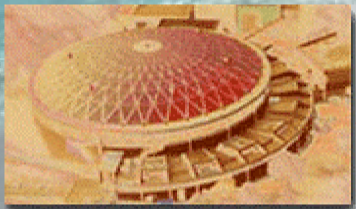
LATTICE QCD AND KAON PHYSICS

Vittorio Lubicz

- $K - \bar{K}$ **Mixing**: ε_K and B_K
- $K \to \pi\pi$: $\Delta I=1/2$ and ε'/ε
- $K \to \pi l \nu$: Measurement of $\sin\theta_C$
- $K_L \to \pi^0 e^+ e^-$: FCNC Kaon Rare Decays

ROMA TRE



Workshop on
$e^+ e^-$ in the 1-2 GeV range:
Physics and Accelerator Prospects
ICFA Mini-workshop - Working Group on High Luminosity e+e- Colliders
10-13 September 2003, Alghero (SS), Italy

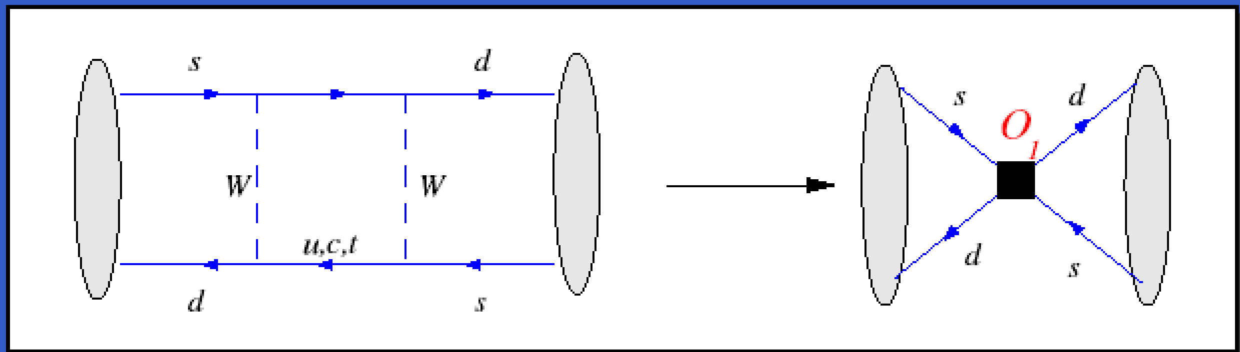
KAON PHYSICS:
A REMARKABLE HISTORY

- **1949**: Powell et al., DISCOVERY OF K MESONS
  "Strange" particles. $t_{Coll.} \approx 10^{-23}$ s (Strong), $t_{Dec.} \approx 10^{-10}$ s (Weak)
- **1956**: Lee, Yang, PARITY VIOLATION
  θ–τ puzzle: $\theta^+ \to \pi^+\pi^0$, $\tau^+ \to \pi^+\pi^+\pi^-$
- **1963**: Cabibbo, UNIVERSALITY OF WEAK INTERACTIONS
  $\Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)$, $\Gamma(K \to \pi l \nu)/\Gamma(\pi \to \pi l \nu)$
- **1964**: Cronin, Fitch et al., CP VIOLATION $K \to \pi\pi$
  ⟹ **1972**: Kobayashi, Maskawa
- **1970**: Glashow, Iliopoulos, Maiani, GIM & CHARM PREDICTION
  Suppression of $K \to \mu^+\mu^-$, $K \to \pi l^+ l^-$. Estimate of m_c from Δm_K

$K - \bar{K}$ Mixing: ε_K and B_K

$$K_L \sim \overset{CP=-1}{(K^0 - \overline{K}^0)} + \varepsilon_K \overset{CP=+1}{(K^0 + \overline{K}^0)}$$

The Effective ΔS=2 Hamiltonian

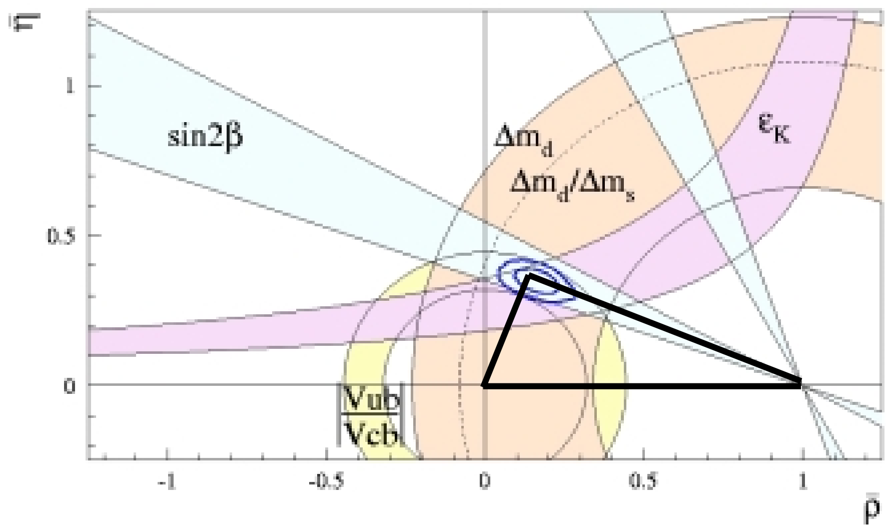


$$\varepsilon_K \sim \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C(\mu) \cdot \langle \bar{K}^0 | \overbrace{\bar{s}\gamma_\mu(1-\gamma_5)d \; \bar{s}\gamma_\mu(1-\gamma_5)d}^{O_1(\mu)} | K^0 \rangle$$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

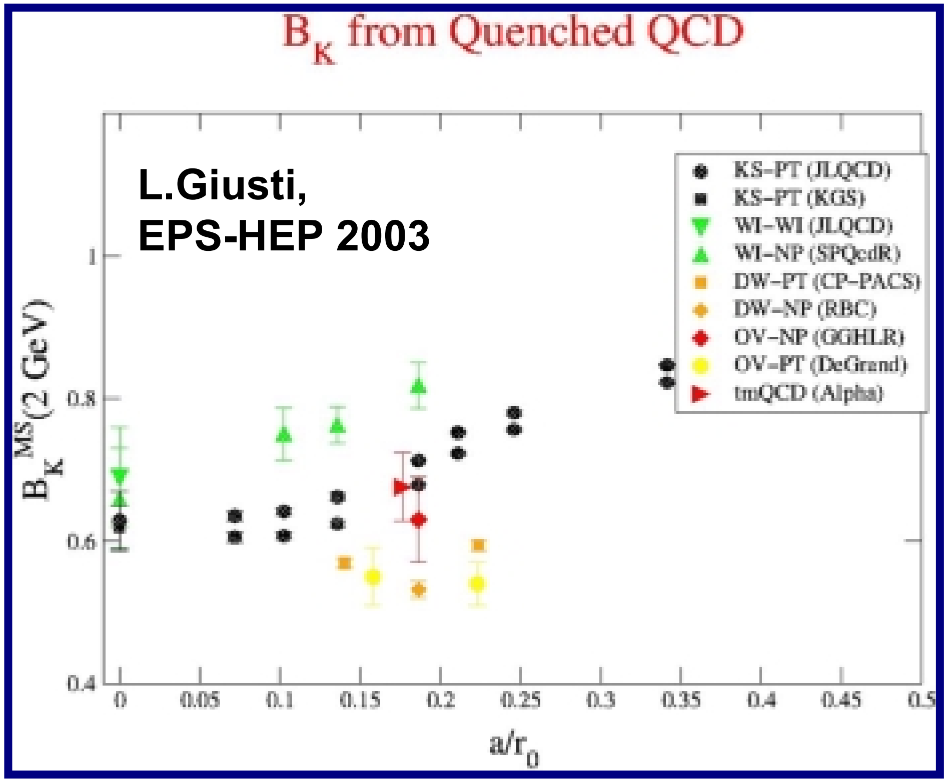
$$\varepsilon_K^{\text{exp.}} = (2.280 \pm 0.013) \times 10^{-3} e^{i\pi/4}$$

B_K and the Unitarity Triangle Analysis



(b→u)/(b→c)	$\bar{\rho}^2 + \bar{\eta}^2$	$\bar{\Lambda}, \lambda_1, f_+, \ldots$
ε_K	$\bar{\eta}[(1-\bar{\rho}) + P]$	B_K
Δm_d	$(1-\bar{\rho})^2 + \bar{\eta}^2$	$f_B^2 B_B$
$\Delta m_d / \Delta m_s$	$(1-\bar{\rho})^2 + \bar{\eta}^2$	ξ
$A(J/\psi K_S)$	$\sin(2\beta)$	—

Lattice Results for B_K



- ✓ High level of accuracy
- ✓ Discretization effects not negligible
- ✓ Estimate of quenching error from ChPT ≤ 15% (Sharpe)

$\hat{B}_K = 0.87 \pm 0.06 \pm 0.13$ ← **Quench. Appr.**

[D. Becirevic, Plenary talk @ LATT03]

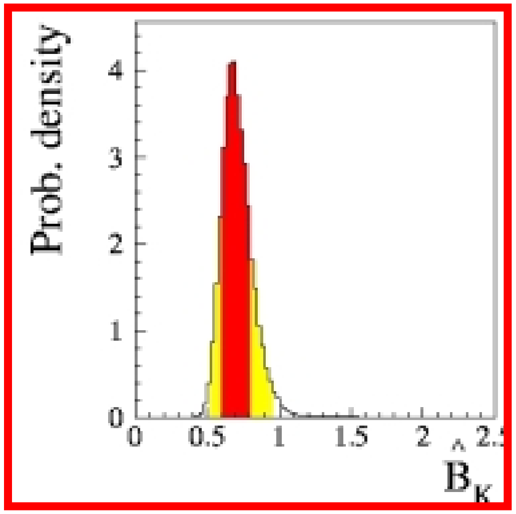
Comparison with Other Results

▲ LATTICE (Average)	$\hat{B}_K = 0.87 \pm 0.06 \pm 0.15$	[Becirevic, LATT03]
▲ QCD SR (Average)	$\hat{B}_K = 0.4 - 0.9$	[Colangelo, Khodjamirian]
▲ $1/N_C$ (Chiral Limit)	$\hat{B}_K = 0.38 \pm 0.11$	[Peris, de Rafael, 2001]
▲ $1/N_C$ + ENJL Model	$\hat{B}_K = 0.77 \pm 0.05 \pm 0.05$	[Bijnens, Prades, 1999]

B_K from the UTA

[Ciuchini, Franco, Parodi, VL, Silvestrini, Stocchi, 2003]

$$\hat{B}_K = 0.69^{+0.11}_{-0.09}$$

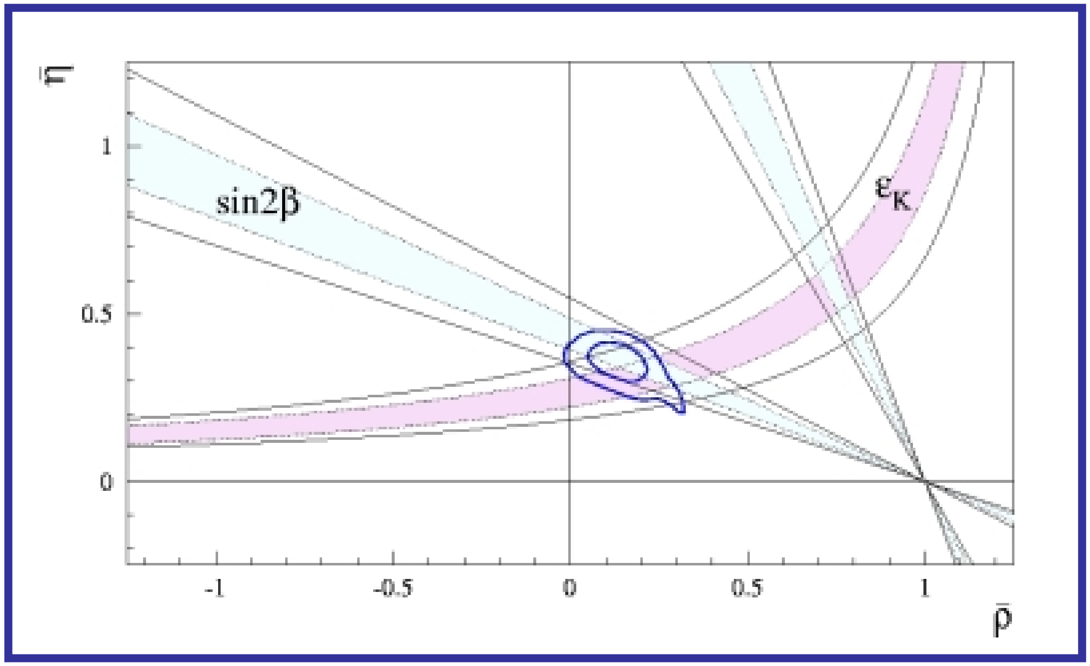


LATTICE PREDICTION (!) $\hat{B}_K = 0.90 \pm 0.20$ [Gavela et al., 1987]

THE CKM MECHANISM OF CP VIOLATION

3 FAMILIES → - Only 1 phase - Angles from Sides

Ciuchini, Franco, Parodi, VL, Silvestrini, Stocchi, 2003



$\sin 2\beta_{UTA}$ = 0.685 ± 0.055

$\sin 2\beta_{J/\psi\ Ks}$ = 0.734 ± 0.054

Prediction (Ciuchini et al., 2000): **$\sin 2\beta_{UTA}$ = 0.698 ± 0.066**

$K \rightarrow \pi\pi$: ΔI=1/2 and ε'/ε

$$\begin{aligned}
\mathcal{A}(K^+ \rightarrow \pi^+\pi^0) &= \sqrt{3/2}A_2 e^{i\delta_2} \\
\mathcal{A}(K^0 \rightarrow \pi^+\pi^-) &= \sqrt{2/3}A_0 e^{i\delta_0} + \sqrt{1/3}A_2 e^{i\delta_2} \\
\mathcal{A}(K^0 \rightarrow \pi^0\pi^0) &= \sqrt{2/3}A_0 e^{i\delta_0} + \sqrt{4/3}A_2 e^{i\delta_2}
\end{aligned}$$

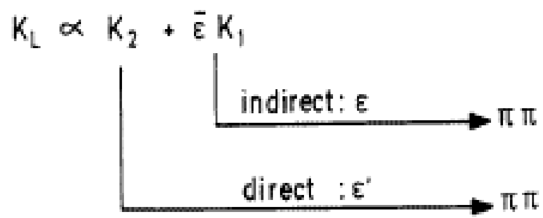
ΔI=1/2

$$(1/\omega)^{\exp} \approx 22 \qquad 1/\omega = \frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$$

Exp. well established

THEORY ??

ε'/ε



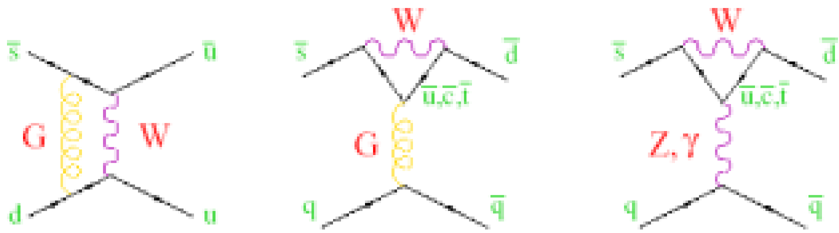
$$\epsilon' = \frac{i e^{i\pi/4}}{\sqrt{2}} \underbrace{\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}}_{\omega} \left(\underbrace{\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}}_{P^2} - \underbrace{\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}}_{P^0} \right)$$

$$\varepsilon'/\varepsilon = (16.6 \pm 1.6) \times 10^{-4}$$

NA48 + KTEV (**KLOE** would be welcome)

THEORY ??

The ΔS=1 Effective Hamiltonian



$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu)\, Q_i(\mu)$$

$$\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{\rm V-A}\ (\bar{u}_\beta d_\alpha)_{\rm V-A} \\
Q_2 &= (\bar{s}u)_{\rm V-A}\ (\bar{u}d)_{\rm V-A} \\
Q_{3,5} &= (\bar{s}d)_{\rm V-A}\ \Sigma_q (\bar{q}q)_{\rm V\mp A} \\
Q_{4,6} &= (\bar{s}_\alpha d_\beta)_{\rm V-A}\ \Sigma_q (\bar{q}_\beta q_\alpha)_{\rm V\mp A} \\
Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{\rm V-A}\ \Sigma_q e_q (\bar{q}q)_{\rm V\pm A} \\
Q_{8,10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{\rm V-A}\ \Sigma_q e_q (\bar{q}_\beta q_\alpha)_{\rm V\pm A}
\end{aligned}$$

$C_i(\mu)$: Short distance dynamics

NLO: Ciuchini et al., Buras et al., 1993

$\langle \pi\pi | Q_i(\mu) | K \rangle$ **:**

Non-perturbative physics

A MAJOR CHALLENGE FOR LATTICE CALCULATIONS:

K → ππ: THE *INFRARED* PROBLEM

1) Maiani, Testa `The NO-GO Theorem`, 1990

- Euclidean correlation functions in Infinite Volume: [Exp(i S)→ Exp(– S)]

$$\langle \Pi_q(t_1)\Pi_{-q}(t_2)J(0)\rangle \longrightarrow \frac{Z_\pi}{2E_q}\, e^{-E_q t_1 - E_q t_2}\,.$$
$$\cdot\left[\frac{1}{2}\Big({}_{out}\langle\pi(q)\pi(-q)|J(0)|0\rangle + {}_{in}\langle\pi(q)\pi(-q)|J(0)|0\rangle \Big) + P_q(t_2)\right]$$

$$|\,{}_{out}\langle\pi(q)\pi(-q)|J(0)|0\rangle\,|\cos\delta(2E_q)$$

Matrix element of interest

$$P_q(t_2) = \mathcal{P}\int_{2m_\pi}^{\infty} dE\, A(E)\, e^{-(E-2E_q)t_2}$$

The "Maiani-Testa" term:
the states with $E < 2\,E_q$ dominate at large t_2

2) Lellouch, Luscher, 2000

- Correlation functions in Finite Volume

• In a finite volume the Maiani-Testa term is a discrete sum with few (1-3) dominant states. These contributions may be controlled by studying the time dependence

• Finite volume matrix elements are related to the physical ones:

For $W_{2\pi} = m_K$:

$$|\langle \pi\pi | H_W(0) | K \rangle|^2 = F_{LL} \, | \, _V\langle \pi\pi | H_W(0) | K \rangle \, |^2 + \mathcal{O}(e^{-mL})$$

Universal "Lellouch-Luscher" Factor

• The phase-shift δ can be computed from $W_{2\pi}$ in a finite volume

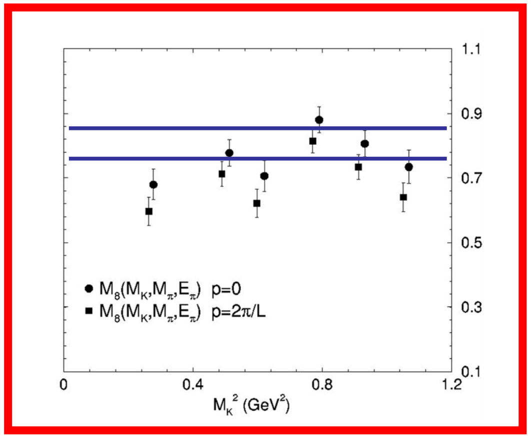
• **THE PROBLEM IS SOLVED IN PRINCIPLE. BUT…** the condition $W_{2\pi} = m_K$ requires L ≈ 5-6 fm which is 3-4 times larger than present lattice sizes

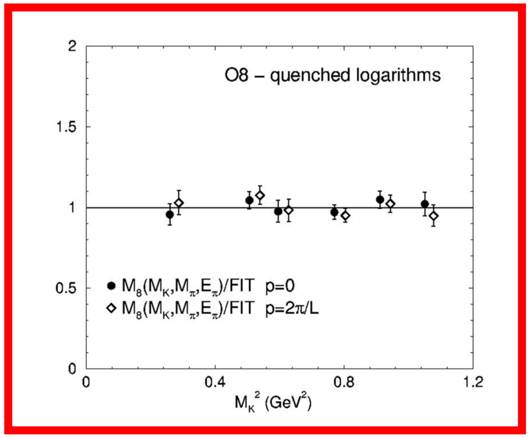
3) Lin, Martinelli, Sachrajda, Testa, 2001

• The LL- formula is extended to the un-physical case $W_{2\pi} \neq m_K$. The unphysical matrix elements, computable on the lattice, can be related to the physical ones by using chiral perturbation theory

4) Lin, Martinelli, Pallante, Sachrajda, Villadoro, 2002

- Complete NLO ChPT calculation of the $K \to \pi\pi$ matrix elements with arbitrary momenta and masses in infinite and finite volume, in full, quenched and partially quenched QCD





SPQ$_{CD}$R Collab.

First Lattice Results with $K \to \pi\pi$ for $\Delta I=3/2$ (2002)

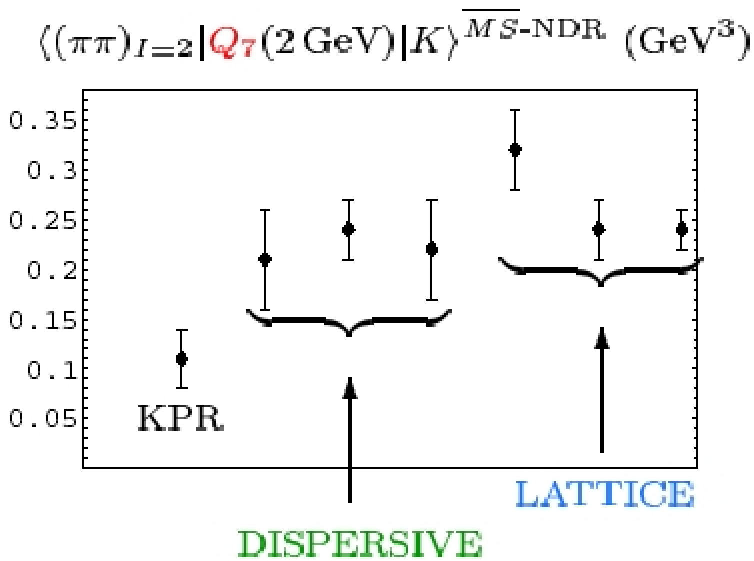
… but the $\Delta I=1/2$ is a more difficult case

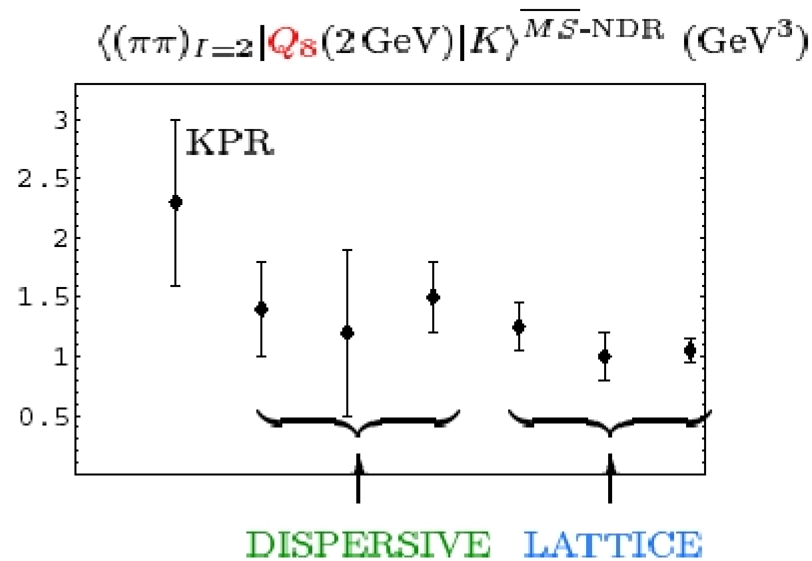
- In the quenched and partially quenched cases **unitarity** is violated:

- FSI are not universal (depend on the operators)
- The LL-formula does not hold anymore
- The time dependence of correlation functions is not universal (depend on the operators)

$\Delta I=3/2$: COMPARISONS WITH OTHER APPROACHES (ChPT, 1/N, Disp.Rels.)

From V. Cirigliano, EPS-HEP 2003





KPR (1/N): Knecht, Peris, de Rafael

DISPERSIVE: - Narison; - Bijnens, Gamiz, Prades; - Cirigliano, Donoghue, Golowich, Maltman

LATTICE: - RBC; - CP-PACS; - SPQ$_{CD}$R

$\Delta I=1/2$: THE $K\to\pi$ METHOD

$K\to\pi\pi$ MATRIX ELEMENTS

LO ChPT in the CHIRAL LIMIT (Soft Pion Theorems)

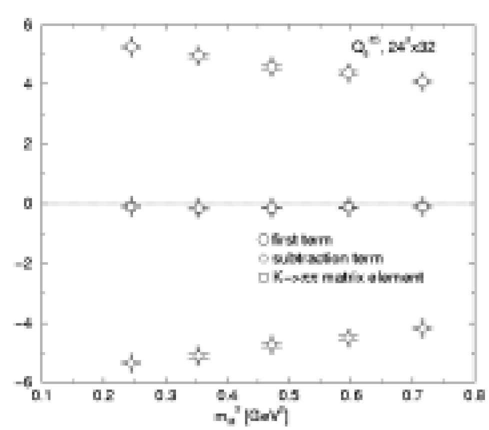
$K\to\pi$ AND $K\to 0$ MATRIX ELEMENTS

(3-pt functions)

	RBC	CP-PACS	EXP.
$\mathrm{Re}(A_0)/\mathrm{Re}(A_2)$	25.3 ± 1.8	$9.5 \pm^{3.2}_{1.8}$	22.2
ε'/ε	$(-4.0 \pm 2.3)\ 10^{-4}$	$(-7.7 \pm 2.0)\ 10^{-4}$	$(16.6 \pm 1.6)\ 10^{-4}$

SYSTEMATIC ERRORS:

- Higher order chiral corrections (FSI)
- The "ULTRAVIOLET" problem: power diverg.
  $\hat{O}_6 = O_6 - (C_P/a^2) O_P$, ...
- Approximate Chiral Symmetry
- Quenched Approximation



O_6 – CP-PACS

V_{us} FROM $K \rightarrow \pi l \nu$ DECAYS

CKM Unitarity: $|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$

Recent analysis by G.Isidori et al., for the CERN-CKM Workshop:

SFT:	$|V_{ud}| = 0.9740 \pm 0.0005$		
N β-dec:	$|V_{ud}| = 0.9731 \pm 0.0015$	$K \rightarrow \pi e \nu$:	$|V_{us}| = 0.2196 \pm 0.0026$
Average	$|V_{ud}| = 0.9739 \pm 0.0005$ ⟷	Unitarity:	$|V_{us}| = 0.2269 \pm 0.0021$

2.2 σ discrepancy!

(From semileptonic hyperon decays: $|V_{us}| = 0.2250 \pm 0.0027$ [Cabibbo et al., July 2003])

➡ **Examine Kl3 decays**

EXPERIMENTS:

- DISCREPANCY OF E685 RESULT.
- POOR CONSISTENCY ($1.8\,\sigma$) BETWEEN K^{+}_{l3} AND K^{0}_{l3} MODES.

→ NEEDS FOR NEW MEASUREMENTS: **KLOE !!**

THEORY:

THE CRUCIAL INGREDIENT IS THE FORM FACTOR AT ZERO MOMENTUM TRANSFER: $f_{+}(0)$

$$f_{+}(0) = 1 + f_2 + f_4 + O(p^8)$$

- 1: Vector Current Conservation
- f_2: $f_2 = -\,0.023$ Independent of L_i (Ademollo-Gatto)
- f_4: LARGEST UNCERTAINTY !!

"Standard" estimate: Leutwyler, Roos (1984) **(QUARK MODEL)**

$$f_4 = -0.016 \pm 0.008$$

NEW: **ChPT, Complete NNLO Calculation**

Post, Schilcher (2001), Bijnens, Talavera (2003)

$$f_4 = 0.014(6) - \frac{8}{F_\pi^4}(C_{12} + C_{34})(m_K^2 - m_\pi^2)^2$$

$$= 0.014(6) - 0.056 \left[\frac{C_{12} + C_{34}}{10^{-5}}\right]$$

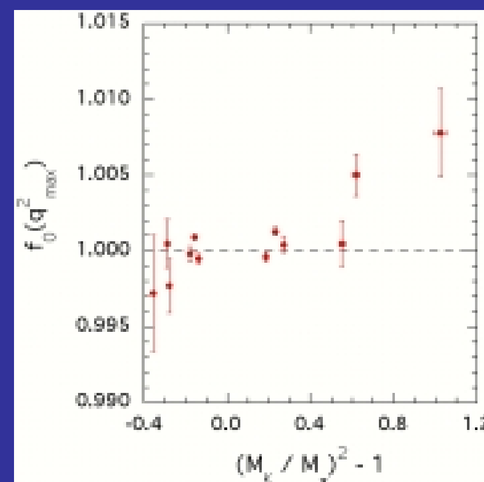
≈ 0.5 Leutwyler & Roos

≈ 1.0 consistent with unitarity

A LATTICE CALCULATION IS IN PROGRESS

[$SPQ_{CD}R$ + Isidori + …]

- Very challenging: 1% of accuracy required
- Use appropriate ratios of correlation functions



RARE KAON DECAYS

THE CKM MECHANISM HAS BEEN ONLY TESTED SO FAR FOR:

- TREE-LEVEL CHARGED CURRENT
- $\Delta F=2$ LOOP INDUCED PROCESSES

BUT:

$\Delta S=1$ (AND $\Delta B=1$) FCNC MAY RECEIVE IMPORTANT CONTRIBUTIONS FROM NEW PHYSICS

THE "GOLDEN MODE" (FREE FROM HADRONIC PARAMETERS) IS $K \to \pi \nu \bar{\nu}$. FOR OTHER PROCESSES LATTICE CALCULATIONS ARE NEEDED.

$K_L \to \pi^0 e^+ e^-$

NEW PHYSICS CONSTRAINTS FROM THE LATTICE

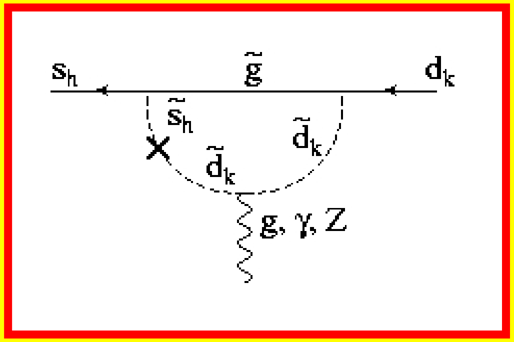
KTeV (2003) $B(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10}$ at 90% C.L.

STANDARD MODEL PREDICTION: $B(K_L \to \pi^0 e^+ e^-) = (3.2^{+1.2}_{-0.8}) \times 10^{-11}$ [Buchalla, D'Ambrosio, Isidori 2003]

CONSTRAINTS ON SUSY MODELS:

First lattice calculation of the electromagnetic operator amplitude:

$$\langle \pi^0 | \bar{s} \sigma_{\mu\nu} F^{\mu\nu} d | K^0 \rangle$$



[Becirevic, VL, Martinelli, Mescia 2001]

KAON PHYSICS HAS REPRESENTED SO FAR A HUGE SOURCE OF INFORMATION FOR PARTICLE PHYSICS. NEW IMPORTANT INSIGHTS ARE EXPECTED IN THE FUTURE.

IT IS CERTAINLY WORTH TO BE STUDIED !!