/\Gamma(K_L \to \pi^\pm e^\mp \nu) = 0.6640 \pm 0.0026$ differ vs the "shape" measurement of the form factors.
- Ratio of $I_K^\mu / I_K^e = 0.6622 \pm 0.0018$ has reduced dependence on the form factor parameterization.

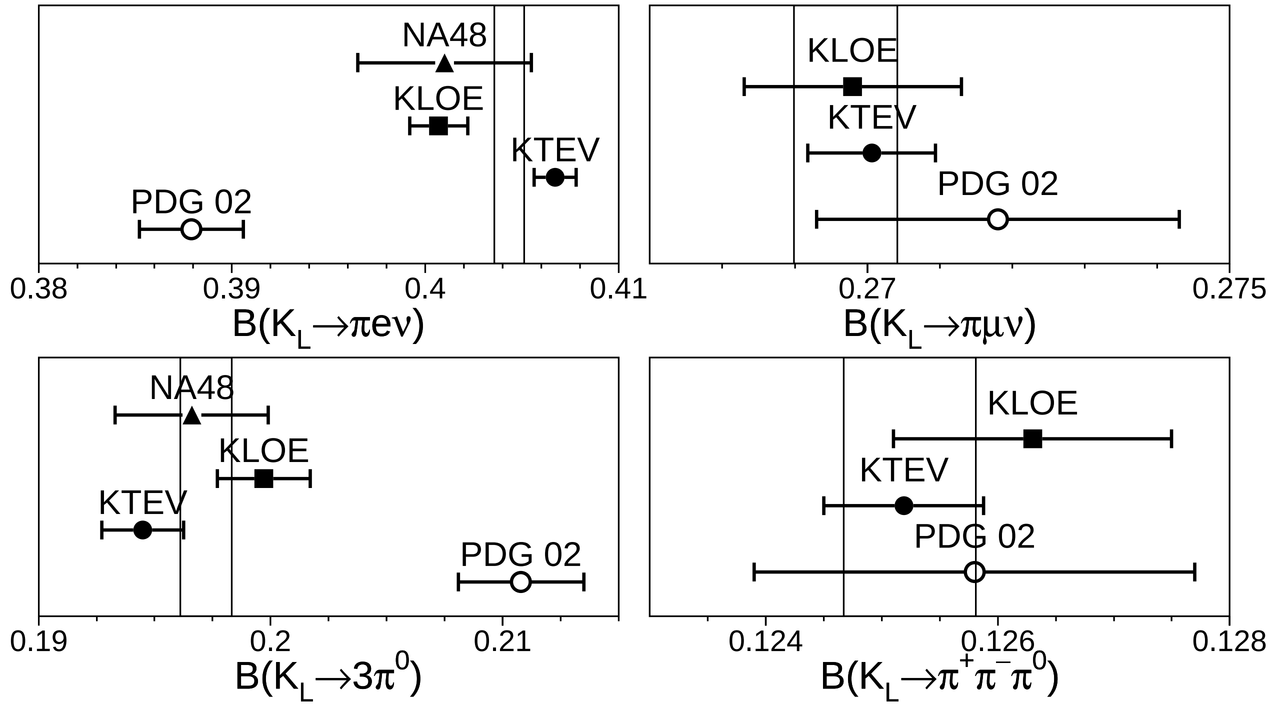
$$(G_F^\mu / G_F^e)^2 = 0.9969 \pm 0.0048$$

NA48 results

NA48 presents new results for

- $B(K_L \to 3\pi^0) = 0.1966 \pm 0.033$ (normalized to $K_S \to 2\pi^0$) — consistent with KTeV
- $B(K_L e3)/B(K_L \to \text{all 2 track}) = 0.498 \pm 0.004$. Using $B(K_L \to 3\pi^0)$ NA48 determines $B(Ke3) = 0.4010 \pm 0.0045$ — again consistent with KTeV.
- $B(K^\pm e3) = (5.14 \pm 0.06)\%$ (using $K^\pm \to \pi^\pm \pi^0$) as normalization mode — consistent with E865.
- Measurement of $K_L e3$ form factor (linear parameterization only) $\lambda_+ = 0.0288 \pm 0.0012$, also in agreement with KTeV (0.0283 ± 0.0006) but with much larger systematic uncertainty. (don't use $t_\perp$ reconstruction method, large uncertainty from unknown kaon momentum spectrum)
- NA48 **does not confirm** non-linear λ'' term in form factor dependence.

KTeV vs KLOE vs NA48



Lepton universality for the average:

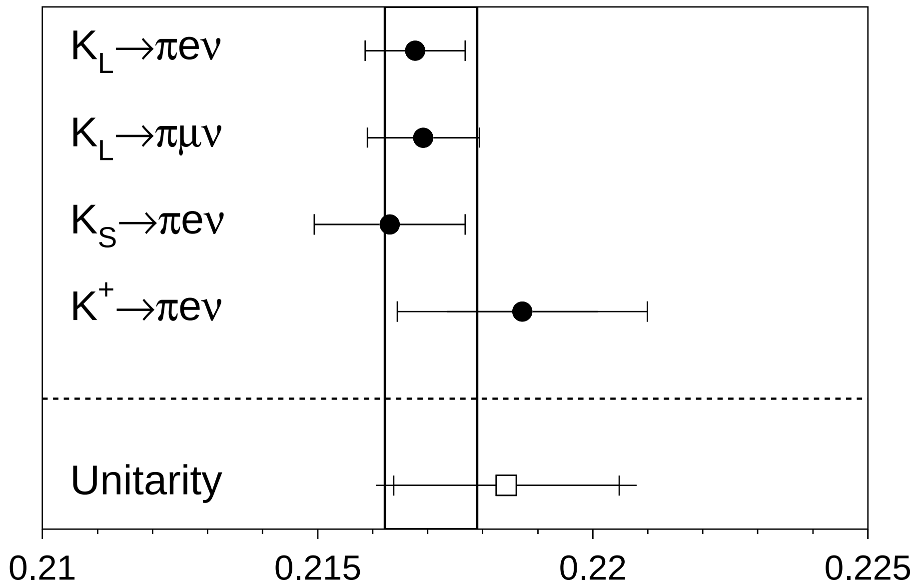
$$(G_F^\mu / G_F^e)^2 = 1.0014 \pm 0.0045$$

Using $Re(\epsilon'/\epsilon)$ compare with KLOE $\Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0)$

KLOE	KTeV (using K_L)	Average (using K_L)
2.2549 ± 0.0054	2.261 ± 0.033	2.218 ± 0.024

$V_{us} f_+(0)$ – Semileptonic Decays

$|V_{us}| f_+(0)$ separates theoretical and experimental errors. Using KTeV values of the phase space integrals and $\tau_L = 51.11 \pm 0.19$,



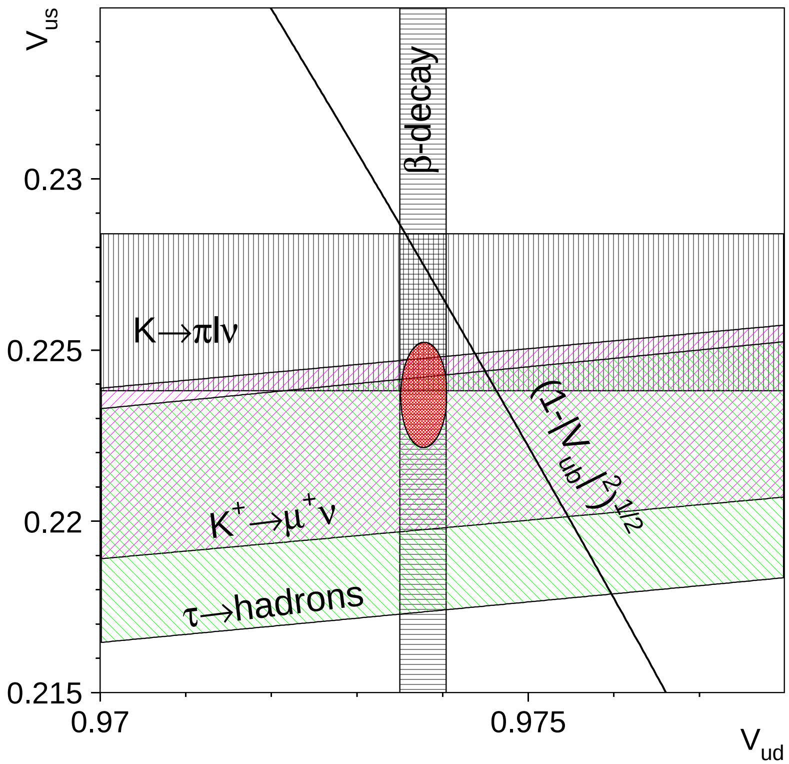
Taking into account large correlation between measurements:

$$V_{us} f_+(0) = 0.2171 \pm 0.0008$$

And using Quenched LQCD value of $f_+(0) = 0.960 \pm 0.009$:

$$|V_{us}| = 0.2261 \pm 0.0009_{\mathrm{exp}} \pm 0.0021_{\mathrm{th.}}$$

V_{us} and V_{ud}



Kaon decay constant measurement and strange τ decays combined with lattice QCD calculation and OPE provide relation between V_{ud} and V_{us}. Combined result:

$$|V_{ud}| = 0.97378 \pm 0.00027 \quad |V_{us}| = 0.22369 \pm 0.00154$$

Conclusions and Outlook

Using new experimental results for V_{ud} and V_{us}, deviation from unitarity $\delta = 0.0017 \pm 0.0009$. From this result, a violation of unitarity is limited to less than 0.35% at 99% *c.l.*

The next V_{us} results from hadron beam experiments include completing of the program with charged kaons (primarily NA48).

Also, already collected high statistics Ke3 samples (0.5×10^9 for KTeV) may help to fix the size of the second order term for $f_+(t)$.