CP, CPT and Rare Decays

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- Based also on work with G. Isidori, N.Paver and A.Pugliese
- Maiani, in the $II\ DA\Phi NE\ Handbook$

Outline

- Motivation
  - CPT violation $\sim$ theory
  - connection with ν's, challenge
- Bell-Steinberger: PDG update
- CPT conserved, CP violation and chiral tests: $K_L \to 3\pi$, $K_L \to \pi^+\pi^-\gamma$
- Conclusions

CPT violation

- Lorentz invariance
- Hermiticity
  of the Hamiltonian
- Locality

$\Longrightarrow$ CPT conservation

- CPT spontaneously broken in string th. Kostelecky et al.
- Non-locality is enough? Barenboim,Lykken

CPT violation: how?

$$\mathcal{L}^{eff}_{SU(3)\times SU(2)\times U(1)} = \frac{1}{2} i\overline{\psi}\Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \overline{\psi} M \psi$$

Spurions break Lorenz sym. Coleman-Glashow,Kostelecky et al.

$$\Gamma^\nu \equiv \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + i f^\nu \gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu}$$

$$M \equiv m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu}\sigma_{\mu\nu}$$

- Free homeotic fermions (only non-locality) Barenboim,Lykken

$$(\not p - m\epsilon(p_0))u_\pm(p) = 0 \qquad u_+(p)=u(p), u_-(p)=v(-p)$$

$C\not{P}T$ in ν's and challenge ($\delta \equiv \Delta m^2_\nu - \Delta m^2_{\bar\nu}$)

- Atmospheric ν's, ν and $\bar\nu$ same maximal mixing, SuperK

$$-7.5 \times 10^{-3} eV^2 < \delta < 5.5 \times 10^{-3} eV^2$$

- SNO, Kamland $|\delta| < 1.3 \times 10^{-3} eV^2 \quad (90\% C.L.)$ Murayama

- Kaons vs ν's

$$|m_{\bar K} - m_K| < 10^{-18} m_K \Longrightarrow |m^2_{\bar K} - m^2_K| < 0.25 eV^2$$

- LSND maybe explained by $C\not{P}T$

$C\not{P}T$ in the K's mass matrix

Diagonalize $\begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$

$$K_{S,L} = \frac{1}{\sqrt{2\left(1 + |\epsilon_{S,L}|^2\right)}} \left[(1 + \epsilon_{S,L})\, K^0 + (1 - \epsilon_{S,L})\, \bar{K}^0 \right]$$

$$\epsilon_{S,L} = \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$

$$= \epsilon_M \mp \Delta$$

$$\epsilon_M \equiv |\epsilon_M| e^{i\varphi_{SW}} \qquad \tan \varphi_{SW} = \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$$

$C\not{P}T$ in semileptonic decays

$$\begin{aligned} A(K^0 \to l^+ \nu \pi^-) &= a + b \\ A(K^0 \to l^- \nu \pi^+) &= c + d \\ A(\bar{K}^0 \to l^- \nu \pi^+) &= a^* - b^* \\ A(\bar{K}^0 \to l^+ \nu \pi^-) &= c^* - d^* \end{aligned}$$

b, d $C\not{P}T$, $\quad c, d \quad \Delta S \neq \Delta Q$ Dell'Agnello

$$\delta_{S,L} = \frac{\Gamma^{l^+}_{S,L} - \Gamma^{l^-}_{S,L}}{\Gamma^{l^+}_{S,L} + \Gamma^{l^-}_{S,L}} = 2\Re(\epsilon_{S,L}) + 2\Re\left(\frac{b}{a}\right) \mp 2\Re\left(\frac{d^*}{a}\right)$$

- $\delta_S - \delta_L \propto \Re\Delta, \Re(\frac{d^*}{a})$

$C\not{P}T$ in $K \to \pi\pi$

$$A(K^0 \to \pi\pi(I)) \equiv (A_I + B_I)e^{i\delta_I}$$
$$A(\bar{K}^0 \to \pi\pi(I)) \equiv (A_I^* - B_I^*)e^{i\delta_I}$$

- B_I is $C\not{P}T$ as $\left(\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}} \quad \eta_{00}=|\eta_{00}|e^{i\phi_{00}}\right)$

$$\phi_{+-} - \phi_{00} = 0.22 \pm 0.45 \qquad \text{KTEV,NA48}$$

Bell-Steinberger relation and $C\not{P}T$

- Even if $C\not{P}T$ unitarity must be valid. Then $|K(t)\rangle = a_S|K_S\rangle + a_L|K_L\rangle$

$$-\frac{d}{dt}|\langle K(0)|K(0)\rangle|^2 = \sum_f |a_S A(K_S \to f) + a_L A(K_L \to f)|^2 \Longrightarrow$$

$$(1 + i\tan\varphi_{SW})\left[\Re(\epsilon_M) - i\Im(\Delta)\right] = \sum_f \alpha_f$$

- $\alpha_f = B^S_{+-}\eta_{+-},\ B^S_{00}\eta_{00},\ B^S_{+-\gamma}\eta_{+-\gamma},\ \frac{\tau_L}{\tau_S}B^L_{000}\eta_{000}, ..$
- $\varphi_{SW},\ \epsilon_M,\ \alpha_{\pi\pi},\ \alpha_{\pi\pi\gamma},\ \alpha_{000} \Longrightarrow \Im(\Delta)$ Maiani,Thomson-Zou,KTEV,NA48

New Limit from NA48

- CPLEAR $\Longrightarrow \Im(\Delta) = (4.5 \pm 5.0) \times 10^{-5}$
- NA48 $\Longrightarrow \Im(\Delta) = (-1.2 \pm 3.0) \times 10^{-5}$
- Improved , but still not close enough to the neutrino sector

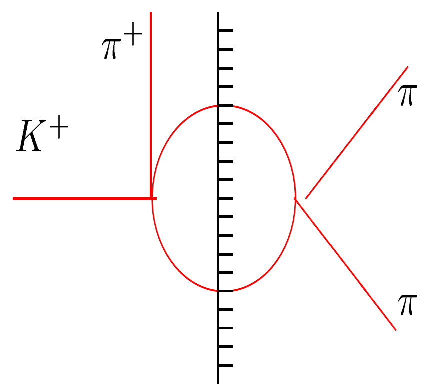
$K \to 3\pi$ and advantages of KLOE/CPLEAR

- Even in the CP limit $K_S \to 3\pi$ in high angular momentum (and $I = 2$)

$$A(K_S \to \pi^0\pi^+\pi^-) = \gamma X(1 + i\delta_2) - \xi XY$$

γ and ξ $\Delta I = 3/2$ transitions, X, Y-Dalitz variables, δ_2 final state interaction. γ predicted by isospin, ξ, δ_2 by ChPT

Final State Interaction



Zeldovich,Grinstein et al

Isidori,Maiani,Pugliese

$K^+ \to 3\pi$ **asymmetries**

$$A(K^+ \to \pi^+\pi^+\pi^-) \quad = \quad a e^{i\alpha_0} + b e^{i\beta_0} Y + \mathcal{O}(Y^2)$$

$$\frac{g_+ - g_-}{g_+ + g_-} = \left[\frac{\Im b}{\Re b} - \frac{\Im a}{\Re a} \right] \sin\left(\alpha_0 - \beta_0\right) ,$$

$K \to 3\pi$ interferences

Since $\delta_2 \sim 0.1\ Br(K_S \to \pi^0\pi^+\pi^-)$ not very sensitive to δ_2.

$$\int_+ I(\pi^+\pi^-\pi^0, l^\pm\pi^\mp\nu; t) d\phi_{3\pi}\, d\phi_{l\pi\nu} - \int_- I(\pi^+\pi^-\pi^0, l^\pm\pi^\mp\nu; t) d\phi_{3\pi}\, d\phi_{l\pi\nu}$$

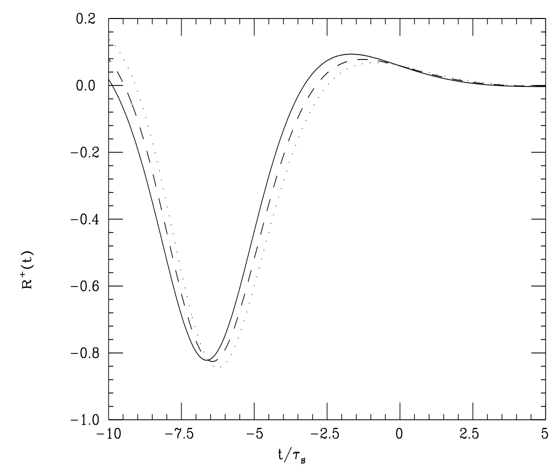
$$\pm 2 \int_+ \Re\left(A_L^{+-0} A_S^{+-0*}\right) d\phi_{3\pi} \left[\cos(\Delta m t) + \widetilde{\delta} \sin(\Delta m t)\right]$$

Kin. cut to select A_S^{+-0*}

CPLEAR $B^S_{+-0} = (3.2^{+1.2}_{-1.0}) \times 10^{-7}$
(ChPT 2.4 ± 0.7)

$\widetilde{\delta} \simeq \delta_{1S} - \delta_2 \qquad 0, 0.2, 0.4$

G.D.,Isidori,Paver,Pugliese



$K_L \to \pi^+\pi^-\gamma$ interferences

$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons $\dfrac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$\downarrow$ Low Theorem $\Rightarrow E_{IB} \sim \dfrac{1}{E^*_\gamma} + c$ $\quad E_D$, M ChPT

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.472 \pm 0.077)10^{-5}$ E787	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is also measured in $K_L \to \pi^+\pi^-e^+e^-$

$K_L \to \pi^+\pi^-\gamma$ **interferences**

- NA48 and KLOE will measure $C\not{P}$ interf. $E1^*EB$ in K^+

$$\frac{\Gamma(K^+ \to \pi^+\pi^0\gamma) - \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma) + \Gamma(K^- \to \pi^-\pi^0\gamma)}$$

- KLOE also with $I(\pi^\pm l^\mp \nu, \pi^+\pi^-\gamma)$ sensitive to $E1^*EB$ in K_L

$$\left[\Re\langle E\rangle_{int}\cos(\Delta m|t|) + \Im\langle E\rangle_{int}\sin(\Delta m|t|)\right]$$

$$\langle E\rangle_{int} \sim \eta_{+-\gamma} = \frac{A(K_L \to \pi^+\pi^-\gamma)_{IB+E1}}{A(K_S \to \pi^+\pi^-\gamma)_{IB+E1}}$$

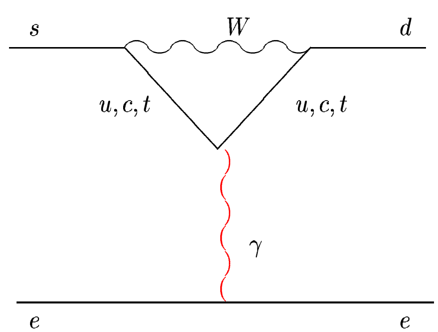
Conclusions

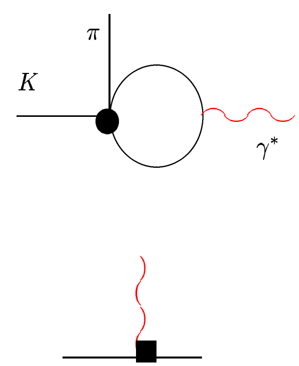
- Has $C\not{P}T$ a benchmark by ν's that a high luminosity φ-factory can attack?
- Bell-Steinberger to improve
- $C\not{P}$ in $K_L \to \pi^+\pi^-\gamma$ testable and interesting (even if NA48 will succeed in K^+)
- Other channells ($K_L \to \pi^+\pi^- e^+ e^-$,..) to study

$$K^{\pm}(K_S) \to \pi^{\pm}(\pi^0)\ell^+\ell^-$$

- short distance << long distance

LD described by form factor W





$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\bar{\mu})$, slopes

- $a_i \quad O(p^4)$ Ecker, Pich, de Rafael

- $b_i \quad O(p^6)$ G.D., Ecker, Isidori, Portoles

- $a_+,\ b_+$ in general not related to $a_S,\ b_S$

- Expt. E865

  $K^+ \to \pi^+ e^+ e^- : \quad a_+ = -0.586 \pm 0.010 \quad b_+ = -0.655 \pm 0.044$

  confirmed in $K^+ \to \pi^+ \mu\overline{\mu}$

- HyperCP has confirmed E865 (02) $(K^+ \to \pi^+ \mu\overline{\mu})$ and put a bound on the CP asymmetry (≤ 0.1)

Problems:	a_i	b_i	same phenomenological size
	p^4	p^6	different theoretical order

Probably explained by large VMD. Then we can just parameterize

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 4.6 \times 10^{-9} a_S^2$$

not predicted but dynamically interesting: $a_S \sim \mathcal{O}(1)$ (?): NA48 Good luck!

$K_S \to \pi^0 e^+ e^-$ at NA48/1 Collaboration at CERN

- 7 events observed (with 0.15 expected background events)

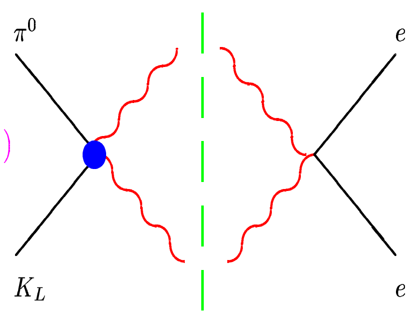
$$B(K_S \to \pi^0 e^+ e^-)_{m_{ee} > 165 \text{ MeV}} = \left(3.0^{+1.5}_{-1.2} \pm 0.2\right) \times 10^{-9}$$

$$|a_S| = 1.08^{+0.26}_{-0.21}$$

$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \le 5 \cdot 10^{-10}$ KTeV

1)

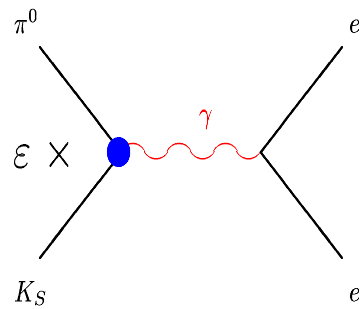


CP conserving NA48

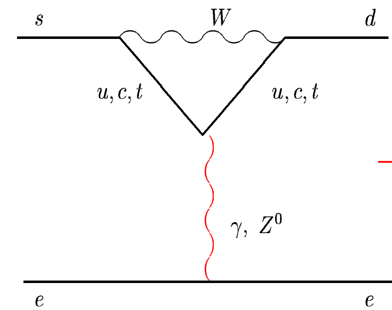
$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \sim 10^{-12}$

V-A⊗ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s}d)_{V-A} (\bar{e}e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon \times$



$+$ 3)



$\longrightarrow \mathrm{Im}\ \lambda_t$

$\lambda_t = V_{td} V_{ts}^*$

$\uparrow$ $\mathrm{B}(K_s \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\ a_S^2 - 6.8\frac{Im\lambda_t}{10^{-4}}\ a_S + 2.8\left(\frac{Im\lambda_t}{10^{-4}}\right)^2\right] \cdot 10^{-12}$$

$$[17.7 \pm \qquad 9.5 + \qquad 4.7] \quad \cdot 10^{-12}$$

- The large slope for $K^+ \to \pi^+ e^+ e^-$ calls for large VMD

- $K^+ \to \pi^+ e^+ e^-$ receives substantial $\pi\pi$-loop, contrary to $K_S \to \pi^0 e^+ e^-$ (~ 0),

- if we split

$$\left(\frac{a_i^{\rm VMD}}{1 - z m_K^2/m_V^2} + a_i^{\rm nVMD} \right) \approx \left[\left(a_i^{\rm VMD} + a_i^{\rm nVMD} \right) + a_i^{\rm VMD} \frac{m_K^2}{m_V^2} z \right]$$

Then we can determine both terms from expt.

$$a_+^{\rm VMD} = \frac{m_V^2}{m_K^2} b_+^{\rm exp} = -1.6 \pm 0.1 \ , \qquad a_+^{\rm nVMD} = a_+^{\rm exp} - a_+^{\rm VMD} = 1.0 \pm 0.1$$

- Also we can hope $a_i^{\rm VMD}$ obey a short distance relation since i) VMD is a larger scale and ii) NOT affected by $\pi\pi$-loop

- The only operator at short distances is $Q_7 = \bar{s}\gamma^\mu(1-\gamma_5)d\,\bar{\ell}\gamma_\mu\ell$,

$$\mathcal{H}_{eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \Big[\sum_{i=1}^{6,7V} (z_i(\mu) + \tau y_i(\mu)) Q_i(\mu) + \tau y_{7A}(M_W) Q_{7A}(M_W)$$

$\tau = -V_{ts}^* V_{td}/V_{us}^* V_{ud}$. The Wilson coefficients $z_7(\mu)$ and $\tau y_7(\mu)$ determine the CPC CPV amplitudes and their relative sign. The isospin structure of Q_{7V} leads

$$(a_S)_{\langle Q_{7V}\rangle} = -(a_+)_{\langle Q_{7V}\rangle}$$

- If this relation is obeyed by the full VMD amplitude

$$(a_S^{\mathrm{VMD}})_{\langle Q_{7V}\rangle} = -a_+^{\mathrm{VMD}} = 1.6 \pm 0.1$$

in good agreement with NA48 $(|a_S| = 1.08^{+0.26}_{-0.21})$