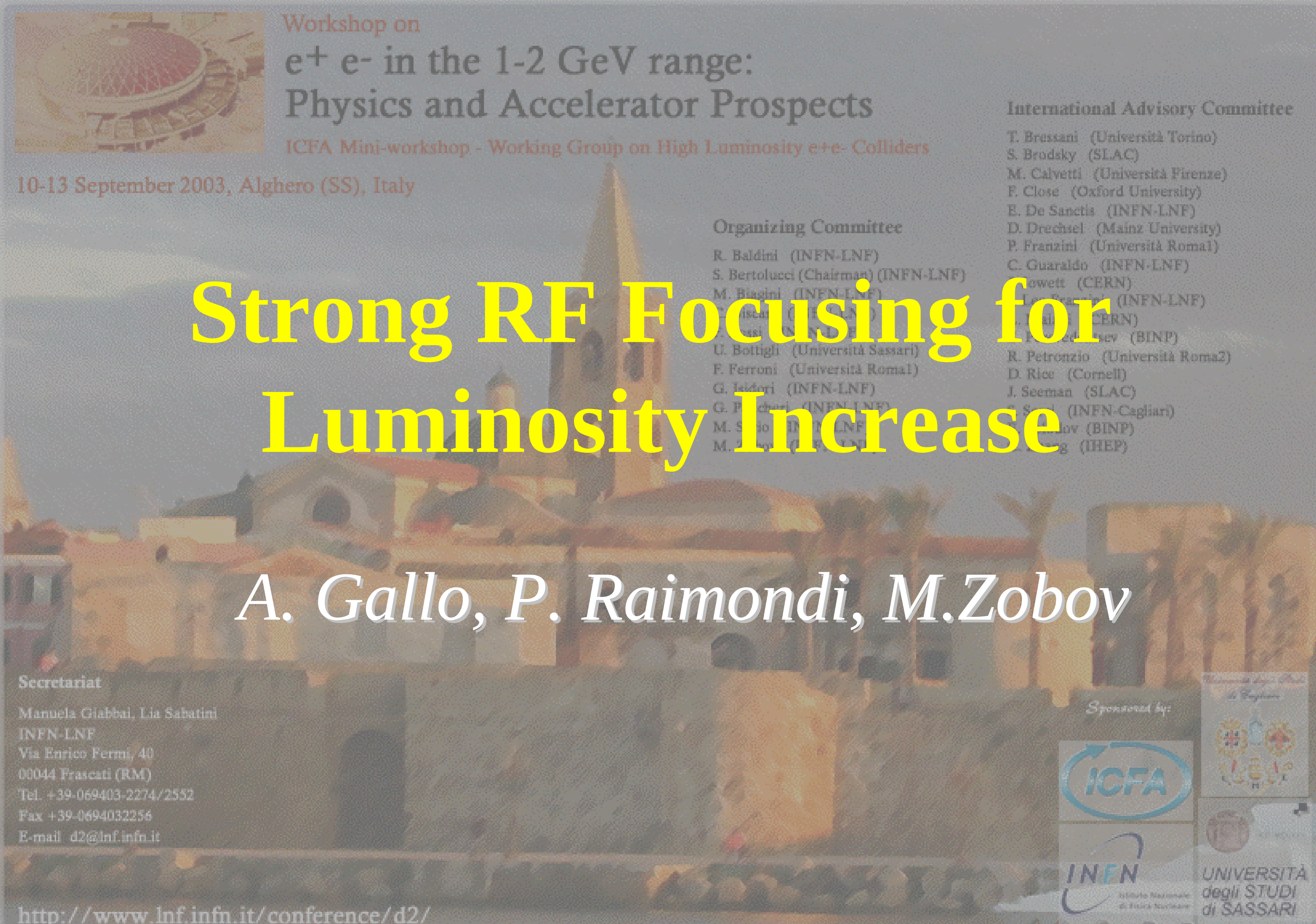
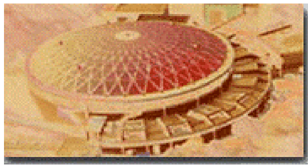
Strong RF Focusing for Luminosity Increase

A. Gallo, P. Raimondi, M.Zobov



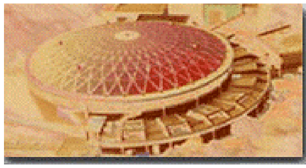


- The minimum value of the vertical beta-function β_y at the IP in a collider is set by the hour-glass effect and it is almost equal to the bunch length σ_z. Reducing the bunch length in storage rings is therefore one of the most promising way to make a step forward in the achievable Luminosity

$$\boxed{L \propto \frac{1}{\sigma_x \sigma_y} \propto \frac{1}{\sqrt{\beta_x \beta_y}}}$$

- It may be seen that by only scaling the horizontal and vertical beta functions β_x and β_y at the IP as the bunch length σ_z, the linear tune shift parameters $\xi_{x,y}$ are unaffected, while the luminosity scales as:

$$\boxed{L \propto \frac{1}{\sqrt{\beta_x \beta_y}} \propto \frac{1}{\sigma_z}}$$



The rms length σ_z of the equilibrium charge distribution in the bunch, neglecting the lengthening process coming from the short-range wakefields, is given by:

$$\sigma_z = c\ \sigma_\tau = c\,\frac{\sigma_E}{E}\,\frac{\alpha_c}{\omega_s} = L_{ring}\,\frac{\sigma_E}{E}\sqrt{\frac{\alpha_c (E/e)}{2\pi h\ V_{RF}}}$$

Anyway, to be effective in terms of luminosity, the bunch has to remain short up to the design current. The Boussard criterion can be used to estimate the µ-wave threshold:

$$I_{th} = \alpha_c (2\pi)^{3/2} \left(\frac{\sigma_E}{E}\right)^2 \frac{E/e}{(Z/n)_{eff}}\ \frac{\sigma_z}{L_{ring}}$$

For current values beyond the threshold, assuming purely inductive ring impedance, the bunch length is given by:

$$\sigma_z = L_{ring}\sqrt[3]{\frac{2}{(2\pi)^{5/2}}\,\frac{I}{hV_{RF}\cos\phi_s}(Z/n)_{eff}}$$

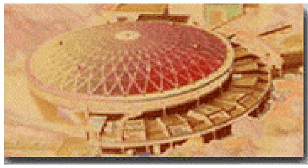
Short bunches beyond μ-wave threshold

To keep the bunch short at currents exceeding the μ-wave threshold, very high RF gradients together with small values of the ring impedance are required. For typical values of a Φ-factory collider parameters, and even considering a RF voltage as high as 10 MV, one gets in the end:

$$\boxed{\sigma_z \geq 4 \ \ mm}$$

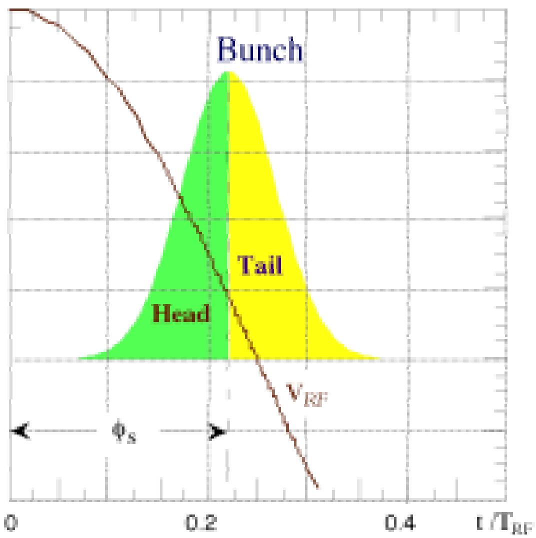
Bunch lengths significantly smaller than this can be hardly obtained, due to the presence of the cubic root in the σ_z expression. Moreover, beyond the μ-wave threshold the longitudinal dynamics may be critical since the wakefields are likely to drive various forms of bunch longitudinal instability.

<u>This way will not be followed in the rest of the presentation</u>.



Strong RF Focusing: Short bunches at the IP

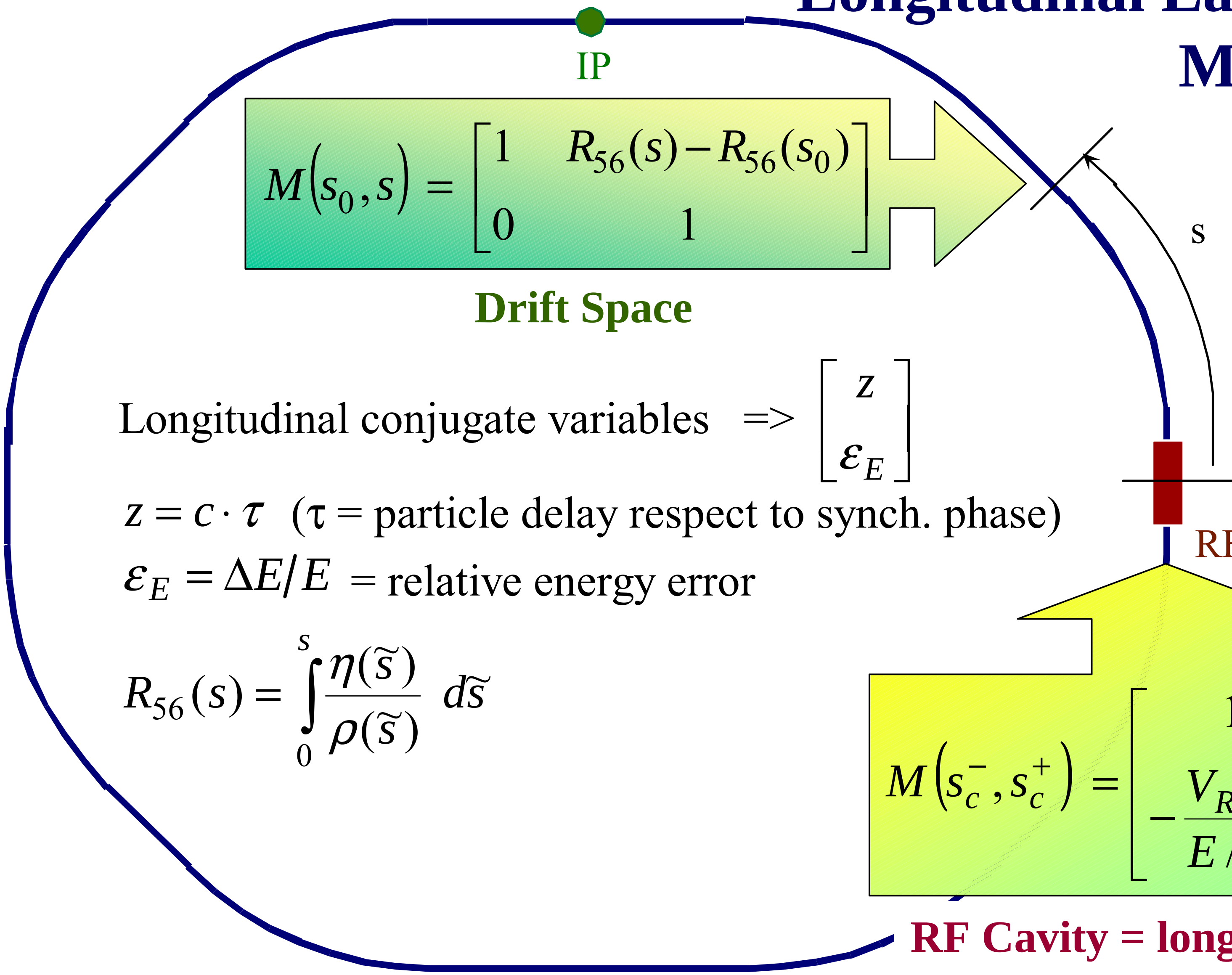
Let's consider a storage ring model where the accelerating fields of the RF cavities are represented as an overall accelerating voltage located at some azimuthal position (longitudinal thin lens model).



The image in the longitudinal phase space of a bunch emerging from the longitudinal thin lens shows a certain degree of correlation since the energy gained by the bunch head differs from that gained by the bunch tail (and is higher for positive values of the ring momentum compaction and lower for negative values).

This correlation can be used to differentiate the particle path lengths to the IP by means of the lattice $R_{56}(s)$ parameter to compress the bunch just like in a magnetic compressor. To describe quantitatively this effect a linear model of the longitudinal lattice has to be considered.

Longitudinal Lattice Model



$$M(s_0, s) = \begin{bmatrix} 1 & R_{56}(s) - R_{56}(s_0) \\ 0 & 1 \end{bmatrix}$$

Drift Space

Longitudinal conjugate variables => $\begin{bmatrix} z \\ \varepsilon_E \end{bmatrix}$

$z = c \cdot \tau$ (τ = particle delay respect to synch. phase)

$\varepsilon_E = \Delta E / E$ = relative energy error

$$R_{56}(s) = \int_0^s \frac{\eta(\tilde{s})}{\rho(\tilde{s})} \, d\tilde{s}$$

$$M(s_c^-, s_c^+) = \begin{bmatrix} 1 & 0 \\ -\frac{V_{RF}}{E/e} \frac{2\pi}{\lambda_{RF}} & 1 \end{bmatrix}$$

RF Cavity = long. thin lens

The one-turn longitudinal transfer matrix

Taking the cavity position as the origin ($s_c = 0$), the one-turn longitudinal transfer matrix is given by:

$$M(s, s+L) = \begin{bmatrix} 1 - 2\pi \dfrac{R_{56}(s)}{\lambda_{RF}} \dfrac{V_{RF}}{E/e} & \alpha_c L \left(1 - 2\pi \dfrac{R_{56}(s)}{\lambda_{RF}} \left(1 - \dfrac{R_{56}(s)}{\alpha_c L} \right) \dfrac{V_{RF}}{E/e} \right) \\ -\dfrac{V_{RF}}{E/e} \dfrac{2\pi}{\lambda_{RF}} & 1 + 2\pi \dfrac{R_{56}(s)}{\lambda_{RF}} \left(1 - \dfrac{\alpha_c L}{R_{56}(s)} \right) \dfrac{V_{RF}}{E/e} \end{bmatrix}$$

The one-turn synchrotron phase advance and the synchrotron tune are given by:

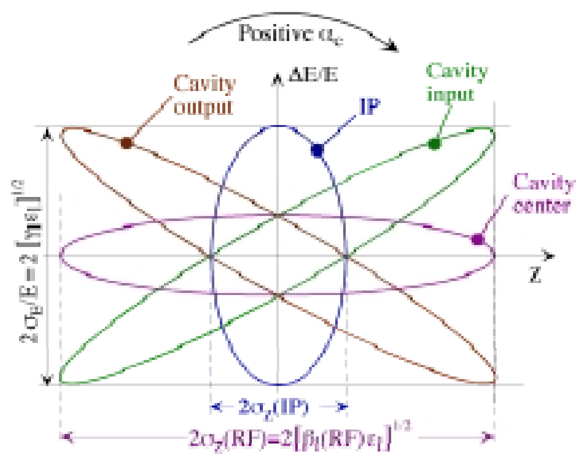
$$\cos\mu = \frac{1}{2} Tr\left[M\left(s_0, s_0 + L\right) \right] = 1 - \pi \frac{\alpha_c L}{\lambda_{RF}} \frac{V_{RF}}{E/e}; \quad \nu_s = \frac{1}{2\pi} \mathrm{A}\cos\left[1 - \pi \frac{\alpha_c L}{\lambda_{RF}} \frac{V_{RF}}{E/e} \right]$$

leading to the following stability condition:

$$|\cos\mu| \le 1 \;\Rightarrow\; \mu \le \pi \;\Rightarrow\; \nu_s \le 1/2 \;\Rightarrow\; V_{RF} \le \frac{2}{\pi} \frac{\lambda_{RF}}{\alpha_c L} E/e = V_{RF_{Max}}$$

The one-turn transfer matrix can be put in canonical form:

$$M(s,s+L) = \cos\mu \cdot \hat{I} + \sin\mu \cdot \hat{J} = \cos\mu \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\mu \cdot \begin{bmatrix} \alpha_l & \beta_l \\ -\gamma_l & -\alpha_l \end{bmatrix}$$ with:



$$\alpha_l(s) = \frac{1-\cos\mu}{\sin\mu}\left[1 - \frac{2R_{56}(s)}{\alpha_c L}\right]$$

$$\beta_l(s) = \frac{\alpha_c L}{\sin\mu}\left[1 - (1-\cos\mu)\frac{2R_{56}(s)}{\alpha_c L}\left(1 - \frac{R_{56}(s)}{\alpha_c L}\right)\right]$$

$$\gamma_l(s) = \frac{1-\cos\mu}{\sin\mu}\frac{2}{\alpha_c L}$$

Since γ_l does not depend upon s, the vertical size of the ellipse (i.e. the normalized energy spread σ_E/E of the equilibrium distribution) does not vary along the ring. The longitudinal emittance ε_l is related to σ_E/E according to:

$$\frac{\sigma_E}{E} = \sqrt{\varepsilon_l \gamma_l} \;\; \Rightarrow \;\; \varepsilon_l = (\sigma_E / E)^2 \frac{\sin\mu}{2(1-\cos\mu)}\alpha_c L$$

On the contrary, since β_l <u>does</u> depend upon s, the horizontal size of the ellipse (i.e. the bunch length σ_z) varies along the ring according to:

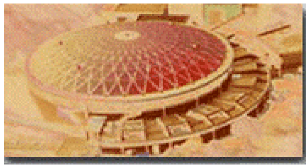
$$\sigma_z(s) = \sqrt{\varepsilon_l \beta_l(s)} = \frac{\sigma_E}{E}\alpha_c L \sqrt{\frac{1}{2(1-\cos\mu)} - \frac{R_{56}(s)}{\alpha_c L}\left(1-\frac{R_{56}(s)}{\alpha_c L}\right)} =$$

$$= \sigma_z(0)\sqrt{1-2(1-\cos\mu)\frac{R_{56}(s)}{\alpha_c L}\left(1-\frac{R_{56}(s)}{\alpha_c L}\right)}$$

where $\sigma_z(0)$ is the bunch length at $s=0$ (i.e. at the cavity position). It may be noticed that $\sigma_z(0)= \sigma_{z,cav}$ is the maximum value of the bunch length along the ring. On the other hand, there is a minimum value of σ_z corresponding to the s_m position where $R_{56}(s_m)= \alpha_c L/2$. If the minimum position corresponds to the IP one gets:

$$\sigma_z(IP) = \frac{\sigma_E}{E}\frac{\alpha_c L}{2}\sqrt{\frac{1+\cos\mu}{1-\cos\mu}} = \sigma_z(Cav)\sqrt{\frac{1+\cos\mu}{2}}$$

As μ approaches π, the ratio $\sigma_z(IP)/\sigma_z(Cav)$ goes to zero!



Equilibrium Energy Spread

In order to compute exactly the bunch size along the ring one needs to know the equilibrium energy spread (or, equivalently, the longitudinal emittance). It has been derived following an analytical approach based on the computation of the second momenta of the bunch equilibrium distributions using the eigenvectors of the longitudinal one-turn transfer matrix[*] that gives the following result:

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{1}{1+\cos\mu}\,\frac{55}{48\sqrt{3}}\,\frac{r_e\hbar}{m_e}\,\frac{\gamma^5\tau_d}{L}\oint\left[1-(1-\cos\mu)\frac{2R_{56}(s)}{\alpha_c L}\left(1-\frac{R_{56}(s)}{\alpha_c L}\right)\right]\frac{ds}{|\rho(s)|^3}$$

where r_e and m_e are the electron classical radius and rest mass, τ_d is the longitudinal damping time and $\gamma = E/(mc^2)$ is the relativistic factor.

[*] A. W. Chao, "Evaluation of Beam Distribution Parameters in an Electron Storage Ring", Journal of Applied Physics 50: 595-598, 1979.

Asymptotic Behavior of the Equilibrium Energy Spread

The equilibrium energy spread can be conveniently rewritten as follows:

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{1}{1+\cos\mu}\,\frac{55}{48\sqrt{3}}\,\frac{r_e\hbar}{m_e}\,\frac{\gamma^5\tau_d}{L}\oint\frac{\beta_l(s)}{\beta_l(0)}\,\frac{ds}{|\rho(s)|^3} = \left(\left.\frac{\sigma_E}{E}\right|_0\right)^2 \frac{2}{1+\cos\mu}\,\frac{\oint\frac{\beta_l(s)}{\beta_l(0)}\,\frac{ds}{|\rho(s)|^3}}{\oint\frac{ds}{|\rho(s)|^3}}$$

It may be noticed that the previous expression is diverging as μ tends to π in such a way that the bunch length remains finite at the IP (which is a nice physical result), while at low tunes it tends to the value $\left.\frac{\sigma_E}{E}\right|_0$:

$$\left(\left.\frac{\sigma_E}{E}\right|_0\right)^2 = \frac{55}{96\sqrt{3}}\,\frac{r_e\hbar}{m_e}\,\frac{\gamma^5\tau_d}{L}\oint\frac{ds}{|\rho(s)|^3}$$

which is the expression commonly reported in the accelerator physics textbooks.

In the simplified assumption of constant bending radius ρ and $R_{56}(s)$ linearly growing in the arcs the equilibrium energy spread expression reduces to:

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{2}{3}\left(\left.\frac{\sigma_E}{E}\right|_0\right)^2 \frac{2+\cos\mu}{1+\cos\mu}$$

Different results may be obtained if the ring has variable bending radii and/or the $R_{56}(s)$ function does not grow linearly in the arcs.

Under the previous assumptions the longitudinal emittance ε_l and the bunch lengths at the RF cavity and IP are given by:

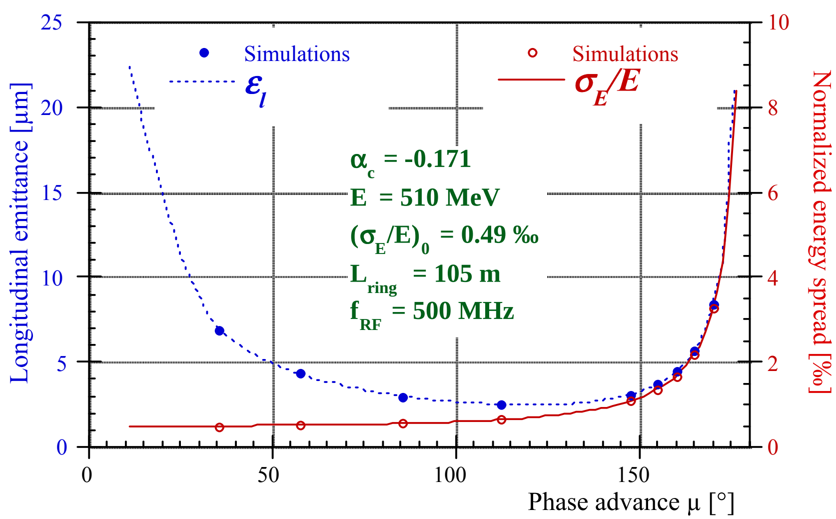
$$\varepsilon_l = \frac{\alpha_c L}{3}\left(\left.\frac{\sigma_E}{E}\right|_0\right)^2 \frac{2+\cos\mu}{\sin\mu}$$

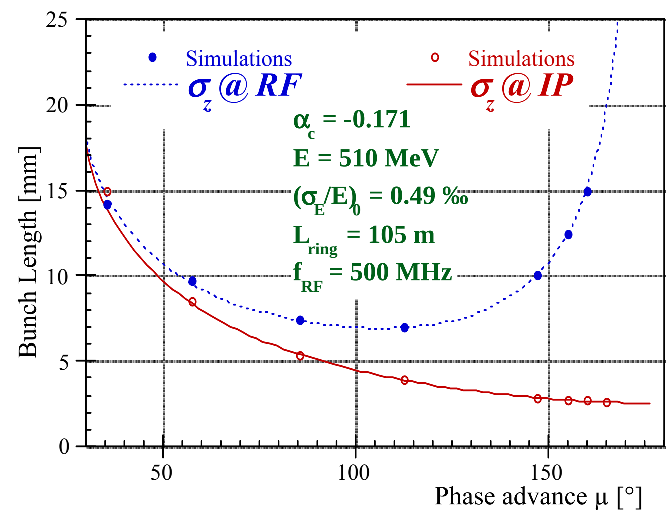
$$\sigma_z(Cav) = \frac{\alpha_c L}{\sin\mu}\left(\left.\frac{\sigma_E}{E}\right|_0\right)\sqrt{\frac{2+\cos\mu}{3}}; \quad \sigma_z(IP) = \alpha_c L\left(\left.\frac{\sigma_E}{E}\right|_0\right)\sqrt{\frac{2+\cos\mu}{6(1-\cos\mu)}}$$

The emittance and the bunch length at the RF cavity, as well as the energy spread, diverge as μ approaches π, while the bunch length at the IP remains finite.

Comparison with Numerical Results:

These analytical results have been compared with multi-particle tracking simulations of the bunch longitudinal dynamics in a strong RF focusing configuration. Uniform R_{56} growth and emission rate in the arcs have been assumed in the tracking. The agreement is evident.

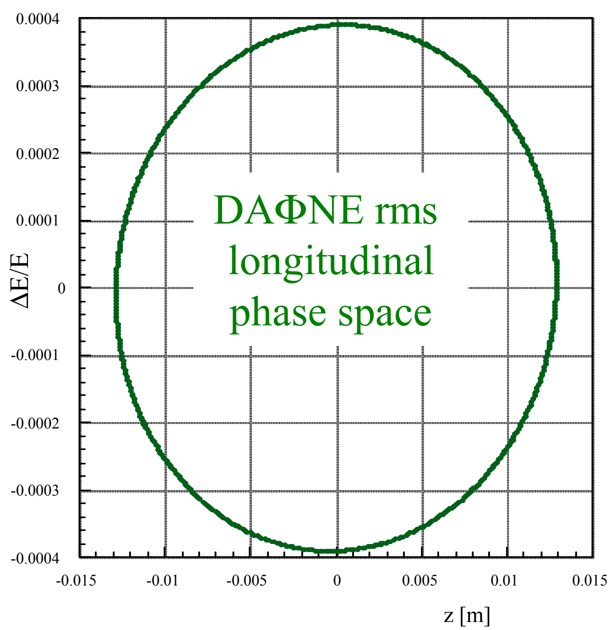


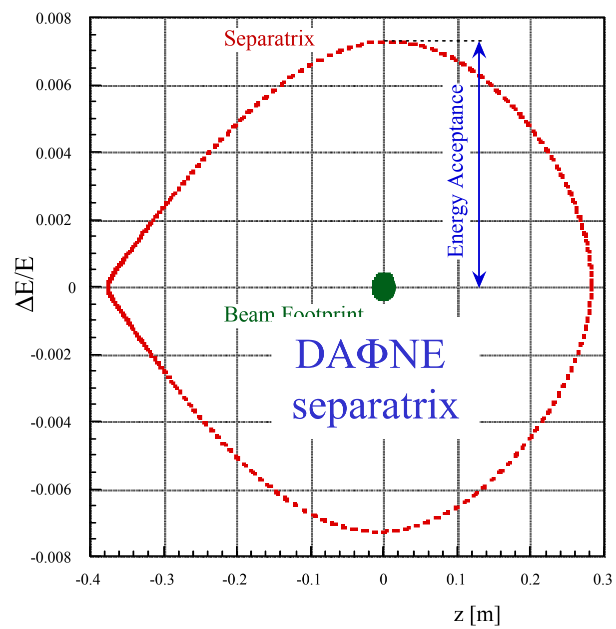


Energy Acceptance in a Strong RF Focusing Configuration:

The ring energy acceptance has to guarantee a sufficient beam lifetime. In a standard, low ν_s ring the conformation of the beam longitudinal phase space does not appreciably change along the ring. The energy acceptance is defined as the half-height of the separatrix curve, and it is given by:

$$\left.\frac{\Delta E}{E}\right|_{\max} = \frac{\sqrt{2\Phi_{\max}}}{\alpha_c} \quad \text{with} \quad \Phi(z) = \frac{\alpha_c}{E_0 L}\int_0^z \left[eV_{RF}(\tilde{z}) - U_0\right] d\tilde{z}$$



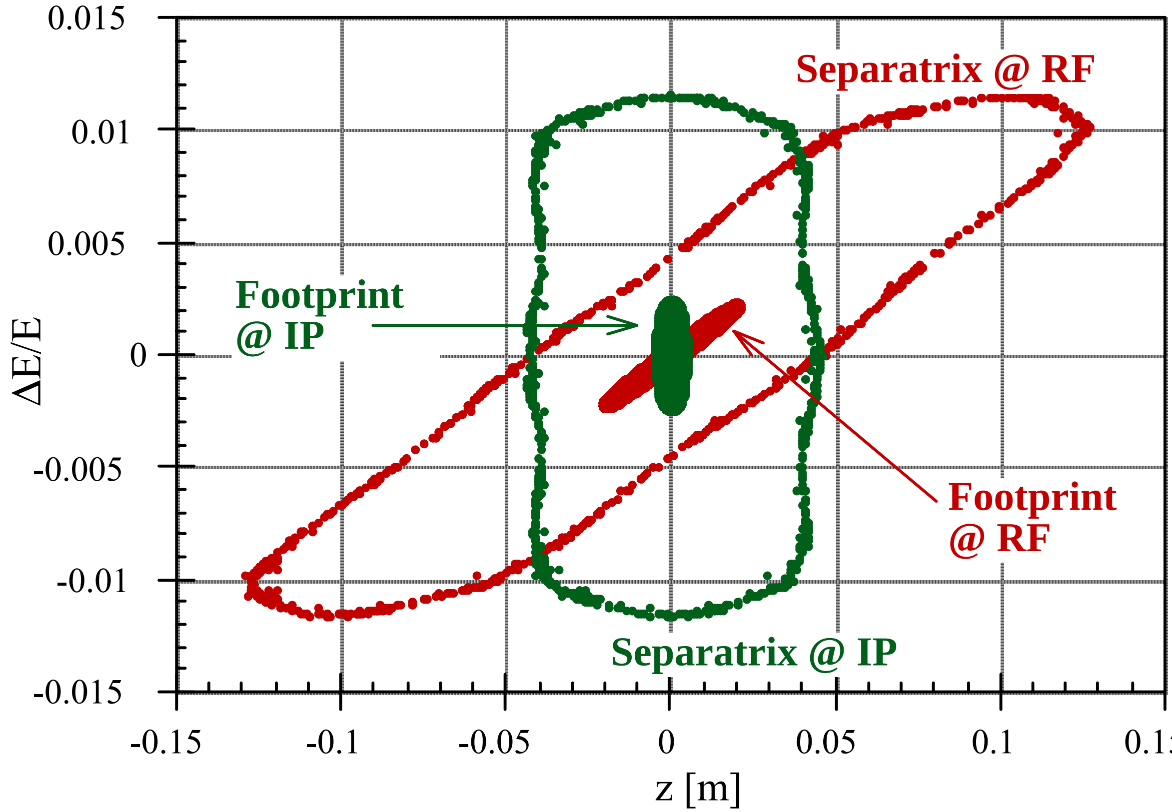


Energy Acceptance in a Strong RF Focusing Configuration:

In the strong RF focusing case, the conformation of the longitudinal phase space changes along the ring, and the same does the separatrix. The energy acceptance, defined as the half-heigth of the separatrix section at $z = 0$, becomes a function of the azimuth ***s*** and this must be taken into account in lifetime evaluations.

$$\left.\frac{\Delta E}{E}\right|_{\max} \approx 0.45\ \%\ \ @\ \ RF$$

$$\left.\frac{\Delta E}{E}\right|_{\max} \approx 1.10\ \%\ \ @\ \ IP$$



Strong RF Focusing: Beam Dynamics with Wakes

1) For short bunches wake potentials are comparable with an external RF voltage:
   → Potential Well Distortion;

2) Local slope of the wake potential can be higher than that of the RF voltage:
   → Microwave Instability;

3) The wake potentials are strongly dependent on the bunch length ($\sim\sigma_z^{-3}$ for the inductive impedance):
   → Beam Dynamics Depends on Impedance Generating Elements Location;

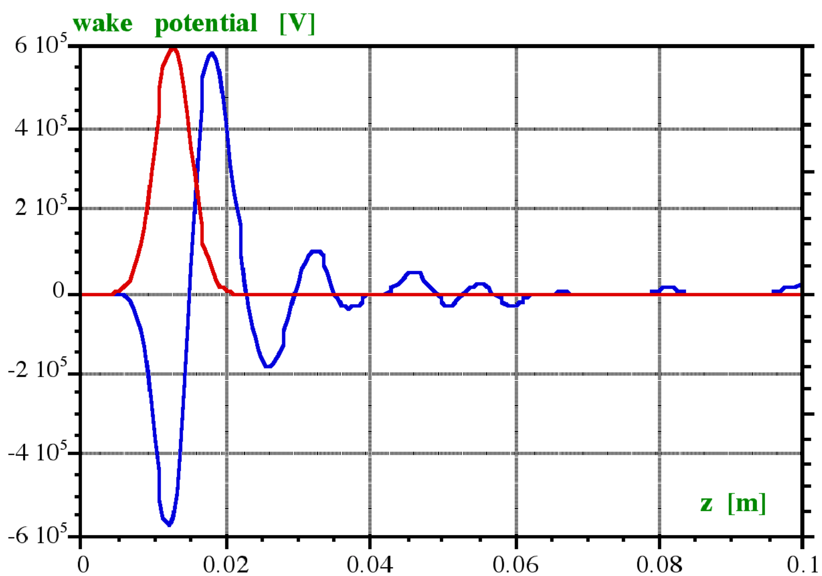
4) Working at synchrotron tunes close to 180^0:
   → Beam Dynamics is Dramatically Sensitive to RF Voltage + Wake Potentials;

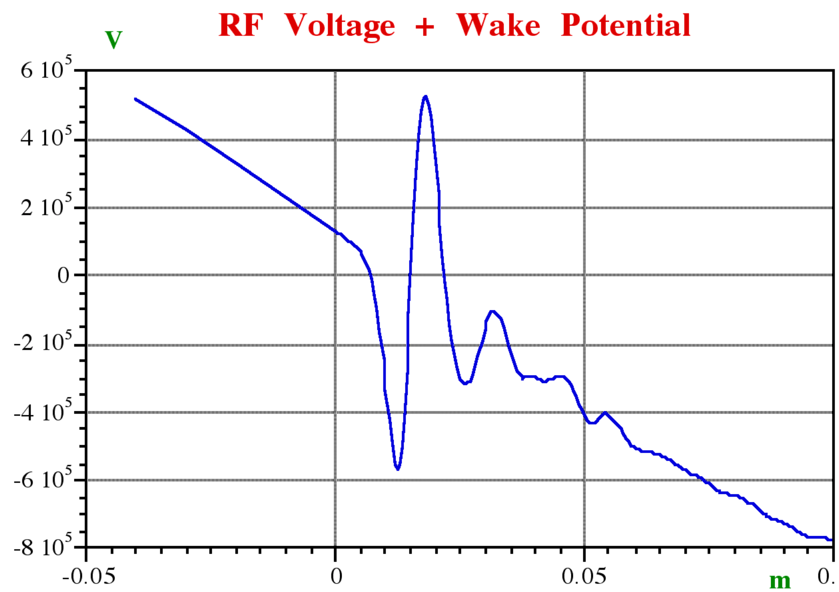
5) SR emission point locations are also very important

6) The momentum compaction sign is yet another important parameter.

WAKE POTENTIAL BASED ON DAΦNE ESTIMATES

($N = 3\text{x}10^{10}$, $\sigma_z = 2.5$ mm)



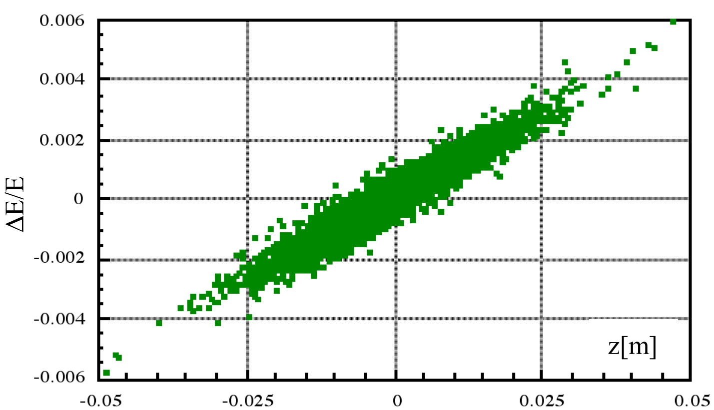


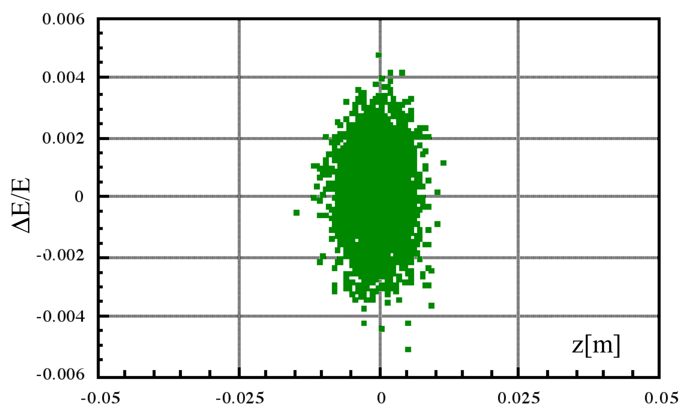
BUNCH CHARGE DISTRIBUTION AFTER RF CAVITY AND AT IP

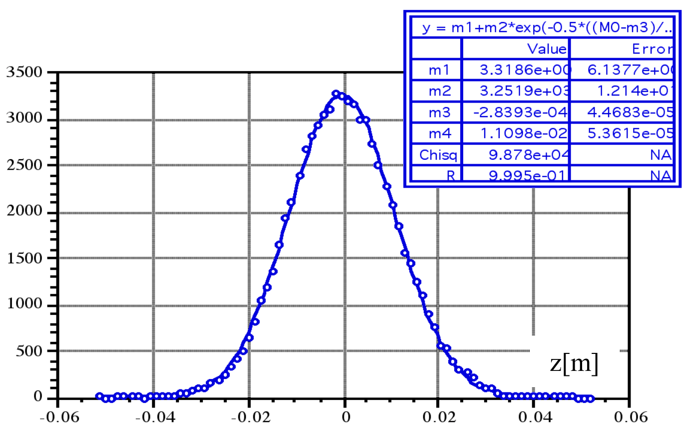
(all wake concentrated at the RF straight)

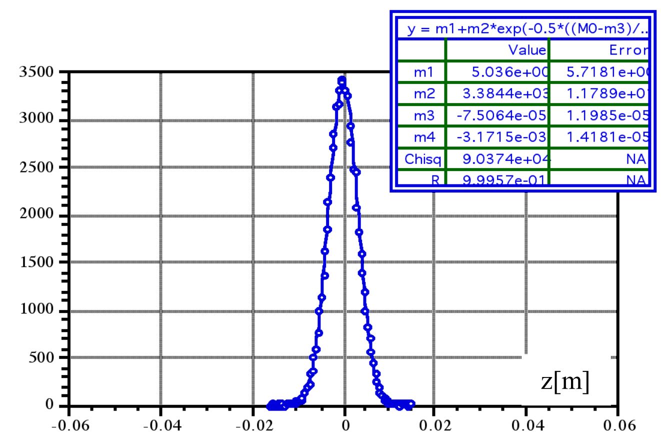
($N_p = 3x10^{10}$, $\alpha_c = - 0.1708$, $V_{rf}= 10.67$ MV $\rightarrow \mu = 165°$)

RF | **IP**





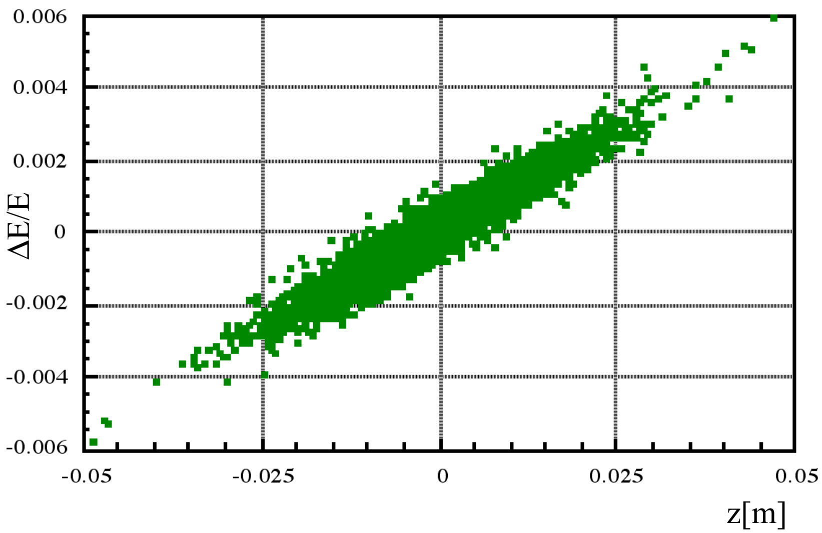


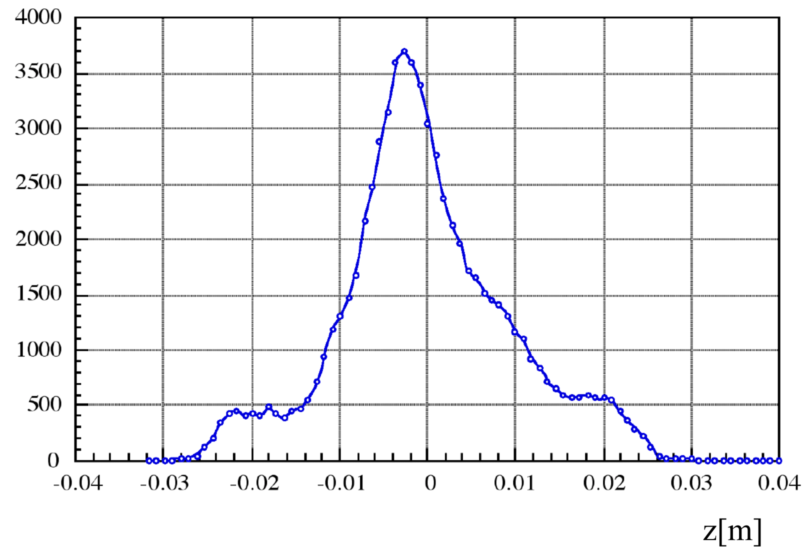


BUNCH CHARGE DISTRIBUTION AT IP

(all wake concentrated at the IP)

(α_c = -0.1708, V_{rf} = 10.67 MV $\rightarrow$ μ = 165 ^{0})

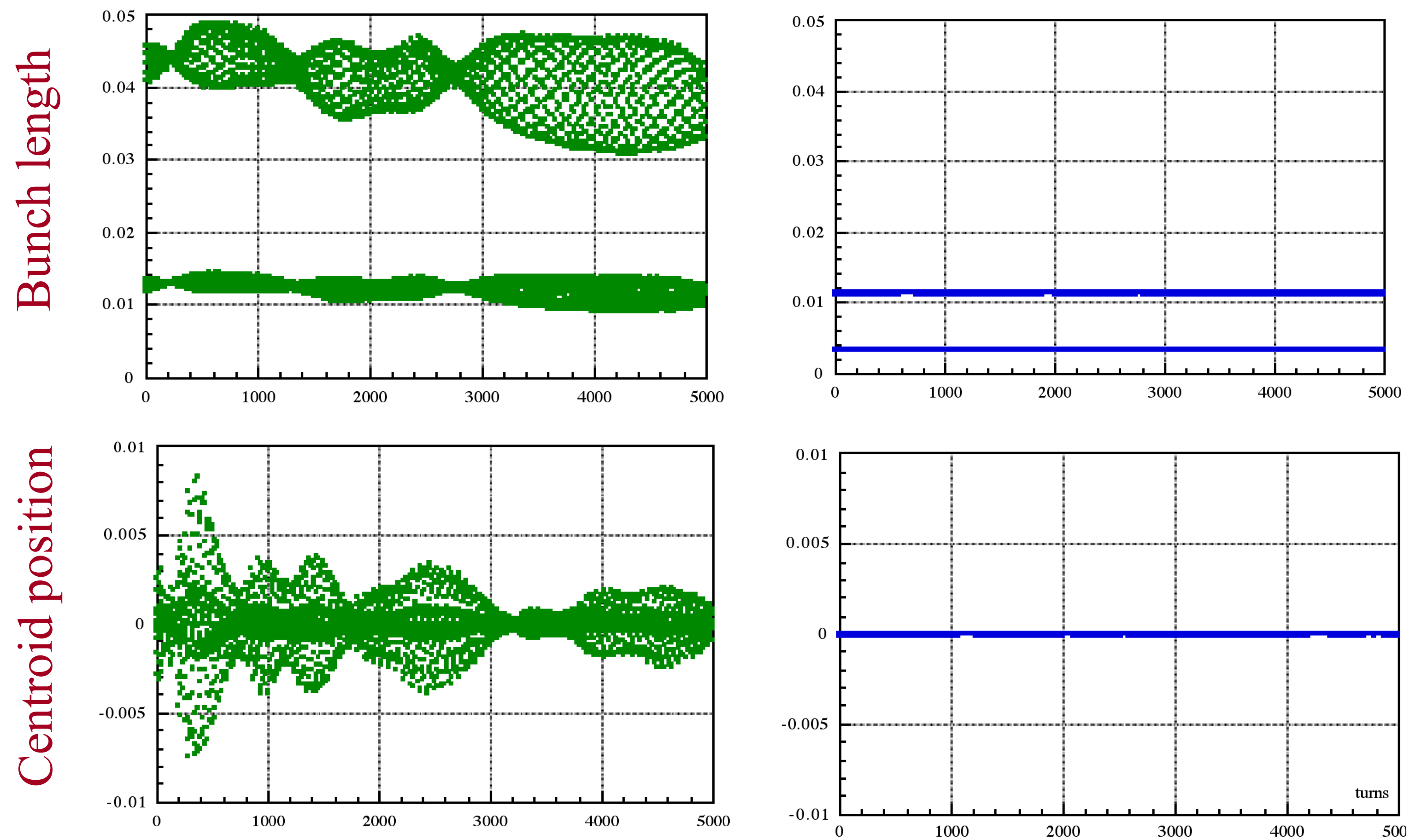




BUNCH LENGTH AND CENTROID OSCILLATIONS

(α_c = -0.1708, V_{rf} = 10.67 MV → μ = 165 ^{0})

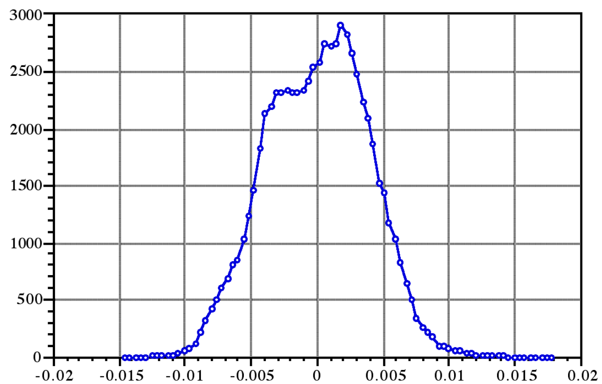
WAKE at IP **WAKE at RF CAVITY**



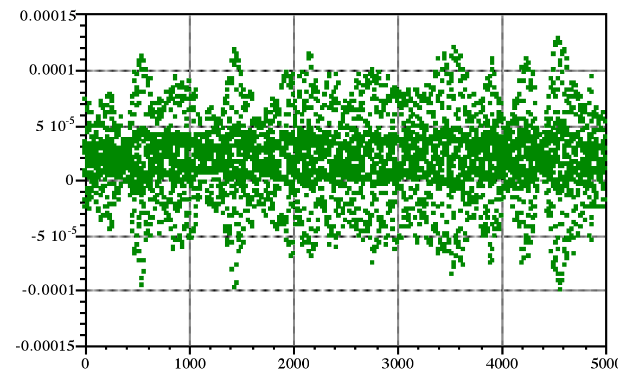
BUNCH CHARGE DISTRIBUTION AND OSCILLATIONS

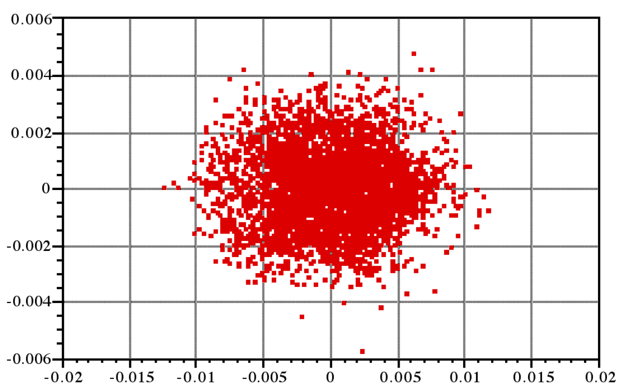
Positive Momentum Compaction - All wake at the RF

Charge distribution @ IP

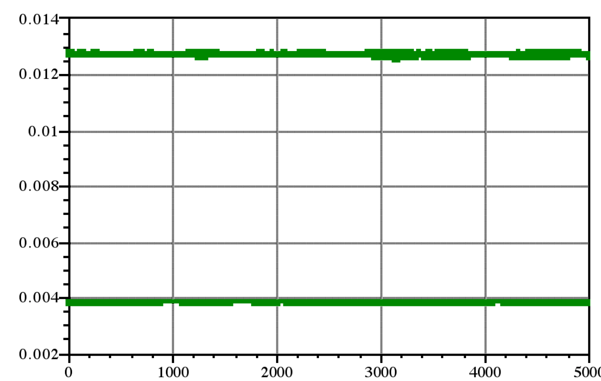


Centroid Position @ IP and RF





Phase Space @ IP



Bunch length @ IP and RF

A Possible Working Point for a Φ-Factory (E_{ring}=0.51 GeV) with σ_z(IP)=2 mm:

Reference Expressions:

$$\sigma_z(IP) = \alpha_c L \left(\left. \frac{\sigma_E}{E} \right|_0 \right) K[\rho(s), \beta_l(s)]$$

with

$$K[\rho(s), \beta_l(s)] = \sqrt{\frac{2 + \cos\mu}{6(1 - \cos\mu)}}$$

if ***ρ(s)=kost***. and ***R_{56}(s)*** grows linearly in the arcs.

$$\sigma_z(IP) = \sigma_z(RF)\sqrt{\frac{1+\cos\mu}{2}}$$

$$\sigma_z(IP) = 2\,mm;\ \sigma_z(RF) = 10\,mm \ \Rightarrow \mu = 155°$$

$$K[\rho(s), \beta_l(s)] = 0.27 \qquad \left. \frac{\sigma_E}{E} \right|_0 = 4.5 \cdot 10^{-4};$$

$$L = 100\,m; \qquad \alpha_c = 0.16; \qquad f_{RF} = 500\,MHz;$$

$$V_{RF}(\mu = 180°) = 12.2\,MV;$$

$$V_{RF}(\mu = 155°) = 11.6\,MV;$$

$$\frac{\sigma_E}{E}(\mu = 155°) = 1.1 \cdot 10^{-3};$$

$$\left. \frac{\Delta E}{E} \right|_{\max} @\ RF = 4.5 \cdot 10^{-3};$$

$$\left. \frac{\Delta E}{E} \right|_{\max} @\ IP = 1.1 \cdot 10^{-2}$$

CONCLUSIONS:

- The strong RF focusing concept has been illustrated and analyzed by means of the linear transfer matrix formalism;

- Longitudinal optical functions have been derived, showing that the bunch length varies along the ring and may be minimized at the IP;

- The longitudinal emittance and energy spread of the bunch equilibrium distribution, whose analytical expressions have been validated by comparison with results of multiparticle tracking simulations, diverge as the phase advance approaches 180°;

- The energy acceptance is no longer an invariant. It is a function of the azimuth and assumes its maximum value in the longitudinal waist (IP)

CONCLUSIONS (cnt'd):

- To avoid bunch lengthening and μ-wave instability, the location of the impedance is the crucial point. Good results can be obtained if the ring wake is almost completely concentred near the RF cavity, where the bunch is longest.

In the end

- bunch lengths of 2 mm at the IP at the Φ-factory energy seem reasonably obtainable (1 mm appears ambitious at the moment ...)

But

- many aspects of beam physics (such as bunch lengthening, Touschek lifetime, dynamic aperture, beam-beam effect, …) need to be deeply studied to establish whether or not a collider may efficiently work in the strong longitudinal focusing regime.

A possible candidate cavity

Table 1: Parameters of SCC module

Frequency	508.887 MHz
Gap length	243 mm
R/Q	93 Ω
Geometrical factor	251 Ω
Esp / Eacc	1.84
Hsp/Eacc	40.3 Gauss/(MV/m)
Loaded Q	6-8 x 10^4
Cryostat LHe volume	290 liter

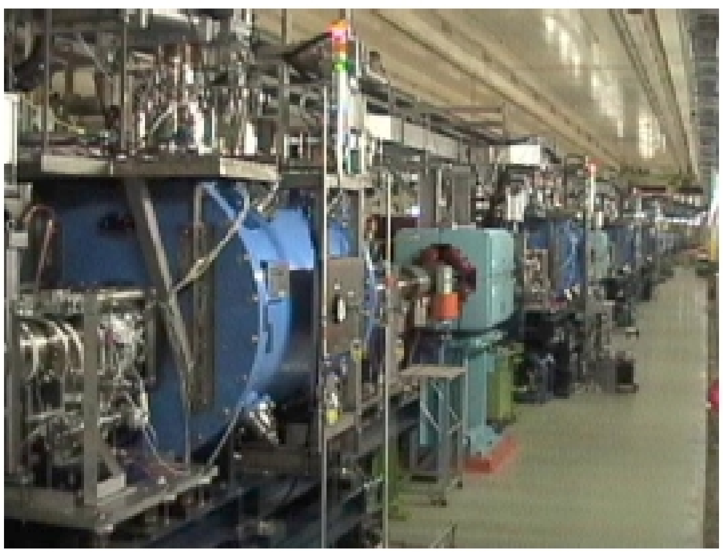


Figure 2: A picture of four SCC modules in Nikko-D11 tunnel

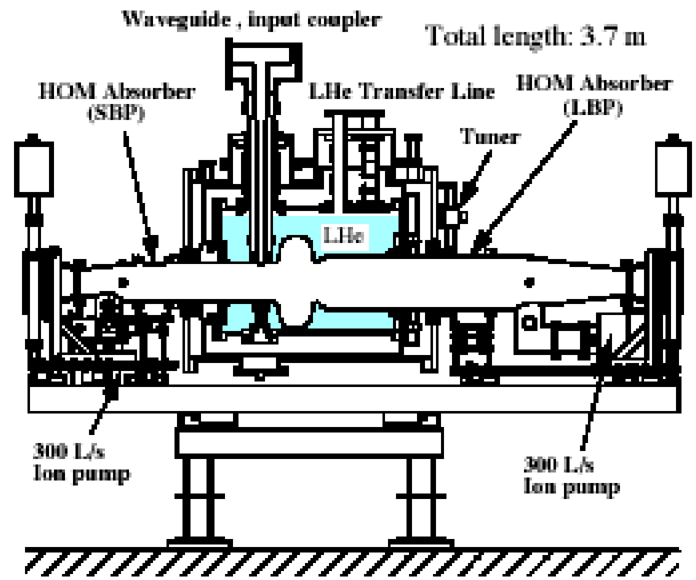


Figure 1: Superconducting cavity module for KEKB

Tested beyond 2.5 MV/module
Operated in high beam loading
Length ≈ 2.2 m /module