X-Ray Backward Transition Radiation from Periodical Target

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The theory of UV and soft X-ray backward transition radiation is developed. The spectral and angular characteristics of backward TR in X-ray range from the periodical target are obtained.

1. Introduction.

Transition radiation (TR) arises when a charged particle transverses the boundary between two media [1-3]. The radiation source in this case is the polarization current which is created by the charged particle field. Effective size of the boundary area participated in this process is directly proportional to the particle Lorenz-factor and the boundary area effective size can be compared with the boundary inhomogeneity. By this reason TR intensity can depend on the surface profile.

On the other way, TR is well known to be a very good tool for diagnostics of relativistic particles energy [4]. Other possibilities to use TR as a tool for beam diagnostics are connected with using backward TR, i.e. radiation emitted in the direction of mirror-reflection for trajectory of the beam. The best resolution have been reached in this way is connected with using optical range and is about 2 micrometers [5]. This is not sufficient for detecting of sub-microns bunches, which are necessary for modern accelerators, like XFEL or ILC. On the other hand, there is the Rayleigh criterion for resolution, which does not allow getting to sub-microns resolution with optical rays. The way of improving of the diagnostics scheme based on the backward TR may be achieved using more high frequencies, UV or soft X-ray [6, 7].

The transition radiation usually is considered in the frames of macroscopic electrodynamics with the help of boundary conditions. In the case of complicated boundary profile application of the boundary conditions lead to considerable calculating difficulties. However, in the UV and soft X-ray region the TR consideration as the radiation due to polarization currents can simplify this problem considerably [8].

So, it is of interest to estimate the spectral and angular characteristics of backward TR in UV and soft X-ray range of frequencies. We will consider the case of the periodical profile surface, because this case is of special interest for the experiment (there might some sharp peaks to be expected) and it is of convenience to check result expressions by going to the well-known analytical expressions of X-ray forward TR for the planar surface [2].

2. Polarization current in the high frequencies region.

The Fourier transform of polarization current density $\boldsymbol{J}(\mathbf{r},\omega)$ linked with the media permittivity $\varepsilon(\omega)$ and the electric field Fourier transform $\boldsymbol{E}(\mathbf{r},\omega)$ as

$$\boldsymbol{J}(\mathbf{r},\omega) = (i\omega/4\pi)\{1 - \varepsilon(\mathbf{r},\omega)\}\boldsymbol{E}(\mathbf{r},\omega), \qquad (1)$$

In the case of homogeneous media the permittivity co-ordinate dependence linked only with the media surface profile. Let us consider the case, when the media surface is independent of coordinate y. Then the homogeneous media occupied the space region $z < ax + b\cos Qx$ and the boundary media-vacuum is the surface

$$z = ax + b\cos Qx\ ,$$

and the dielectric function has the form

$$\varepsilon(\mathbf{r},\omega) = \varepsilon(\omega)\theta(ax + b\cos Qx) \quad (\theta(u) \equiv [u + |u|]/2|u|). \qquad (2)$$

Let us consider the charge e fly into the media from vacuum with the velocity $\boldsymbol{v}$ in the opposite direction to axis z. In this case the radiation is determined by the polarization current density. For the frequencies that exceed the atomic ones, the permittivity can be written in the form

$$\varepsilon_0(\omega) = 1 - \chi(\omega); \qquad \chi(\omega) = (4\pi e^2 n_0/m\omega^2) << 1 \qquad (3)$$

where n_0 is the electron number density in media, m is the electron mass. Accordingly with our method (see [8], and, also Ch. 4 in [9]) in (1) the field of particle taken as in vacuum $\boldsymbol{E}_0$ rather than of the particle field in media $\boldsymbol{E}$. Taking this into account and passing over co-ordinate Fourier transforms it is easy to obtain

$$\boldsymbol{J}(\boldsymbol{q},\omega) = (i\omega/4\pi)\chi(\omega)\int d^3s\boldsymbol{E}_0(\boldsymbol{q}\text{-}\boldsymbol{s},\omega)(1/8\pi^3)\int d^3r\ exp(i\boldsymbol{rs})\theta(ax+b\ cosQx\ -z) \qquad (4)$$

The field of the charged particle with energy $E = \gamma\ mc^2$, moving in vacuum with velocity $\boldsymbol{v}$ along the opposite z axis direction has a form

$$\boldsymbol{E}_0(\boldsymbol{r},t) = \int d^3p\int d\omega\ \boldsymbol{E}_0(\boldsymbol{p},\omega)exp(i\boldsymbol{pr}\text{-}i\omega\ t); \quad \boldsymbol{E}_0(\boldsymbol{p},\omega) = \boldsymbol{E}_0(\boldsymbol{p})\delta(\omega + p_z v). \qquad (5)$$

$$\boldsymbol{E}_0(\boldsymbol{p}) = \frac{ie\{\omega v\boldsymbol{e}_z + \boldsymbol{p}c^2\}}{2\pi^2\{p^2c^2 - \omega^2\}}. \qquad (6)$$

and one hasinstead of (4)

$$\boldsymbol{J}(\boldsymbol{k},\omega) = (\omega/16\pi^3)\ \chi(\omega)\int dp_x\ \boldsymbol{E}_0(p_x,k_y,\omega/v)\times$$
$$\times\{1/(\omega+k_z v)\}\int_{-\infty}^{\infty} dx\ exp\{i(p_x\text{-}k_x - ak_z - a\omega/v)x - b(k_z+\omega/v)cos\ Qx\} \qquad (7)$$

Using the well-known expression [10]

$$\int_{-\infty}^{\infty} dx(1;x)\{1/(x^2+G^2)\}exp(ixb) = (1;-iGsgnb)(\pi/G)\ exp(-|b|G) \qquad (8)$$

and (6) it is easy to get

$$\int dp_x\ exp(ip_x x)\boldsymbol{E}_0(p_x,k_y,\omega/v) =$$
$$= (ie/2\pi)\{i\boldsymbol{e}_x sgnx + (\boldsymbol{e}_y k_y + \boldsymbol{e}_z u)/G\}\ exp(-G|x|) \qquad (9)$$

where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ – are unit vectors of co-ordinate axis , $G = (k_y^2 + \omega^2/v^2\gamma^2)^{1/2}$,

$u = (\omega/v)(1+v^2/c^2)$. From (7) and (9) it follows

$$\boldsymbol{J}(\boldsymbol{k},\omega) = \frac{e\omega\,\chi(\omega)}{32\pi^4(\omega + k_z v)} \times \tag{10}$$

$$\times\int_{-\infty}^{\infty} dx\, \{\boldsymbol{e}_x \operatorname{sgn} x - i(\boldsymbol{e}_y k_y + \boldsymbol{e}_z u)/G\} \exp\{-|x|G - ik_x x - i(k_z + \omega/v)(ax + b\cos Qx)\}$$

3. Frequency-angular distribution of radiation.

The frequency-angular distribution of radiation, generated by arbitrary current density

$$\boldsymbol{J}(\boldsymbol{r},t)=\int d^3q\int d\omega\, \boldsymbol{J}(\boldsymbol{q},\omega) \exp(i\boldsymbol{qr}-i\omega t), \tag{11}$$

can be written as

$$d^2E(\boldsymbol{n},\omega) = (1/c)(2\pi)^6|[\boldsymbol{kJ}(\boldsymbol{k},\omega)]|^2 d\omega\, d\Omega \tag{12}$$

In the case considered we have

$$[\boldsymbol{kJ}(\boldsymbol{k},\omega)] = \frac{e\omega\,\chi(\omega)}{32\pi^4(\omega + k_z v)} \times \tag{13}$$

$$\times\int_{-\infty}^{\infty} dx\{[\boldsymbol{ke}_x] \operatorname{sgn} x - i([\boldsymbol{ke}_y]k_y + [\boldsymbol{ke}_z]u)/G\} \exp\{-|x|G - ik_x x - i(k_z + \omega/v)(ax + b\cos Qx)\}$$

Using the well-known expression [9]

$$\exp(iu\cos Qx) = \Sigma_s\, i^s J_s(u) \exp(isQx) \tag{14}$$

it is possible to written (13) as ($L = k_x + a(k_z+\omega/v)$; $M = b(k_z+\omega/v)$)

$$[\boldsymbol{kJ}(\boldsymbol{k},\omega)] = \frac{e\omega\,\chi(\omega)}{16\pi^4(\omega + k_z v)} \sum_s (-i)^s J_s(M) \times \tag{15}$$

$$\times \frac{\{[\boldsymbol{ke}_x](L+sQ)^2 + ([\boldsymbol{ke}_y]k_y + [\boldsymbol{ke}_z]u)G\}}{G^2 + (L+sQ)^2}$$

The value $[\boldsymbol{kJ}(\boldsymbol{k},\omega)]$ has a maximum for small $G=(k^2\sin^2\theta\,\sin^2\varphi+\omega^2/v^2\gamma^2)^{1/2}$ and $(L+sQ)$. In the case of ultrarelativistic particles G can be small if $\theta\sim 1/\gamma<<1$ and $L=k\sin\theta\,\cos\varphi+a(k\cos\theta+\omega/v)\approx -sQ$. Therefore for fixed values of a,Q,k,θ and φ exists only one value of $s=S$ corresponded to maximum of (15) and it is possible to neglect the another part of sum. After this the distribution of transition radiation can be written as

$$\frac{d^2E(\boldsymbol{n},\omega)}{d\omega d\Omega}=\frac{|e\omega\,\chi(\omega)|^2}{c4\pi^2(\omega+k_z v)^2}|J_S(M)|^2\times$$

$$\times\frac{\{[\boldsymbol{k}\boldsymbol{e}_x](L+SQ)^2+([\boldsymbol{k}\boldsymbol{e}_y]k_y+[\boldsymbol{k}\boldsymbol{e}_z]u)G\}}{G^2+(L+SQ)^2} \tag{16}$$

For example if $a(1+\omega/kv)=-SQ/k$ and $\theta<<1$, then

$$G^2+(L+SQ)^2=k^2\theta^2+\omega^2/v^2\gamma^2+ak^2\theta^3\cos\varphi,$$

the denominator in (16) is small, and if $a\theta\cos\varphi<1$ the radiation intensity will be proportional to $\gamma^2=E^2/m^2c^4$ and increase quadric with the particle energy.

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