

QCD Studies in

Low Energy e^+e^- Annihilation

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SLAC

Workshop on e^+e^- in the 1-2 GeV Range

Physics and Accelerator Issues

Alghero, Sardinia, Italy

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New Physics Opportunities

Low Energy e^+e^- Annihilation

High \mathcal{L} : Small σ
asymmetries
new states

Photo Detection:

$$\gamma^* \rightarrow \gamma X$$

New physics
regime

Timelike DCS

Exotica: $(\gamma\bar{\gamma}\gamma\bar{\gamma})_{I=2} \dots$


Polarization:

polarized beams

final state P, \dots

Non-zero crossing angle, moving CM

Atomic Beamline

- $e^+e^- \rightarrow$ hadrons / leptons 

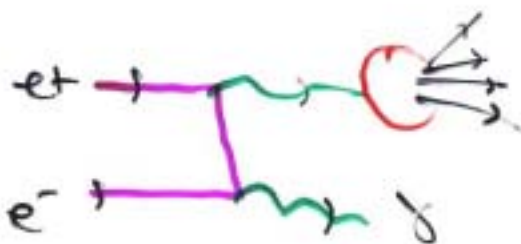
- Illustrious History : Adone!

CEA, SPEAR, Novosibirsk, Adone, BES
 Orsay, PEP, BABAR, BELLE ...
 SLR, LEP

- Low energy regime

- high luminosity machines like Adone

- radiative return at Babar, Belle



$$\sigma \approx 2\sigma < \sigma = \frac{4\pi r_e^2}{3}$$

Much remains to do:

✗ Exclusive channels

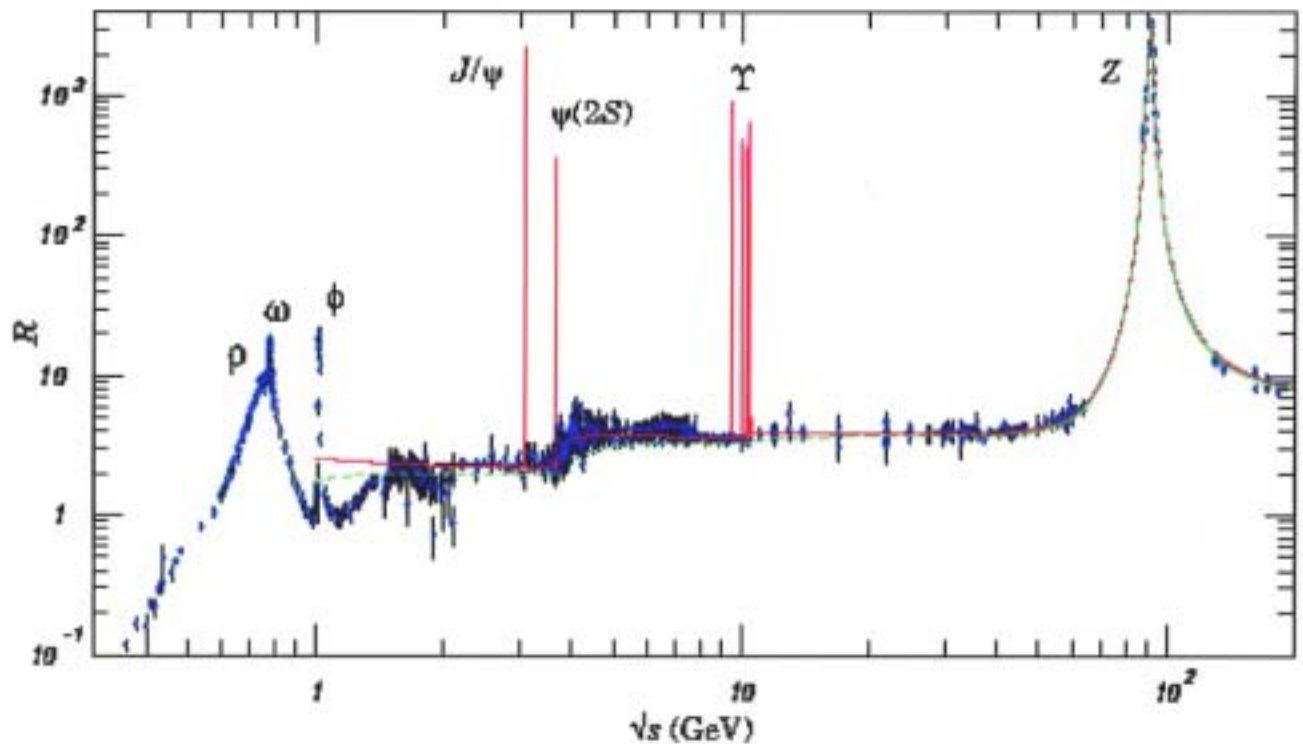
✗ Polarization

✗ Asymmetries

✗ Resonances

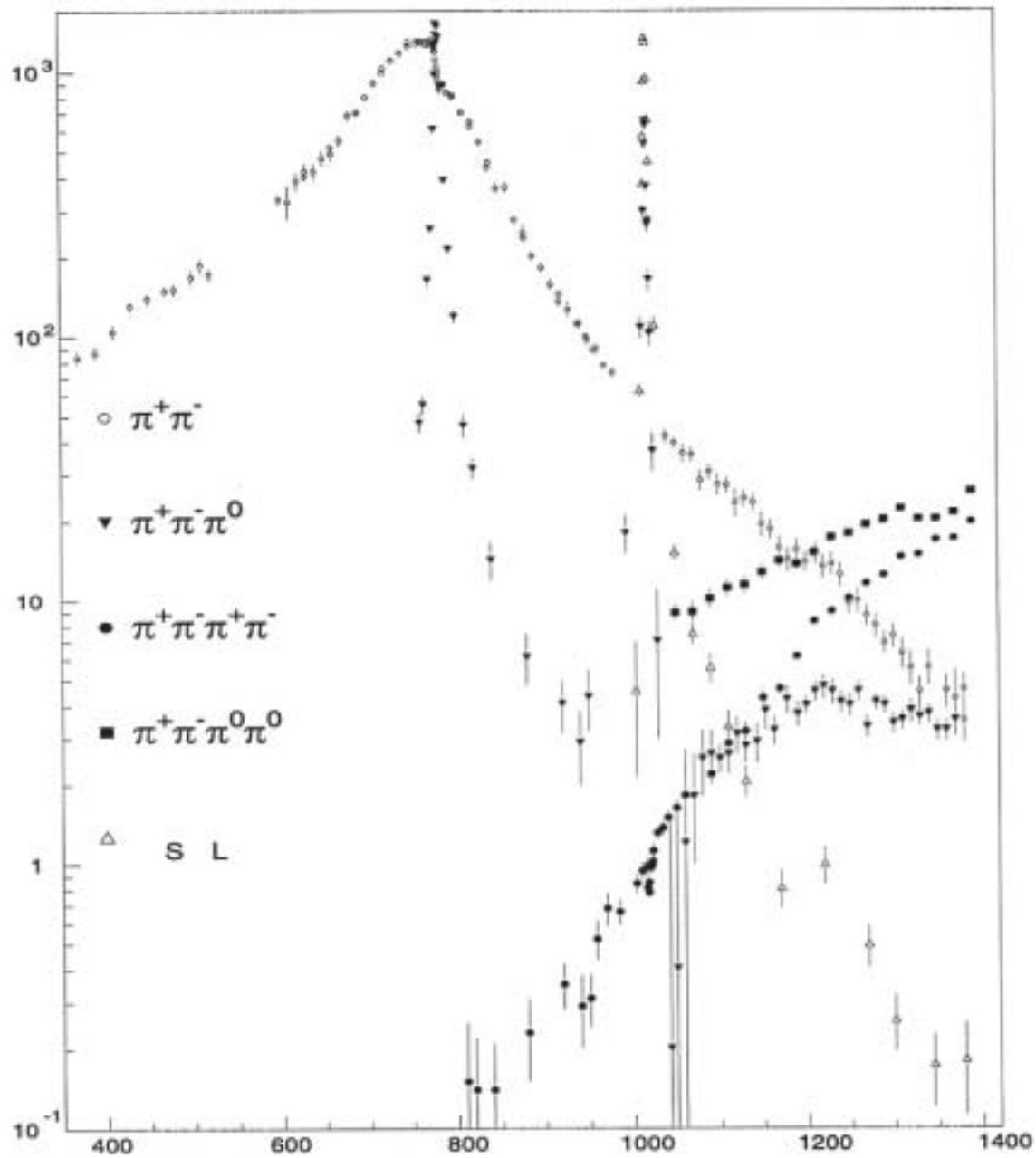
✗ $\tau \rightarrow \nu \ell \ell$: discrepancy with $\tau \rightarrow \nu \ell \ell$

R measurements



$$R = \sigma(e^+e^- \rightarrow \text{Hadrons}) / \sigma_0(e^+e^- \rightarrow \mu^+\mu^-)$$

*A large number of measurements scattered at various energy ranges,
over the last 3 decades.*

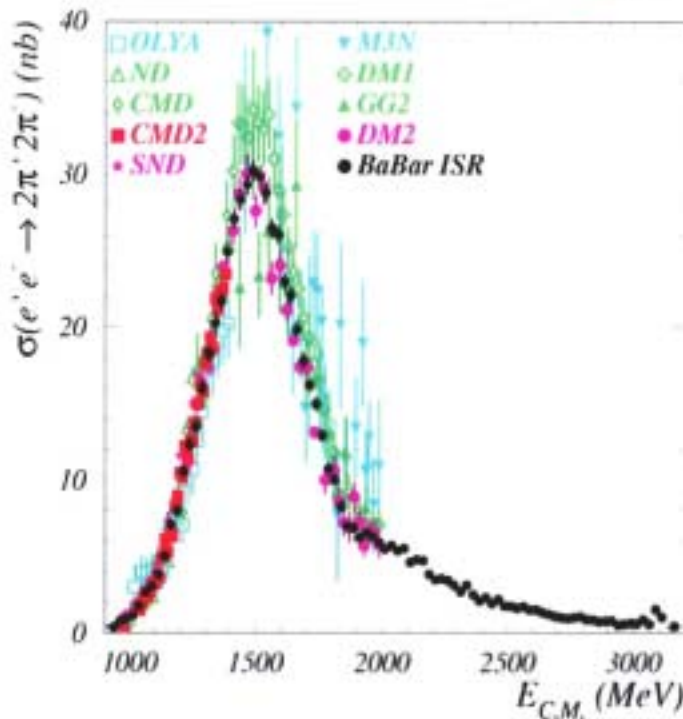


$e^+e^- \rightarrow \text{hadrons}$ for $E_{CM} < 1400$ MeV; recent data from Novosibirsk VEPP-2M collider [27].



$\pi^+\pi^-\pi^+\pi^-$ cross section

Systematic Errors can be under control



- $\mu\mu\gamma$ ISR luminosity 3%
- background subtraction 1%
(10-15% for $M_{4\pi} < 1.0$ GeV)
- χ^2 MC-DATA cut difference 2%
- radiative corr. accuracy 1%
- MC-DATA track losses diff. 2%
- model dependent acceptance 2%

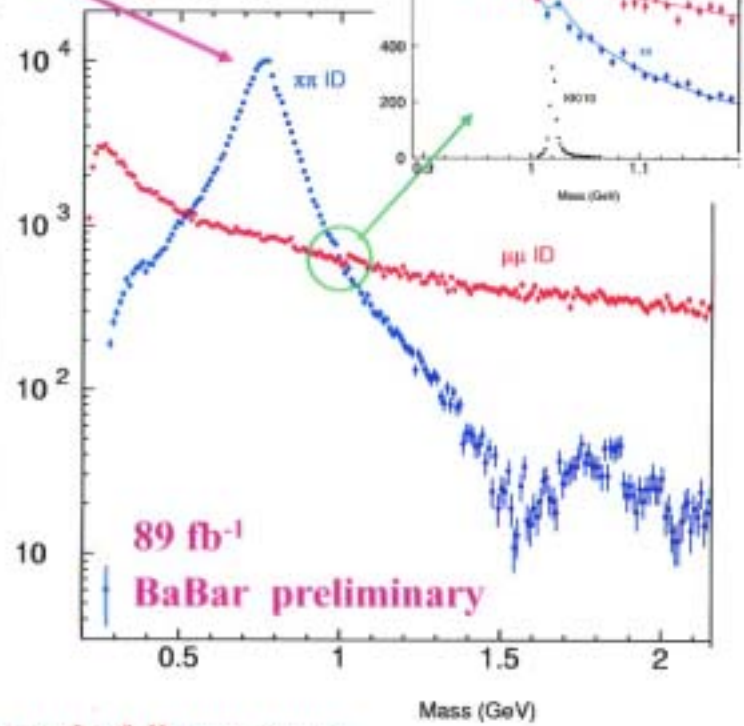
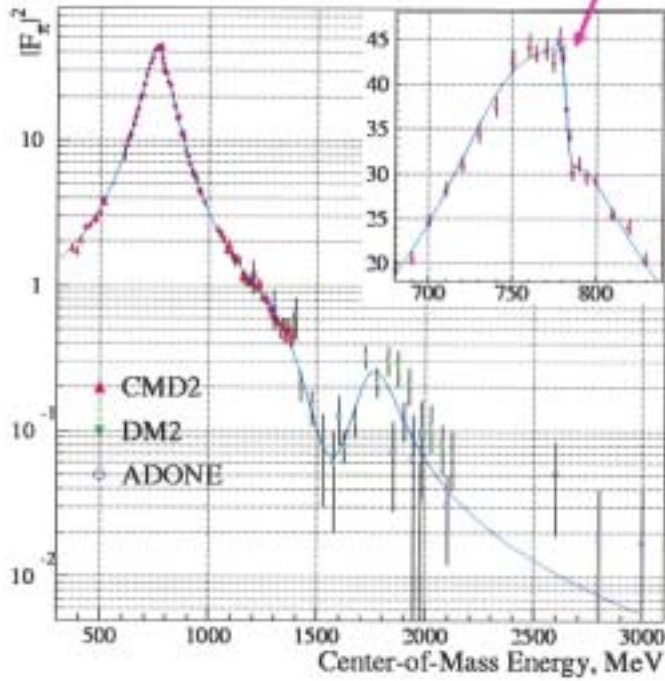
Estimated total systematic error 5%

- **Very competitive statistical uncertainties**
- **Babar is the only experiment which covers whole range**
- no point-to-point normalization problem

Details are in dedicated talk by Roberto Stroili

$$\pi^+ \pi^- \gamma$$

Pion Form Factor with $\rho - \omega$ interference



BaBar data covering the full mass range



ISR Cross Section

$$\frac{d\sigma(s, x)}{dx} = W(s, x) \sigma_f[s(1-x)]; \quad x = \frac{2E_\gamma^*}{\sqrt{s}}$$

In Born approximation:

M. Benayoun *et al.* Mod.Phys.Lett. A Vol.14, No. 37(1999)2605.

$$W(s, x) = \frac{2\alpha}{\pi \cdot x} \cdot \left(2 \ln \frac{\sqrt{s}}{m_e} - 1\right) \cdot \left(1 - x + \frac{x^2}{2}\right)$$

$W(s, x)$ can be calculated with < 1% accuracy

Cross Section for final state f (normalized to radiative dimuons)

$$\sigma_f(s') = \frac{dN_{f\ell}}{dN_{\mu\mu\gamma}} \cdot \epsilon_{\mu\mu} \cdot (1 + \delta_{FSR}^{\mu\mu}) \cdot \sigma_{e'e^- \rightarrow \mu'\mu'}(s') \cdot \epsilon_f \cdot (1 + \delta_{FSR}^f)$$

"effective c.m. energy squared" = $M_{inv}^{2f} = s(1-x)$

$dL(s')$
ISR luminosity

Corrections for final state radiation

Detection efficiencies

ISR statistics compared to low energy e^+e^-

BaBar integrated $\mathcal{L} = 125 \text{ fb}^{-1}$

+ folding in estimated event selection and photon/muon efficiencies

$E_{\text{cm}} = 0.61\text{--}0.96 \text{ GeV}$:

	Luminosity	Hadrons
CMD2 (published)	180K Bhabha	114K (more stat on ρ peak)
BaBar $ \cos\theta_\gamma^* < 0.80$	176K $\mu\mu\gamma$	970K

*(comparable overall statistics, but BaBar statistics stronger off ρ peak – specially at $s^{1/2} = 0.8\text{--}1 \text{ GeV}$ where CMD-2 & τ data most discrepant
CMD-2 also has x5 more data now being analyzed)*

$E_{\text{cm}} = 2\text{--}5 \text{ GeV}$:

	Hadrons
BES	~85K
BaBar $ \cos\theta_\gamma^* < 0.80$	1760K

(the large sample from BaBar ISR is equivalent to 30 points in $s^{1/2}$ at 100 MeV step with 0.4% stat. precision each)

Inclusive R Measurements

What about using the ISR photon energy + hadrons opposite to it?
Resolution enough? Efficiency under control? Does it matter?

Hadronic contributions to α_{em} :

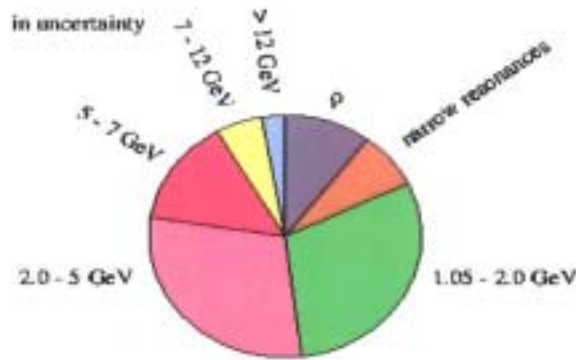
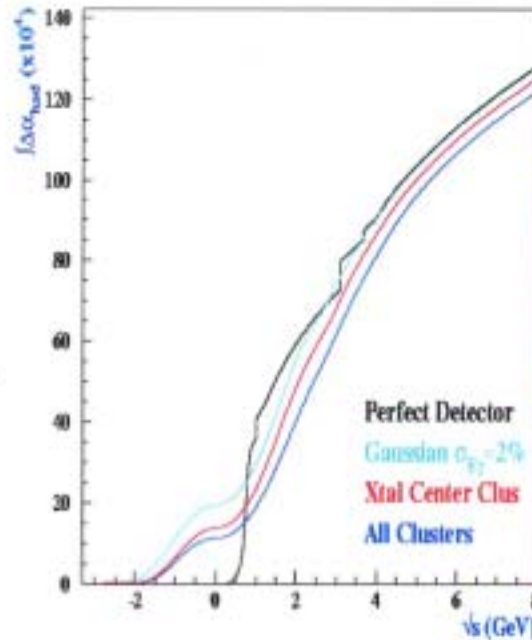


Fig. 2. Relative contributions to $\Delta\alpha_{had}^{\text{had}}(m_s^2)$ in magnitude and uncertainty.



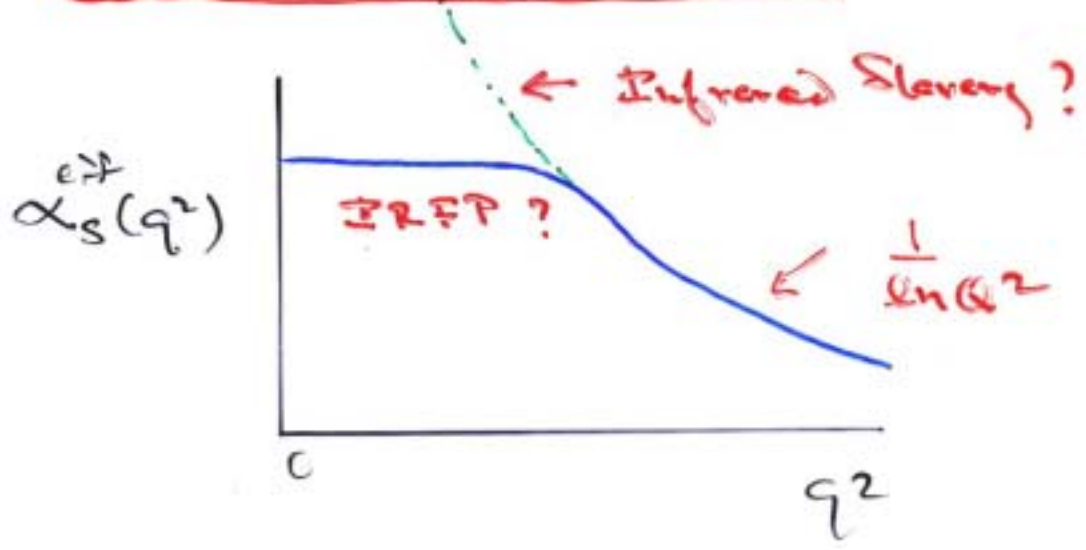
The absolute total effect of integral at 7 GeV Compared to perfect detector:

Gaussian $\sim 0.5\%$
Crystal center $\sim 3\%$
All clusters $\sim 7\%$

We can surely calibrate
From data to a small fraction of the 3%!
Looks very promising.

Can work for $\Delta\alpha_{had}$ up to $\sqrt{s} \sim 7-8 \text{ GeV}$ before photon background from normal non-radiative hadronic events becoming significant.

QCD in the Infrared



$\beta > 0$ in IR?

Corbett
 Petronio
 Parisi
 Alkof
 Roberts
 Marcell

Near conformal behavior of QCD
 ADS/CFT correspondence *Polyakov
 Strocchi*
 Avoids renormalization resummation
 Counting Rules of Exclusive Processes
BF, ANT

Natthigk
 Skarvin
 Heide Harro
 Rothstein SJB

Peter
 $\rightarrow H^2$

Why study $\mathcal{R}_{\text{ete}}(s)$ at low energies?

* 1. Evidence for IR-Fixed Point of Effective coupling

$$\mathcal{R}_{\text{ete}}(s) = \frac{\mathcal{V}_{\text{ete} \rightarrow \text{hadrons}}}{\mathcal{V}_{\text{ete} \rightarrow \mu^+\mu^-}} = \sum_f e_f^2 \left[1 + \frac{\alpha_R(s)}{\pi} \right]$$

$\alpha_R(s) \sim \text{constant}$ at low energies

Mattingly + Stevenson

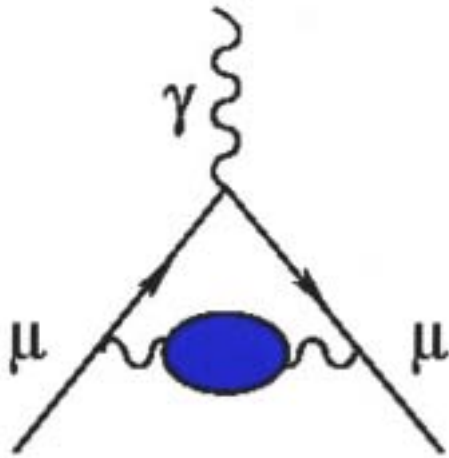
Compare with $\alpha_R(m_f^2)$

Near conformal interactions

* 2. Generalized Crewther Relation

* 3. $\alpha_{\mu}^{\text{had}}$

The Need for R in Muon g-2



$$a_\mu = (g-2)/2$$

	$a_\mu \times 10^{11}$
QED	11658406 ± 3
Hadronic (LO)	$\sim 7000 \pm \sim 60$
Hadronic (NLO)	101 ± 6
Hadronic (light-by-light)	80 ± 40
Weak	152 ± 4

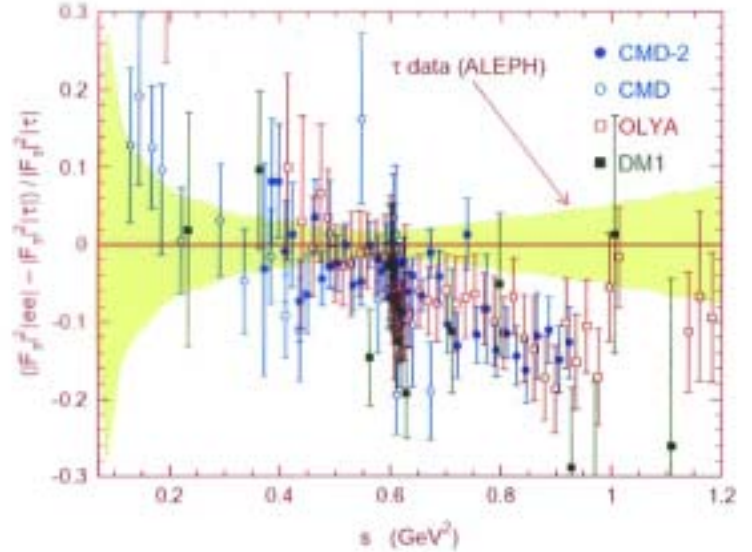
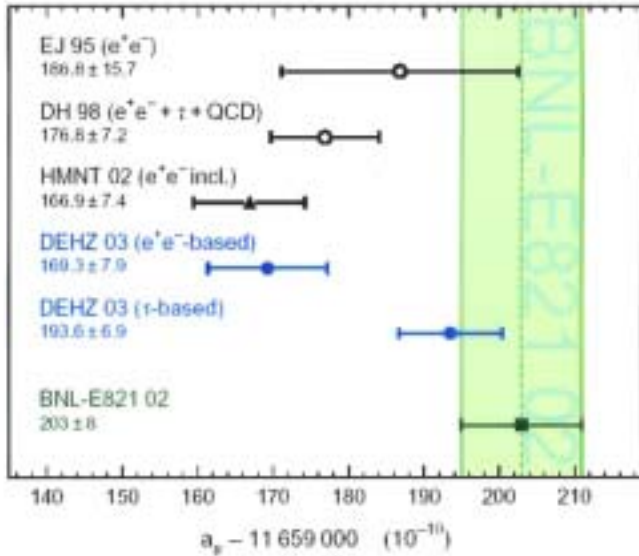
The leading order hadronic correction cannot be calculated from perturbative QCD => Need experimental R(s) measurement.

$$\Delta a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2}^{\infty} \frac{R_{\text{had}}(s') K(s')}{s'^2} ds'$$

Most contributions and uncertainty come from $s' < 3 \text{ GeV}^2$.
 Low energy measurements traditionally done by summing exclusive modes, dominated by $\pi^+\pi^-$ mode contribution.

The status of a_μ

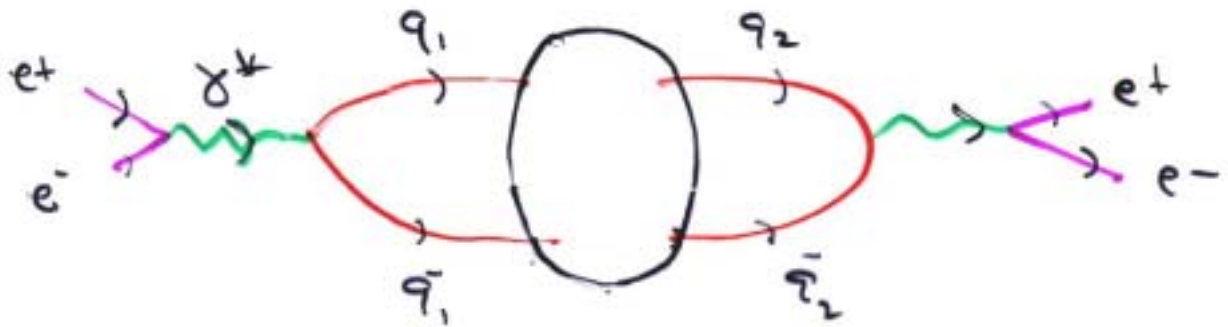
Aug/02 review by: Davier, Eidelman, Hoecker, Zhang hep-ph/0208177



At least one conclusion can be drawn:

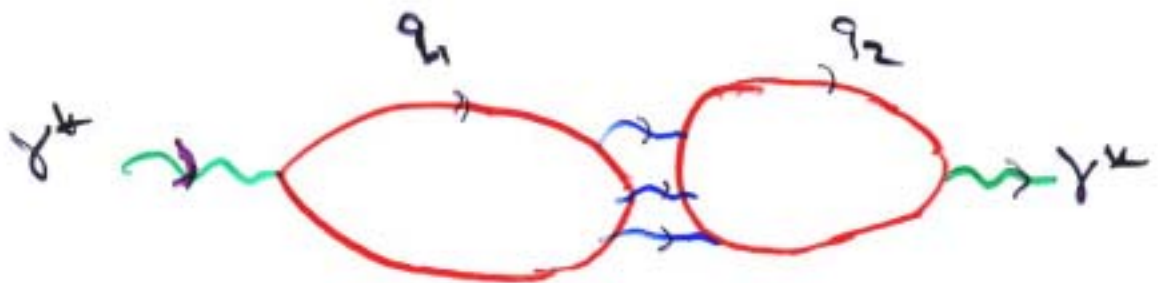
New $e^+e^- R$ measurements would be very desirable !

Interference between different quark currents



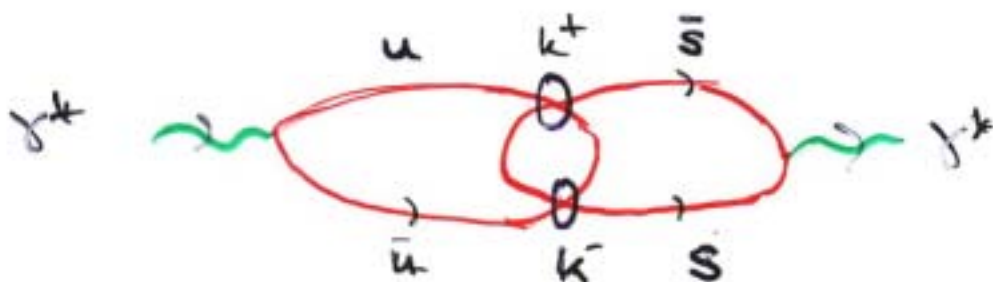
u, s interference: $\gamma^k \rightarrow k^+ k^-$

Rete- (s) at leading twist:



$$\left(\sum_i e q_i \right)^2 \propto \alpha_s^3$$

channel
by
channel



$$R(Q) \equiv 3 \sum_f Q_f^2 \left[1 + \frac{\alpha_R(Q)}{\pi} \right]$$

$\kappa \rightarrow Q$

$$\begin{aligned} \frac{\alpha_R(Q)}{\pi} = & \frac{\alpha_{\overline{MS}}(Q)}{\pi} + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi} \right)^2 \left[\left(\frac{41}{8} - \frac{11}{3} \zeta_3 \right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3} \zeta_3 \right) f \right] \\ & + \left(\frac{\alpha_{\overline{MS}}(Q)}{\pi} \right)^3 \left\{ \left(\frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2 \right) C_A^2 \right. \\ & + \left(-\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5 \right) C_A C_F - \frac{23}{32} C_F^2 \\ & + \left[\left(-\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2 \right) C_A \right. \\ & + \left. \left. \left(-\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5 \right) C_F \right] f \right. \\ & + \left. \left(\frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2 \right) f^2 \right. \\ & + \left. \left(\frac{11}{144} - \frac{1}{6} \zeta_3 \right) \frac{d^{abc} d^{abc} \left(\sum_f Q_f \right)^2}{C_F d(R) \sum_f Q_f^2} \right\} \end{aligned}$$

ASSUMES
 $\kappa = Q$

\uparrow
light-by-light

$$d(R) = N$$

$$C_A = N$$

Gorishny
Kataev
Larin

$$d^{abc} d^{abc} = \frac{16}{3}$$

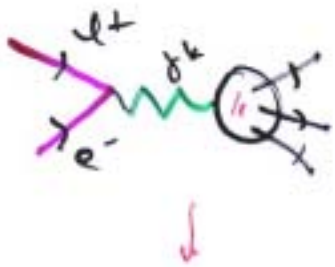
$$C_F = \frac{N^2 - 1}{2N}$$

$$\sum_f Q_f = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

Surguladze
Samud

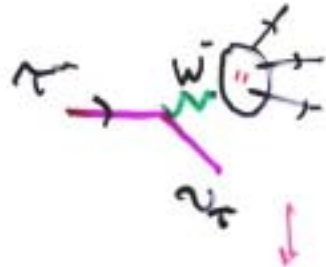
$$T = \frac{1}{2}$$

Compare $e^+e^- \rightarrow X$ and $\tau \rightarrow \nu_\tau X$



$$\langle 0 | \int \gamma^\mu | H^0 \rangle$$

$I = 0, 1$
 $I_z = 0$



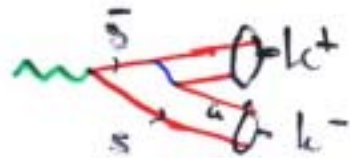
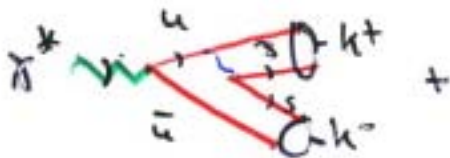
$$\langle 0 | \int W^- | H^- \rangle$$

$I = 1$
 $I_z = -1$

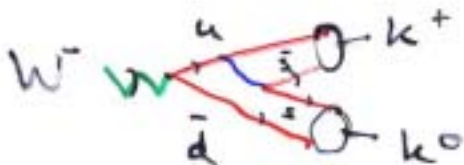
- $I = 1$ Vector currents related by isospin

* Davier, Hocker : Fails at $s > 0.6 \text{ GeV}^2$

Possible source of difference



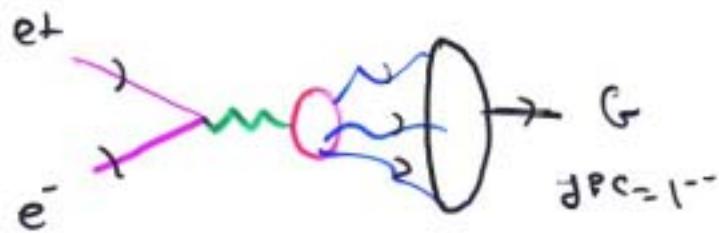
Interfere!



no analog

Need analysis channel by channel

Search for glueballs in $e^+e^- \rightarrow \gamma\gamma$

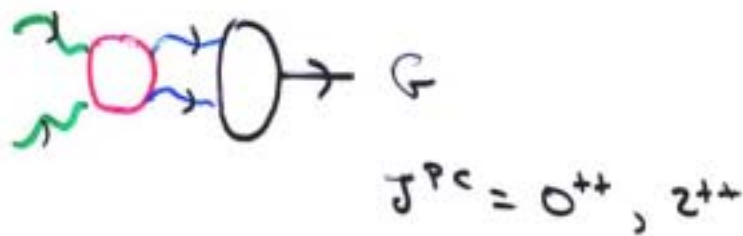


resonance
at $\sqrt{s} \sim M_G^2$

Lattice: $M_G^{1--} \sim 3.6 \text{ GeV}$

$$\int_{\sqrt{s} \sim M_G^2} d\sqrt{s} \sigma_{e^+e^- \rightarrow f} = 2\pi^2 (2J+1) \frac{\Gamma_{e^+e^-} \Gamma_f}{\Gamma_{\text{TOT}}}$$

α_s^2 suppressed



$J^{PC} = 0^{++}, 2^{++}$

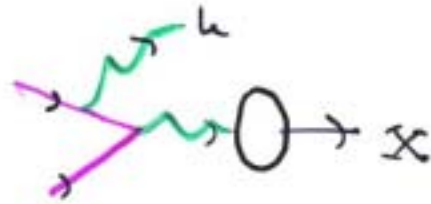
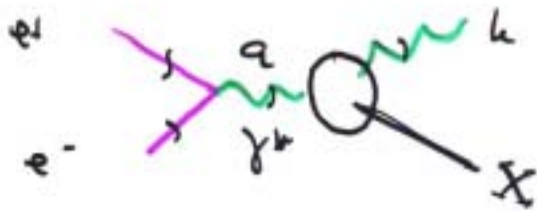
$$\int_{\sqrt{s} \sim M_G^2} d\sqrt{s} \sigma_{\gamma\gamma \rightarrow f} = 2\pi^2 (2J+1) \frac{\Gamma_{\gamma\gamma} \Gamma_f}{\Gamma_{\text{TOT}}}$$

α_s^2 suppressed

Search for Exotic $C=+$ States

$$e^+e^- \rightarrow \gamma X$$

$$m_X^2 = s - 2\omega_e \sqrt{s} \quad (CA)$$

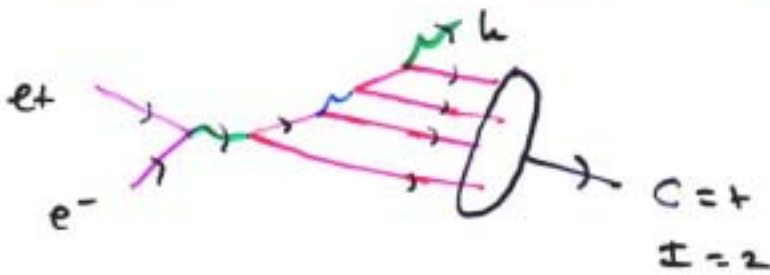


$(9\bar{9}), (9\bar{9}8), (9\bar{8})$
 $(9\bar{9})8$

$C=+$
 $I=0, 1, 2$

$C=-$
 $I=0, 1$

Example: $C=+$, $I=2$, $4q$ state



$$\delta_C \times \delta_C = 1_C$$

Avoid ISR: Θ_{μ}^{cm} large angle photon

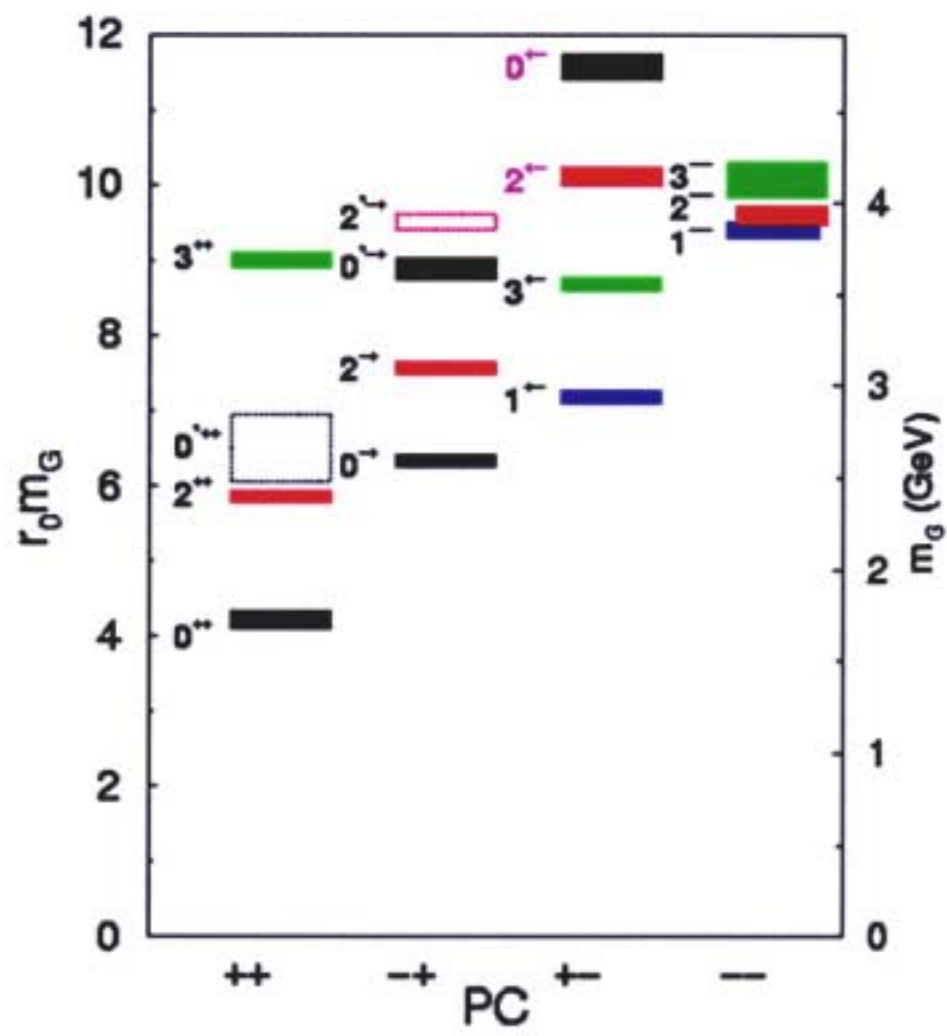
Interfere with ISR

Charge asymmetry

$$\text{Re } T_{\mu\nu}^\dagger F$$

Single-spin asymmetry

$$\text{Im } T_{\mu\nu}^\dagger F$$



Morimoto et al

$$e^+ e^- \rightarrow \phi G_J$$

Polarized e^- Beam

More observables

$$e^-_T e^+ \rightarrow \phi_T G_J$$

Beam
polarization

$$\left\{ \begin{array}{l} \vec{s}_e \cdot (\vec{P}_\phi \times \vec{P}_e) \\ \vec{s}_e \cdot \vec{S}_\phi \end{array} \right.$$

SSA

Spin Transfer

$$* \vec{S}_\phi \cdot (\vec{P}_\phi \times \vec{P}_e)$$

SSA

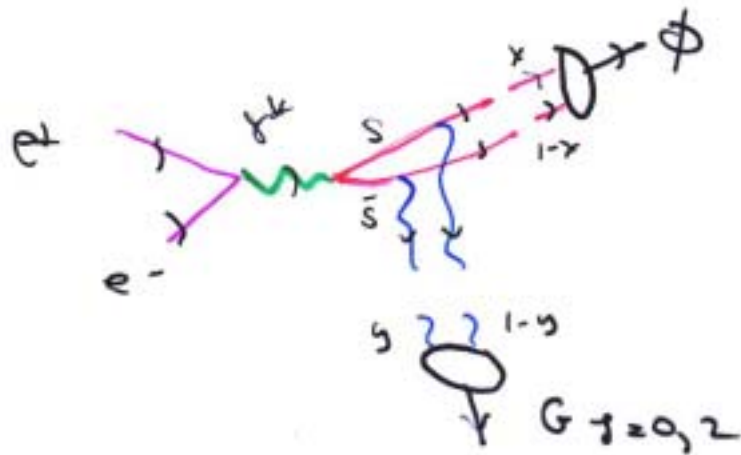
$$* (\vec{P}_e \times \vec{P}_{e^+}) \cdot (\vec{P}_\phi \times \vec{P}_G)$$

non-zero
crossing
angle

SSA needs FSI $\sim \phi, G_J$

$$\frac{dR}{d\cos\theta} (e^+ e^- \rightarrow \phi G_J) \sim \begin{cases} \frac{1}{s^2} \sin^2 \theta & J=0 \\ \frac{1}{s^3} (1 + \cos^2 \theta) & J=2 \end{cases}$$

Search for glueballs in $e^+e^- \rightarrow \phi G$



J. Lee
F. Godshoven
CJ3

Glue Rich
High "Strehness"
Chomats

$$M = \int_0^1 dx \int_0^1 dy \mathcal{T}_H(e^+e^- \rightarrow s\bar{s}g\bar{g}) \phi_\phi^{(x)} \phi_{G_J}^{(y)}$$

$$\phi_\phi(x) = \int d^4k_L \Psi_{s\bar{s}/\phi}^{LF}(x, k_L)$$

$$\phi_G(y) = \int d^4k_L \Psi_{g\bar{g}/G}^{LF}(y, k_L)$$

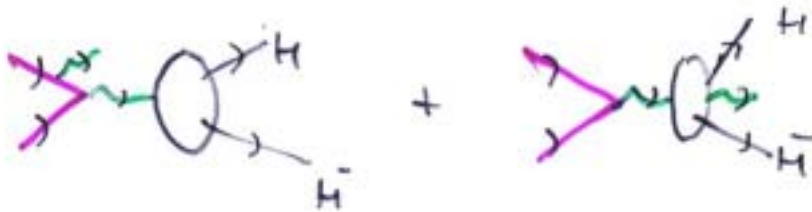
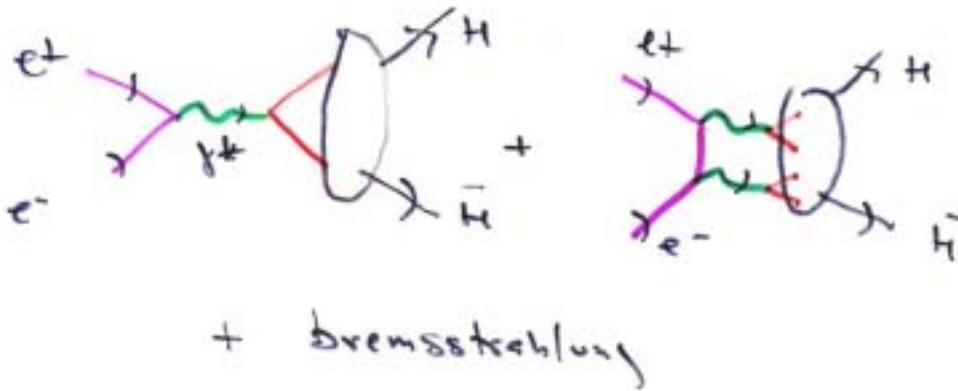
$$\langle 0 | G_{\mu\nu} G^{\mu\nu} | G_J \rangle$$

$$dR(e^+e^- \rightarrow \phi G_0) \sim \alpha_s^2 \left(\frac{m_s^2}{s}\right)^2 \alpha \frac{e_s^2}{N_c^2}$$

$$\left[\frac{3}{4}(1-\cos^2\theta) + \frac{8m_s^2}{s} \cdot \frac{3}{8}(1+\cos^2\theta) \right]$$

$$\leftarrow J_{\frac{\phi}{2}} = 0$$

Charge asymmetry is $e^+e^- \rightarrow H\bar{H}$

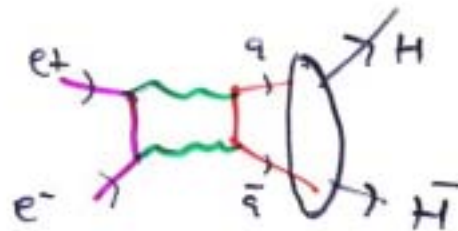


dom by
ext line radiation
for $\Delta E/E \rightarrow 0$

Compare to asymmetry in QED

$$R_A = \frac{A_{e^+e^- \rightarrow H\bar{H}}}{A_{e^+e^- \rightarrow H^+H^-}} = R_A(\theta_{cm}, \phi, S)$$

Simplest model:
($J=0$ from p-1c)



*

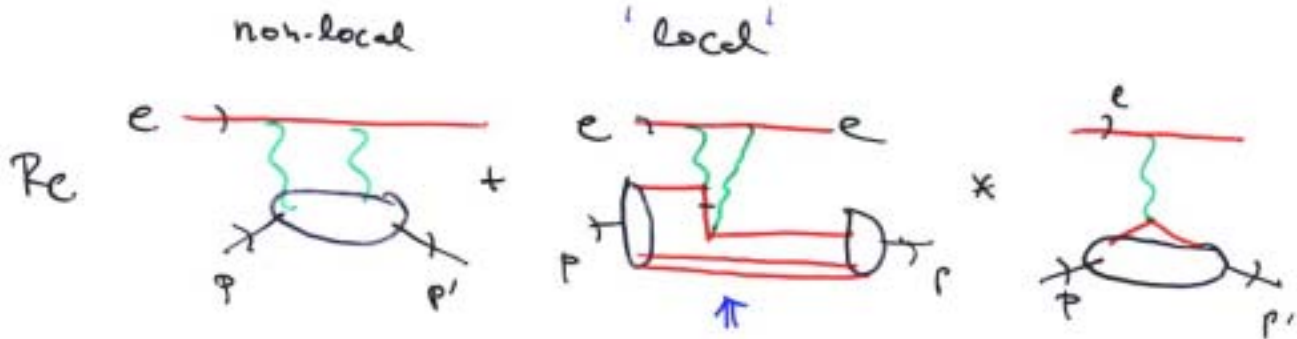
$$R_A = \sum_{q \in H} e_q^2 \langle \frac{1}{x_q} \rangle$$

indep of
 θ, ϕ, S

Afonsev
Caution
S03

Guides Vervorbenen
Blunden, Melutovich, et al

$$e^+ p \rightarrow e^+ p \quad \text{Asymmetry}$$



$$\sum_{\epsilon \in p} e_{\epsilon}^2 \left\langle \frac{1}{x_{\epsilon}} \right\rangle$$

" J=0 " Fixed Pole
 $M \sim S^0 F(+)$

π^2 enhancement in space-like scattering:

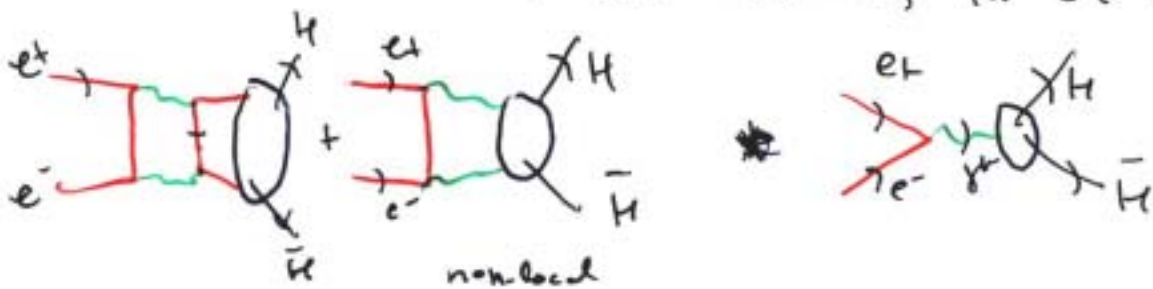
$$\delta M_p^2 = \delta M_p^2 \left\langle \frac{1}{x_{\epsilon}} \right\rangle$$

Some moment.

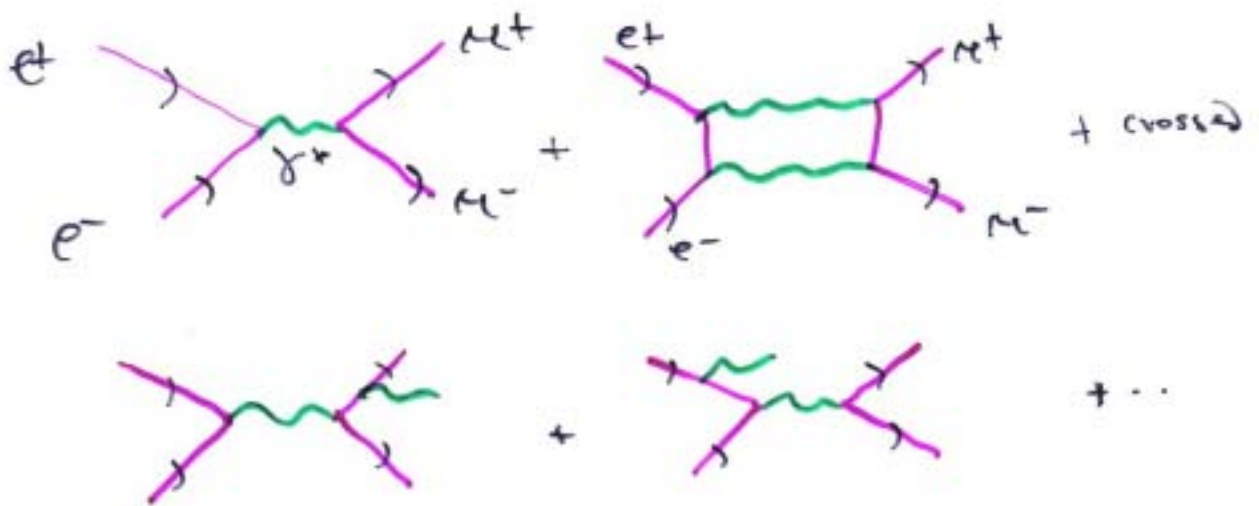
Afanasev

Carlson

Test in front-back asymmetry in $e^+ e^- \rightarrow H \bar{H}$



Charge asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2\theta) [1 + \delta]$$

$$A = \frac{d\sigma(\theta, \phi) - d\sigma(\pi - \theta, \phi)}{d\sigma(\theta, \phi) + d\sigma(\pi - \theta, \phi)}$$

$$= \delta_{MC}^A + \delta_{MC}^B$$

Meister + Jennie

R.W. Brown, et al

PL

130, 1210 (1962)

PLB

43, 403 (1973)