

# Touscheck beam lifetime

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Workshop on  $e^+e^-$  in the 1-2 GeV Range: Physics and  
Accelerator Prospects

Alghero 11/9/03

## DAFNE lifetime

DAFNE beam lifetime is dominated by Touschek effect.

The average residual gas pressure is well below  $10^{-9}$  Torr and the contributions of beam gas interactions are negligible.

Increasing the luminosity by 2 orders of magnitude is done by squeezing the beams and therefore reduces the Touschek lifetime.

This is a preliminary estimate of beam lifetime for a machine with an extremely short bunch length  $\sigma_z \approx 2.5 \div 4\text{mm}$  at a luminosity of  $10^{34}$ .

# Touschek lifetime

The beam lifetime due to single Touschek scattering is proportional to the third power of the energy, and therefore it is the main limitation for low energy storage rings like DAΦNE.

The Touschek half-lifetime is calculated according to the formula given by H. Brook[3]:

$$\frac{1}{\tau} = \frac{\sqrt{\pi} r_0^2 c N}{\gamma^3 \sigma'_x \varepsilon^2 (4\pi)^{3/2} \sigma_1 \sigma_x \sigma_y} C(u_{min})$$

where:

$r_0$  = classical electron radius

$c$  = velocity of light

$\gamma$  = electron energy in units of its rest mass

$N$  = number of electrons per bunch

$\sigma'_x$  = angular divergence of the beam

$(4\pi)^{3/2} \sigma_1 \sigma_x \sigma_y$  = beam volume

and

$$C(u_{min}) = \int_{u_{min}}^{\infty} \frac{1}{u^2} \left[ u - u_{min} - \frac{1}{2} \ln\left(\frac{u}{u_{min}}\right) \right] e^{-u} du$$

with

$$u_{min} = \left( \frac{\varepsilon}{\gamma \sigma'_x} \right)^2$$

$$\sigma'_x = \left[ \varepsilon_x / \beta_x + \sigma_p^2 (D'_x + D_x \alpha_x / \beta_x)^2 \right]^{1/2}$$

and  $\varepsilon$  = limiting acceptance for the relative momentum deviation of a particle which undergoes a large angle Touschek scattering. It is the minimum between the RF acceptance and the momentum acceptance due to the transverse aperture, either physical or dynamic.

# Touschek lifetime

Neglecting  $C(u_{\min})$  which is a slowly varying function of  $\varepsilon$ :

$$\frac{1}{\tau} \propto \frac{N}{\gamma^3 \sigma_x \varepsilon^2 \sigma_x \sigma_y \sigma_l}$$

$\tau$  is proportional to  $\varepsilon^2$  and to the bunch density.

$\varepsilon$  is the energy acceptance of the ring and is the minimum between:

- RF acceptance
- Aperture limitation
- Dynamic aperture

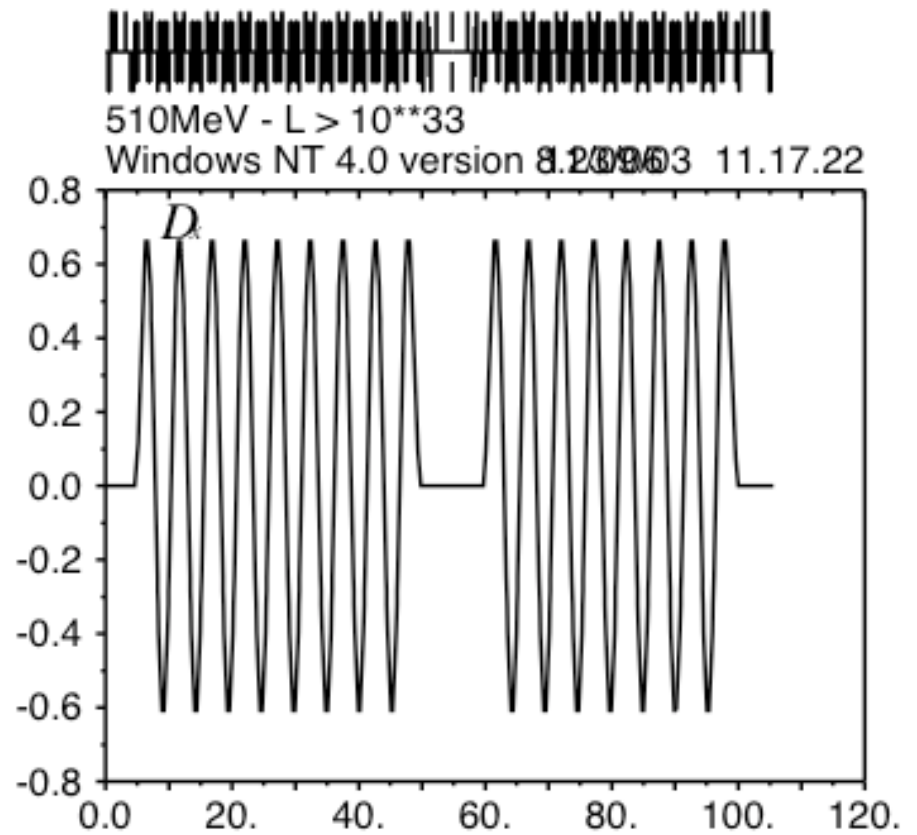
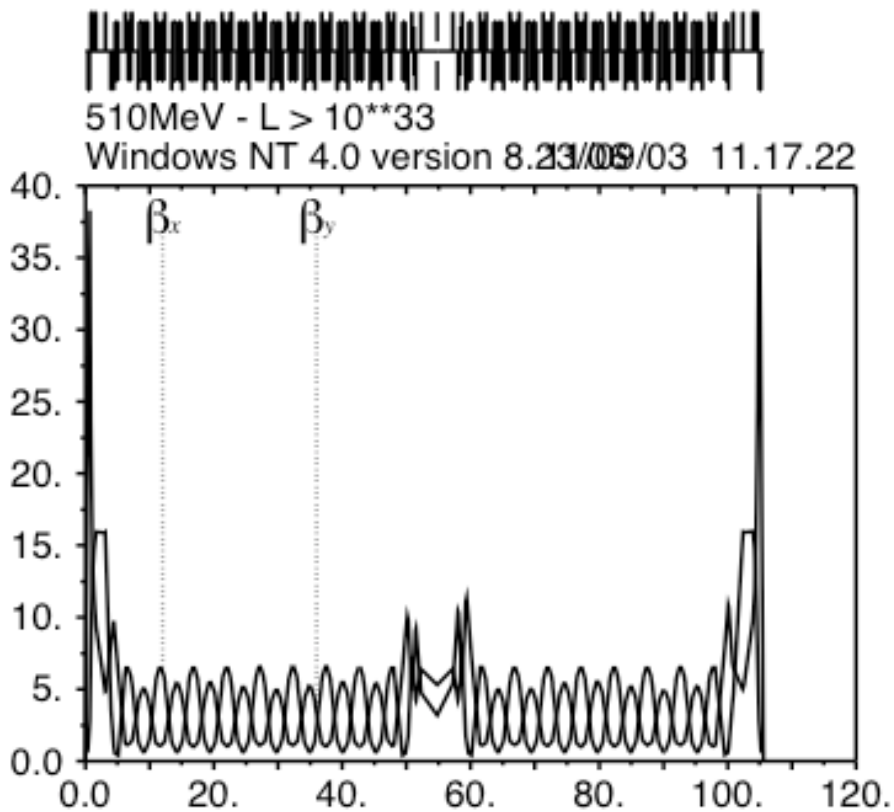
# Energy acceptance

Particles are lost when the oscillation amplitude is equal to the ring acceptance:

$$\varepsilon(\sqrt{H\beta_x} + D_x) = A_x$$

$$H = D^2\gamma + 2\alpha DD' + D'^2\beta$$

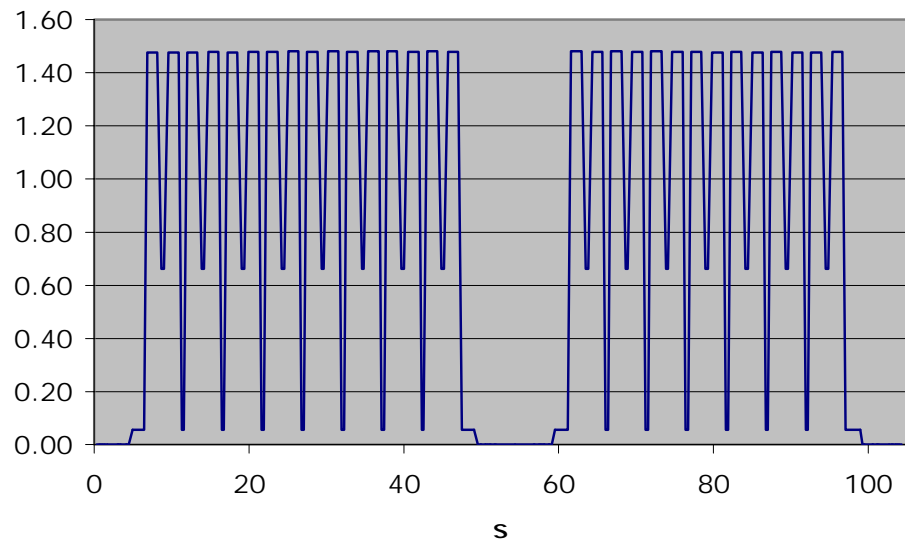
The machine aperture has to be chosen to have the energy acceptance required by beam lifetime.



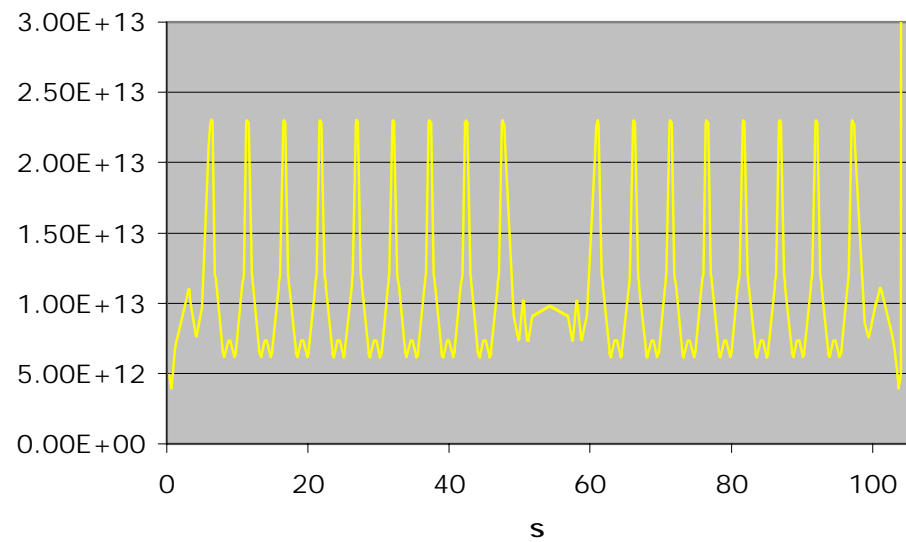
## Lattice Parameters

Circumference 104 m  
 Emittance .19  $\mu\text{m}$   
 $\alpha_c$  -.17  
 N cells 15  
 $Q_x, Q_y$  9.18, 7.24  
 $D_x \text{ max}, \beta_x \text{ max}$  .64, 7.3  
 H 1.5

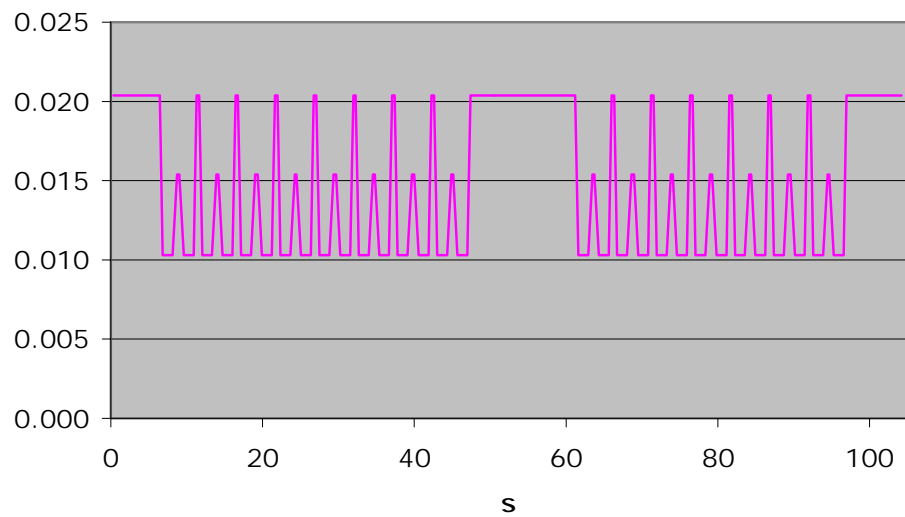
**H**



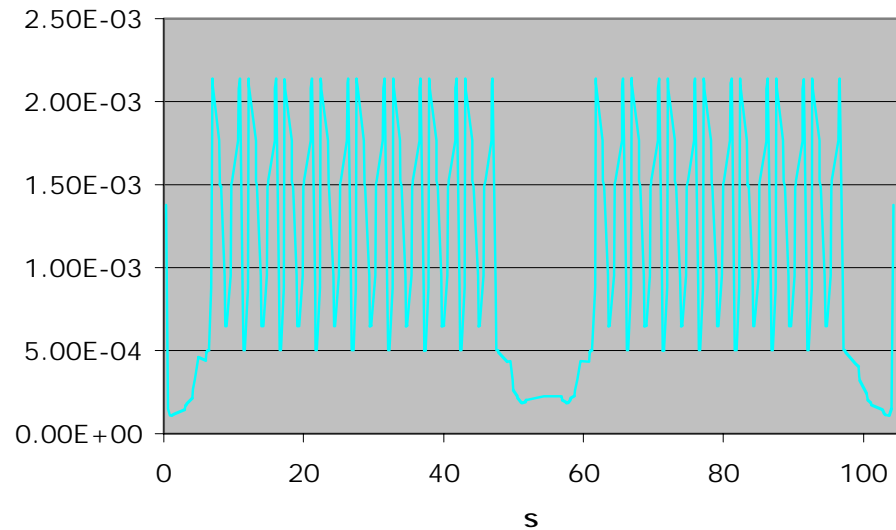
**1 / (σx'σxσt)**



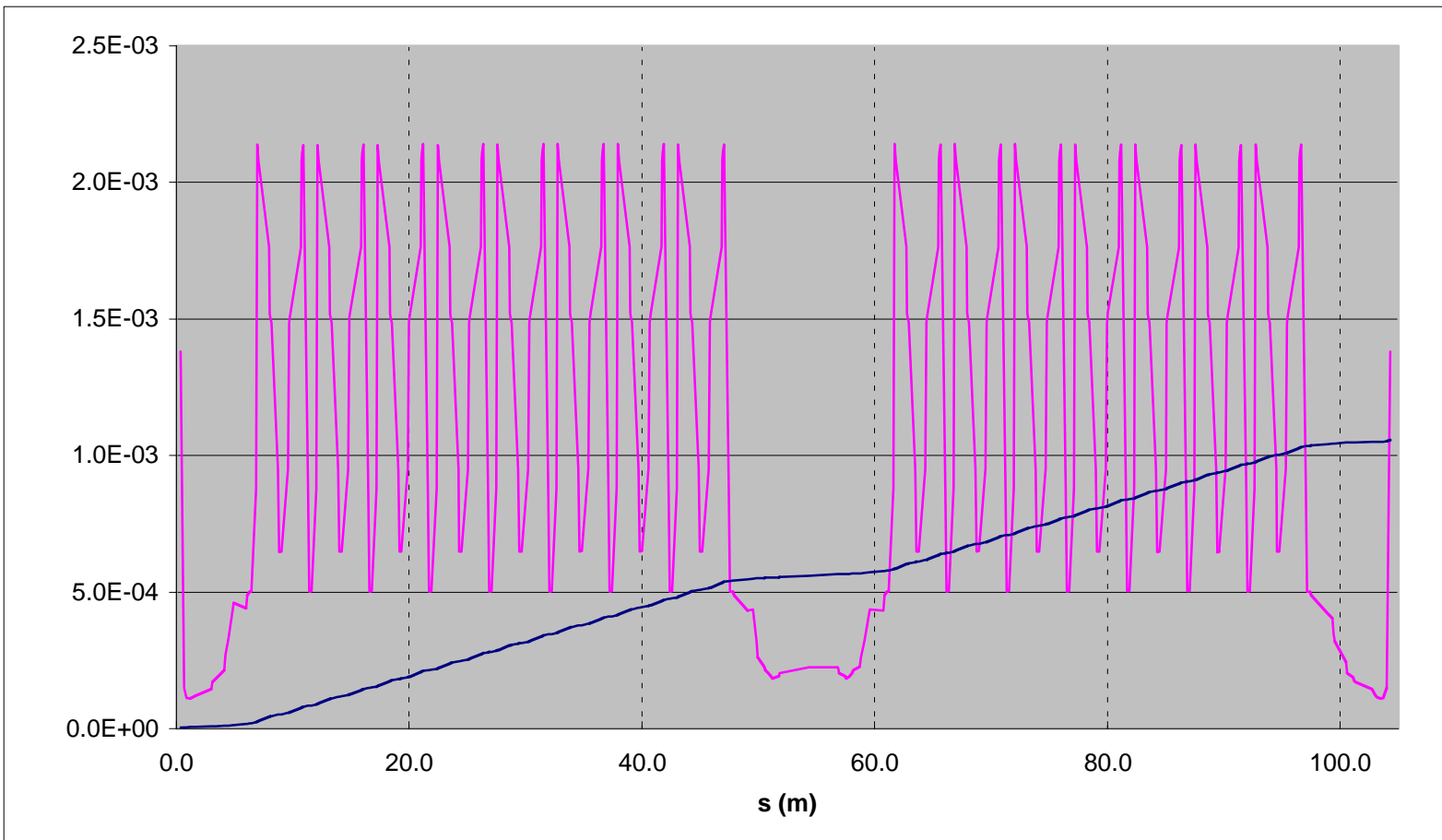
**εlim - Aperx = 5cm**



**1 / τ**



$1/\tau$  and  $(1/L)\int 1/\tau ds$





Constant bunch length, no vacuum chamber aperture

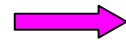
3 sets of parameters for  $10^{34}$

$\beta_x/\beta_y$	2.6/.026	4.0/.04	4.0/.04
Bunch frequency (MHz)	500	500	500
Emittance (mm mrad)	.19	.29	.19
$\kappa$	.01	.01	.01
I (mA)	16	24.5	20
$\sigma_l$ (mm)	2.6	4.0	4.0
$\sigma_p$	$5.4e-4$	$5.4e-4$	$5.4e-4$
$\epsilon_{RF}$	$1.0e-2$	$1.0e-2$	$1.0e-2$
X	.083	.083	.105
$\tau_{TOU}$ (s)	461 (7.7')	707 (12')	568 (9.5')

## Very short bunch length

$$L \propto 1/\beta_y \quad ; \quad \beta_y \sim \sigma_l$$

To increase luminosity  
a very short bunch  
length is needed.



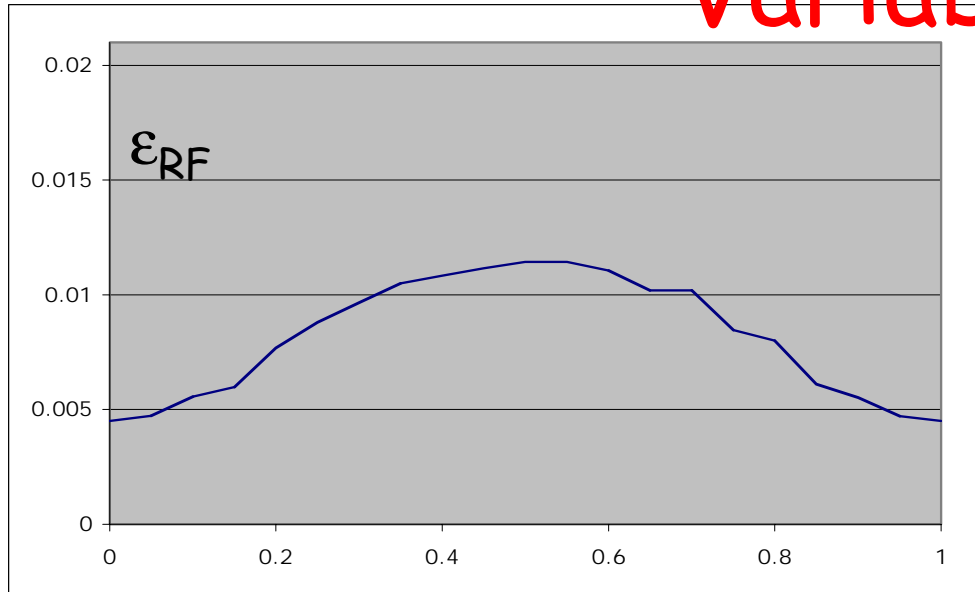
Lifetime reduction  
Larger Impedance

A longitudinal phase advance near  $180^\circ$  gives a strong  
variation of  $\sigma_l$  along the ring.



$\sigma_l$  is very small only at the IP

# Variable $\sigma_I$

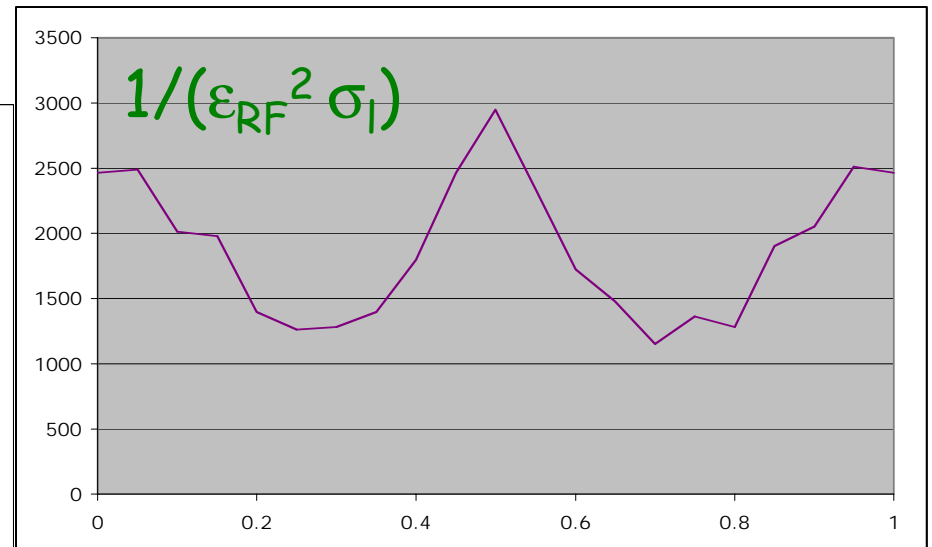
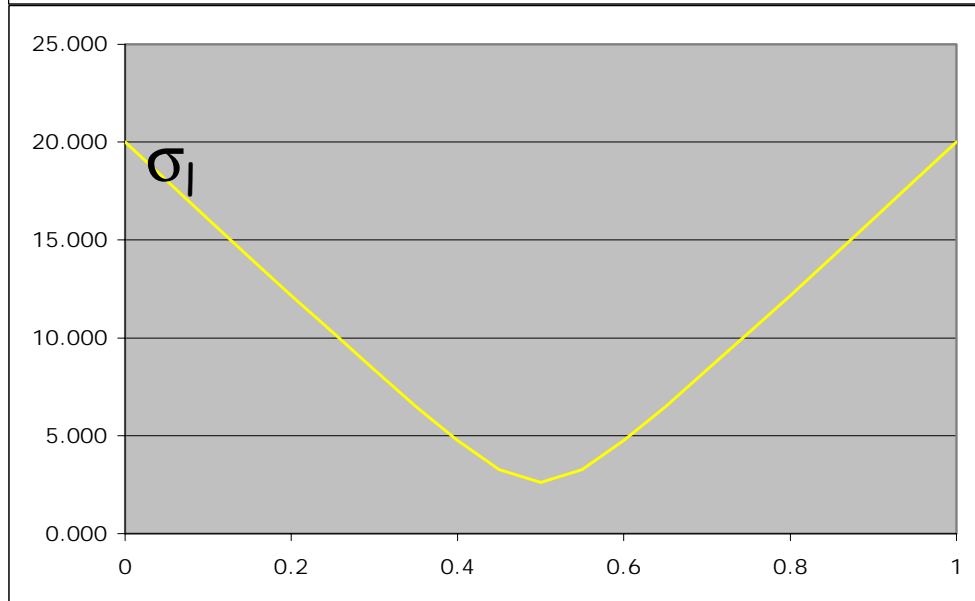


$$\mu_{\text{long}} = 165^\circ$$

$$\alpha_c = -.17, V_{RF} = 10.68 \text{ MV}$$

$$\sigma_p = 2.2 \cdot 10^{-3}$$

$$1/\tau \propto \langle 1/(\epsilon_{RF}^2 \sigma_I) \rangle = 1890 \text{ mm}^{-1}$$



$$\sigma_I^{\text{IP}} = 2.6 \text{ mm}, \sigma_I^{\text{RF}} = 20 \text{ mm}$$

To calculate  $1/\tau$  we substitute the value of  $1/(\epsilon_{RF}^2 \sigma_I)$  with its average along the ring.

No vacuum chamber aperture, variable  $\sigma_I$

$\mu_I$	165°
Emittance (mm mrad)	.19
$\kappa$	.01
I (mA)	16
$\alpha_c$	-.17
VRF (MV)	10.68
$\sigma_I^{IP}$ (mm)	2.5
$\sigma_I^{RF}$ (mm)	20.0
$\sigma_p$	2.2e-3
$\epsilon_{RF}$ at IP	1.1e-2
$\epsilon_{RF}$ at RF	4.5e-3
Luminosity/csi	1e34/.083
$\tau_{TOU}$ (s)	1050 (17.5')
$\tau_{quantum}$ (s)	86 (1.4') !!

# Quantum lifetime

$\varepsilon_{RF} \geq 6 \sigma_p$  for quantum lifetime.

The set of longitudinal parameters has to be optimized.

A smaller  $sp$  is obtained with a slightly smaller  $\mu_{long}$ .

For example  $\mu = 150$  gives:  $sp = 1e-3$

$\sigma_I^{IP} = 2.8 \text{ mm}$  ,  $\sigma_I^{RF} = 10.9 \text{ mm}$

$\varepsilon_{RF}$  at IP =  $1.1e-2$  ,  $\varepsilon_{RF}$  at RF =  $4.5e-3$

The physical and dynamic aperture has to be large enough to have 1.1% energy acceptance.

# Variable $\sigma_I$

$\mu_I$	165	150
Emittance (mm mrad)	.19	.19
$\kappa$	.01	.01
I (mA)	16	16
$\alpha_c$	-.17	-.17
VRF (MV)	10.68	10.15
$\sigma_I^{IP}$ (mm)	2.5	2.8
$\sigma_I^{RF}$ (mm)	20.0	10.9
$\sigma_p$	$2.2e-3$	$1.2e-3$
$\epsilon_{RF}$ at IP	$1.1e-2$	$1.1e-2$
$\epsilon_{RF}$ at RF	$4.5e-3$	$4.5e-3$
Luminosity/csi	$1e34/.083$	$1e34/.083$
$\tau_{TOU}$ (s)	1050 (17.5')	550 (9.2')
$\tau_{quantum}$ (s)	86 (1.4') !!	$6.1e14$

# Conclusions

Strong RF focusing (bunch length variation along the ring) seems promising to get very short bunch length at the IP.

Touschek lifetime has been calculated with a preliminary set of longitudinal parameters. A further optimization is possible.

Anyway at  $L = 10^{34}$  lifetimes are of the order of 10 minutes:

- continuous injection is needed
- a setup for Luminosity optimization with rapidly decreasing currents has to be provided.