$(g_\mu - 2)/2$ and hadronic e^+e^- cross sections

Simon Eidelman

Budker Institute of Nuclear Physics,
Novosibirsk, Russia

Outline

1. Why $(g_\mu - 2)/2$?
2. $(g_\mu - 2)/2$ and $e^+e^- \to$ hadrons
3. $(g_\mu - 2)/2$ and τ Decays
4. Prospects
5. Conclusions

Muon Anomalous Magnetic Moment

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = (g-2)/2.$$

a_μ is measured with a $5 \cdot 10^{-7}$ relative accuracy:

G.W. Bennett et al., 2004 $\quad a_\mu = (11659208 \pm 6) \cdot 10^{-10}$.

$a_{\rm e}$ is measured with a $4 \cdot 10^{-9}$ accuracy, but a_μ is much more sensitive to new physics effects: the gain is usually $\sim (m_\mu/m_{\rm e})^2 \approx 4.3 \cdot 10^4$.

Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates new physics beyond the Standard Model.

$$a_\mu^{\rm th} = a_\mu^{\rm SM} + a_\mu^{\rm non-SM}, \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}.$$

QED and EW Contributions to a_μ

For the QED term the analytical calculation of the α^3 terms, numerical calculation of the α^4 terms and estimation of some of the α^5 terms gives (V. Hughes and T. Kinoshita, 1999):

$$a_\mu^{\rm QED} = (11658470.6 \pm 0.3) \cdot 10^{-10}.$$

A recent calculation of 469 diagrams of the α^4 terms gave the new value (T. Kinoshita and M. Nio, 2004) for the QED term:

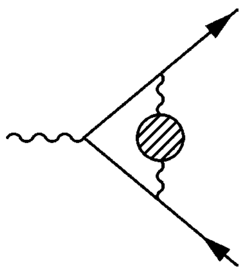
$$a_\mu^{\rm QED} = (11658471.9 \pm 0.2) \cdot 10^{-10}.$$

For the EW term the most recent estimation including all one- and two-loop terms as well as part of three-loop terms gives (A. Czarnecki, W.J. Marciano and A. Vainshtein, 2002):

$$a_\mu^{\rm EW} = (15.4 \pm 0.1 \pm 0.2) \cdot 10^{-10}.$$

Hadronic contribution a_μ^{had}

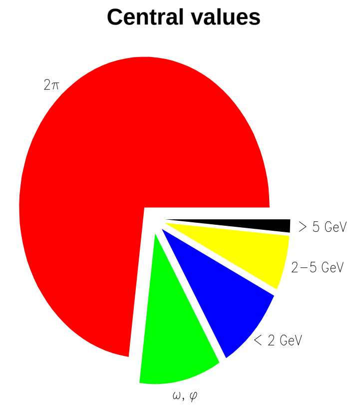
$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$

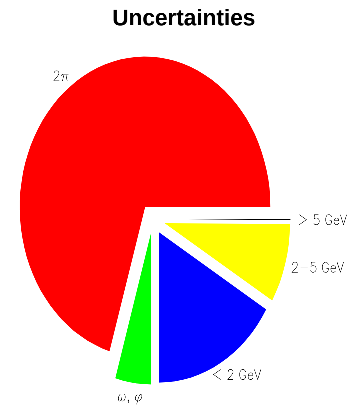


$$a_\mu^{\text{had,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2},$$

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$, $1/s^2$ emphasizes the role of low energies, particularly important is the reaction $e^+e^- \to \pi^+\pi^-$ with a large cross section below 1 GeV.





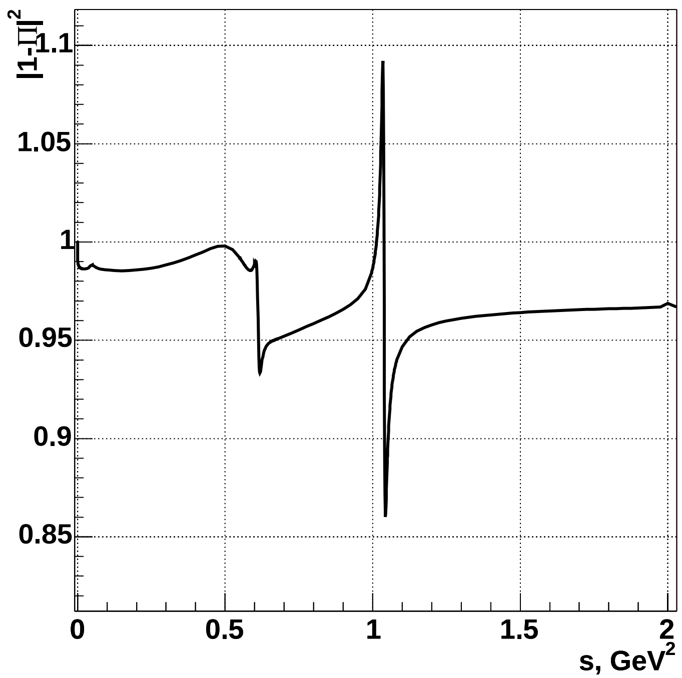
Calculations of $a_\mu^{\mathrm{had,LO}}$

Authors	Year	$a_\mu^{\mathrm{had,LO}}$, 10^{-10}
C.Bouchiat, L.Michel	1961	$\simeq 648$
T.Kinoshita, R.J.Oakes	1967	$\simeq 750$
M.Gourdin, E. de Rafael	1969	650 ± 50
A.Bramon et al.	1972	680 ± 90
V.Barger et al.	1975	660 ± 100
J.Calmet et al.	1977	702 ± 80
T.Kinoshita et al.	1985	707 ± 18
S.Eidelman, F.Jegerlehner	1995	702 ± 15
R.Alemany et al.	1998	701.1 ± 9.4
M.Davier, A.Höcker	1998	692.4 ± 6.2

Measurement of R(s)

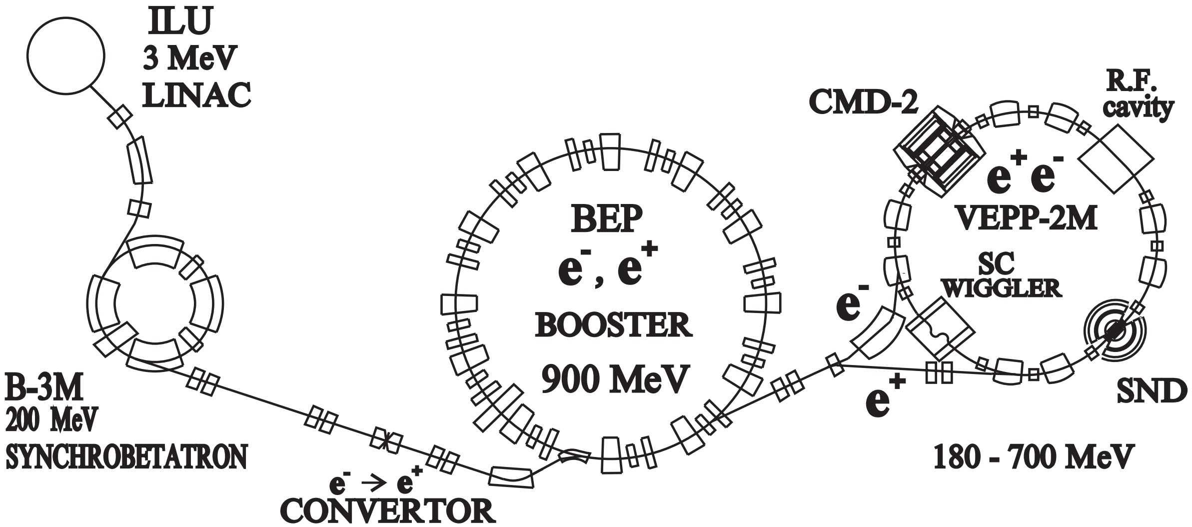
- $\sqrt{s} < 2$ GeV – exclusive modes $(\pi^+\pi^-, \pi^+\pi^-\pi^0, \ldots, K\bar{K}, \ldots)$
- Possibly missing (small σ, undetected) final states
- Above 2 GeV – total R (all multihadronic events)
- Initial state radiation (ISR), vacuum polarization (VP), final state radiation (FSR): M. Drees, K. Hikasa, 1990

$$\sigma_{\text{bare}} = \sigma_{\text{dressed}} |1 - \Pi(s)|^2$$



$$|1 - \Pi(s)|^2$$

VEPP-2M Collider in Novosibirsk



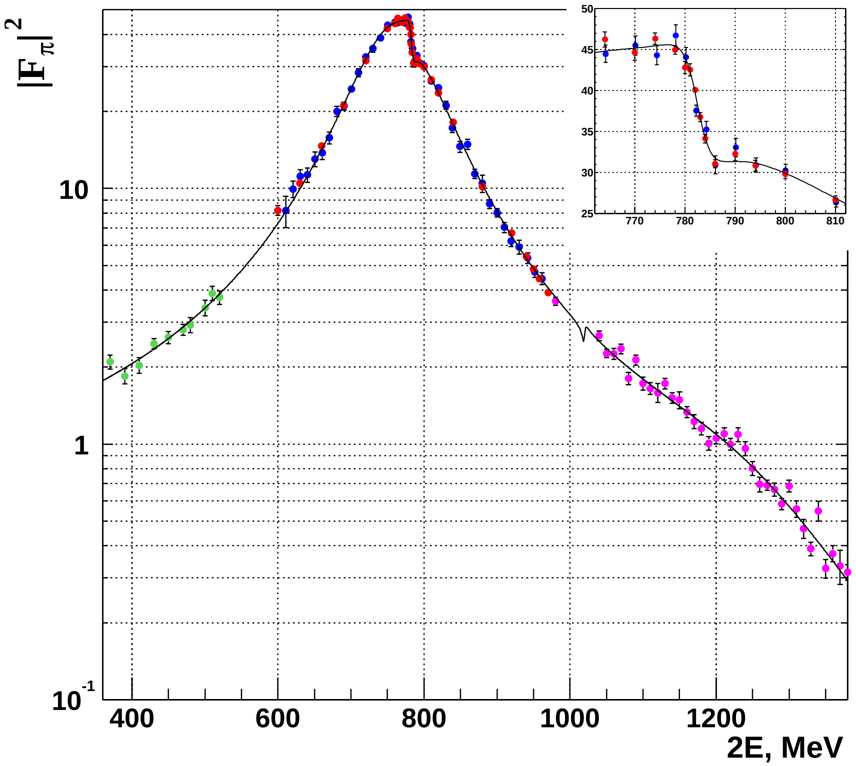
- Experiments: 1974 – 2000
- Energy range: $0.36 < \sqrt{s} < 1.40$ GeV
- Peak luminosity: $L_{\text{peak}} = 3 \cdot 10^{30}$ cm^{-2} s^{-1}
- Integrated luminosity ≈ 100 pb^{-1} in Novosibirsk below 1.4 GeV compared to ≈ 6 pb^{-1} in Orsay and Frascati at $1.4 < \sqrt{s} < 3.0$ GeV!

Study of $e^+e^- \to \pi^+\pi^-$ at CMD-2

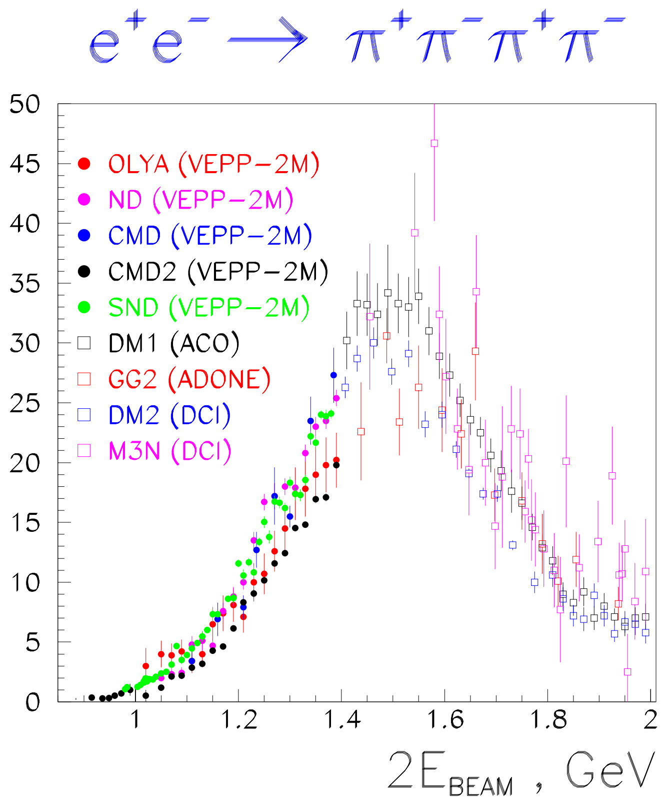
This process dominates $a_\mu^{\rm had,LO}$ ($\sim 73\%$). CMD-2 analyzed 114k events in the crucial ρ meson region from 600 to 960 MeV. Analysis of the full data sample (more than 1M events) will soon be completed.

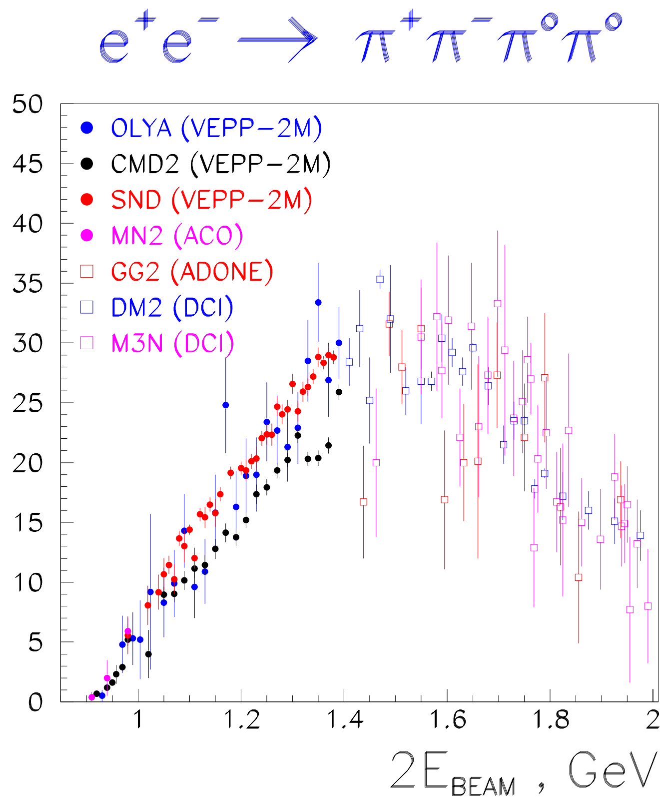
Systematic error:

- 1.2% at $\sqrt{s} < 600$ MeV
- 0.6% at $600 < \sqrt{s} < 960$ MeV (small sample) and 1% (big sample)
- (1.3–5.0)% at $\sqrt{s} > 1040$ MeV



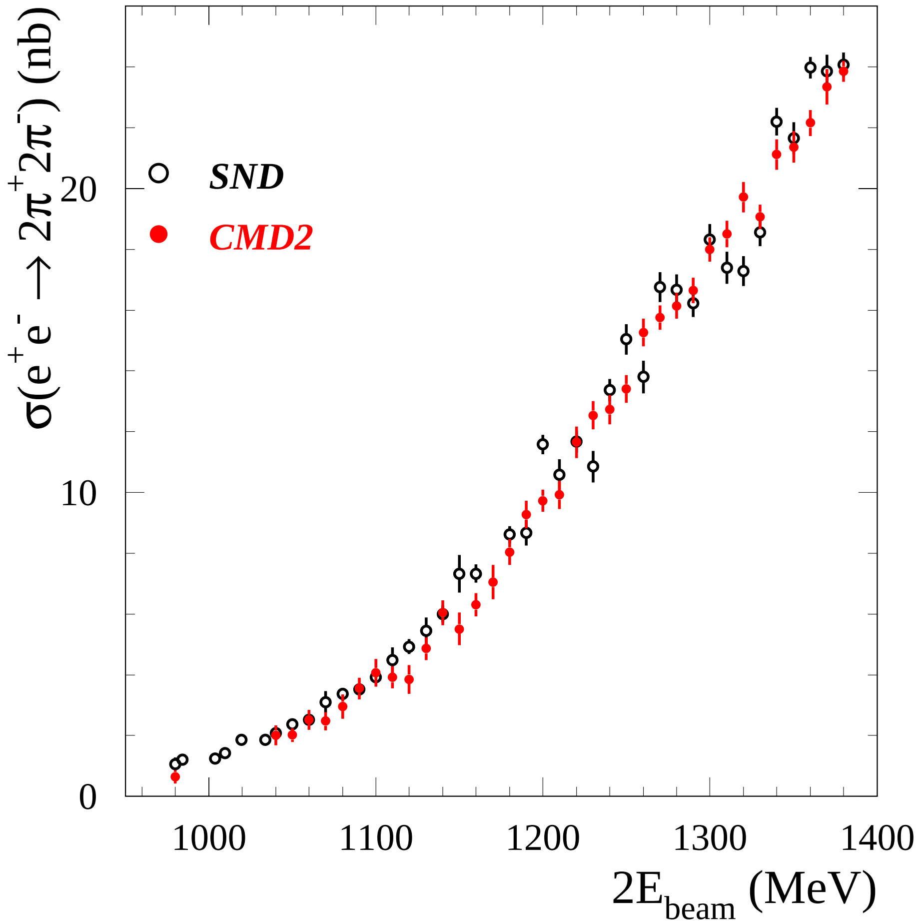
Study of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$

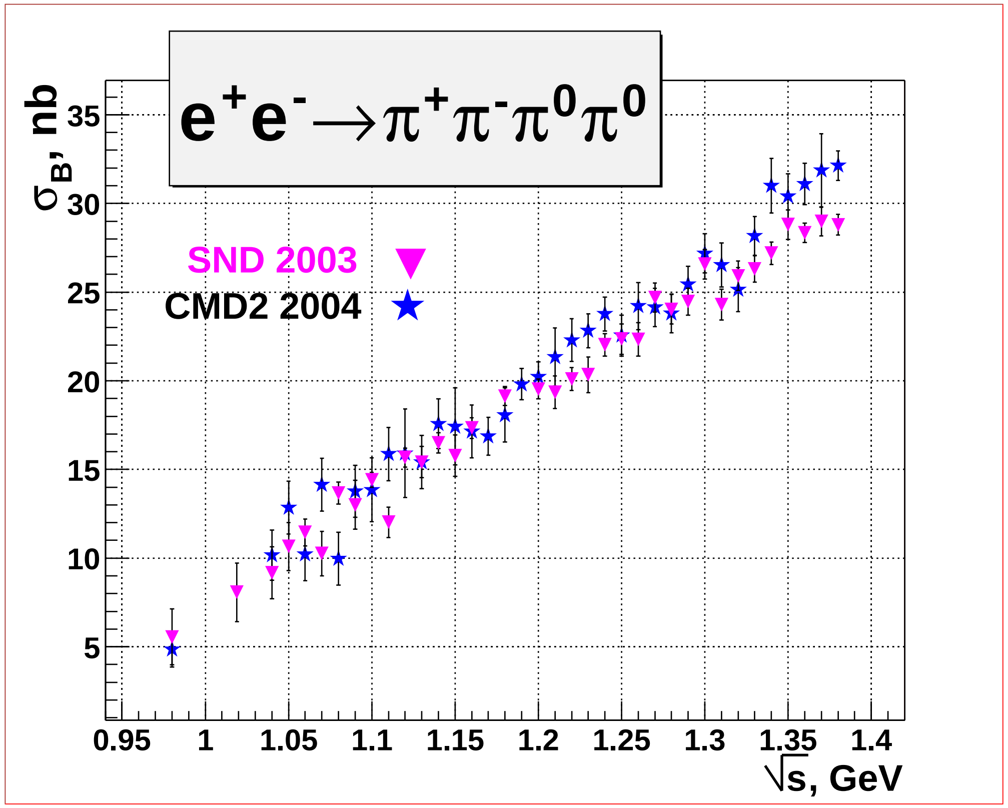




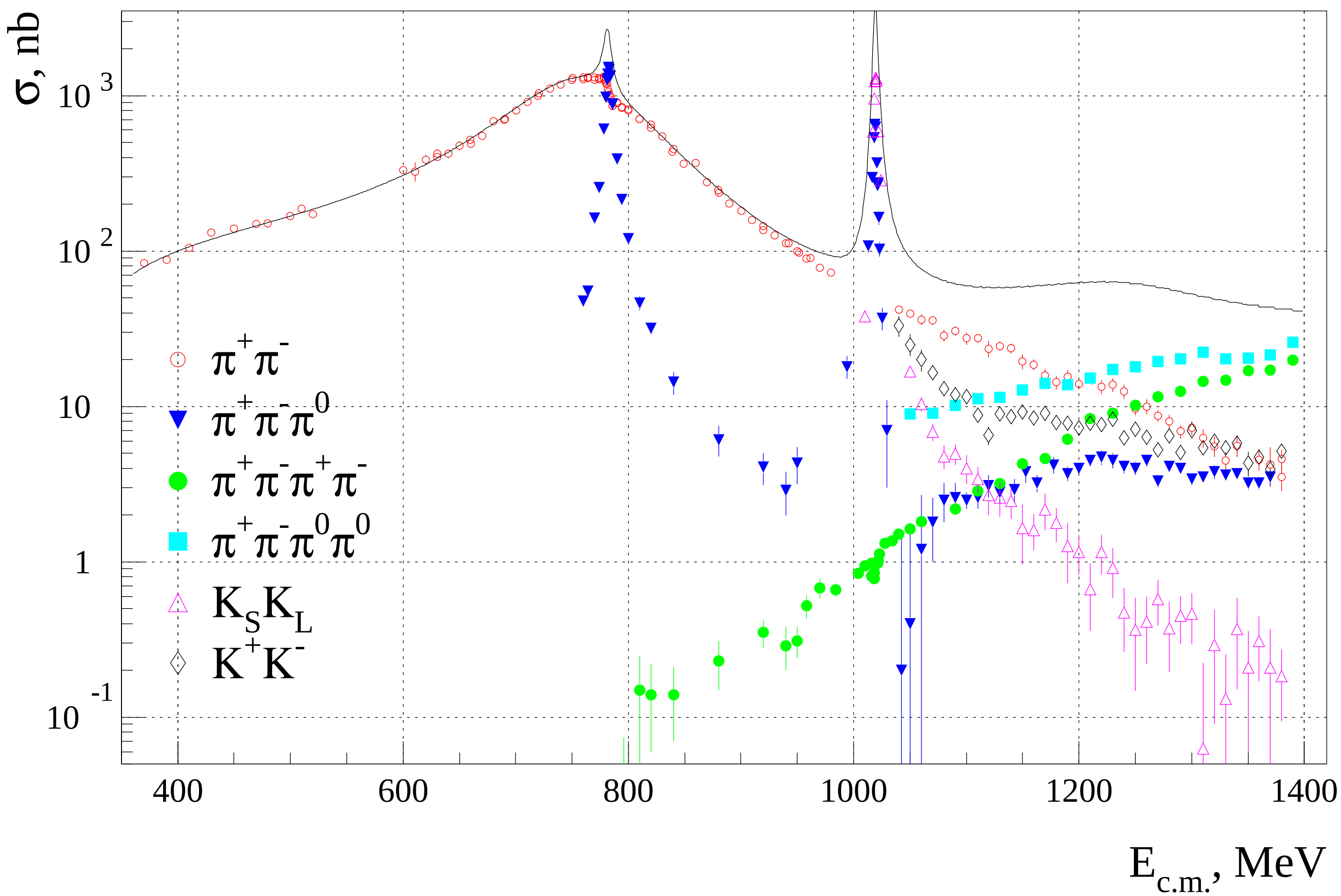
Large data scatter above 1.4 GeV!

Study of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$, $\pi^+\pi^-\pi^0\pi^0$ with CMD-2 and SND



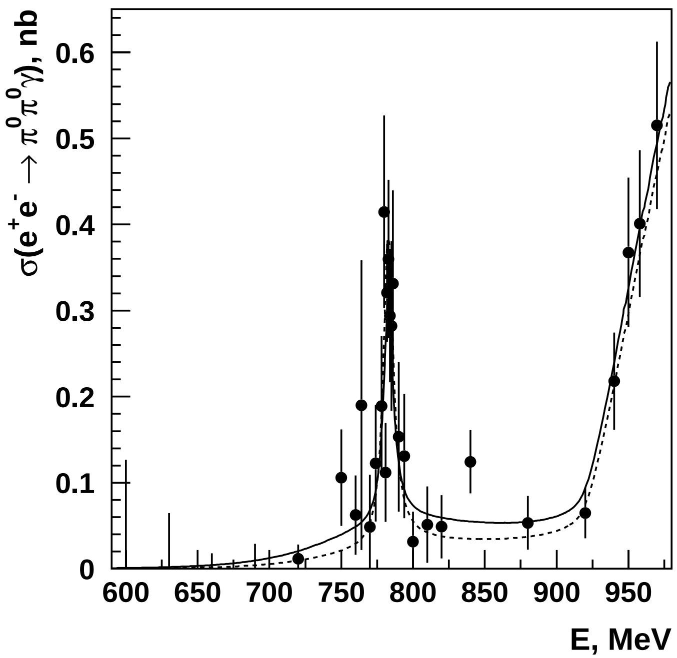


Hadronic Cross Sections at CMD-2



Study of Neutral Final States at VEPP-2M

- $e^+e^- \to \pi^0\gamma$ (SND, CMD-2)
- $e^+e^- \to \eta\gamma$ (SND, CMD-2)
- $e^+e^- \to \pi^0\pi^0\gamma$ (SND, CMD-2)
- $e^+e^- \to \eta\pi^0\gamma$ (CMD-2)



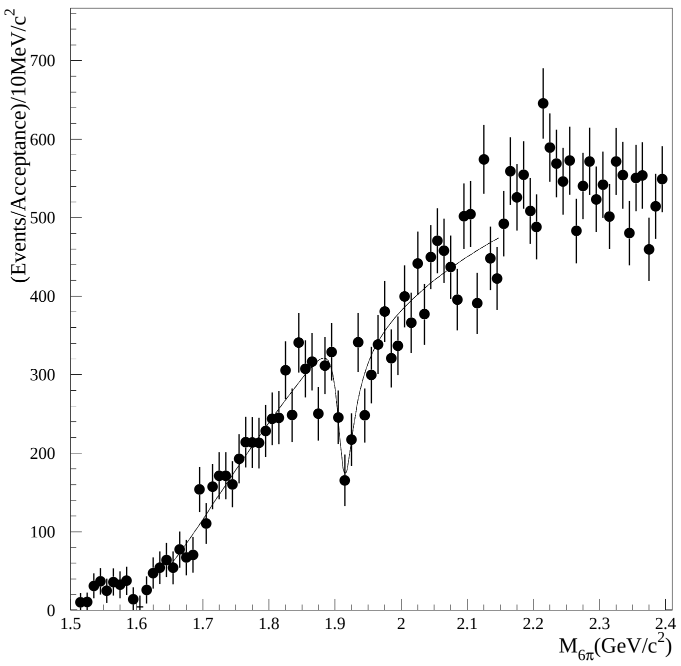
$\rho-, \omega-, \phi-$mesons dominate the cross sections.

From upper limits on nonresonant cross sections $a_\mu^{\rm rad,LO} < 0.7 \cdot 10^{-10}$.

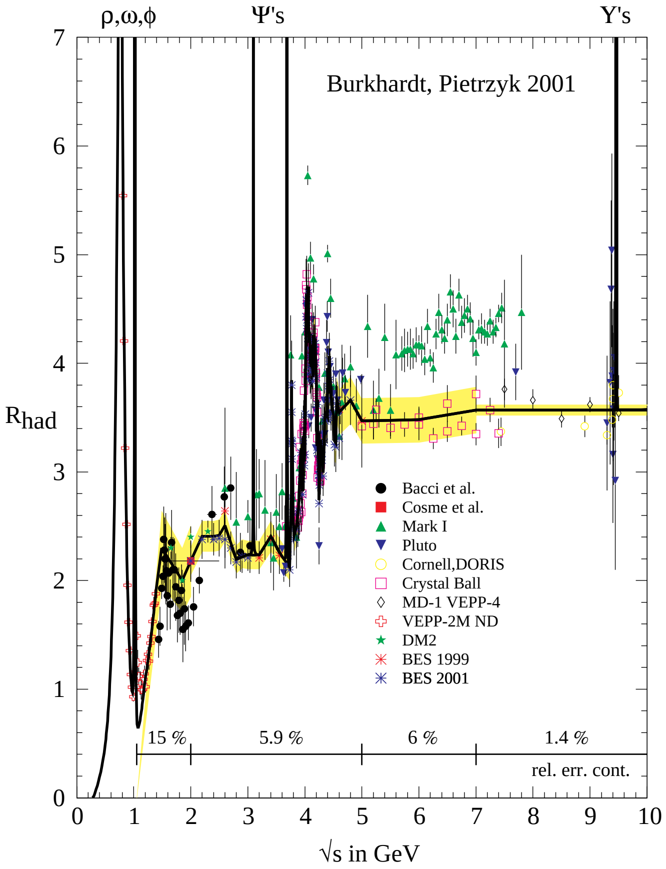
Measurements at 1.4 GeV $< \sqrt{s} <$ 2 GeV

- 5 resonances ($2\rho'$,$2\omega'$,ϕ') with badly known properties
- Mixing of $q\bar{q}$ with hybrids?
- In 2001 E687 (FNAL) observed a narrow dip in $\gamma p \to 3\pi^+3\pi^- p$, M$= 1911 \pm 4 \pm 1$ MeV, $\Gamma = 29 \pm 11 \pm 4$ MeV
- Earlier observed in e^+e^-: DM2 (1988) - $e^+e^- \to 6\pi$, FENICE (1996) - $e^+e^- \to$ hadrons
- A hybrid or $N\bar{N}$ state?

$\gamma p \to 3\pi^+3\pi^- p$



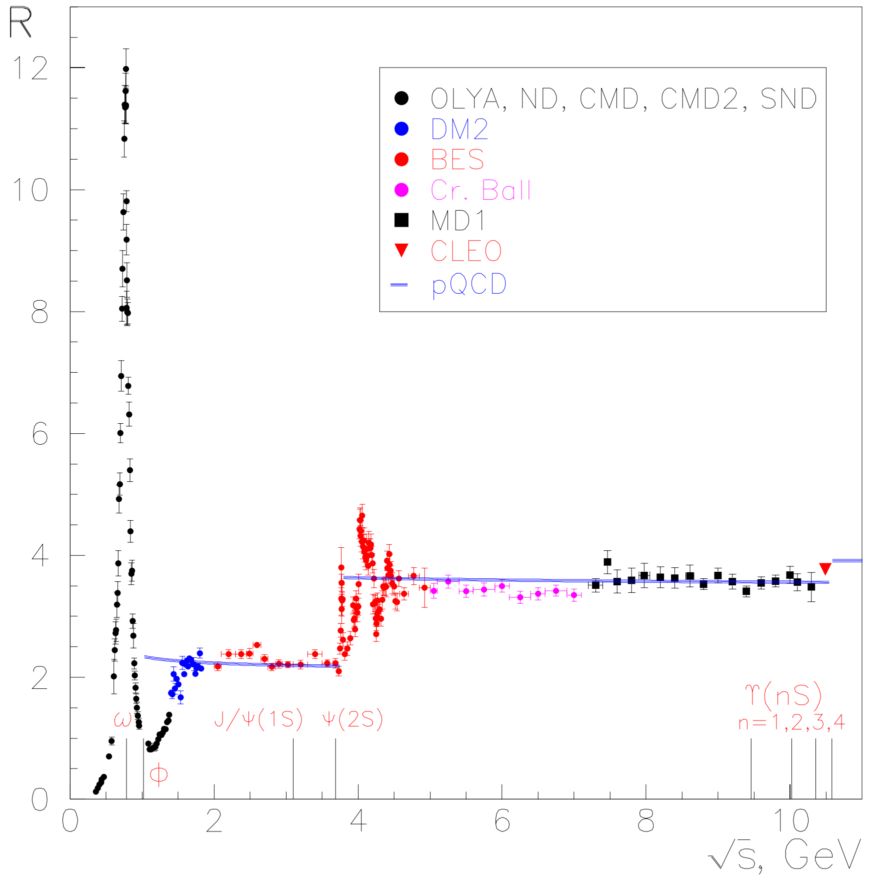
R Measurements at $\sqrt{s} < 10$ GeV



$\gamma\gamma 2$ vs. BES

Detector	$\gamma\gamma 2$	BES
$\sqrt{s}$, GeV	2.0-3.1	2.0-3.0
Acceptance, %	19-23	50-68
Syst. error, %	21	5.9-8.4
$\int Ldt$, nb^{-1}	130	990
Data sample	920	18500

R Measurements at $\sqrt{s} < 10$ GeV

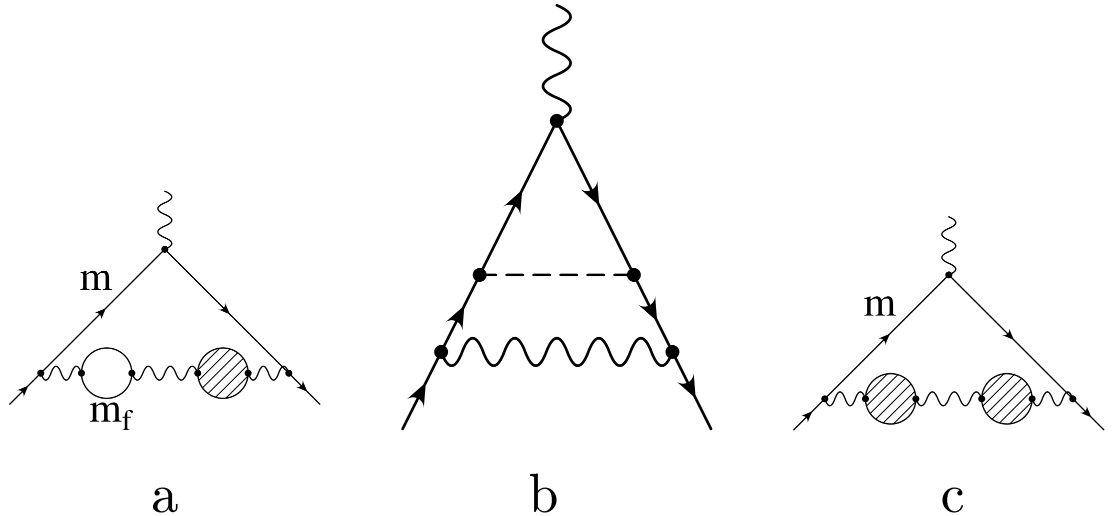


New e^+e^- Data Based Calculation of $a_\mu^{\rm had,LO}$

$\sqrt{s}$, GeV	$a_\mu^{\rm had,LO}, 10^{-10}$	$a_\mu^{\rm had,LO}$, %
2π	$508.20 \pm 5.18 \pm 2.74$	72.99
ω	$37.96 \pm 1.02 \pm 0.31$	5.45
ϕ	$35.71 \pm 0.84 \pm 0.20$	5.13
$0.6 - 2.0$	$63.18 \pm 2.19 \pm 0.86$	9.07
$2.0 - 5.0$	$33.92 \pm 1.72 \pm 0.03$	4.87
$J/\psi, \psi'$	$7.44 \pm 0.38 \pm 0.00$	1.07
> 5.0	$9.88 \pm 0.11 \pm 0.00$	1.42
Total	$696.3 \pm 6.2 \pm 3.6$	100.0

Higher accuracy of e^+e^- data makes the $a_\mu^{\rm had,LO}$ error 2 times smaller!

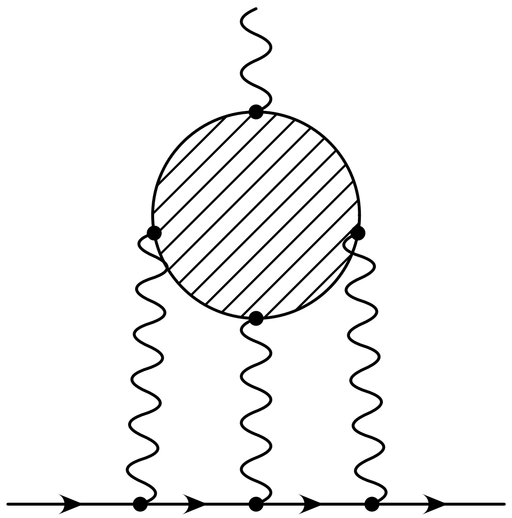
Higher Order Hadronic Contributions $a_\mu^{\mathrm{had,HO}}$



The contributions of all 3 graphs can be calculated in terms of the $\int R(s)G(s)ds/s^{2(3)}$, where $G(s)$ is a smooth function of s, so that the low energy range again dominates the integral. Several calculations agree. The accepted value is (B. Krause, 1997; R.Alemany, M.Davier, A.Höcker, 1998):

$$a_\mu^{\mathrm{had,HO}} = (-10.0 \pm 0.6) \cdot 10^{-10}.$$

Light-by-Light Scattering



Various approaches used:

- Vector Dominance and Chiral models
- Data on $\gamma\gamma^* \to \pi^0, \eta, \eta'$ (single-tag)
- Effective field theory approach

M. Knecht and A. Nyffeler, 2002: the correct sign!

Authors	Year	$a_\mu^{\rm lbl}, 10^{-10}$
J. Bijnens et al.	1996 (2002)	8.3 ± 3.2
M. Hayakawa and T. Kinoshita	1998 (2002)	9.0 ± 1.5
Average	2002	8.6 ± 3.5
K. Melnikov and A. Vainshtein	2003	13.6 ± 2.5

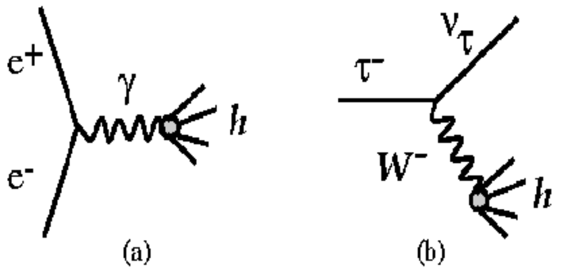
Theory vs Experiment (January 2004)

Contribution	$a_\mu, 10^{-10}$
Experiment	11659208 ± 6
QED	11658470.6 ± 0.3
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	694.9 ± 7.9
Theory	11659180.9 ± 8.0
Exp.–Theory	$27.1 \pm 10.0\ (2.7\sigma)$

Recent theory progress: $a_\mu^{\rm exp} - a_\mu^{\rm th} = (20.8 \pm 9.7) \cdot 10^{-10}\ (2.1\sigma)$

How can the theoretical error be improved?

CVC. $e^+e^- \to X^0$ and $\tau^- \to \nu_\tau X^-$



Allowed $I^G J^P = 1^+1^-$:

$X^- = \pi^-\pi^0, (4\pi)^-, \omega\pi^-, \eta\pi^-\pi^0, K^-K^0, (6\pi)^-, \ldots$

$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ud}|^2 S_{EW}}{32\pi^2 m_\tau^3} f_{kin} v_1(q^2)$ with

$v_1(q^2) = \frac{q^2 \sigma^{I=1}_{e^+e^-}(q^2)}{4\pi\alpha^2}$.

CVC tests showed good agreement of the τ branchings predicted from e^+e^- with τ data (N. Kawamoto and A. Sanda, 1978, F. Gilman and D. Miller, 1978, S. Eidelman and V. Ivanchenko, 1991, 1997).

The very first application of τ data to $a_\mu^{\rm had,LO}$ improved the accuracy by a factor of 1.5 (R. Alemany, M. Davier, A. Höcker, 1998)!

Branchings of $\tau^- \to X^- \nu_\tau$ Decay,%

Hadronic State X	Experiment, 2002	CVC Prediction	$\mathcal{B}_{\rm exp} - \mathcal{B}_{\rm CVC}$
$\pi^-\pi^0$	25.31 ± 0.18	24.76 ± 0.25	0.55 ± 0.31
$\pi^-3\pi^0$	1.08 ± 0.10	1.07 ± 0.05	0.01 ± 0.11
$2\pi^-\pi^+\pi^0$	4.19 ± 0.23	3.84 ± 0.17	0.35 ± 0.29
$\omega\pi^-$	1.94 ± 0.07	1.82 ± 0.07	0.12 ± 0.10
Total	31.59 ± 0.31	30.28 ± 0.34	1.31 ± 0.46

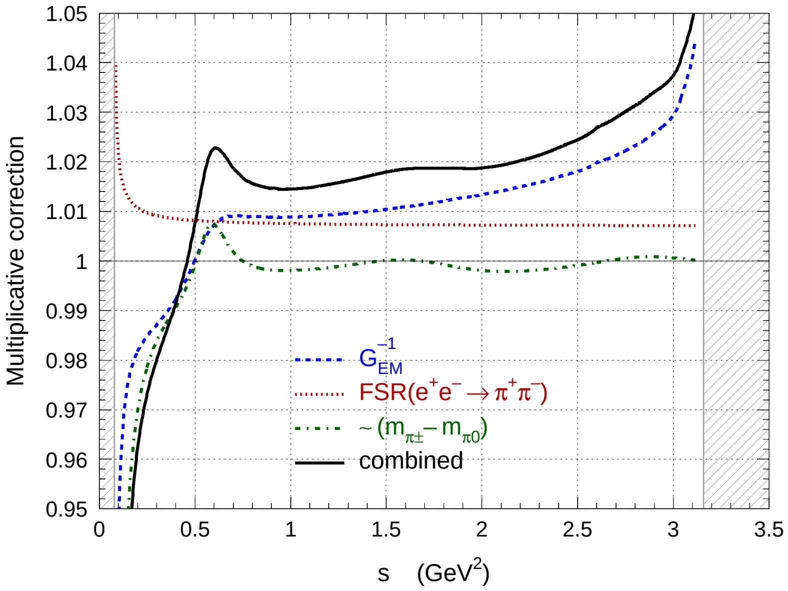
With more accurate data some deviations have been observed.

Corrections to the τ Spectral Functions

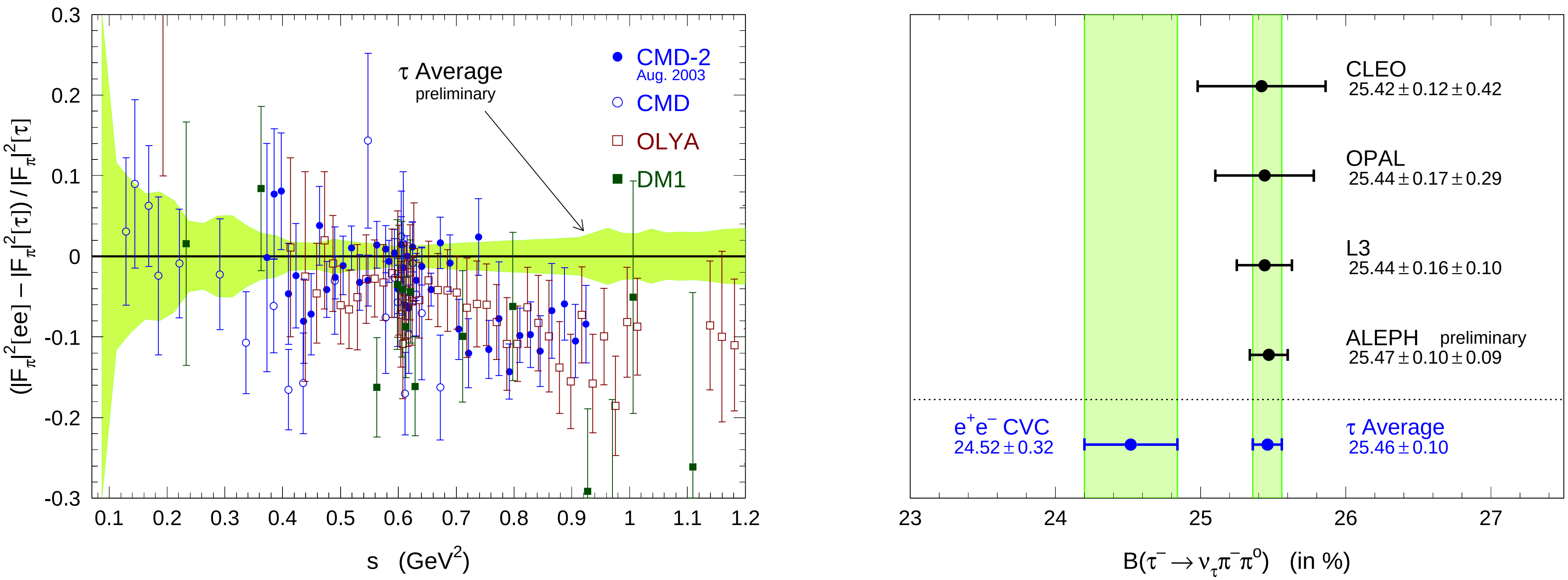
- $S_{EW} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $m_{\pi^\pm} \neq m_{\pi^0}$
  (phase space, Γ_ρ)
- $m_{\rho^\pm} \neq m_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays
  $(\pi\pi\gamma, \pi(\eta)\gamma, l^+l^-)$
- $m_u \neq m_d$
  and 2 class currents

V. Cirigliano, G. Ecker, H. Neufeld, 2002

M. Davier, S. Eidelman, A. Höcker, Z. Zhang, 2002



CVC in the 2π Channel. e^+e^- vs. τ



The branching from all groups is systematically higher than the CVC prediction: $\mathcal{B}_\tau - \mathcal{B}_{ee} = (0.94 \pm 0.32)\%$!

Contributions to $a_\mu^{\text{had,LO}}$ from e^+e^- and $\tau, 10^{-10}$

Mode	e^+e^-	τ	$\Delta(e^+e^- - \tau)$
$\pi^+\pi^-$	$508.20 \pm 5.18 \pm 2.74$	$520.06 \pm 3.36 \pm 2.62$	-11.9 ± 6.9
$\pi^+\pi^-2\pi^0$	$16.76 \pm 1.31 \pm 0.20$	$21.45 \pm 1.33 \pm 0.60$	-4.7 ± 1.8
$2\pi^+2\pi^-$	$14.21 \pm 0.87 \pm 0.23$	$12.35 \pm 0.96 \pm 0.40$	$+1.9 \pm 2.0$
Total	$539.17 \pm 5.41 \pm 3.17$	$553.86 \pm 3.74 \pm 3.02$	-14.7 $\pm$ 7.9

The difference of 1.86σ makes averaging meaningless: a scale factor of 1.82 makes a final error equal to $6.94 \cdot 10^{-10}$ ($7.20 \cdot 10^{-10}$ from e^+e^- only), i.e. no gain at all!

Why are e^+e^- and τ Spectral Functions Different?

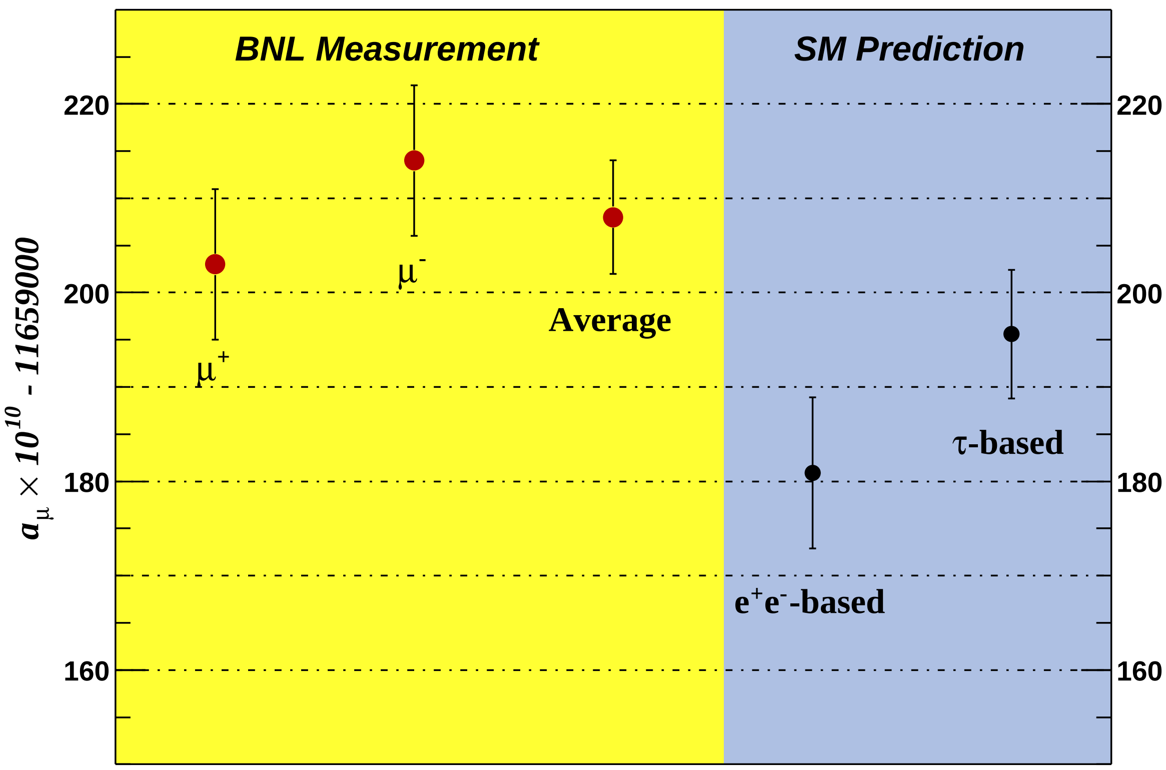
- Problems with data
- Problems with SU(2) breaking corrections
- Non-(V–A) contribution to weak interactions (problems with the $\pi^+ \to e^+\nu_e\gamma$ decay)
- Difference in mass (width) of ρ mesons ($m_{\rho^\pm} > m_{\rho^0}$ by a few MeV). Current experiments indicate equality within a few MeV.

Recent Calculations of $a_\mu^{\mathrm{had,LO}}$

Authors	Data	$a_\mu^{\mathrm{had,LO}}$, 10^{-10}
M. Davier et al.	e^+e^-	$696.3 \pm 6.2_{\mathrm{exp}} \pm 3.6_{\mathrm{rad}}$
K. Hagiwara et al.	e^+e^-	$692.4 \pm 5.9_{\mathrm{exp}} \pm 2.4_{\mathrm{rad}}$
S. Ghozzi and F. Jegerlehner	e^+e^-	694.8 ± 8.6
V. Ezhela et al.	e^+e^-	$699.6 \pm 8.5_{\mathrm{exp}} \pm 1.9_{\mathrm{rad}} \pm 2.0_{\mathrm{proc}}$
M. Davier et al.	τ	$711.0 \pm 5.0_{\mathrm{exp}} \pm 0.8_{\mathrm{rad}} \pm 2.8_{\mathrm{SU(2)}}$

All $\boldsymbol{e^+e^-}$ based calculations agree!

Theory vs Experiment



The e^+e^- and τ based predictions are below the experimental value by 2.7σ (2.1σ) and 0.7σ, respectively!

Possible Progress for $a_\mu^{\mathrm{LO,had}}$

Experiments are planned at the new machine VEPP-2000 (VEPP-2M upgrade) with 2 detectors (CMD-3 and SND) up to $\sqrt{s}$=2 GeV with $L_{\max} = 10^{32}$ cm^{-2}s^{-1}.

$\sqrt{s}$, GeV	$a_\mu^{\mathrm{LO,had}}$ in 2003 (2007), 10^{-10}	2003+2007, 10^{-10}
$2\pi, < 2$	508.4 ± 5.5 (4.0)	3.2
ω	38.0 ± 1.1 (0.4)	0.4
ϕ	35.7 ± 0.9 (0.4)	0.3
0.6–2.0	62.9 ± 2.5 (1.3)	1.2
Total	645.0 ± 6.2 (4.2)	3.5

The total error of $a_\mu^{\mathrm{had,LO}}$ falls from $7.2 \cdot 10^{-10}$ to $3.9 \cdot 10^{-10}$. Other measurements are welcome (DAΦNE-II)!

Radiative Return (ISR)

The idea: a photon with E_γ energy emitted by initial $e^\pm$ allows a study of hadron production at smaller energy: $2E' = 2E\sqrt{1 - E_\gamma/E}$. A smaller cross section is compensated by a much higher luminosity.

Already today there is a large data sample with small systematic errors:
KLOE – 11k/pb^{-1} vs. 360k/pb^{-1} with CMD-2, but 1.5M $\pi^+\pi^-$ events in total!
BaBar – $150 \cdot 10^3$ exclusive events between 1 and 3 GeV per 100 fb^{-1}.
Belle – starting.

ISR is an independent source of high precision R measurements in the energy range $2m_\pi < \sqrt{s} < 3$ GeV, important for $a_\mu^{\rm had}$ and $\alpha(M_Z^2)$;
it also provides an invaluable input for hadronic spectroscopy and QCD.

Future of $(g_\mu - 2)/2$

1. Experiment
   - Today a_μ is known with a $5 \cdot 10^{-7}$ relative accuracy.
   - If funded, the BNL group can reach $2.5 \cdot 10^{-7}$ at BNL and $6 \cdot 10^{-8} - 1 \cdot 10^{-7}$ at J-PARC.
2. Theory
   - Today: QED, EW – $2.5 \cdot 10^{-8}$, Hadr. LO – $6.7 \cdot 10^{-7}$
   - We'll badly need at least one order of magnitude improvement in the hadronic contribution accuracy
   - It is equivalent to measuring R (τ) to a 10^{-3} accuracy (???)
   - Or $a_\mu^{\rm had}$ calculation from 1st principles (QCD, Lattice). Recently from Lattice: $a_\mu^{\rm had} = (446 \pm 23) \cdot 10^{-10}, \ (545 \pm 65) \cdot 10^{-10}$.

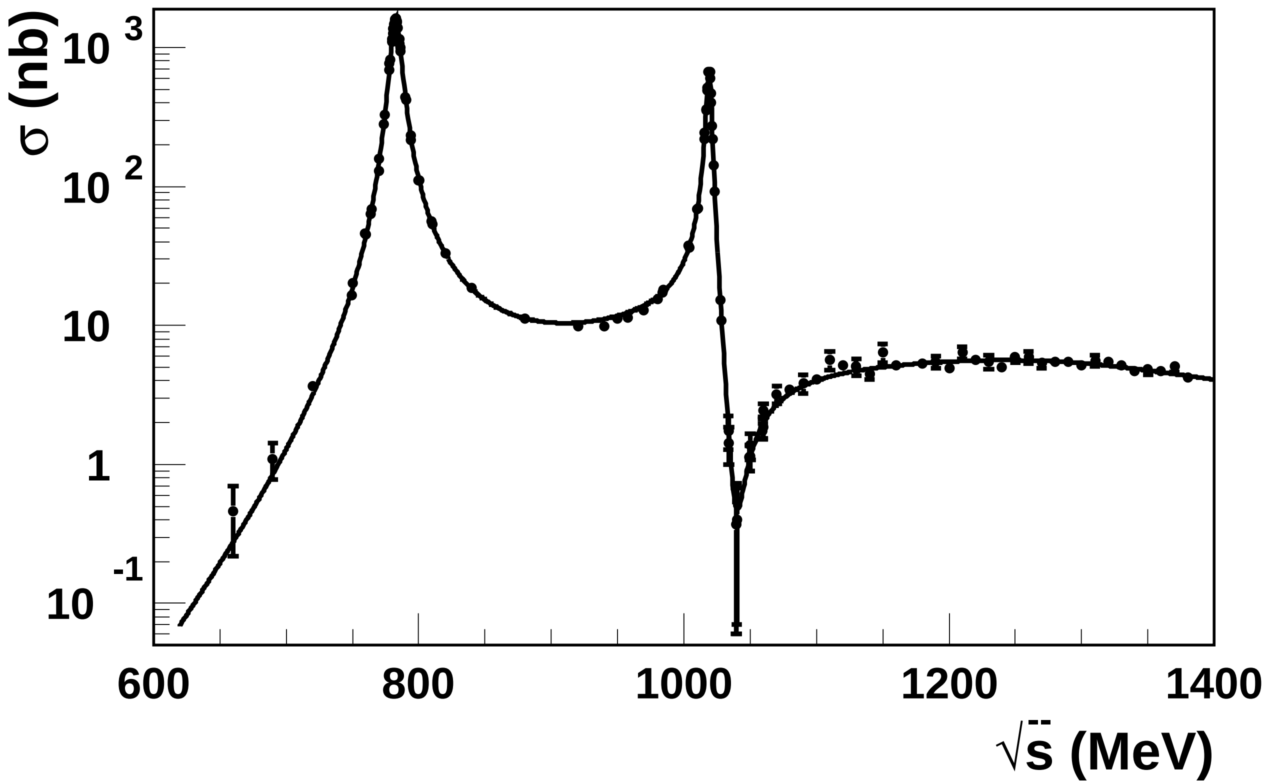
Conclusions

- BNL success stimulated significant progress of e^+e^- experiments and related theory
- Improvement of e^+e^- data (VEPP-2M and BEPC) decreased an error of $a_\mu^{\rm had,LO}$ by a factor of 2, but the experimental accuracy of a_μ is still better
- τ data could further improve the accuracy by 1.5 but e^+e^- and τ data differ
- Further improvement in $a_\mu^{\rm had,LO}$ by a factor of 2–3 will be possible after VEPP-2000, DAΦNE-II, CESRc and $(c-\tau)$ factory as well as with ISR at DAΦNE and B-factories
- $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ differs from 0 by (2.1–2.7) $\sigma \Rightarrow$ SM is still alive!

New Physics and a_μ

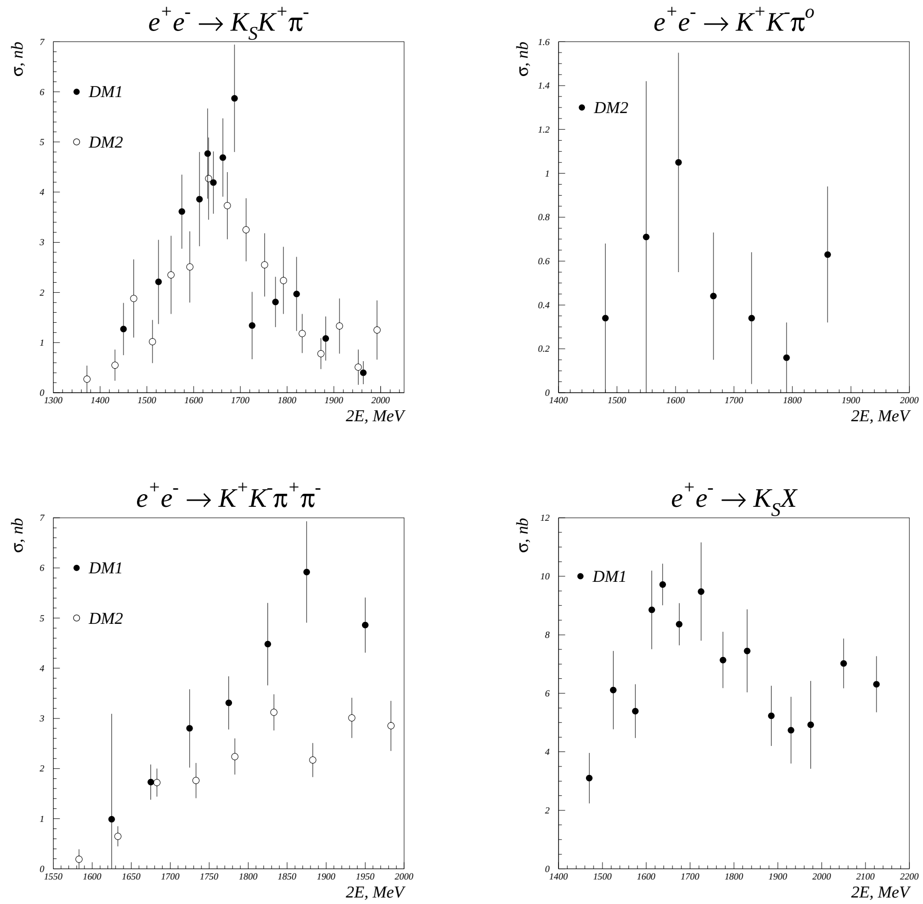
- Supersymmetry ($\tilde{m} \simeq 200$ GeV–1 TeV)
- Cold dark matter and neutralino
- Tachyons ($m^2 < 0$)
- Radiative m_μ generation (new scale $\Lambda \simeq 1 - 2$ TeV)
- Technicolor
- Leptoquarks
- New gauge bosons (Z′ etc.)
- Large extra dimensions ($M > 600$ GeV)
- Lepton flavor violation ($\mu \to e\gamma$, μ edm, $\tau \to \mu\gamma, \ldots$)
- $b \to s\gamma$
- Composite quarks and leptons

Study of $e^+e^- \to \pi^+\pi^-\pi^0$ at SND



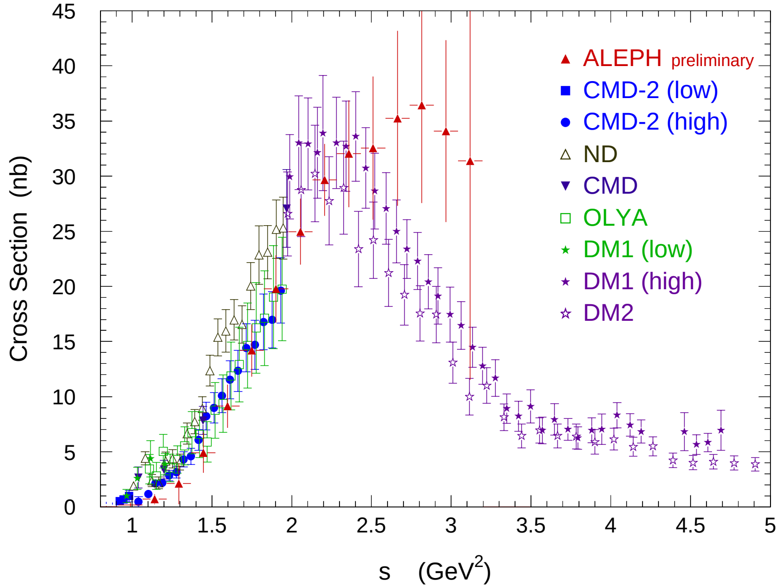
Processes with Kaons

- $e^+e^- \to K^+K^-, K_SK_L$:
  measurement of branchings,
  separation of I=0,1
- $e^+e^- \to K\bar{K}\pi$:
  2 of 3 (no $e^+e^- \to K^0\bar{K}^0\pi^0$),
  mechanism ($V\pi, K^*K$)
- $e^+e^- \to K\bar{K}\pi\pi$:
  $e^+e^- \to K^+K^-\pi^+\pi^-$ only,
  total σ 4.5 times bigger?

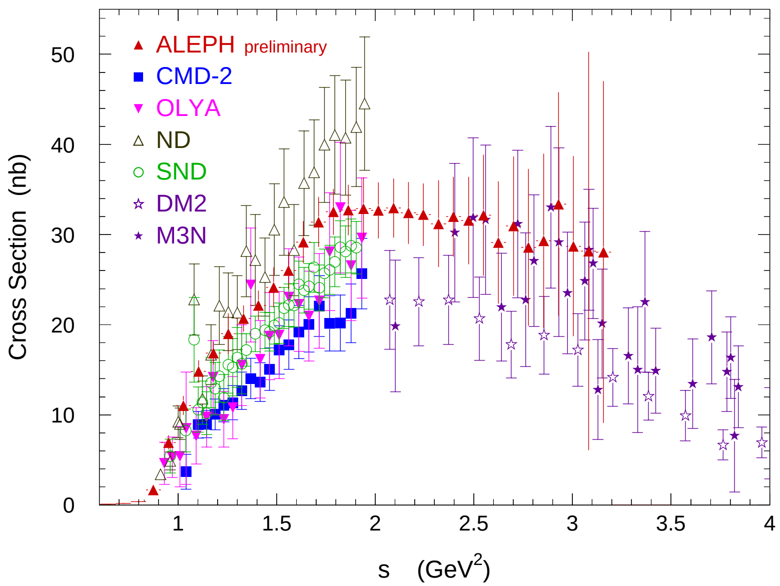


4π Spectral Functions from e^+e^- and τ

$2\pi^+2\pi^-$



$\pi^+\pi^-2\pi^0$



Branchings of τ Decays into 2 and 4 Pions

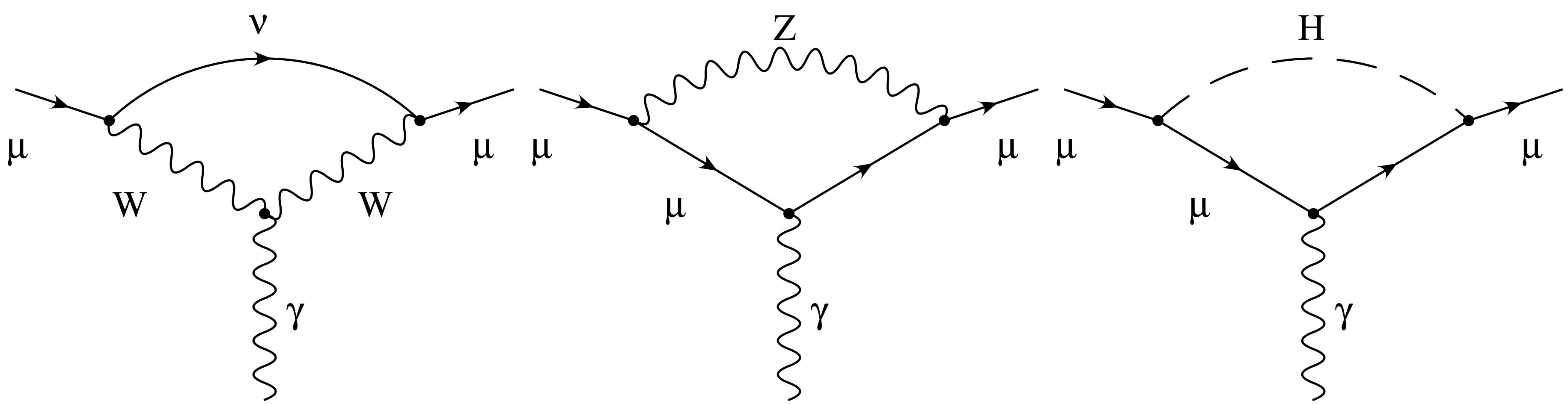
Mode	τ data	Branching Ratios, % e^+e^- and CVC	$\mathcal{B}_\tau - \mathcal{B}_{ee}$
$\nu_\tau \pi^- \pi^0$	25.46 ± 0.10	$24.52 \pm \underbrace{0.26_{\rm exp} \pm 0.11_{\rm rad} \pm 0.12_{\rm SU(2)}}_{0.31}$	$+0.94 \pm 0.32$
$\nu_\tau \pi^- 3\pi^0$	1.01 ± 0.08	$1.09 \pm \underbrace{0.06_{\rm exp} \pm 0.02_{\rm rad} \pm 0.05_{\rm SU(2)}}_{0.08}$	-0.08 ± 0.11
$\nu_\tau 2\pi^- \pi^+ \pi^0$	4.54 ± 0.13	$3.63 \pm \underbrace{0.19_{\rm exp} \pm 0.04_{\rm rad} \pm 0.09_{\rm SU(2)}}_{0.21}$	$+0.91 \pm 0.25$

The observed excess is 2.94σ for the $\pi^-\pi^0$ and 3.64σ (or 25%) for the $2\pi^-\pi^+\pi^0$ mode! Serious discrepancies are observed.

Higher Order Hadronic Contributions $a_\mu^{\rm had,HO}$

Authors	Year	a	b	c	Total, 10^{-10}
J.Calmet et al.	1976	11.0 ± 1.4	-20.7 ± 3.0	0.2 ± 0.1	-9.5 ± 3.2
T.Kinoshita et al.	1985	10.7 ± 0.3	-19.9 ± 0.4	0.2 ± 0.1	-9.0 ± 0.5
B.Krause	1997	10.7 ± 0.2	-21.1 ± 0.5	0.3 ± 0.1	-10.1 ± 0.6
R.Alemany et al.	1998	10.6 ± 0.2	-20.9 ± 0.4	0.3 ± 0.1	-10.0 ± 0.6

Electroweak contribution a_μ^{EW}



One-loop electroweak contributions

Authors	Year	a_μ^{EW}, 10^{-10}
... , ... , ...	1972	19.5
A. Czarnecki et al.	1996	15.2 ± 0.4
A. Czarnecki et al.	2002	$15.4 \pm 0.1 \pm 0.2$