Recent Measurements of $V_{us}$

A. Glazov

DESY
$V_{us}$ and Unitarity check

CKM matrix describes the quark mixing:

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\] (1)

$V_{us}$ is the oldest known mixing element (Cabibbo angle). Yet many exciting developments have happened in the last two years!

Unitarity of CKM matrix requires:

\[
1 - (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = \delta = 0
\] (2)

Largest contribution comes from $|V_{ud}|$, next from $|V_{us}|$, negligible from $|V_{ub}|$.

According PDG-02, $\delta = 0.0043 \pm 0.0019$, about 2.2σ deviation from unitarity, with uncertainty from $V_{us}$ of 0.0010.
Methods to extract $V_{us}$

The most accurate approach to extract $V_{us}$ is to use rate of semileptonic kaon decays:

$$\Gamma_{K\ell3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta^\ell_K) C^2 |V_{us}|^2 f_+^2(0) I^\ell_K,$$

(3)

Here:

- $S_{EW}, \delta^\ell_K$ – universal short- and mode dependent long-distance radiative corrections.
- $C = 1$ for $K_L$ and $C = 1/2$ for $K^{\pm}$.
- $f_+^2(0)$ is calculated in theory form factor value for $t = 0$
- $I^\ell_K$ are mode and form factor ($f_+(t)$ for $Ke3$ and $f_+(t), f_0(t)$ for $K\mu3$) dependent decay phase space integrals.
Situation before 2004

Apart from unitarity problem, $V_{us}$ seemed to be well understood before the new data has arrived:

- Measured with $K_L e^3 \, (0.2182 \pm 0.0012_{\text{exp}})$, $K^\pm e^3 \, (0.2208 \pm 0.0016_{\text{exp}})$ and Hyperon decays $\, (0.2176 \pm 0.0026)$. The most precise measurement came from $K_L e^3$ decays.
- $K_L e^3$ branching fraction is extracted from various measurements of 36 different experiments performed between 1967-1995, they show good internal agreement
- $f_+(t)$ form factor is measured by $\sim 10$ experiments, well described by linear $\lambda^+$ term. The value of $\lambda^+$ is consistent between $K^\pm \, (0.028 \pm 0.003)$ and $K_L \, (0.030 \pm 0.002)$ as well as with theory (chiral QCD) expectations ($\sim 0.028$).
- $f_+(0)$ is calculated by Leutweyler and Roos in 1984, their analysis shows that $K^\pm e^3$ and $K_L e^3$ data are consistent.

The only problem in this picture was BNL E865 determination of $V_{us}$ based on $K^\pm e^3$ data (PRL 91 261802, published on 31 Dec 2003) which triggered a lot of new experimental activity.
Consistency check: $Ke3$ vs $K\mu3$

$V_{us}$ measured with $Ke3$ should be equal to $V_{us}$ measured with $K\mu3$ ("lepton universality"). Also, $f_+^{Ke3}(t) = f_+^{K\mu3}(t)$. For a linear parameterization of $f_0(t)$ this allows to extract $\lambda_0$ from $Br(K\mu3)/Br(Ke3)$:

- unsatisfactory experimental situation.
- theory (which is used for $f_+(0)$) largely disagree with BR result.
Ke3 vs Kμ3 – long standing problem

“For λ₀ measurements, χ²/DF = 88/16 ... In view of large χ²/DF, the fit results should be taken with a grain of salt.”
T. Tripp, PDG82

“Concerning λ₀ experimental situation is not clear. The value λ₀ = 0.019 ± 0.004 obtained in a high statistics experiment in 1974 (Donaldson et al) confirmed the theoretical expectations. ... More recent measurements of this parameter however disagree with the above value. The muonic phase space integrals are quite sensitive to λ₀ (If λ₀ is increased from 0.019 to 0.046 (Cho et al, 1980) the phase space integrals for $K^+_\mu3$ and $K^0_{\mu3}$ increase by 6%).”

Key measurements to resolve this issue would be λ₀ and $Br(K\mu3)/Br(Ke3)$, the latter is rather easy for Re($\epsilon'/\epsilon$) hadron beam experiment (0.4% error for 1 day low intensity KTeV run), could have been done in late 80s – early 90s.
KTeV measurement of $K_L$ branching fractions

Since there is no way to tag the kaon, measure all six largest decay modes in terms of five branching fraction ratios and use the constraint that the remaining width is just 0.03%. Use external $\tau_L$ to convert branching fractions into partial widths.

The five measured ratios are:

\[
\begin{align*}
\frac{\Gamma_{K\mu3}}{\Gamma_{Ke3}} &\equiv \frac{\Gamma(K_L \to \pi^\pm\mu^\mp\nu)}{\Gamma(K_L \to \pi^\pm e^\mp\nu)} \quad (4) \\
\frac{\Gamma_{+0}}{\Gamma_{Ke3}} &\equiv \frac{\Gamma(K_L \to \pi^+\pi^-\pi^0)}{\Gamma(K_L \to \pi^\pm e^\mp\nu)} \quad (5) \\
\frac{\Gamma_{000}}{\Gamma_{Ke3}} &\equiv \frac{\Gamma(K_L \to \pi^0\pi^0\pi^0)}{\Gamma(K_L \to \pi^\pm e^\mp\nu)} \quad (6) \\
\frac{\Gamma_{+-}}{\Gamma_{Ke3}} &\equiv \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^\pm e^\mp\nu)} \quad (7) \\
\frac{\Gamma_{00}}{\Gamma_{000}} &\equiv \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_L \to \pi^0\pi^0\pi^0)}, \quad (8)
\end{align*}
\]

The ratios are formed between charged (2-track), neutral (0-tracks) decay modes to cancel systematic uncertainties. The “mixed” ratio $\frac{\Gamma_{000}}{\Gamma_{Ke3}}$ is selected to have a common trigger.
• Acceptance is different for different modes but well described by MC

• Special effort to minimize effects from different particle types (e.g. $\mu$ vs $\pi$). For example, $\mu$ system is not used in the main $K\mu3$ analysis and $\pi^0$ decay products are ignored for $\pi^+\pi^-\pi^0$. 
KTeV results for $K_L$ Branching Fractions

Large change compared to PDG for 4 out of 6 decay modes. In particular, $K\epsilon 3$ is about 5% higher. But $K\mu 3$ is consistent with older values.
For all experiments: $\chi^2/dof = 83/34$

Excluding Cho80, NA31: $\chi^2/dof = 42/31$
KTeV measurement of semileptonic form factors

Since kaon energy is unknown (2-fold ambiguity) use boost invariant transverse-\(t\) determined using \(p_\perp\) of the particles.
Form factors: non-linear term

Parameterization of the form factors:

\[
\begin{align*}
  f_+(t) &= f_+(0) \times \left[ 1 + \lambda' \frac{t}{M_\pi^2} + \frac{1}{2} \lambda'' \frac{t^2}{M_\pi^4} \right] \\
  f_0(t) &= f_+(0) \times \left[ 1 + \lambda_0' \frac{t}{M_\pi^2} \right] 
\end{align*}
\] (9)

KTeV sees improvement in the fit to \( t_\perp \) distribution using the quadratic parameterization for \( f_+(t) \):

\[ \chi^2/dof = 33.5/16 \]

\[ \chi^2/dof = 13.7/16 \]

→ the second order fit changes \( I_K \) integrals by about \(-1\%\)
# Form factor results

![Graphs showing form factor results](image)

### Table of Results

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\lambda'_+ \times 10^{-3}$</th>
<th>$\lambda''_+ \times 10^{-3}$</th>
<th>$\lambda_0$ (for $\lambda_+ = 0.0277$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KTeV</td>
<td>$20.64 \pm 1.75$</td>
<td>$3.20 \pm 0.69$</td>
<td>$16.5 \pm 1.1$</td>
</tr>
<tr>
<td>ISTRA+</td>
<td>$23.24 \pm 1.55$</td>
<td>$1.68 \pm 0.82$</td>
<td>$18.3 \pm 1.1$</td>
</tr>
<tr>
<td>NA48</td>
<td>$28.0 \pm 2.4$</td>
<td>$0.2 \pm 0.4$</td>
<td></td>
</tr>
<tr>
<td>KLOE</td>
<td>$25.5 \pm 1.5$</td>
<td>$1.4 \pm 0.7$</td>
<td></td>
</tr>
</tbody>
</table>
KTeV check: lepton universality

$V_{us}$ measured with $K e 3$ and $K \mu 3$ should be the same – lepton universality. More directly, the ratio of the Fermi coupling constants for electrons and muons must be the same:

$$
\left( \frac{G_{F}^{\mu}}{G_{F}^{e}} \right)^{2} = \frac{\Gamma(K_{L} \rightarrow \pi^{\pm} e^{\mp} \nu)}{\Gamma(K_{L} \rightarrow \pi^{\pm} e^{\mp} \nu)} / \left( \frac{1 + \delta_{K}^{\mu}}{1 + \delta_{K}^{e}} \cdot \frac{I_{K}^{\mu}}{I_{K}^{e}} \right)
$$

(10)

- Theoretical uncertainties in $f_{+}(0)$ cancel for this ratio
- "Matching scale" uncertainties for $\delta_{K}^{\ell}$ are reduced:
  $$(1 + \delta_{K}^{\mu})/(1 + \delta_{K}^{e}) = 1.0058 \pm 0.0010$$
- Uncertainties for the "rate" measurement of
  $\Gamma(K_{L} \rightarrow \pi^{\pm} e^{\mp} \nu)/\Gamma(K_{L} \rightarrow \pi^{\pm} e^{\mp} \nu) = 0.6640 \pm 0.0026$
  differ vs the "shape" measurement of the form factors.
- Ratio of $I_{K}^{\mu}/I_{K}^{e} = 0.6622 \pm 0.0018$ has reduced dependence on the form factor parameterization.

$$
\left( \frac{G_{F}^{\mu}}{G_{F}^{e}} \right)^{2} = 0.9969 \pm 0.0048
$$
NA48 presents new results for

- \( B(K_L \rightarrow 3\pi^0) = 0.1966 \pm 0.033 \) (normalized to \( K_S \rightarrow 2\pi^0 \)) — consistent with KTeV

- \( B(K_L e3)/B(K_L \rightarrow \text{all 2 track}) = 0.498 \pm 0.004 \). Using \( B(K_L \rightarrow 3\pi^0) \) NA48 determines \( B(Ke3) = 0.4010 \pm 0.0045 \) — again consistent with KTeV.

- \( B(K^\pm e3) = (5.14 \pm 0.06)\% \) (using \( K^\pm \rightarrow \pi^\pm \pi^0 \)) as normalization mode — consistent with E865.

- Measurement of \( K_L e3 \) form factor (linear parameterization only) \( \lambda_+ = 0.0288 \pm 0.0012 \), also in agreement with KTeV (0.0283 ± 0.0006) but with much larger systematic uncertainty. (don’t use \( t_\perp \) reconstruction method, large uncertainty from unknown kaon momentum spectrum)

- NA48 **does not confirm** non-linear \( \lambda'' \) term in form factor dependence.
Lepton universality for the average:

\[ \left( \frac{G_{\mu}^u}{G_{\mu}^e} \right)^2 = 1.0014 \pm 0.0045 \]

Using \( Re(\epsilon'/\epsilon) \) compare with KLOE \( \Gamma(K_S \rightarrow \pi^+\pi^-)/\Gamma(K_S \rightarrow \pi^0\pi^0) \)

<table>
<thead>
<tr>
<th>KLOE</th>
<th>KTeV (using ( K_L ))</th>
<th>Average (using ( K_L ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2549 ± 0.0054</td>
<td>2.261 ± 0.033</td>
<td>2.218 ± 0.024</td>
</tr>
</tbody>
</table>
$V_{us} f_{+}(0) - $ Semileptonic Decays

$|V_{us}| f_{+}(0)$ separates theoretical and experimental errors. Using KTeV values of the phase space integrals and $\tau_L = 51.11 \pm 0.19$,

![Graph showing $K_L \rightarrow \pi e\nu$, $K_L \rightarrow \pi \mu\nu$, $K_S \rightarrow \pi e\nu$, $K^+ \rightarrow \pi e\nu$, and Unitarity]

Taking into account large correlation between measurements:

$$V_{us} f_{+}(0) = 0.2171 \pm 0.0008$$

And using Quenched LQCD value of $f_{+}(0) = 0.960 \pm 0.009$:

$$|V_{us}| = 0.2261 \pm 0.0009_{\text{exp}} \pm 0.0021_{\text{th.}}$$
Kaon decay constant measurement and strange $\tau$ decays combined with lattice QCD calculation and OPE provide relation between $V_{ud}$ and $V_{us}$. Combined result:

$$|V_{ud}| = 0.97378 \pm 0.00027 \quad |V_{us}| = 0.22369 \pm 0.00154$$
Conclusions and Outlook

Using new experimental results for $V_{ud}$ and $V_{us}$, deviation from unitarity $\delta = 0.0017 \pm 0.0009$. From this result, a violation of unitarity is limited to less than 0.35% at 99% c.l.

The next $V_{us}$ results from hadron beam experiments include completing of the program with charged kaons (primarily NA48).

Also, already collected high statistics Ke3 samples ($0.5 \times 10^9$ for KTeV) may help to fix the size of the second order term for $f_+(t)$. 