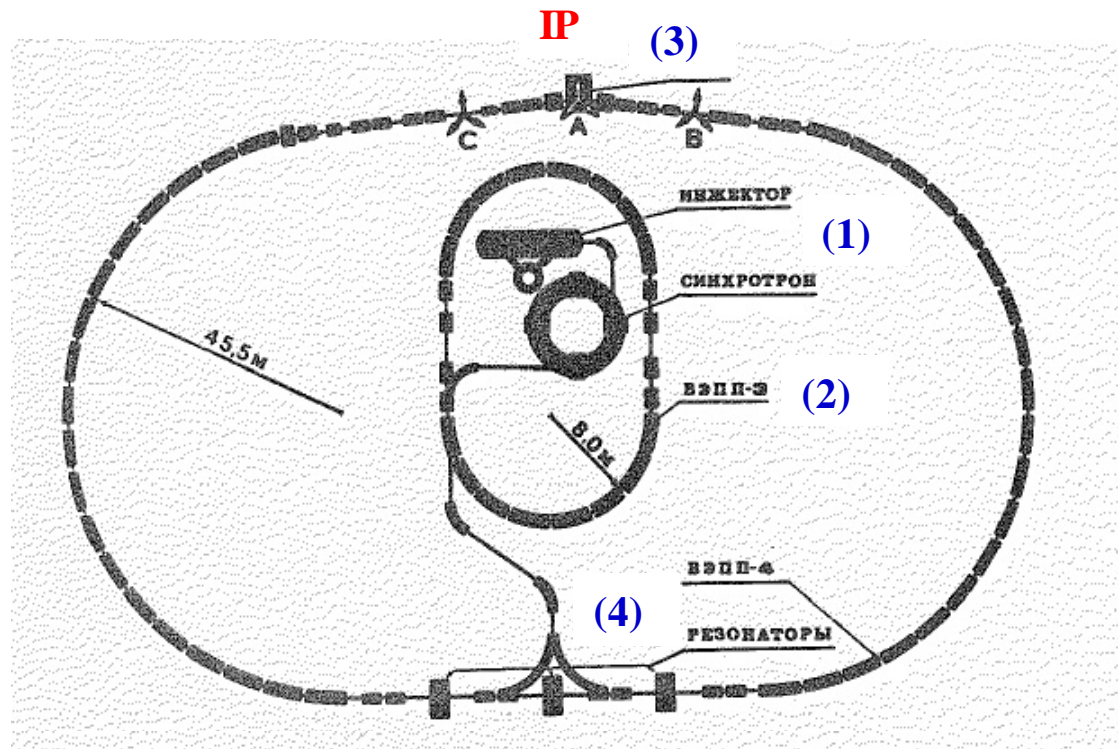


Study of Beam-Beam interaction at VEPP-4: Tune Plane Appearance and Cubic Non-linearity Effect

ICFA Mini-workshop – Working Group on High Luminosity e+e-
Colliders, 10-13 September 2003, Alghero (SS), Italy

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VEPP-4 collider layout

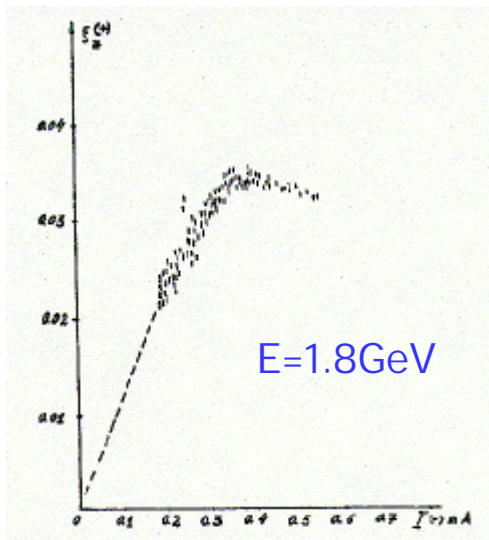


1. Injector and synchrotron
2. Booster (VEPP-3 storage ring)
3. Detector MD-1 (with vertical magnetic field)
4. RF cavities
5. VEPP-4 storage ring, ~400m circumference

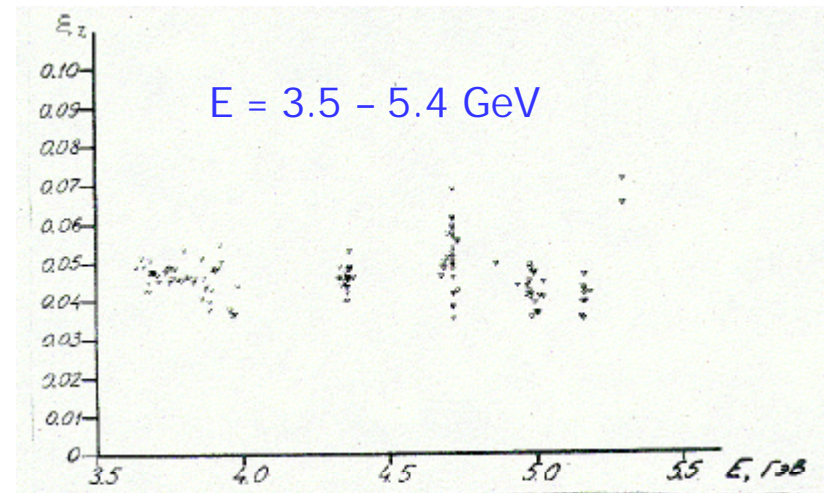
General information

1. Time of operation: 1978 – 1985
2. Operation energy: from 1.8GeV (J/Psi) to 5.5GeV (Y")
3. Most significant experiments: J/Psi and Y mesons mass calibration.
4. 1x1 bunch operation
5. Achieved luminosity: $\sim 1.3 \times 10^{28}$ 1/cm²/sec at 1.8GeV and $\sim 6.0 \times 10^{30}$ 1/cm²/sec at 5.5GeV.

Vertical beam-beam tune shift from luminosity:



Beam-beam tune shift vs bunch current at 1.8GeV, ξ_z max ~ 0.037



Beam-beam tune shift vs beam energy, $\xi_z \sim 0.045 - 0.07$

Tune scan experiment: measurement setup

Measured parameters:

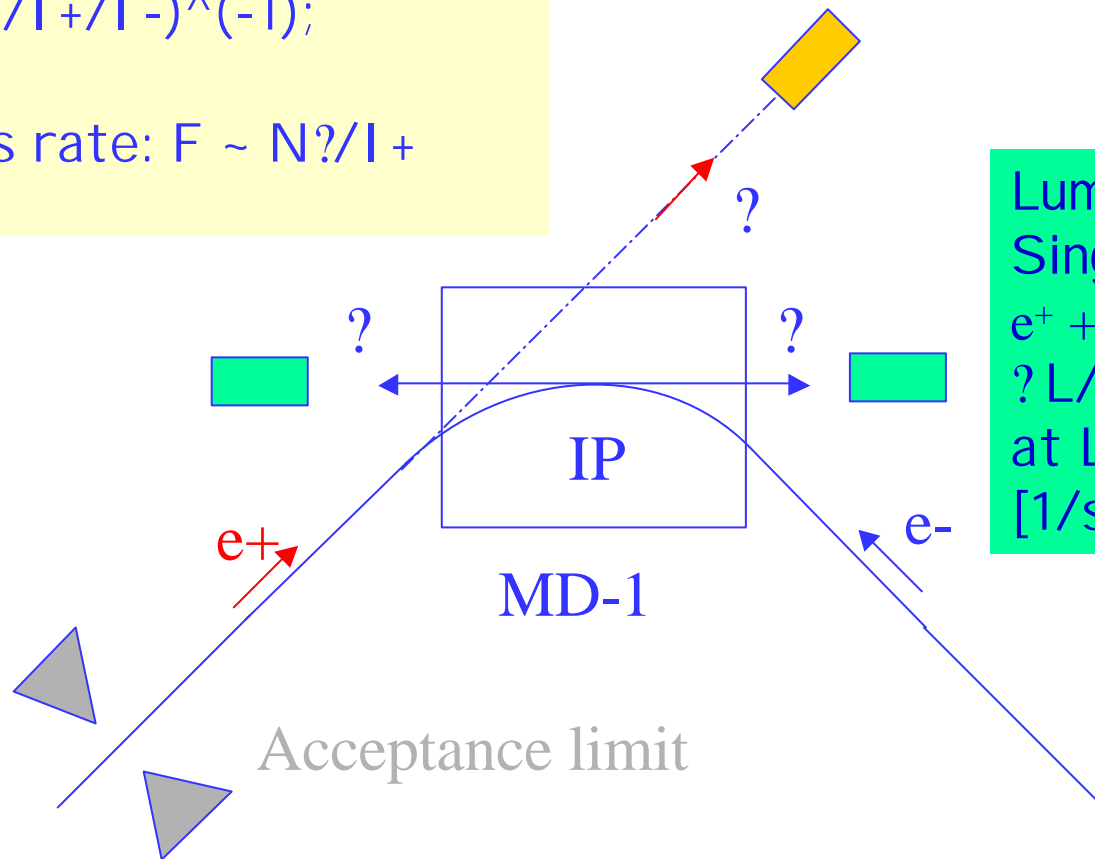
1. Vertical beam size from luminosity:

$$? y \sim (L/I_+/I_-)^{-1};$$

2. e^+ loss rate: $F \sim N?/I_+$

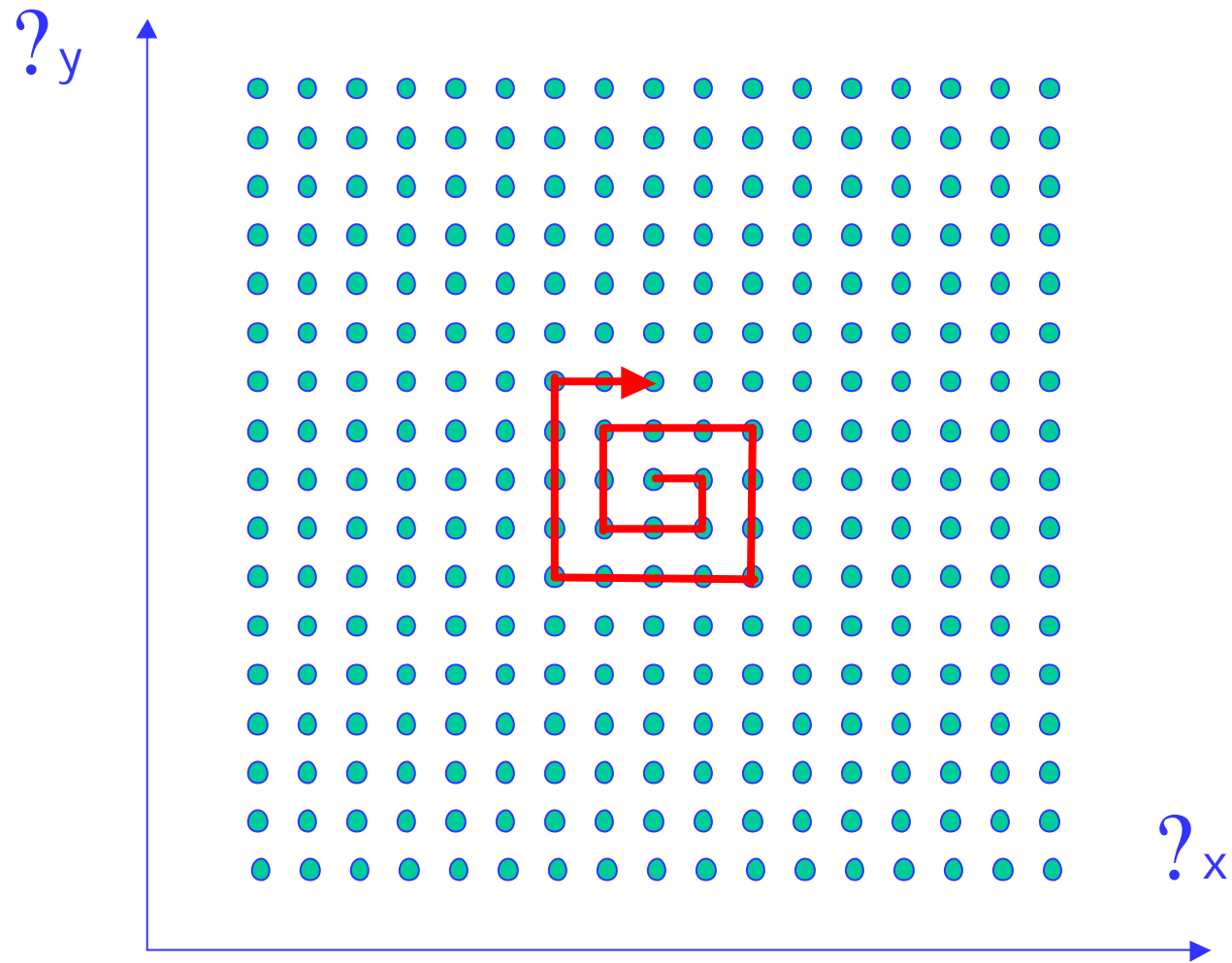
Radiation monitor, $N?$

Luminosity monitors:
Single beamstrahlung
 $e^+ + e^- \Rightarrow e^+ + e^- + ?$
 $? L/L \sim 0.5\%$ at 100ms
at $L = 1e30$
[1/sec/cm²]



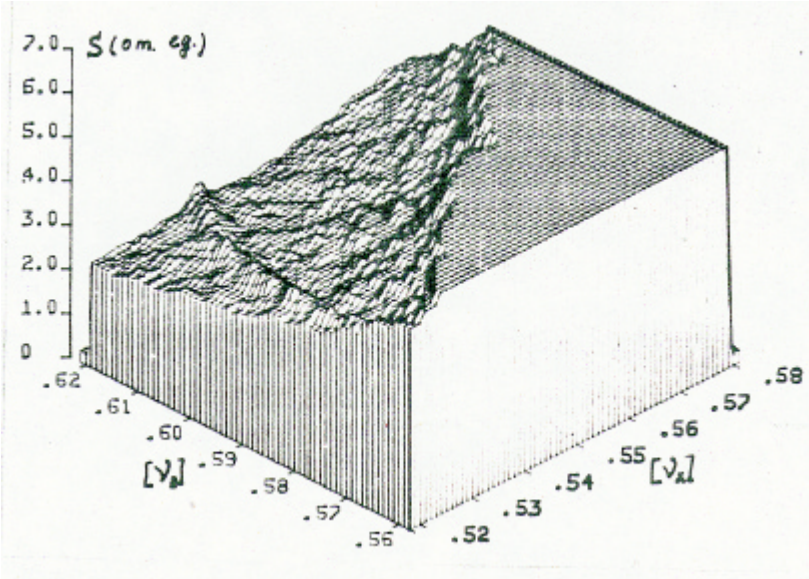
Tune scan experiment: working point trajectory

64x64 grid, ~100ms in each point, ~15min total time



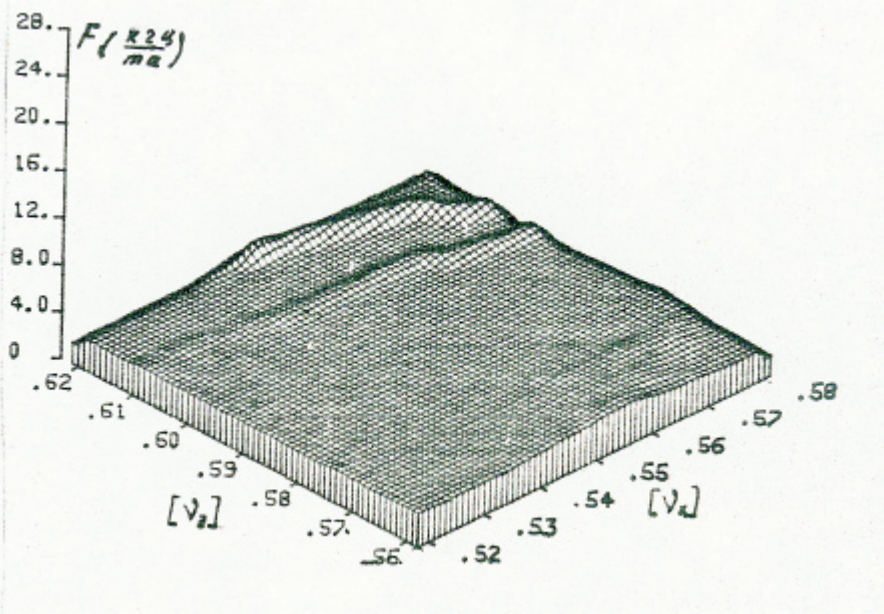
Tune plane appearance: machine resonances

Vertical beam size from luminosity (r.u.)

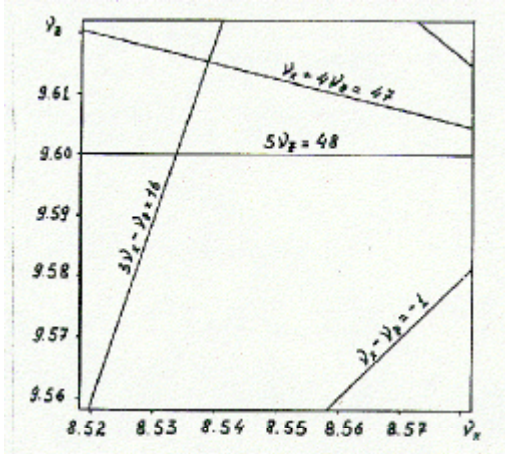


$I_+ \sim 0.72\text{mA}$, $I_- \sim 1.3\text{mA}$
 $\sigma_x = 0.002$, $\sigma_y = 0.007$

Particle loss rate from positron beam

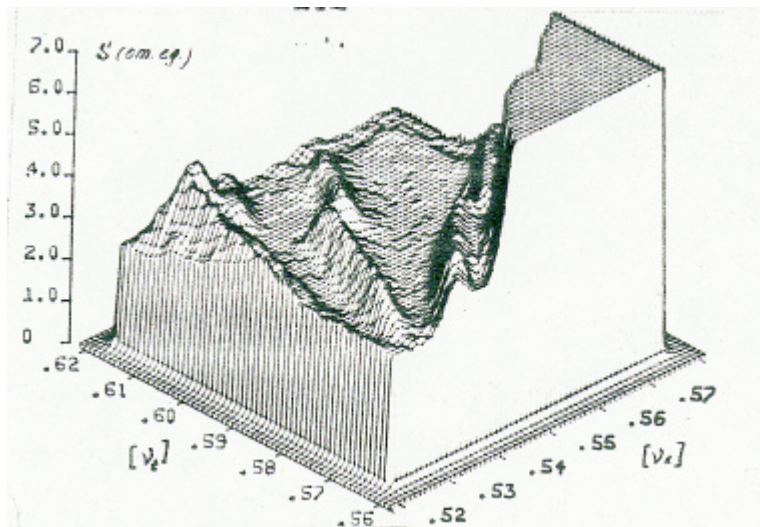


Tune plane with up to 5-th order resonances



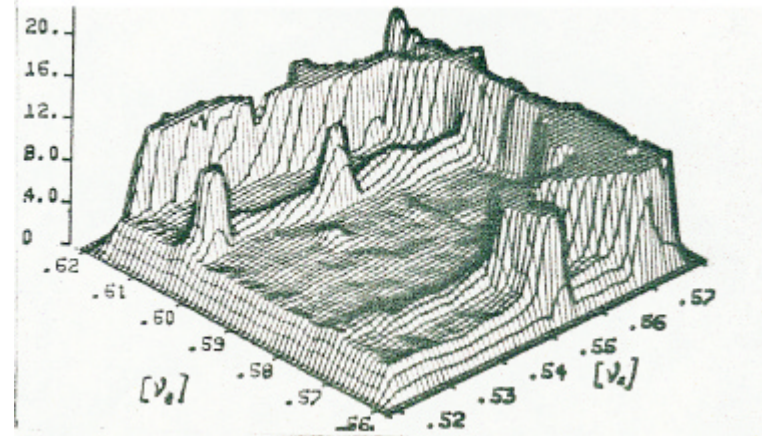
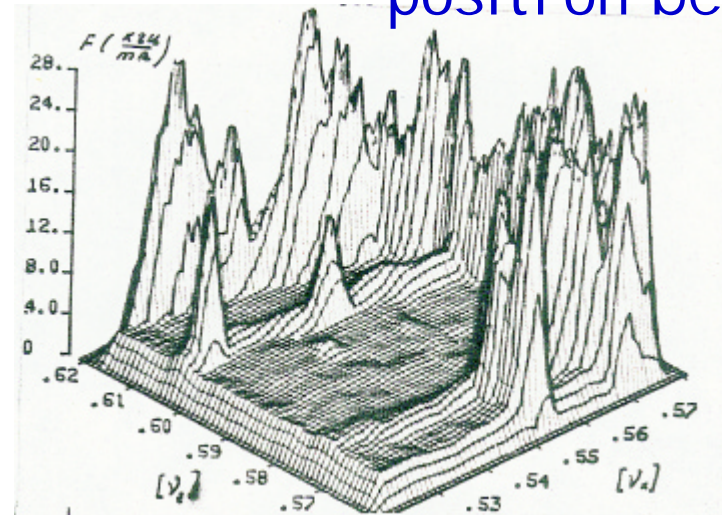
Tune plane appearance: beam-beam interaction

Vertical beam size from
luminosity (r.u.)



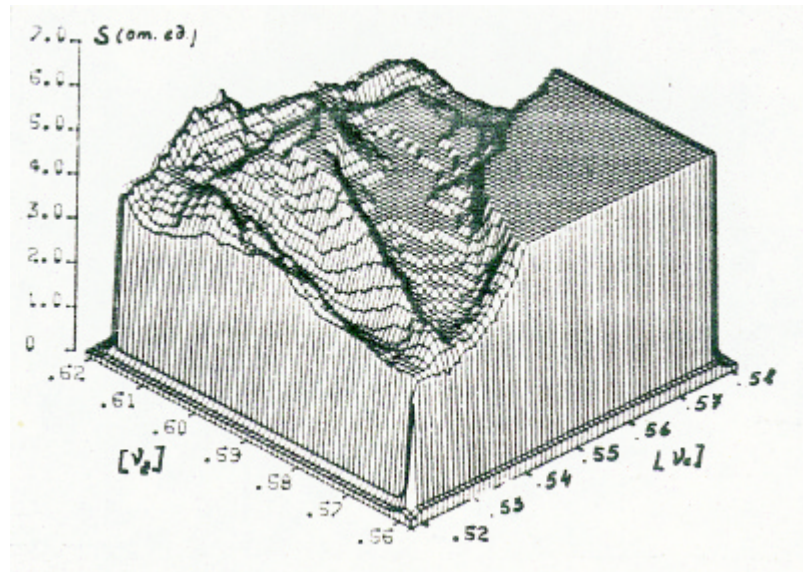
$I_+ \sim 1.2\text{mA}$, $I_- \sim 5.8\text{mA}$
 $?x = 0.009$, $?y = 0.035$

Particle loss rate from
positron beam

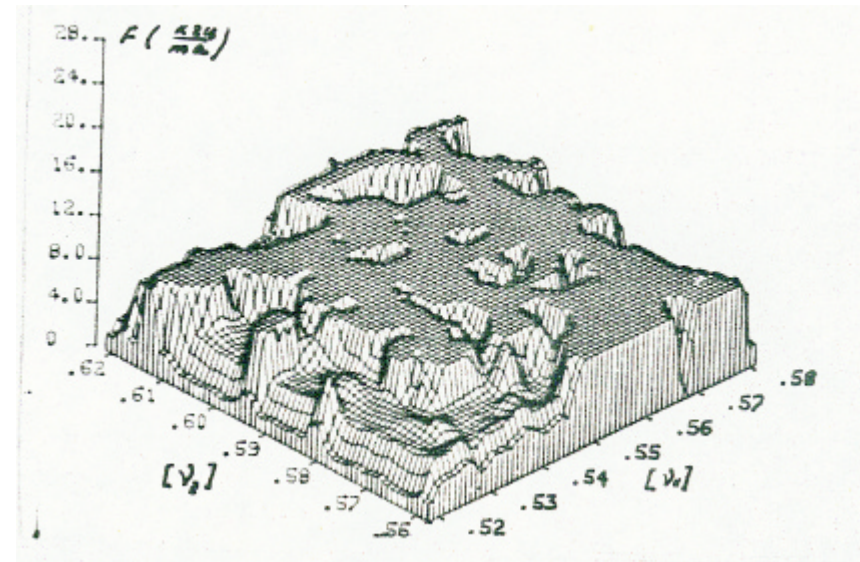


Tune plane appearance: beam-beam interaction

Vertical beam size from
luminosity (r.u.)



Particle loss rate from
positron beam



$I_+ \sim 6.2\text{mA}$, $I_- \sim 10.2\text{mA}$
 $?\text{x} = 0.015$, $?\text{y} = 0.060$

Summary

- 2-D tune scan technique has been used to explore tune plane.
- Variety of resonances excited by machine non-linearity and beam-beam interaction and affecting on vertical beam size and beam life time was observed and identified.
- Tune scan result at high $Q_{x,y}$ suggested optimal working point close to half-integer resonance.

Effect of machine cubic non-linearity on a beam-beam interaction (theory and experiments)

2D-Theory: model with one IP and one octupole lens

$$H = H_0 + V(x, y)(s = s_1) + W(x, y)(s = s_2)$$

Beam-beam
interaction

$$V(x, y) = \frac{r_e N}{\sigma} \int_0^1 dt \frac{1 + \exp\left[-\frac{x^2}{2(\sigma_x^2 + t)}\right] - \frac{y^2}{2(\sigma_y^2 + t)}}{(\sigma_x^2 + t)^{1/2} (\sigma_y^2 + t)^{1/2}}$$

Octupole magnet

$$W(x, y) = \frac{2}{3} B (x^4 + y^4 + 6x^2 y^2)$$

Cubic non-linearity effect: theory

After transformation to "action-angle" variables:

$$x, y = \sqrt{2J_{x,y}} \cos \theta_{x,y};$$

$$p_{x,y} = \sqrt{\frac{2J_{x,y}}{m_{x,y}}} \sin \theta_{x,y} - \frac{J'_{x,y}}{2} \cos \theta_{x,y}$$

And Fourier expansion one can write:

$$H = J_x \omega_x + J_y \omega_y + V_0(J_x, J_y) + W_0(J_x, J_y)$$

$$+ \sum_{m,n,k} V_{2m,2n}(J_x, J_y) \cos(2m\theta_x + 2n\theta_y + k\theta)$$

$$+ \sum_{m,n,k} W_{m,n}(J_x, J_y) \cos(m\theta_x + n\theta_y + k\theta)$$

Cubic non-linearity effect: theory

$$V_0(J_x, J_y) = \frac{r_e N}{2} \int_0^1 dt \frac{1 - \exp\left\{ -\frac{J_x}{I_0} \left(\frac{J_x}{I_0} t \right)^2 - \frac{J_y}{I_0} \left(\frac{J_y}{I_0} t \right)^2 \right\}}{\left(\frac{J_x}{I_0} t \right)^{1/2} \left(\frac{J_y}{I_0} t \right)^{1/2}};$$

$$\beta_{x,y}^* = \frac{J_{x,y}^*}{2 \left(\frac{J_x}{I_0} t \right)}$$

$$W_0(J_x, J_y) = B \left[J_x^2 \beta_{x,y}^2 + J_y^2 \beta_{x,y}^2 + 4 J_x J_y \beta_{x,y} \right]$$

$\beta_{x,y}^*$ - beta function at IP, $\beta_{x,y}$ - beta function at octupole location.

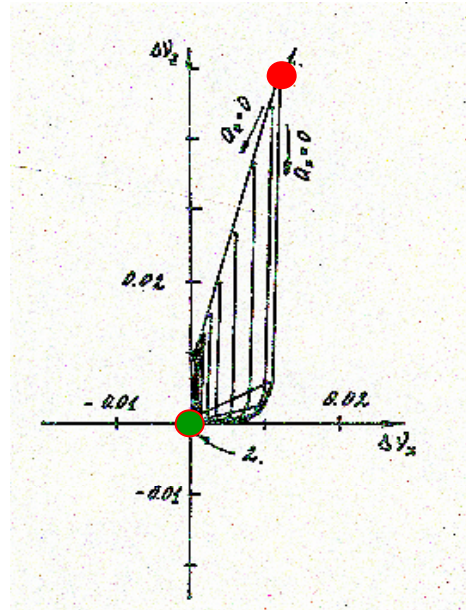
Tune shift as a function of amplitude:

$$\Delta \nu_{x,y}(J_x, J_y) = \frac{\partial V_0(J_x, J_y)}{\partial J_{x,y}} + \frac{\partial W_0(J_x, J_y)}{\partial J_{x,y}};$$

Cubic non-linearity effect: theory

"Foot print" for Δx vs Δy / Δx vs Δy

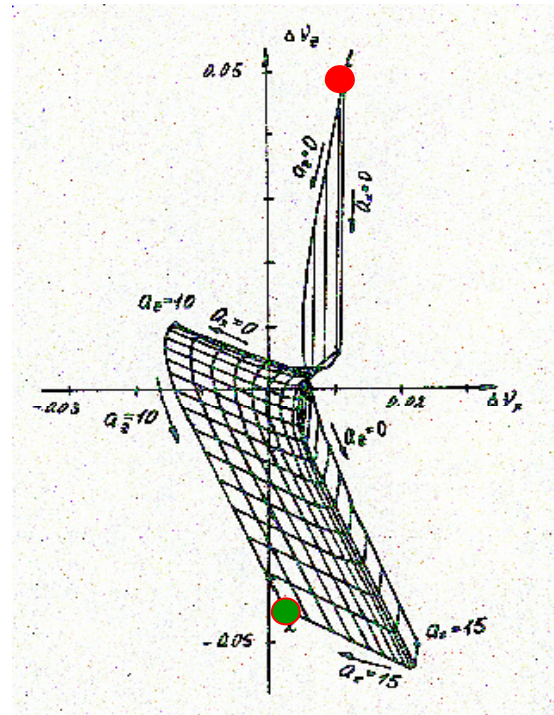
- beam center, $a_{x,y} = 0$
- beam tail, $a_x = 0.015m, a_y = 0.01m$ at location with $\Delta x, \Delta y = 12m$



$$2B_{x,y}^2 \approx 0$$

Zero cubic non-linearity
Minimum tune spread
Simple topology

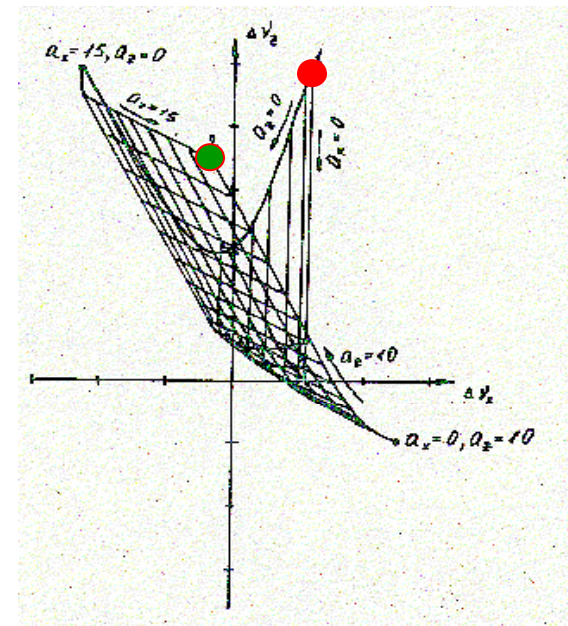
9/11/2003



$$2B_{x,y}^2 \approx -1200 [1/m]$$

Negative cubic non-linearity
Large tune spread
Not a simple topology

e+e- workshop, Italy, Sept
10-14 2003

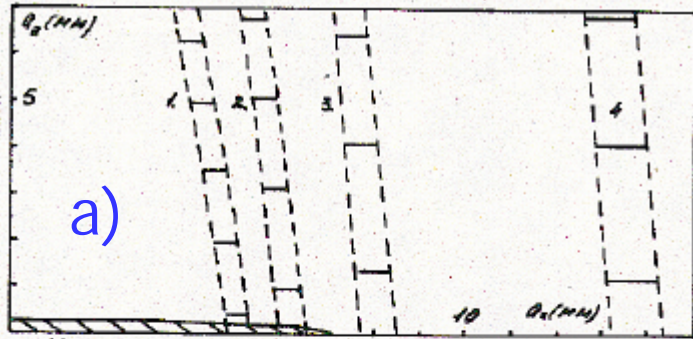


$$2B_{x,y}^2 \approx 1200 [1/m]$$

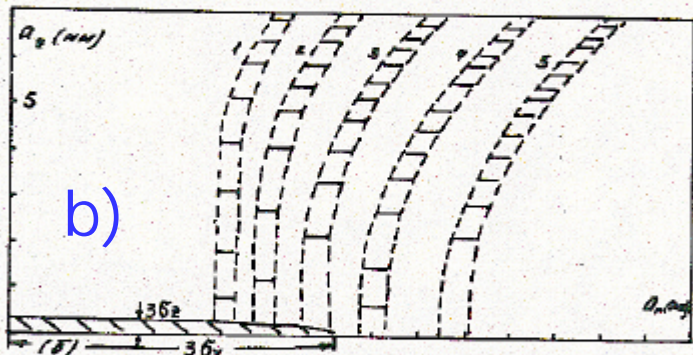
Positive cubic non-linearity
Large spread
Folder type topology

13

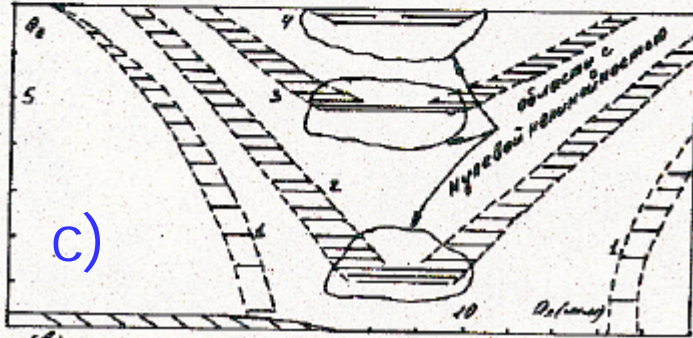
Model calculation: $14 \omega_x = 28$ resonance lines and resonance vectors in amplitude space for various machine cubic nonlinearity.



a)



b)



c)

$\omega_x \omega_y \omega_z \omega_w \omega_v \omega_u \omega_t \omega_s \omega_r \omega_q \omega_p \omega_o \omega_n \omega_m \omega_l \omega_k \omega_j \omega_i \omega_h \omega_g \omega_f \omega_e \omega_d \omega_c \omega_b \omega_a$

Resonance lines:

?? $\omega_x = 8.568$, (2) $\omega_x = 8.569$

?? $\omega_x = 8.570$, (4) $\omega_x = 8.571$

?? $\omega_x = 8.572$

a) $2B \omega_{x,y}^2 \approx 0 [1/m]$

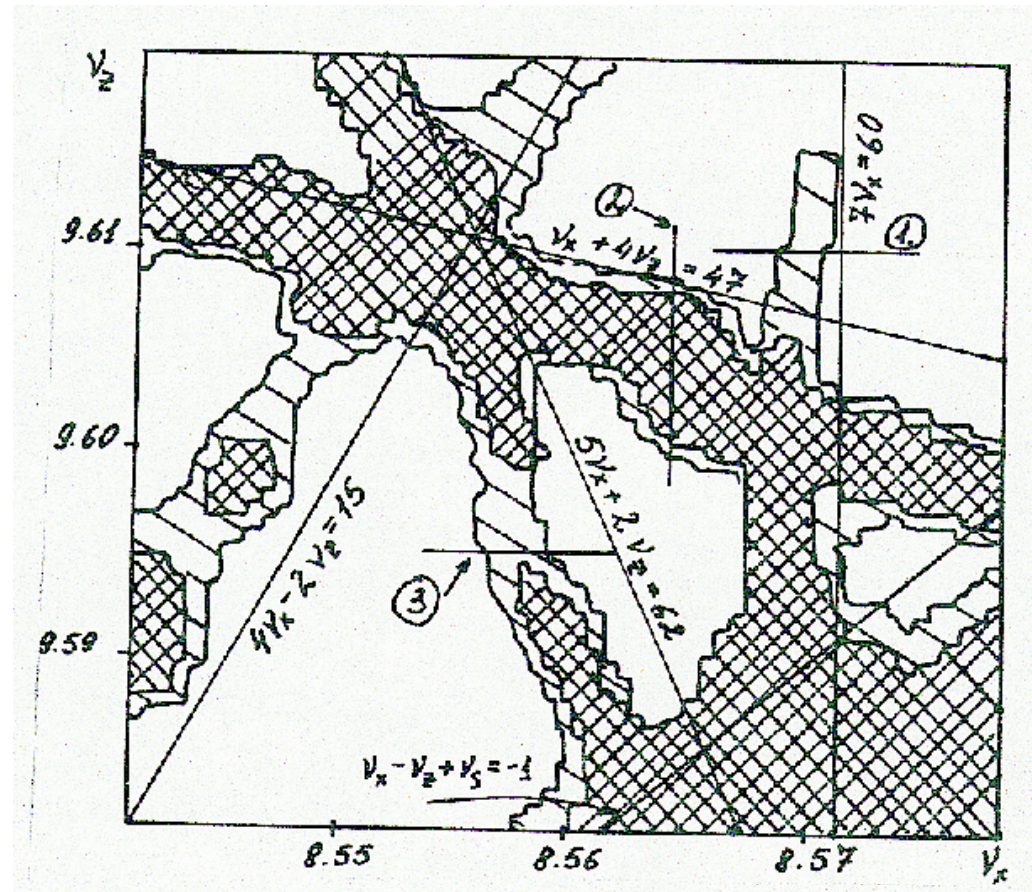
b) $2B \omega_{x,y}^2 \approx 1200 [1/m]$

c) $2B \omega_{x,y}^2 \approx 1200 [1/m]$

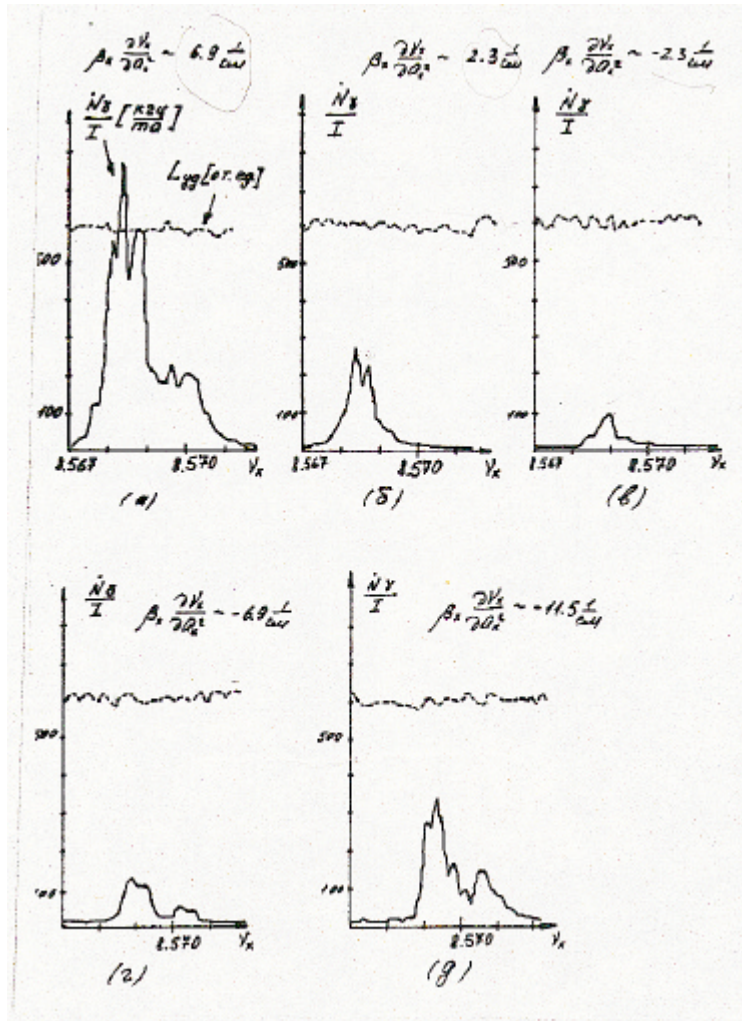
Cubic non-linearity effect: experimental study

2-D tune scan to identify resonances.

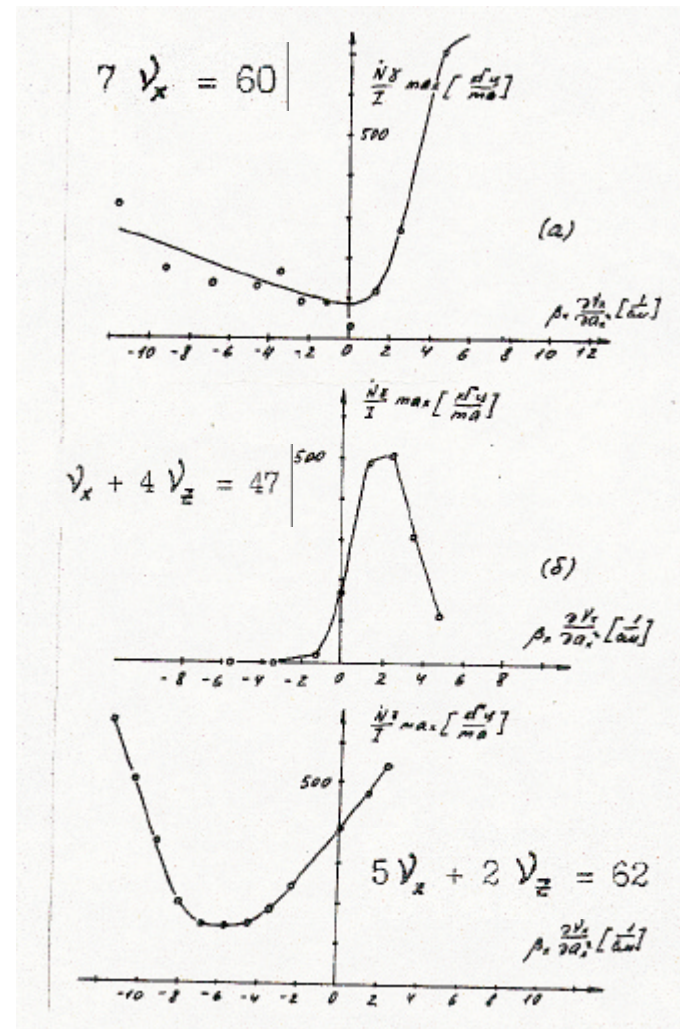
$\Delta x = 0.008$, $\Delta y = 0.030$



E+ loss rate and specific luminosity
 (L/I+/I-) on resonance $7\nu_x = 14$,
 measured for various machine cubic non-
 linearity, $\nu_x = 0.009$, $\nu_y = 0.035$.



Maximum E+ loss rate on $7\nu_x = 14$, $\nu_x + 4\nu_y = 47$ and $5\nu_x + 2\nu_y = 62$ resonances as
 function of machine cubic non-linearity,
 $\nu_x = 0.009$, $\nu_y = 0.035$



Summary

- Machine cubic non-linearity may dramatically change beam-beam interaction dynamics
- Cubic non-linearity must be kept close to "zero"
- Negative non-linearity may be little bit less harmful than positive