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# Storage ring RF and longitudinal dynamics and feedback

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Workshop on  $e^+e^-$  in the 1-2 GeV range

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## Motivation

It is obvious that storage ring RF system has major impact on longitudinal dynamics. What is less obvious is which parameters are important for proper longitudinal feedback operation.

Experimental determination of these parameters is costly - takes a lot of machine study time and, ultimately, requires changes in the RF system or the feedback.

Even if the machine is expected to be stable by design it is important to know whether feedback is feasible. Two examples:

- Longitudinal coupled-bunch feedback at the ALS after harmonic cavity installation
- Longitudinal quadrupole coupled-bunch feedback in DAΦNE

In course of commissioning and operating longitudinal bunch-by-bunch feedback systems at 5 machines (ALS, BESSY-II, DAΦNE, PEP-II, PLS) we gained some understanding of these important factors.

This talk is an attempt to summarize our experience with examples from different machines.

As an exercise I will analyze the proposed “DAΦNE with strong RF focusing” from the LFB point of view.



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## Outline

- 1 Longitudinal multi-bunch dynamics
- 2 Overview of bunch-by-bunch longitudinal feedback system
- 3 Stability factors
  - Dipole and quadrupole coupled-bunch instability growth rates
  - Maximum controllable growth rates
  - Synchronous gap transients
  - Mode 0 frequency shifts due to beam loading
- 4 Analysis example: DAΦNE with strong RF focusing
- 5 Summary

## Coupled-bunch instabilities: eigenmodes and impedances

For an even fill pattern the bunch motion can be easily projected into the even-fill eigenmode (EFEM) basis. For  $N$  coupled harmonic oscillators (bunches) there are  $N$  normal modes.

Modal eigenvalues are given by

$$\Lambda_l = -d_r + i\omega_s + \underbrace{\frac{\alpha e f_{\text{rf}}}{2E_0 v_s}}_{\text{Accelerator parameters}} \underbrace{I_0}_{\text{Beam current}} \underbrace{Z^{\text{eff}}(l\omega_{\text{rev}} + \omega_s)}_{\text{Aliased impedance}}$$

$$Z^{\text{eff}}(\omega) = \sum_{p=-\infty}^{\infty} \frac{(p\omega_{\text{rf}} + \omega)}{\omega_{\text{rf}}} F(p\omega_{\text{rf}} + \omega) Z(p\omega_{\text{rf}} + \omega)$$

Real part of the eigenvalue is the exponential growth rate, imaginary part - undamped natural frequency.

**Growth rate is proportional to beam current. Above some threshold current system is unstable.**

Two ways to fight the instabilities: lower the impedance via passive or active techniques or apply feedback damping

Form factor  $F(\omega) = e^{-(\omega\sigma_z)^2}$  defines roll-off of the aliased impedance due to non-zero bunch length.

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## Coupled-bunch instabilities and RF parameters

Instability growth rate is

- Proportional to RF frequency  $f_{\text{rf}}$
- Proportional to momentum compaction  $\alpha$
- Proportional to  $\frac{1}{v_s}$ , thus scales as  $\frac{1}{\sqrt{V_{\text{rf}}}}$
- Shorter bunch length increases form factor  $F(\omega)$  at high frequencies, thus physical impedances at those frequencies become more important.

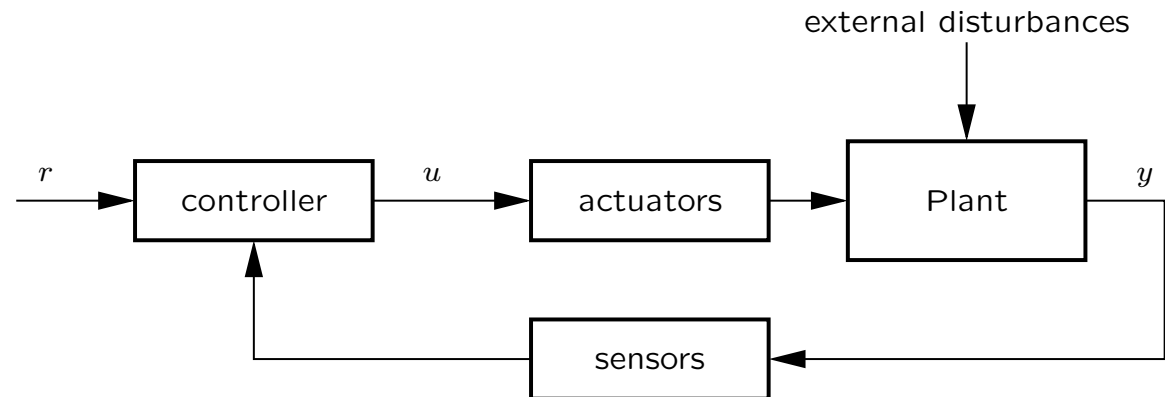
Behavior is similar for the quadrupole coupled-bunch instabilities, however the impedance is aliased with quadratic frequency scaling, so the effect of shorter bunch length is more significant.

## Feedback basics

The objective is to make the output  $y$  of a dynamic system (plant) behave in a desired way by manipulating input or inputs of the plant.

Regulator problem - keep  $y$  small or constant

Servomechanism problem - make  $y$  follow a reference signal  $r$



Feedback controller acts to reject the external disturbances.

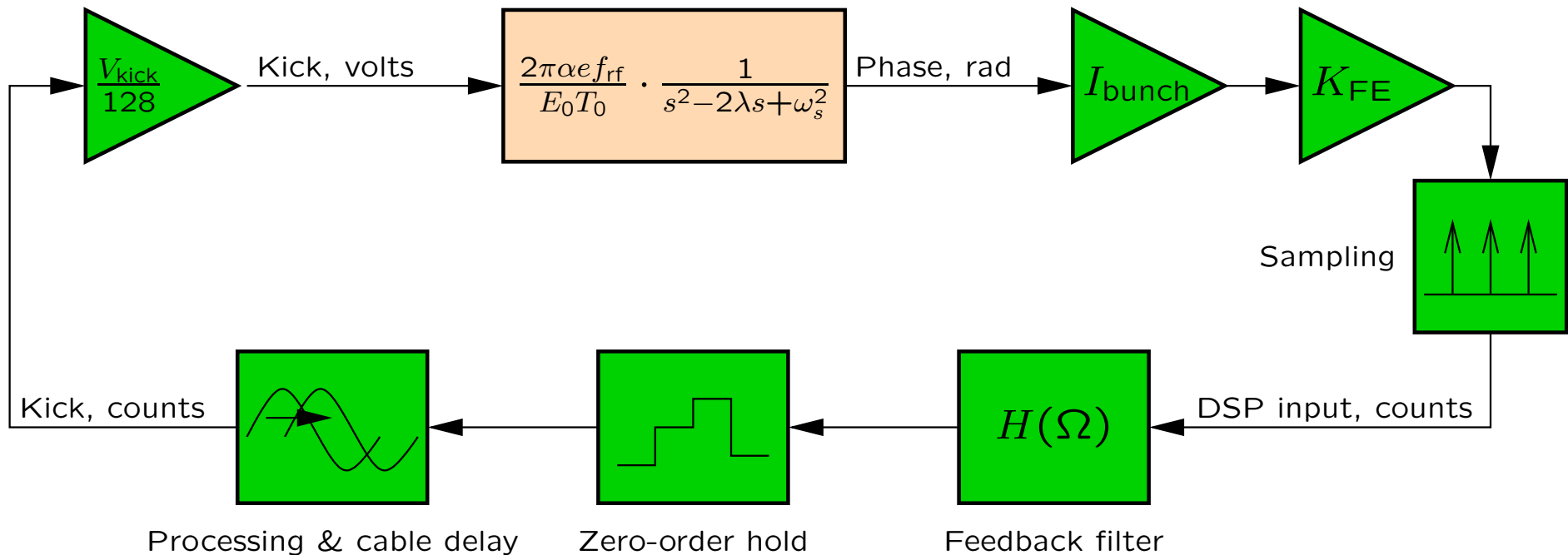
The error between  $y$  and the desired value is the measure of feedback system performance. There are many ways to define the numerical performance metric

- RMS or maximum errors in steady-state operation
- Step response performance such as rise time, settling time, overshoot.

An additional measure of feedback performance is the average or peak actuator effort. Peak actuator effort is almost always important due to the finite actuator range.

**Feedback system robustness** - how does the performance change if the plant parameters or dynamics change? How do the changes in sensors and actuators affect the system?

## A detailed model of the LFB



The same model is valid in the bunch and eigenmode domains since bunch-by-bunch feedback is invariant under coordinate transformations

In LBL/LNF/SLAC-designed systems bunch sampling is done at a subharmonic of the revolution frequency, that is one sample every several turns.

Feedback filter  $H(\Omega)$  provides 90 degree phase shift at the synchrotron frequency, DC rejection, and bandpass filtering to reject out-of-band noise.

# Feedback controller design

## Requirements

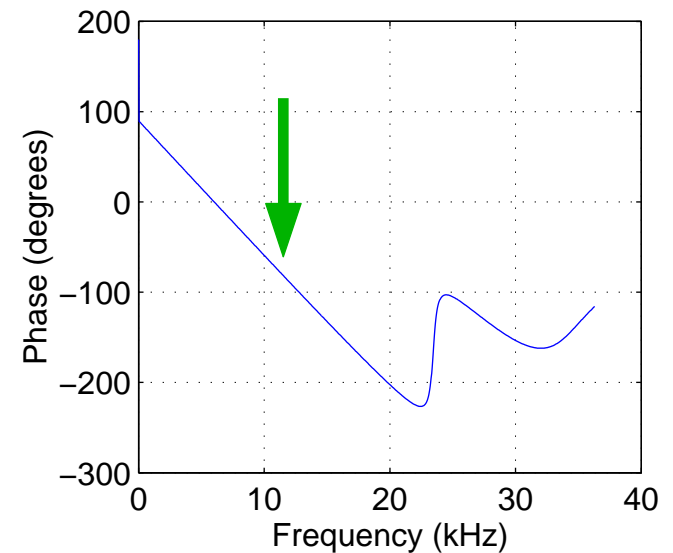
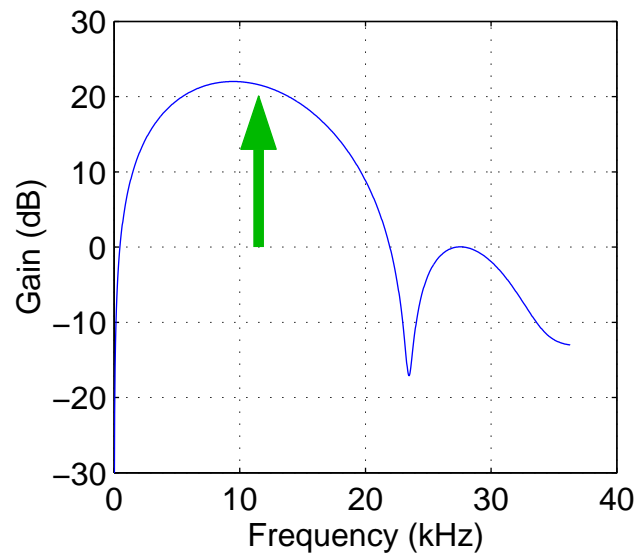
- Control of phase & gain at the synchrotron frequency  $\omega_s$  (90 degree phase shift)
- DC rejection
- Frequency selectivity

FIR digital filter implementation: 
$$y_n = \sum_{k=0}^M b_k x_{n-k}$$

## Design approach

- Let filter impulse response sample a sine wave at the synchrotron frequency.
- Phase and gain adjustments are simple
- Set sum of the impulse response to 0 (DC rejection)

Resulting filter has bandpass characteristic around the  $\omega_s$





## RF voltage and the longitudinal feedback loop

We can rewrite beam response function as follows:

$$G(s) = -\frac{1}{V_{\text{rf}} \cos \phi_s} \cdot \frac{\omega_s^2}{s^2 - 2\lambda s + \omega_n^2}$$

Evaluating the gain of  $G(s)$  at the peak we get

$$|G(j\omega_n)| \approx \frac{\omega_s}{2\lambda V_{\text{rf}} |\cos \phi_s|} \sim \frac{1}{\sqrt{V_{\text{rf}}}}$$

Conclusion: increasing the RF voltage causes the longitudinal feedback gain to drop



## Maximum controllable growth rates

Feedback loop group delay is a fundamental limitation on achievable feedback damping and, therefore, on the controllable growth rate.

Group delay defines a phase slope across the feedback control band around the synchrotron frequency. At some point as we increase the gain the decreased phase margins make the closed-loop response peak around the synchrotron frequency. At this point further gain increases do not improve damping. The closed-loop poles in this case start shifting in imaginary part (frequency) rather than in the real part (damping).

For a conventional system the minimum group delay is one turn. For a small circumference machine cable and amplifier delays can be comparable to the revolution time, so that minimum delay is larger.

From experimental measurements at multiple machines we determined that for a downsampled system the controllable ratio of the oscillation frequency to the growth rate ( $\omega_s/\lambda$ ) is in the range from 20-30.

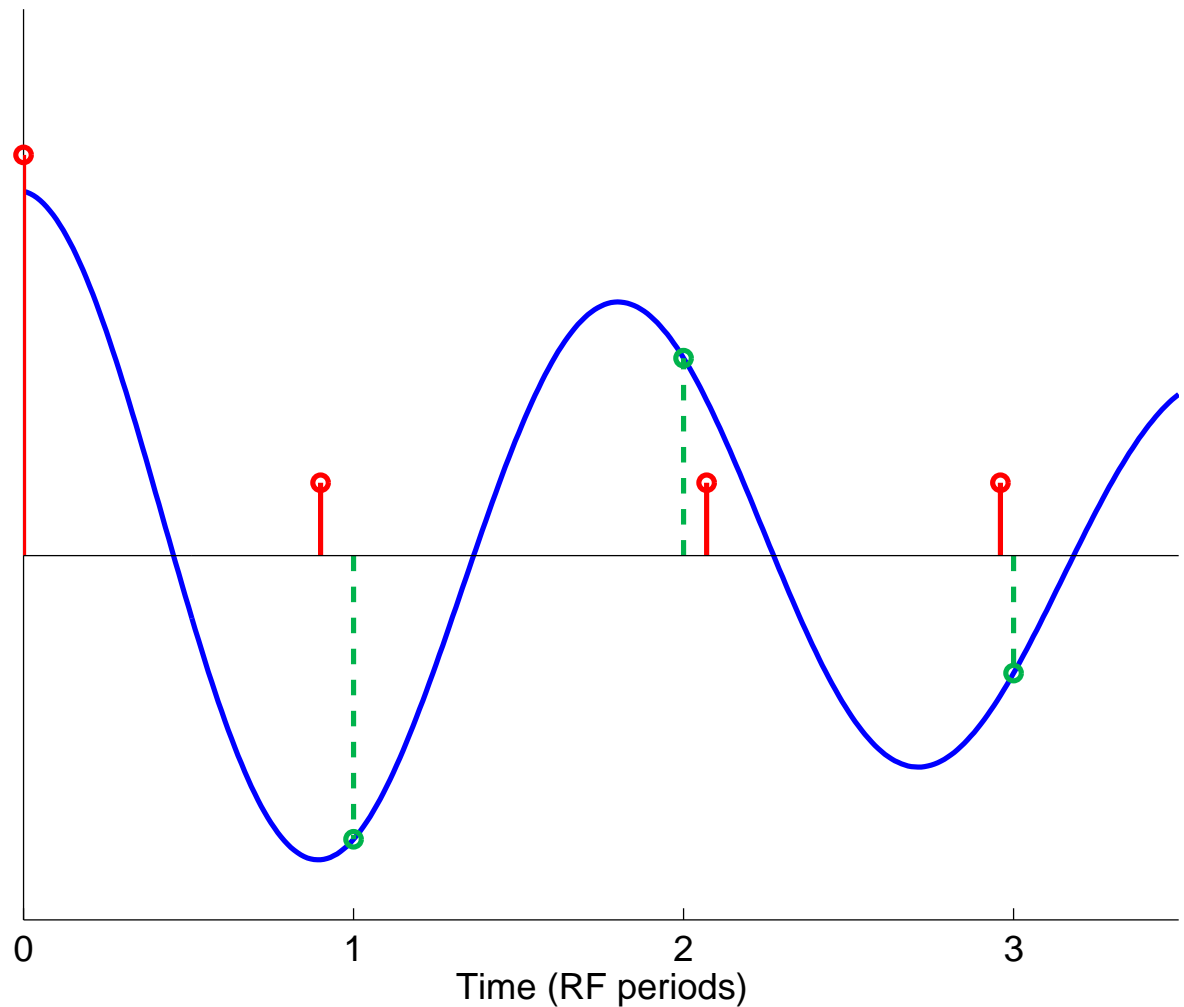
Note that higher synchrotron frequency allows (in the limit) control of faster growth rates.

## Synchronous gap transients

Uneven ring filling patterns (without circular symmetry) give rise to synchronous gap transients.

The effect is illustrated here for 4 bunches, the first of which has much larger charge than others. Passing through a resonant structure the first bunch excites a wakefield. This field is sampled by the following three bunches and sampled voltage determines a synchronous position shift.

For small shifts one can use a simplified linear relationship. As transient amplitudes go up the linear model has larger and larger positive error, i.e. it always predicts bigger transient than in reality.



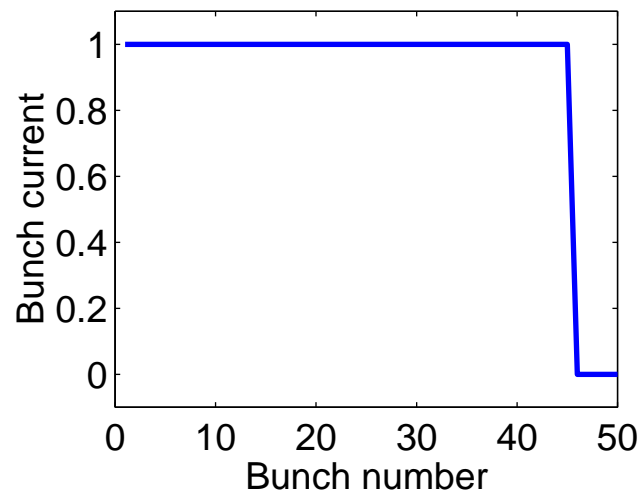
If the linear model predicts more than 10 degrees@RF peak-to-peak transient, use F. Pedersen's small-signal model to get a more precise answer.

## Synchronous Phase Transients

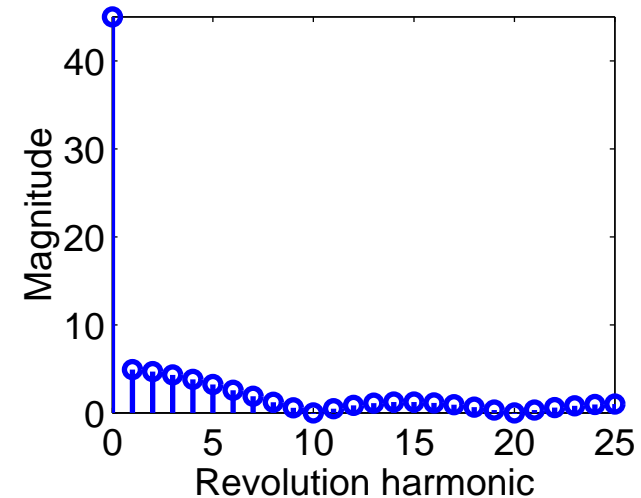
Synchronous positions of bunches are related to aliased longitudinal impedance

$$\Phi_n = -\frac{N}{|V_{\text{rf}} \cos \phi_s|} I_n \tilde{Z}_n \quad \text{where} \quad \tilde{Z}_n = \sum_{m=-\infty}^{\infty} Z(m\omega_{\text{rf}} + n\omega_{\text{rev}})$$

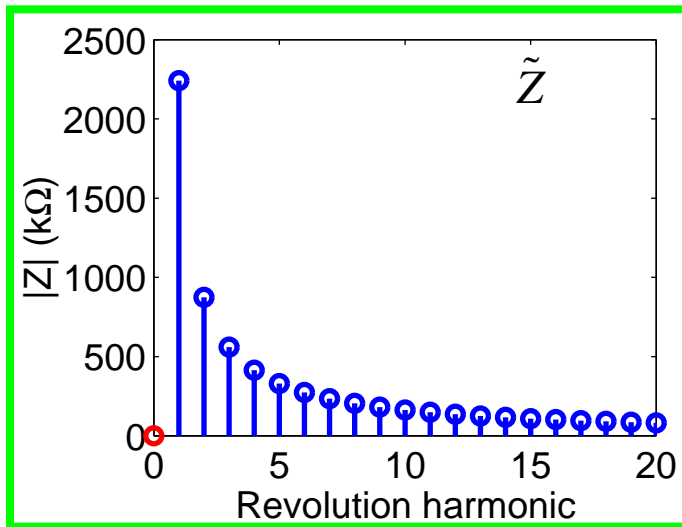
Variable	Description
$\Phi_n$	Discrete Fourier Transform (DFT) of synchronous bunch phases
$I_n$	DFT of bunch currents
$\tilde{Z}_n$	Aliased impedance at the $n^{\text{th}}$ revolution harmonic



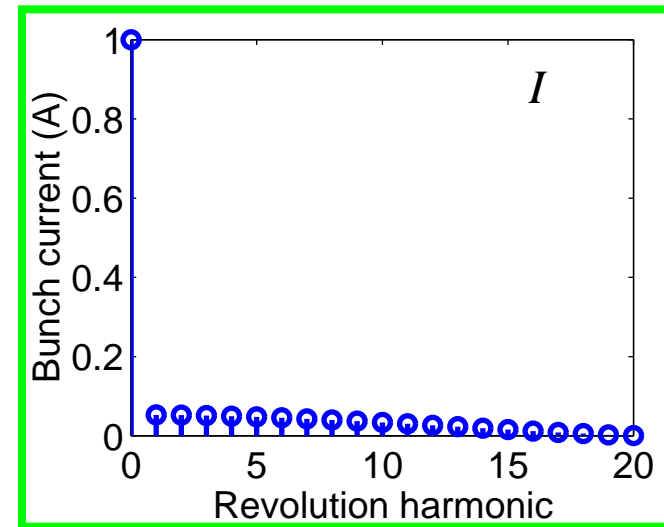
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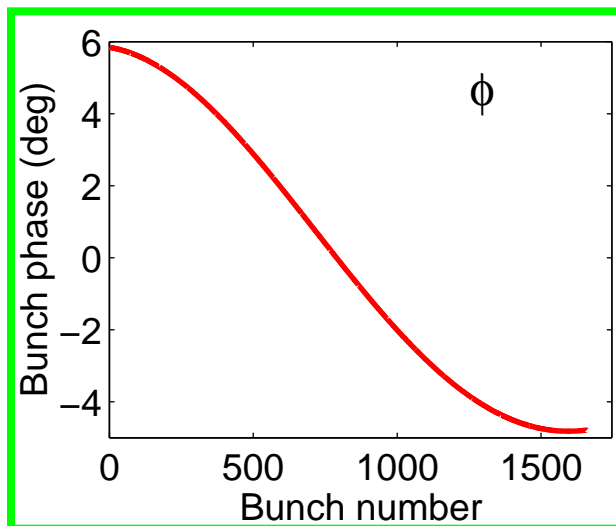
## Effect of the Fundamental Impedance



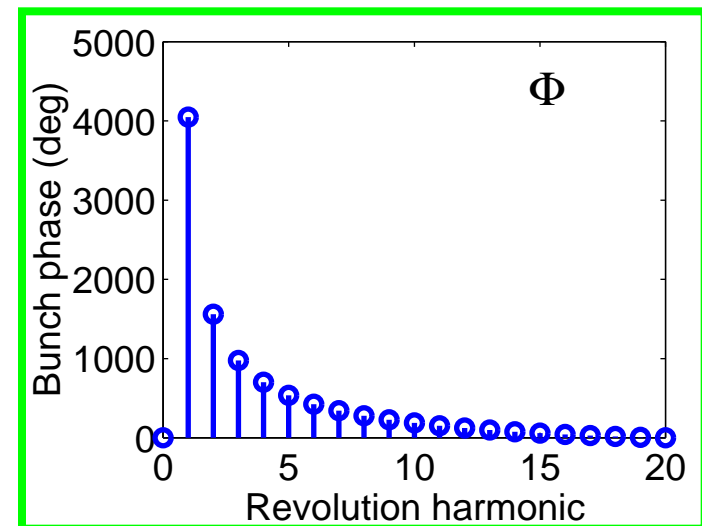
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IDFT



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## Effects of the gap transient

Different bunches see different RF voltage slopes and, therefore, have differing synchrotron tunes - normally a negligible effect.

In the LFB front-end the transient appears as constant DC offsets of individual bunches. This has several consequences:

- Amplitude of the gap transient cannot exceed the full-scale peak-to-peak range of the LFB phase detector (30 degrees@RF for 6<sup>th</sup> RF harmonic detection).
- Largest expected gap transient amplitude sets the feedback front-end gain since we need to properly detect AC motion for the bunches at the extremes of the transient.
- Phase detector gain rolls off as  $\cos(M\phi)$  where  $M$  is the detection harmonic

In the back-end of the LFB the effects of the gap transient are less severe. The main effect is the gain roll off in the kicker at the extremes of the transient, however the effect is smaller due to the lower back-end center frequency.

## Synchronous gap transient: an example

PEP-II Low Energy Ring at 1553 mA

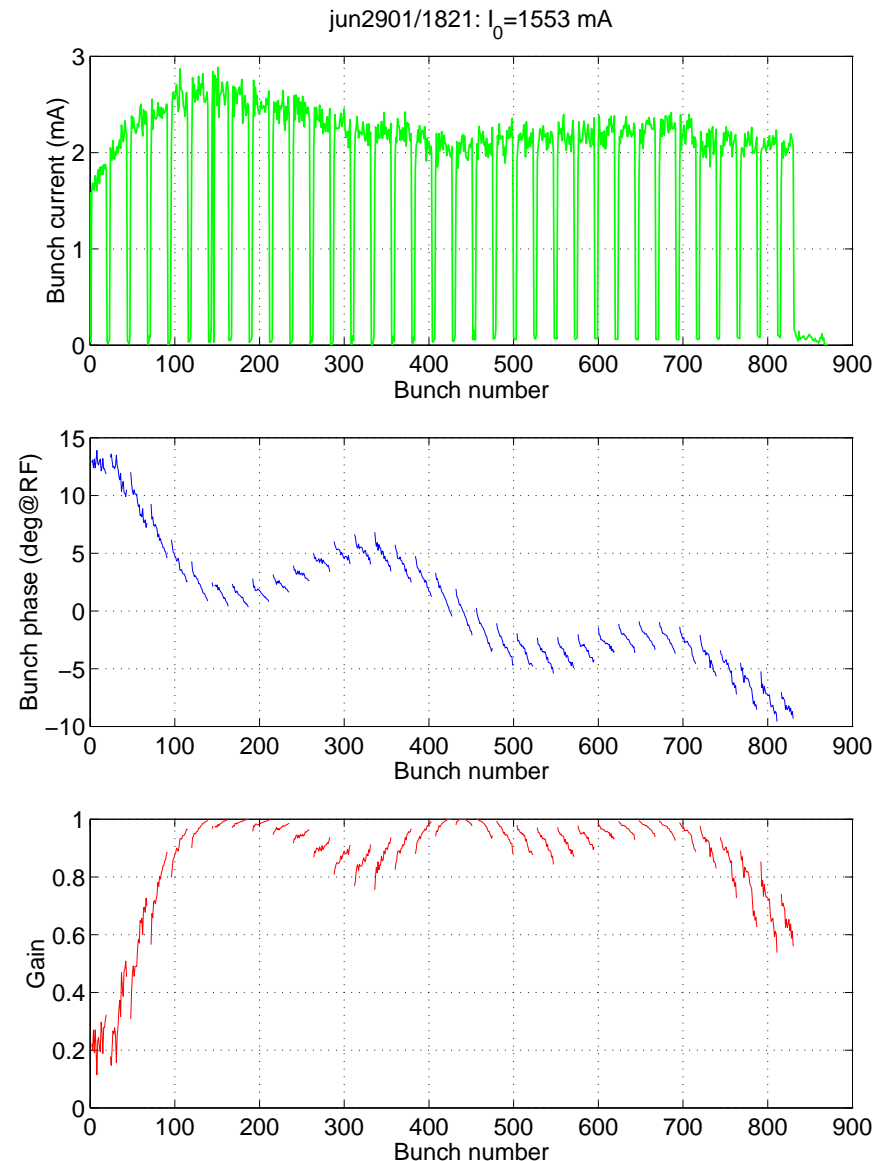
Four RF cavities are powered and two are parked.

Synchronous phase transient includes effects of both active and parked cavities. Cavities parked between 2 and 3 revolution harmonics add oscillatory behavior to the transient.

Overall transient is 23.5 degrees peak-to-peak - this leaves little room for phase drifts.

Bunches at the beginning of the train are offset by 14 degrees! That corresponds to almost 20 dB gain reduction.

In this configuration we keep the tail of the train closer to zero degrees so that the feedback gain at the tail is higher. Since the driving term is larger at the tail of the train we need more gain there.



## Feedback gain limitations

### System group delay

- A fundamental limitation - determines the highest controllable growth rate
- Above certain loop gain feedback does not provide more damping

### Front-end gain

- Limited by the synchronous gap transient as follows:  $K_{FE} = \frac{128}{i_{nom} \sin(M\phi_{max})}$
- Reducing the gap transient helps

### DSP gain

- Limited by the detector noise and the front-end quantization noise. For an 8-bit sampler gains above 50-100 are very hard to achieve. At the gain of 128 two ADC counts of oscillation saturate the back-end
- Increasing sampler resolution is possible, but costly

### Back-end gain

- Limited by the available kicker shunt impedance and the amplifier power.
- Adding gain in the back-end is always very expensive.



## Equivalent model of a cavity

We can model fundamental mode of the cavity as a parallel RLC circuit

$$Z(s) = \frac{2\sigma R s}{s^2 + 2\sigma s + \omega_r^2}$$

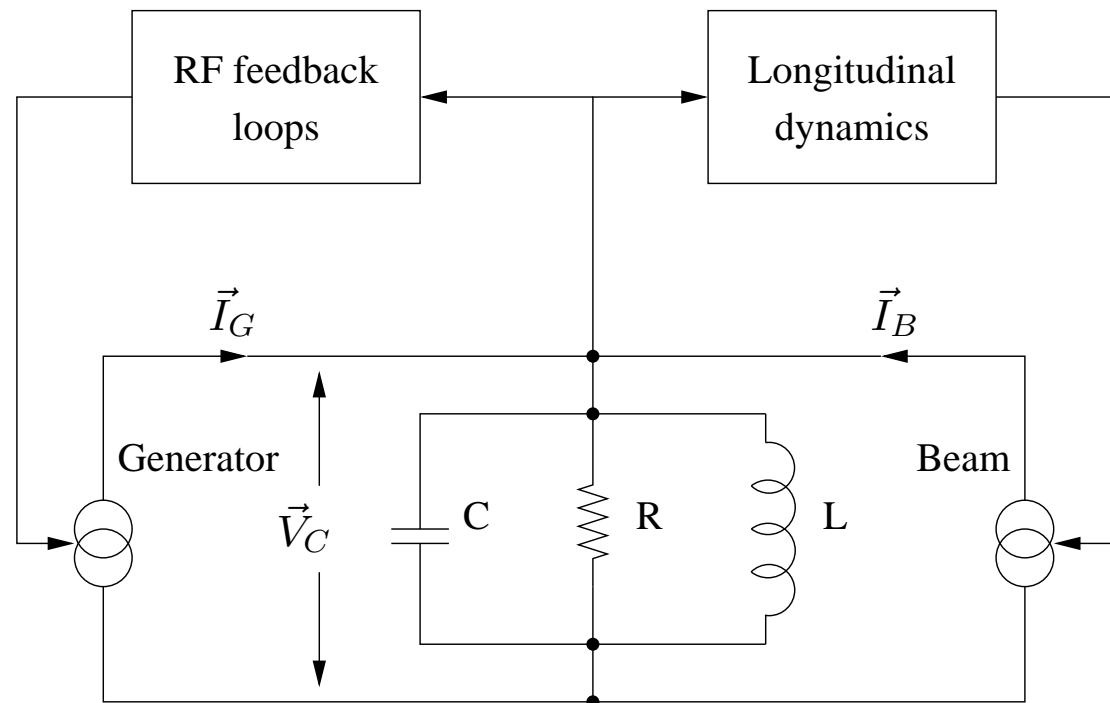
where  $\sigma = \omega_r / (2Q)$  is the damping time of the cavity

The cavity is driven by two current sources: the generator (klystron) and the beam. Total cavity voltage is determined by the sum current and the cavity impedance at  $\omega_{rf}$ .

When the beam current is small relative to the generator current - light beam loading - the cavity voltage is mostly defined by the generator current.

High beam current starts to affect strongly the cavity voltage thus creating a strong interaction between the RF system and the beam.

Think of the interaction as of a “feedback loop”: beam current source is affected by the cavity voltage, while that voltage depends on the beam current.



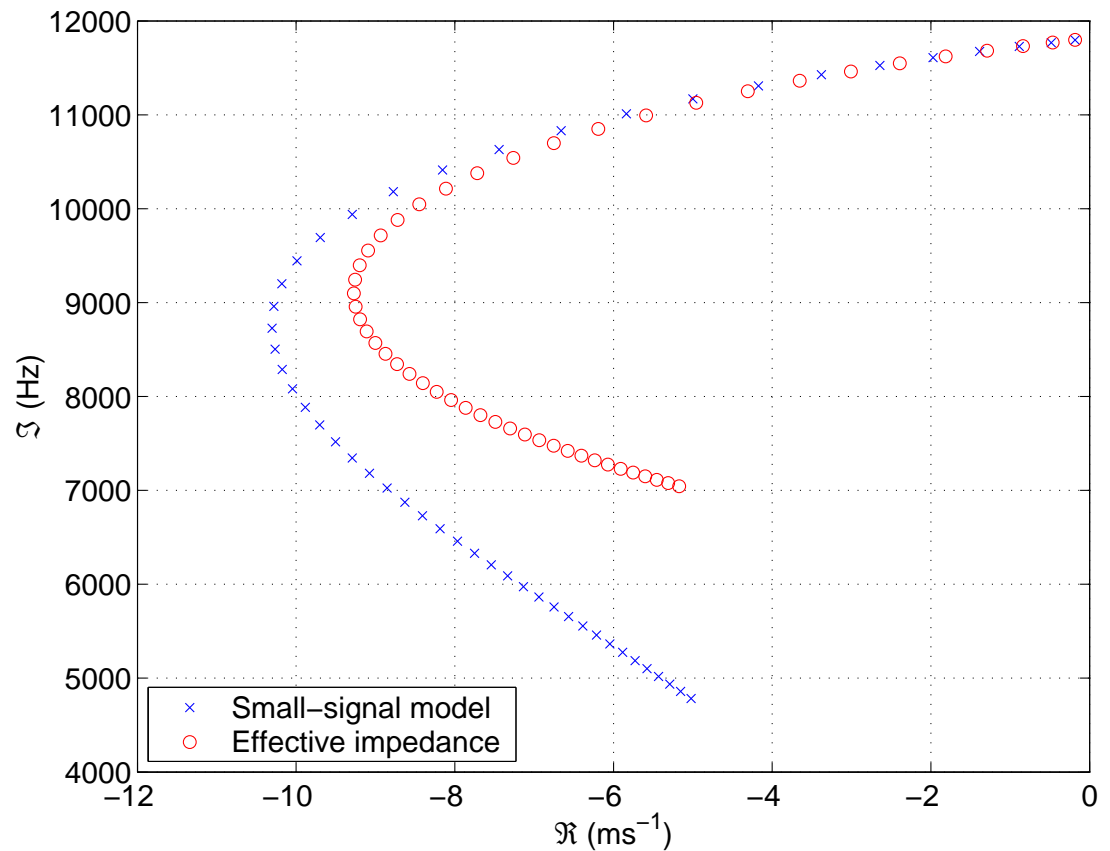
## Mode 0 tune shifts

The lowest frequency synchrotron eigenmode (mode 0) interacts very strongly with the fundamental impedance.

At high beam loading such interaction leads to a significant downward tune shift for mode 0.

Consequently this eigenmode samples the longitudinal feedback controller response at the wrong frequency leading to positive feedback situation

Since mode 0 is stable without feedback we design LFB filters to roll-off quickly at low frequencies so that mode 0 is not excited.



## DAΦNE with strong RF focusing

As an example we will consider the effect of proposed RF configuration on longitudinal feedback

The proposed design has a much higher gap voltage which results in significantly shorter bunches at the IP and higher synchrotron frequency.

Parameter	Current	Proposed
RF frequency ( $f_{rf}$ )	368.25 MHz	500 MHz
Momentum compaction ( $\alpha_c$ )	0.029	-0.171
Circumference ( $L$ )	97.69 m	105 m
Revolution frequency ( $f_{rev}$ )	3.069 MHz	2.857 MHz
Harmonic number	120	175
RF voltage ( $V_{rf}$ )	120 kV	10.677 MV
Synchrotron frequency ( $f_s$ )	30 kHz	1.31 MHz
Revolutions per synchrotron period	~102	2.18
Bunch length ( $\sigma_z$ )	19 - 38 mm	2.6 - 20.4 mm

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## System issues

### Signal processing

- High synchrotron frequency means that we need to process every bunch on every turn. This is addressed by the Gboard architecture described earlier by John Fox.
- Processing for odd harmonic numbers is more difficult to implement than that for even numbers.

### Synchronous gap transients

- $\sim 5 \text{ deg}@RF$  at 1 A, not an issue since their amplitude scales as  $1/V_{rf}$
- Front-end gain can be increased relative to the current setting.

### Loop gain

- Gap voltage increases 89 times, thus effective feedback gain drops by 9.4 for constant growth rates.
- Partially compensated by the higher front-end gain
- Kicker gain for the quadrupole instabilities goes down with the bunch length, need to place the kicker near the RF cavity or design a separate higher frequency quadrupole kicker.

### Mode 0 tune shifts

- Not a significant problem since very high synchrotron tune samples the RF cavity impedance far from resonance.

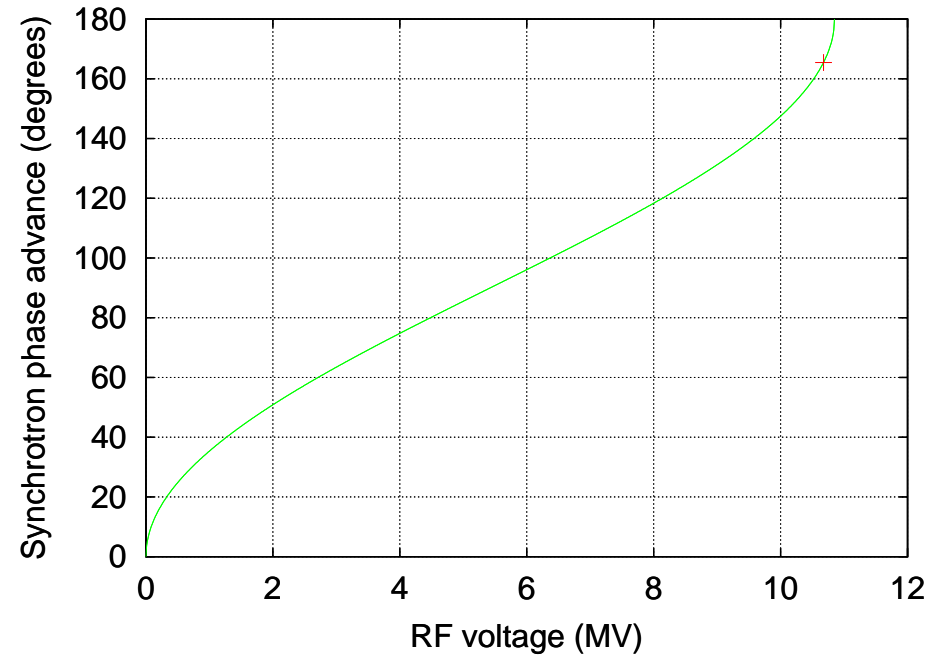
## Noise induced by the RF system

At high phase advance synchrotron tune changes rapidly with the RF voltage.

Therefore noise in the RF will strongly excite the longitudinal motion.

Slope of tune vs. RF voltage is steeper as compared to the current situation.

More importantly, required fractional stability is almost two orders of magnitude better. This is due to the fact that we need to maintain at 10.677 MV the same or better amplitude control as we had at 120 kV.



## Longitudinal coupled-bunch instabilities

Consider the modal eigenvalues  $\Lambda_l$ :

$$\Lambda_l = -d_r + i\omega_s + \frac{\alpha e f_{\text{rf}}}{2E_0 v_s} I_0 Z^{\text{eff}}(l\omega_{\text{rev}} + \omega_s)$$

If the effective impedance and beam current stay the same, the eigenvalues change as  $\frac{6 \cdot 1.36}{47} = 0.17$  where factor of 6 is from change in  $\alpha$ , 1.36 - from  $f_{\text{rf}}$ , and 47 - from  $v_s$ .

These are good news - the growth rates are reduced.

Another advantage is that at higher synchrotron frequency faster growth rates can be controlled.

### Problems

- Shorter bunches sample higher frequency impedances, impedance is aliased with linear (dipole) or quadratic (quadrupole) frequency weighting.
- Achieving the same feedback loop gain is harder

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## Conclusions

There are many interactions between the RF system and the coupled-bunch instability feedback

Experience from operating LBL/LNF/SLAC designed feedback systems at 5 different machines allowed us to carefully characterize these interactions.

Information on the RF parameters together with the impedance data can be used to predict with high degree of confidence the feasibility of the proposed configuration.

Analysis of the proposed strong RF focusing for DAΦNE shows feasibility of the design with respect to longitudinal coupled-bunch feedback with several possible problems:

- Excitation of the beam by the RF noise
- Reduced effective loop gain
- Lower kicker gain for quadrupole control
- High-frequency impedances sampled by a shorter bunch

More analysis needs to be done at the later stages of the design process.

Most importantly, from our experience with the LFB at multiple installations two things stand out

- In operating the machine you almost always find instability surprises not predicted in the design.
- Flexibility of the feedback architecture is critical to effectively control these “surprises”.



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