Alghero/L'Alguer
September 2003

QUANTUM MECHANICS and NEUTRAL KAON INTERFEROMETRY

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quant-ph/0306114 hep-ph/0307047

"If there is any place where we have a chance to test the main principles of quantum mechanics in the purest way — does the superposition of amplitudes work or doesn't it? — this is it."

R. Feynman
"Lectures on Physics"

THE NEUTRAL KAON SYSTEM

1.- $K^0 - \bar{K}^0$ basis $\leftrightarrow$ Strangeness measurements

$$K^0 \sim d\bar{s} \quad S=+1$$
$$\bar{K}^0 \sim s\bar{d} \quad S=-1$$
$$\Big\} \langle K^0 | \bar{K}^0 \rangle = 0$$

$H^{weak}_{non-diag.} \leftrightarrow K^0 \rightleftarrows \bar{K}^0$ oscillations in t

2.- $K_S - K_L$ basis $\leftrightarrow$ lifetime measurements

$$H^{weak}_{diag.} \sim \begin{pmatrix} m_S - \frac{i}{2}\Gamma_S & 0 \\ 0 & m_L - \frac{i}{2}\Gamma_L \end{pmatrix} \leftrightarrow |K_{S,L}(t)\rangle = e^{-i\lambda_{S,L} t} |K_{S,L}(0)\rangle$$

no $K_S - K_L$ oscillations

$$U(t) \sim \begin{pmatrix} e^{-\frac{\Gamma_S}{2}t} e^{+\frac{i}{2}\Delta m t} & 0 \\ 0 & e^{-\frac{\Gamma_L}{2}t} e^{-\frac{i}{2}\Delta m t} \end{pmatrix}$$

$$\Delta m \equiv m_L - m_S \sim \frac{1}{2}\Gamma_S >> \Gamma_L$$

$$\underbrace{\langle K_S | K_L \rangle}_{0} = \frac{\varepsilon + \varepsilon^*}{1+|\varepsilon|^2} \sim 3 \cdot 10^{-3}$$

neglect ~~CP~~

3.- Bohr's complementarity

$$|K^0\rangle \simeq \frac{1}{\sqrt{2}}|K_S\rangle + \frac{1}{\sqrt{2}}|K_L\rangle \qquad |+\rangle_x = \frac{1}{\sqrt{2}}|+\rangle_z + \frac{1}{\sqrt{2}}|-\rangle_z$$

$$|\bar{K}^0\rangle \simeq \frac{1}{\sqrt{2}}|K_S\rangle - \frac{1}{\sqrt{2}}|K_L\rangle \qquad |-\rangle_x = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle_z$$

S-measurements
Γ- "

} "maximally" incompatible

$K^0-\bar{K}^0$ OSCILLATIONS

1.- At $t=0$ $\pi^- p \to \Lambda K^0$

2.- Transition P's at $t \geq 0$

$$\begin{cases} |\langle K^0 | K^0(t)\rangle|^2 = \frac{1}{4}\{e^{-\Gamma_S t} + e^{-\Gamma_L t}\}\{1 + \mathcal{V}_0(t) \cos \Delta m t\} \\ |\langle \bar{K}^0 | K^0(t)\rangle|^2 = \quad " \quad \{1 - \mathcal{V}_0(t) \cos \Delta m t\} \end{cases}$$

with $\mathcal{V}_0(t) = 1/\cosh \frac{1}{2}\Delta\Gamma t \qquad \Delta\Gamma \equiv \Gamma_L - \Gamma_S$

"oscillation visibility"

$$\begin{cases} |\langle K_L | K^0(t)\rangle|^2 = \frac{1}{2}|\langle K_L | K_L(t)\rangle|^2 = \frac{1}{2}e^{-\Gamma_L t} \\ |\langle K_S | K^0(t)\rangle|^2 = \frac{1}{2}|\langle K_S | K_S(t)\rangle|^2 = \frac{1}{2}e^{-\Gamma_S t} \end{cases}$$

DOUBLE-SLIT ANALOGY

Particle-like: Trajectories!

"Which Way" $\underset{WW}{\longleftrightarrow}$ "Which Width" $K_S - K_L$

Wave-like: Interference!

"Fringe Visibility" $\underset{\mathcal{V}}{\longleftrightarrow}$ "Oscillation Visibility" $K^0-\bar{K}^0$

Duality: $\left.\begin{array}{l}\text{Adquisition of WW info} \\ \text{Observed Visibility}\end{array}\right\}$ mutually exclusive

INTERFEROMETRIC DUALITY (I)

1.- B.-G. Englert 1996

"Fringe visibility" $\mathcal{V}_0$ in $I(\varphi)_{\pm} \propto [1 \pm \mathcal{V}_0 \cos\varphi]$

vs

Path "Predictability" $\mathcal{P}$ in $\mathcal{P} \equiv |w_I - w_{II}|$

(a priori WW knowledge) $w_I + w_{II} = 1$

* Greenberger + Yasin 1988

Quantitative duality/complementarity $\} \quad \mathcal{V}_0^2 + \mathcal{P}^2 \leq 1$

(pure states)

(our case)

2.- Strangeness oscillations (int) for (undecayed) neutral kaons

$K^0(0) \to K^0(t)$

$\Delta\Gamma \equiv \Gamma_L - \Gamma_S$

$$\mathcal{P} = |w_S - w_L| = \left| \frac{e^{-\Gamma_S t} - e^{-\Gamma_L t}}{e^{-\Gamma_S t} + e^{-\Gamma_L t}} \right| = \tanh\left|\tfrac{1}{2}\Delta\Gamma t\right| = \mathcal{P}(t)$$

$$\mathcal{V}_0 = \ldots = \frac{2}{\sqrt{2 + e^{\Delta\Gamma t} + e^{-\Delta\Gamma t}}} = \frac{1}{\cosh[\frac{1}{2}\Delta\Gamma t]} = \mathcal{V}_0(t)$$

Thus, $\mathcal{V}_0(t)^2 + \mathcal{P}(t)^2 = 1 \quad \forall t$ t-dependence!

3.- CPLEAR data reinterpreted Phys. Lett. B503 (01) 49

" " B444 (98) 38

$\boxed{\text{CPLEAR DATA}}$ (Single kaon)

Strangeness or K^0–$\bar{K}^0$ oscillations in time τ

P's $\quad |\langle K^0 | K^0(\tau)\rangle|^2, \ |\langle \bar{K}^0 | K^0(\tau)\rangle|^2, \ldots$

1.- CPLEAR (semileptonic decays of K^0 or $\bar{K}^0$)

Phys. Lett. B444 (1998) 38

$$A^{weak}_{\Delta m}(\tau/\tau_S) = \frac{1}{\underbrace{\cosh\frac{\Delta\Gamma}{2}\tau}_{V_0(\tau)}} \cdot \cos\Delta m\tau$$

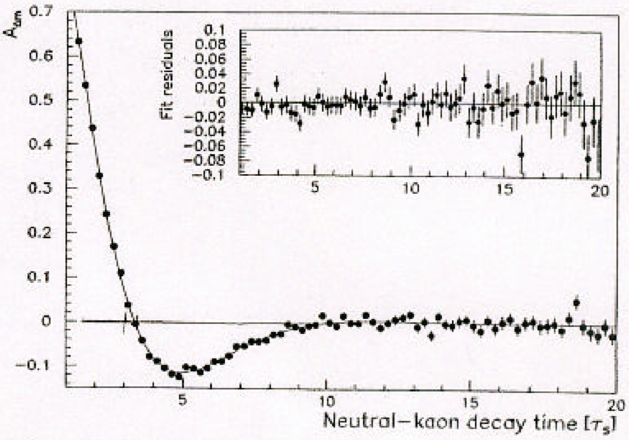


Fig. 2. The asymmetry $A_{\Delta m}$ versus the neutral-kaon decay time (in unit of τ_S). The solid line represents the result of the fit.

2.- CPLEAR (strong K^0, $\bar{K}^0$ interactions on nuclei $K^0 p \to K^+ n$ or $\bar{K}^0 n \to \Lambda\pi^0$)

$\hookrightarrow p\pi^-$

Phys. Lett. B503 (2001) 49

$$A^{strong}_{\Delta m}(\tau/\tau_S) = \frac{2V_0(\tau)\cos\Delta m\tau}{1+V_0(\tau)^2\cos^2\Delta m\tau}$$

CPLEAR Collaboration / Physics Letters B 503 (2001) 49–57

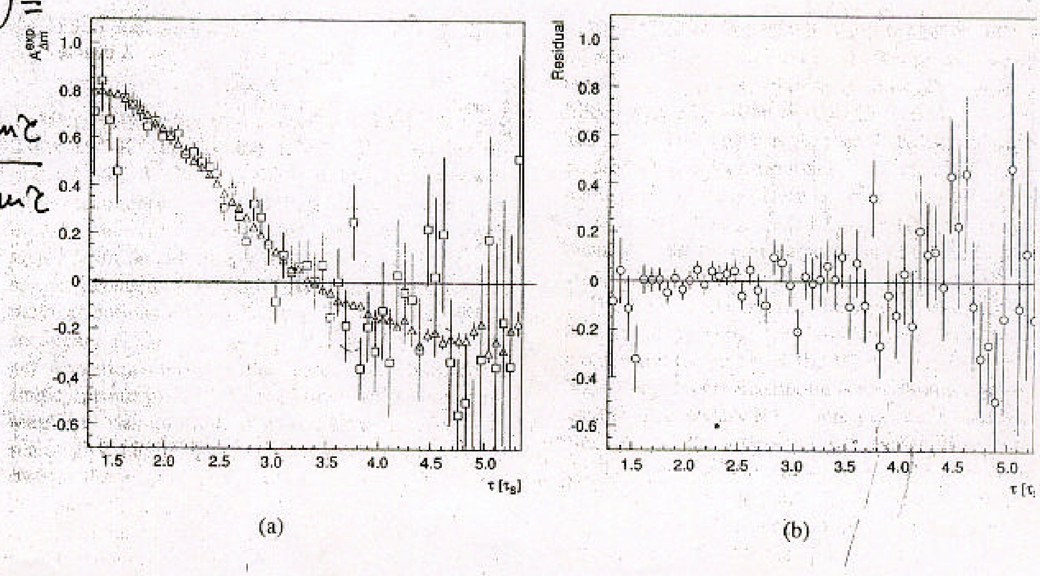


Fig. 5. (a) Best fit to the data points $A^{exp}_{\Delta m}(\tau)$(squares) with the simulated asymmetries (triangles) for the ($K^- + \Lambda$) sample, see text. Distribution of the fit residuals.

A CONJECTURE

both z-dependent

If $I(z) \propto \{1 + V_0(z) \cos[bz]\}$ then

$V_0(z)^2 + P(z)^2 = 1$ is satisfied with

$$V_0(z) = \frac{1}{\cosh[az]} \qquad P(z) = \tanh[az]$$

and

$\#$ of fringes/oscillations $\sim 0.26\, b/a$

1.- Neutral kaons
(CPLEAR data)

$$\frac{b}{a} = \frac{\Delta m}{\frac{1}{2}\Delta\Gamma} \sim 1$$

2.- Wheeler + Bartell ('30)
(double-slit + lens)

$\frac{b}{a}$ introduced

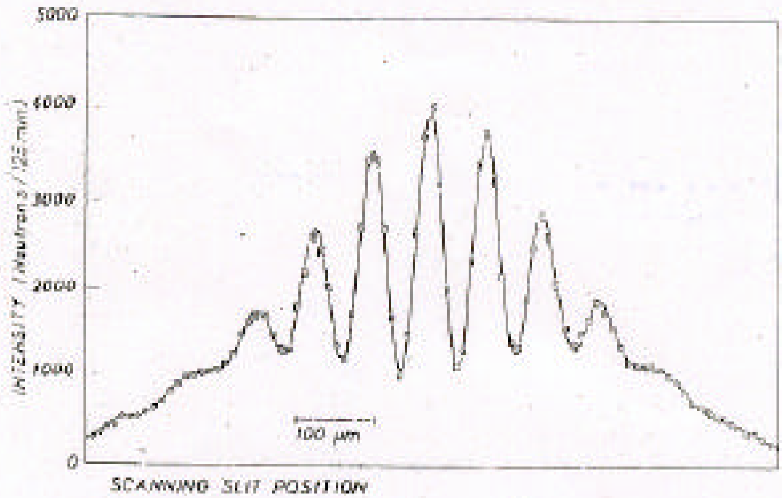
3.- Zeilinger et al (1991)
(double-slit for neutrons)

$$\frac{b}{a} \sim 10$$

4.- Mott : ${}_6C^{12} + {}_6C^{12} \to {}_6C^{12} + {}_6C^{12}$
(PRL 4 (1960) 365 data)
at 5 MeV = E_{CM}

$$\frac{b}{a} = \frac{z_1 z_2 \alpha c}{v_{CM}} \sim 9$$

Neutron data in a double-slit.

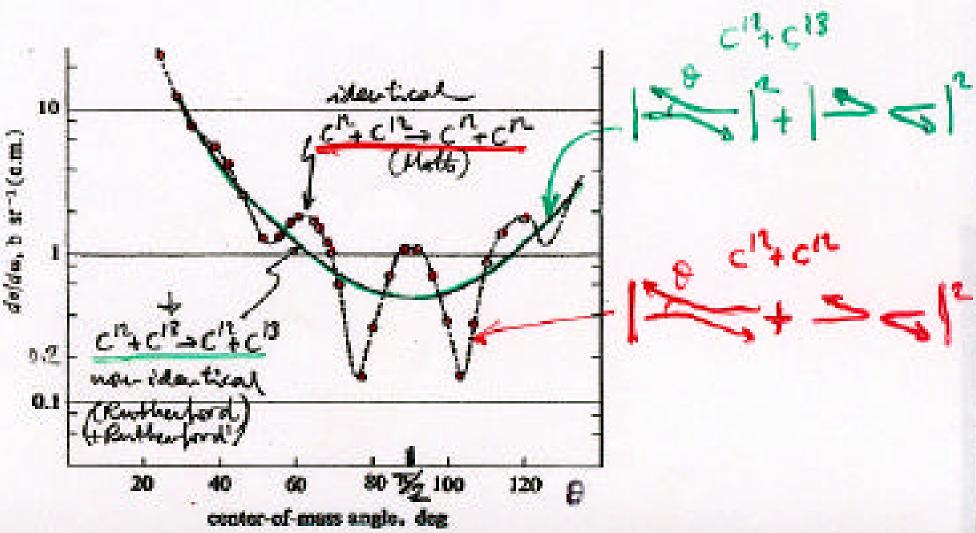


$C^{12} + C^{12}$ elastic scattering (Mott). Data at 5 MeV = E_{CM}

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM}^{Mott} \propto \left(\frac{1}{\sin^4\frac{\theta}{2}} + \frac{1}{\cos^4\frac{\theta}{2}} + \frac{2}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}} \cos\left[b \ln \operatorname{tg}^2\frac{\theta}{2}\right] \right) \qquad b = \frac{Z_1 Z_2 \alpha c}{v_{CM}}$$

$$= \left(1 + \frac{1-\cos^2\theta}{1+\cos^2\theta} \cos\left[b \ln \operatorname{tg}^2\frac{\theta}{2}\right] \right)$$

$$= \left(1 + \frac{1}{\cosh x} \cos[bx] \right) \quad \text{where } e^{\pm x} = \frac{1+\cos\theta}{1-\cos\theta}$$



INTERFEROMETRIC DUALITY II

1.- B.-G. Englert 1996

"Fringe visibility" $\mathcal{V}$ $\qquad [1+\mathcal{V}\cos\varphi]$

vs

"Path or WW knowledge" $\mathcal{K}_W > \mathcal{P}$

(after a W-measurement, W either S or Γ)

Quantitative duality/complementarity $\qquad \mathcal{V}(t)^2 + \mathcal{K}_W(t)^2 \leq 1$

$\uparrow$ pure state

2.- $\varphi \to K\bar{K}$

(left/right kaon = object/meter kaon)

$$t=0 \quad |\phi(0)\rangle = \frac{1}{\sqrt{2}} K^0\bar{K}^0 - \frac{1}{\sqrt{2}}\bar{K}^0 K^0 = \frac{1}{\sqrt{2}} K_L K_S - \frac{1}{\sqrt{2}} K_S K_L$$

$$\Delta t \equiv t_\ell - t_r^0$$

$$|\phi(\Delta t)\rangle = \frac{1}{\sqrt{2}\sqrt{1+e^{\Delta\Gamma\Delta t}}}\Big\{ \overbrace{K^0 K_S - \bar{K}^0 K_L}^{S(\text{left})} - e^{i\Delta m\Delta t} e^{\frac{1}{2}\Delta\Gamma\Delta t} \underbrace{(K^0 K_L + \bar{K}^0 K_S)}_{\Gamma_{S,L}(\text{right})} \Big\}$$

i) W-measurement:

$\Gamma_{S,L}(\text{right}) \to \mathcal{K}_\Gamma = 1,\ \mathcal{V}_\Gamma = 0.$

$$|\phi(\Delta t)\rangle = \frac{1}{\sqrt{2}\sqrt{1+e^{\Delta\Gamma\Delta t}}}\Big\{(1-e^{i\Delta m\Delta t} e^{\frac{1}{2}\Delta\Gamma\Delta t})(K^0K^0 - \bar{K}^0\bar{K}^0) + (1+e^{i\Delta m\Delta t} e^{\frac{1}{2}\Delta\Gamma\Delta t})(K^0\bar{K}^0 - \bar{K}^0K^0)\Big\}$$

S in both sides

ii) W-measurement:

$S(\text{right}) \to$

$$\mathcal{K}_S(\Delta t) = \tanh\left|\tfrac{1}{2}\Delta\Gamma\Delta t\right|$$

$$\mathcal{V}_S(\Delta t) = \left[\cosh\left(\tfrac{1}{2}\Delta\Gamma\Delta t\right)\right]^{-1}$$

QUANTUM ERASER (Scully + Drühl '82)

3.- Data

CPLEAR: $\Delta t = 0$, $\Delta t = 1.2\,\tau_S$ $\qquad$ DAΦNE: ...

PLB 422 (1998) 339