Mechanisms of High Energy Charged Particles Beam Deflection by bent crystals. Analogies

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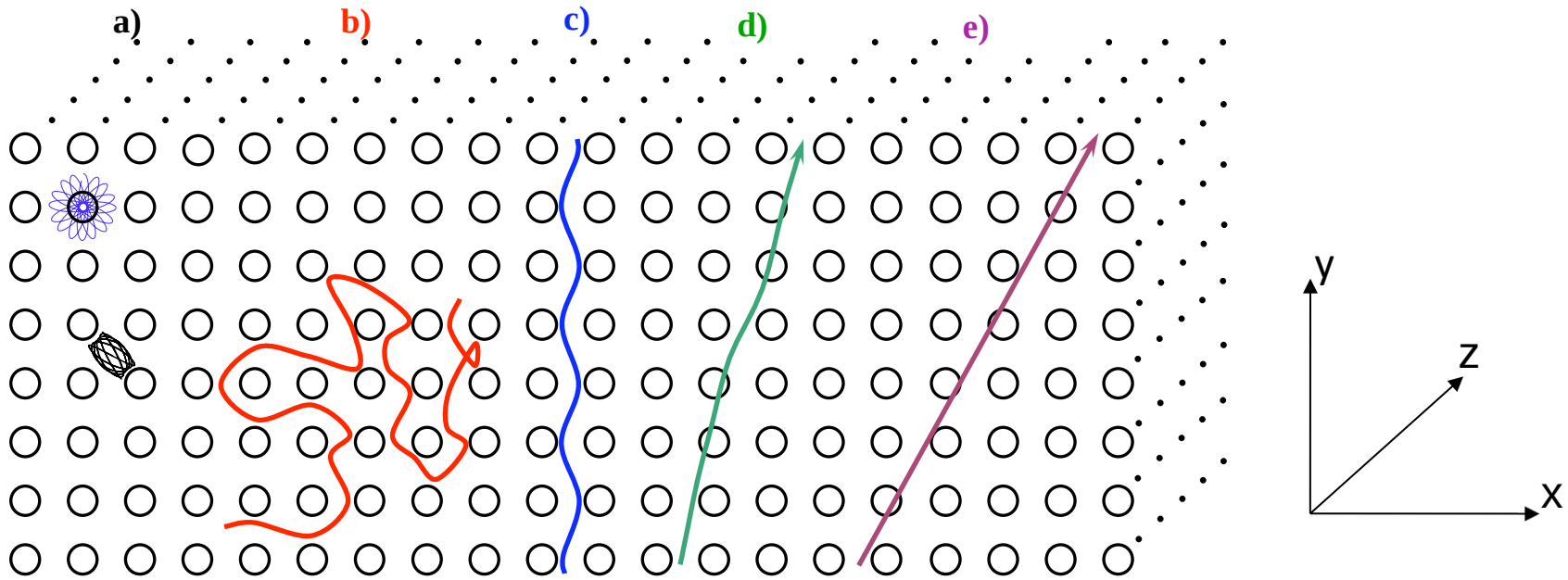
- Planar channeling in a bent crystal
- Volume reflection
- Stochastic mechanism
- CERN Experiments

Mechanisms of Charged Particles Motion near <100> Axis



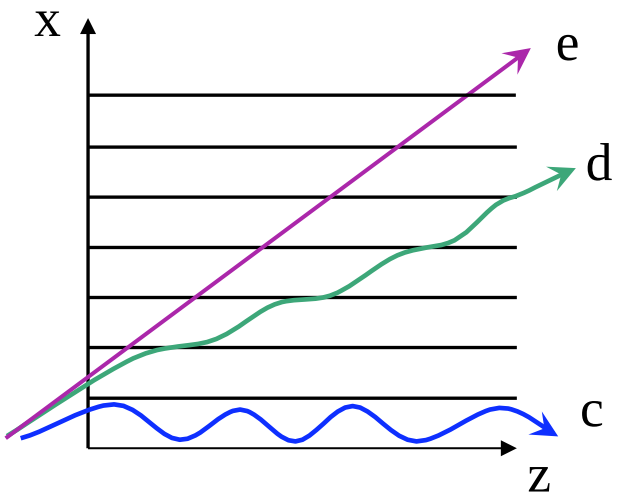
a) Hyperchanneling (e^+, e^-)

b) Stochastic multiple scattering

c) Planar channeling

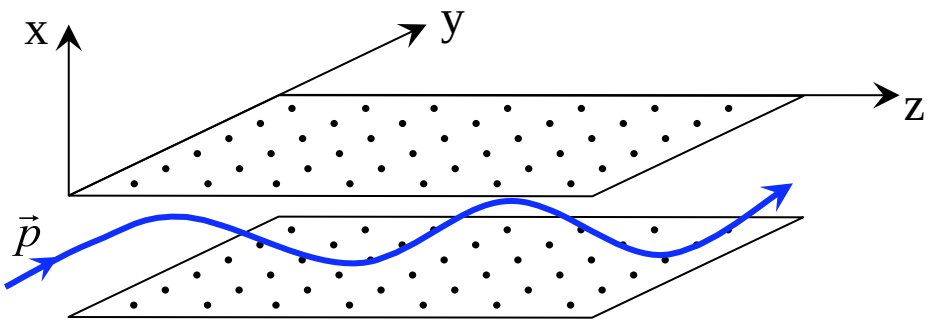
d) Above barrier motion $\varepsilon_\perp \sim U_{pl\max}$

e) Above barrier motion $\varepsilon_\perp >> U_{pl\max}$

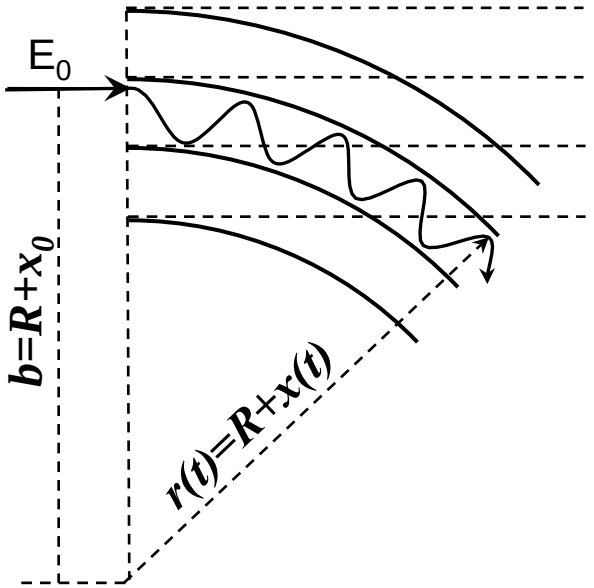


Beam Deflection of fast Charged Particles due to Plane Channeling Effect in Bent Crystal

E.Tsyganov (1976)

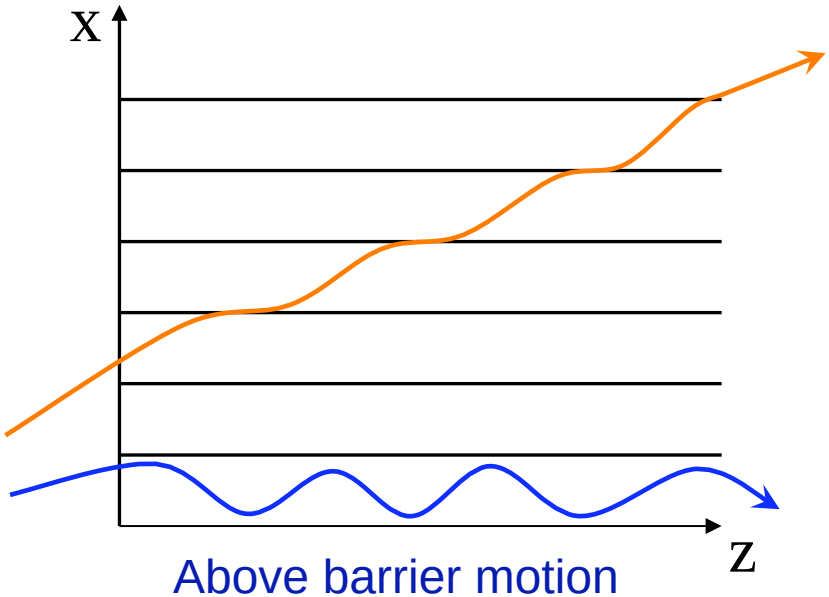


Plane channeling
Lindhard 1965

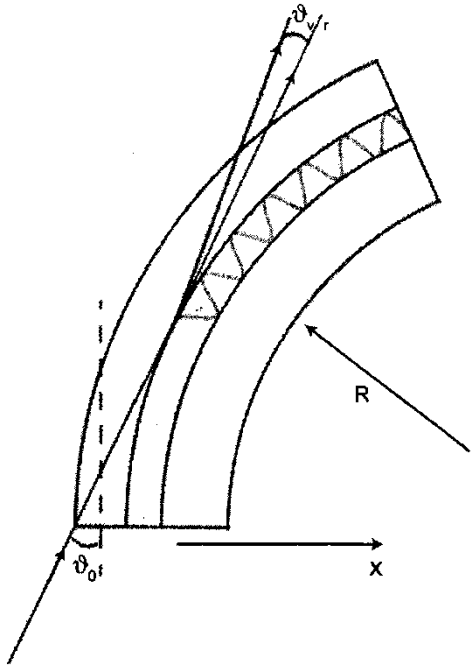


“Volume reflection” effect

A. Taratin, S. Vorobiev 1987

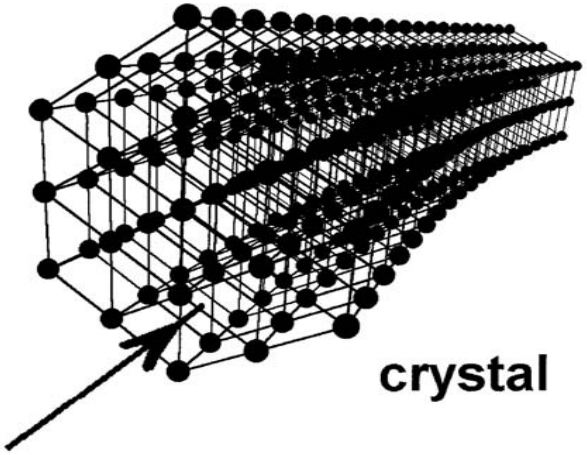


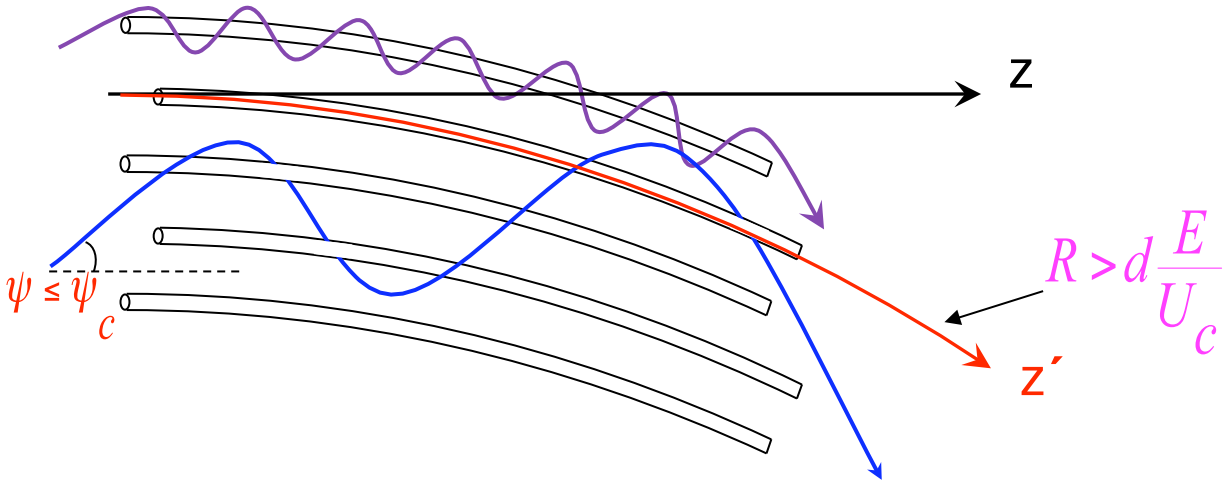
Above barrier motion
Akhiezer, Shul’ga 1978



Stochastic Mechanism of Beam Deflection

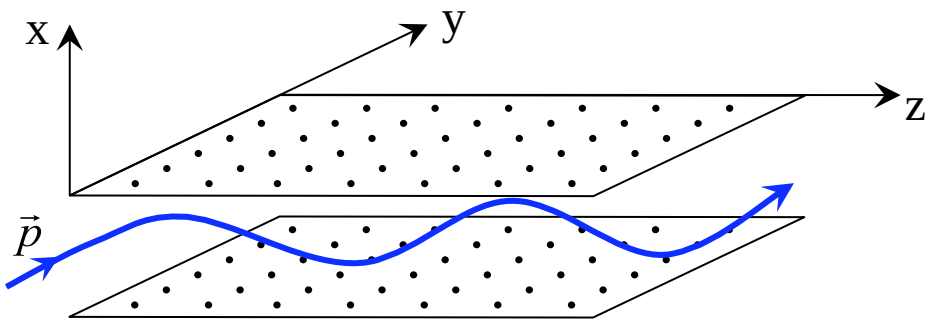
A.Greenenko, N. Shul'ga (1991)





Plane Channeling (Regular Motion)

Lindhard (1965)

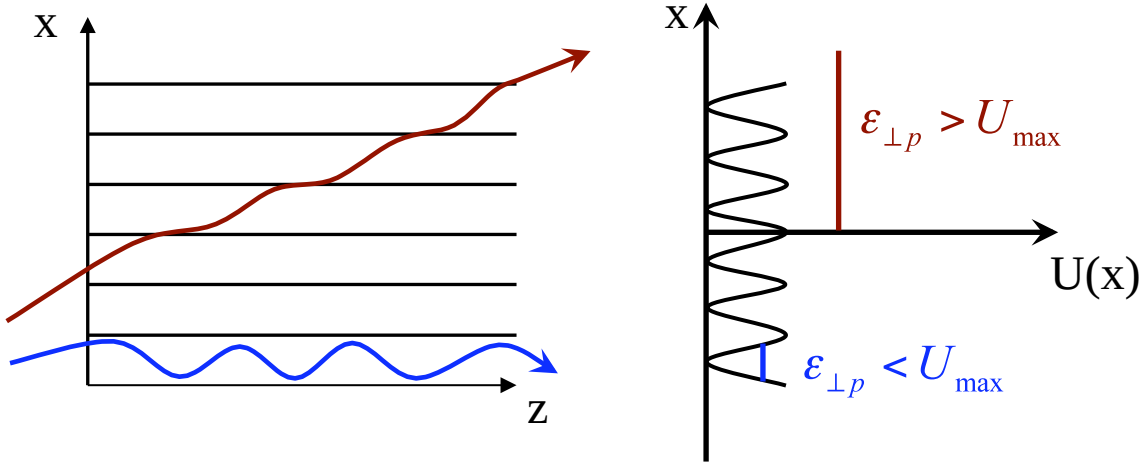


$p_z = const \approx p$

$$\ddot{x} = -\frac{1}{E}\frac{\partial}{\partial x}U_p(x)$$

$$\varepsilon_{\perp p} = \frac{E\,\dot{x}^2}{2} + U_p(x)$$

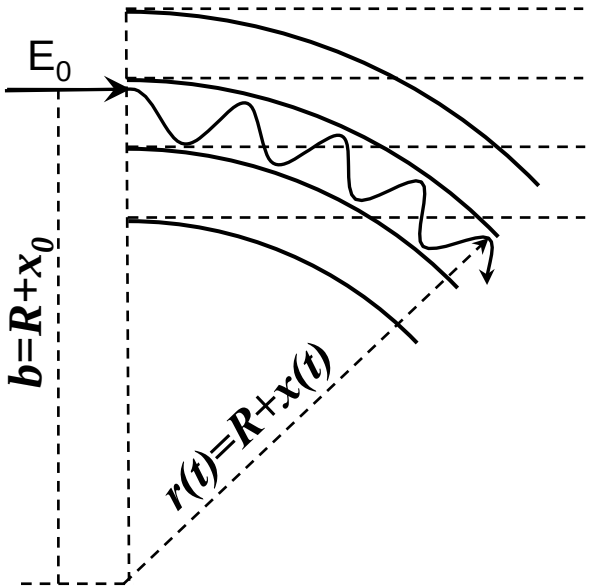
$$U_p(x) = \frac{1}{L_y L_z}\int dy\,dz \sum_n u(\vec{r} - \vec{r}_n)$$

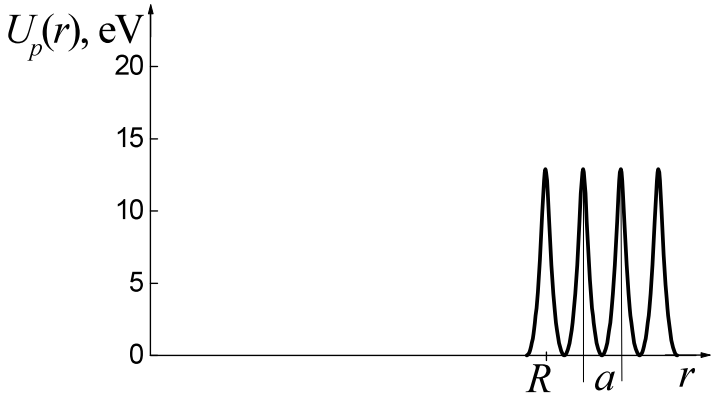


$$\varepsilon_{\perp p} = U_{\max} = \frac{E\,\theta_p^{\,2}}{2}$$

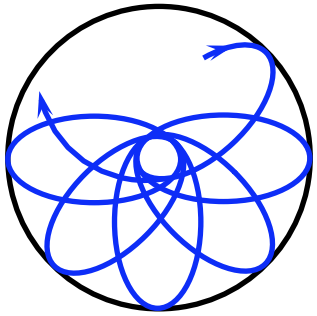
$$\theta_p = \sqrt{\frac{2U_{\max}}{E}}$$

The Motion of Relativistic Particle in Central Field of Bent Crystal Planes





Finite motion, precession



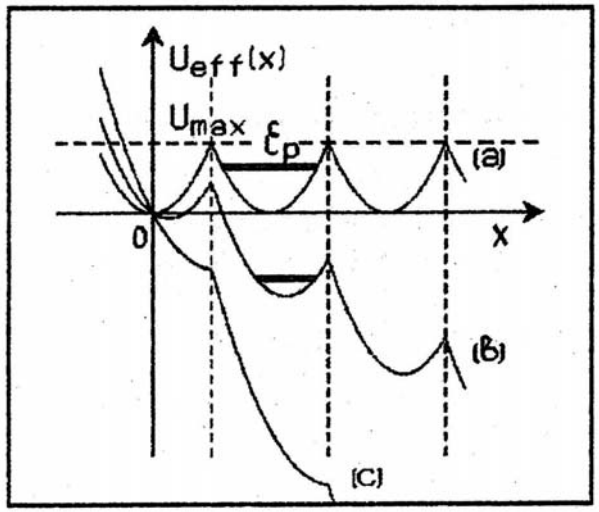
$$\frac{d^2x}{dt^2} = -\frac{1}{E}\frac{\partial}{\partial x}U_{eff}(x)$$

$$U_{eff}(x) = U_p(x) - x\frac{E}{R}$$

A. Akhiezer, N.Shul'ga, A.Greenenko et al., Sov.Phys. Usp. 1995

J. Ellison, Nucl. Phys. B 206 (1982) 205

Critical Radius of Channeling in Bent Crystal



$$U_{eff}(x) = U_{p}(x) - x\frac{E}{R}$$

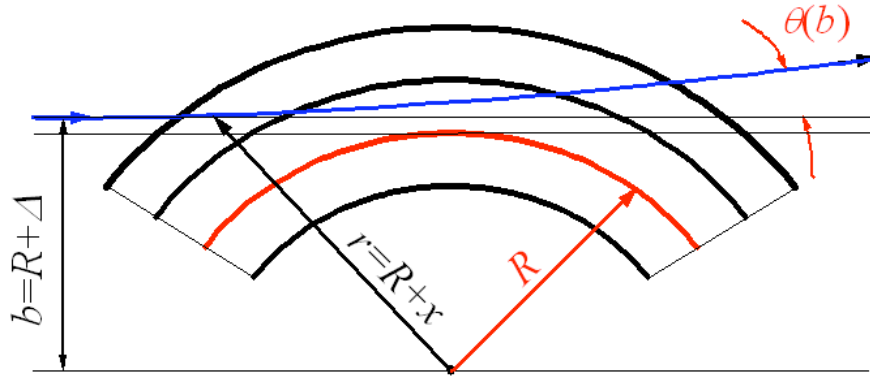
$$\left.\frac{\partial U_{eff}(x)}{\partial x}\right|_{x=d/2} = 0$$

$$R_c = d\frac{E}{2U_{\max}}$$

$$E = 100\,GeV, \quad U_{\max} = 20\,eV, \quad d = 0.2\,nm, \quad \Rightarrow \quad R_c = 25\,cm$$

Beam Reflection from Bent Crystal Planes

N.F. Shul'ga, V.I. Truten', V.V. Boyko, 2009



$$r = R + x, \qquad b = R + \Delta$$

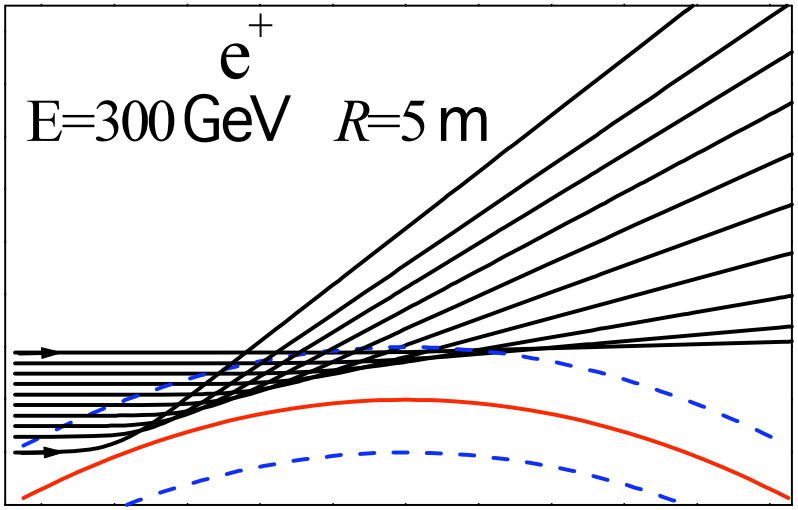
$$R >> x, \quad R >> \Delta, \quad E >> U_p$$

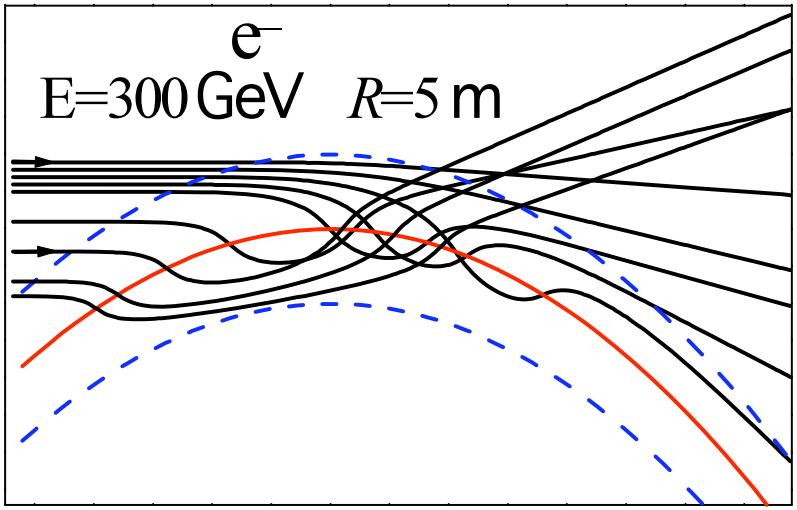
Analogy with scattering in central field

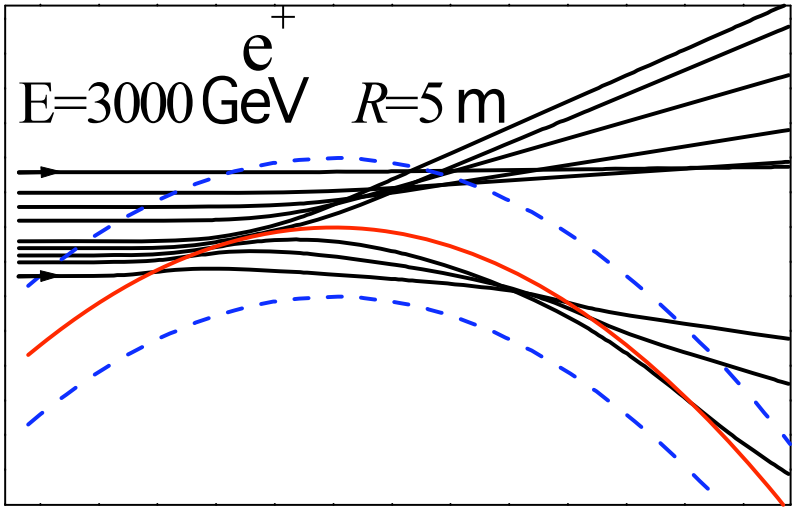
$$\vartheta(b) = \pi - 2M\int_{r_0}^{\infty} \frac{dr/r^2}{\sqrt{(\varepsilon - U_p)^2 - M^2/r^2 - m^2}}, \qquad M = pb$$

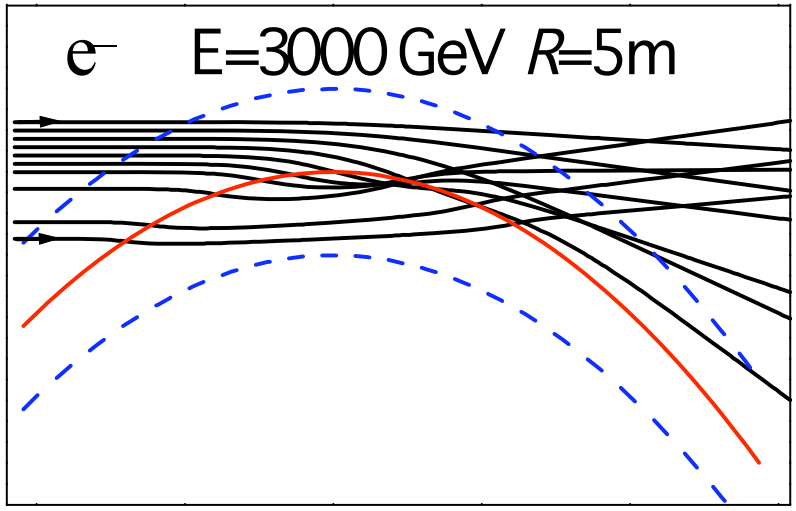
$$\vartheta(\Delta) \sim 2\theta_p, \qquad \theta_p = \sqrt{2U_{\max}/\varepsilon}$$

Scattering on one bent crystal plane





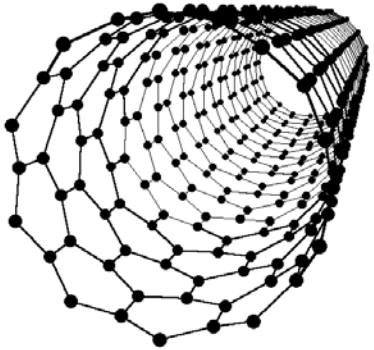




e^+ e^-

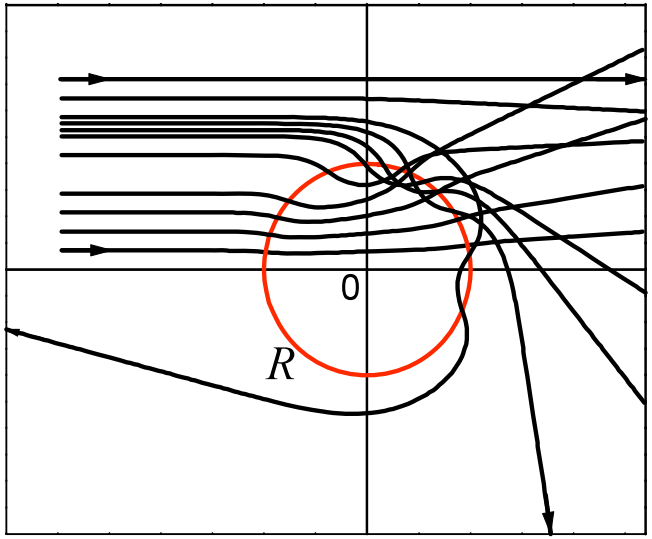
Scattering by Nanotube

N.Shul'ga, V.Truten', (2000)

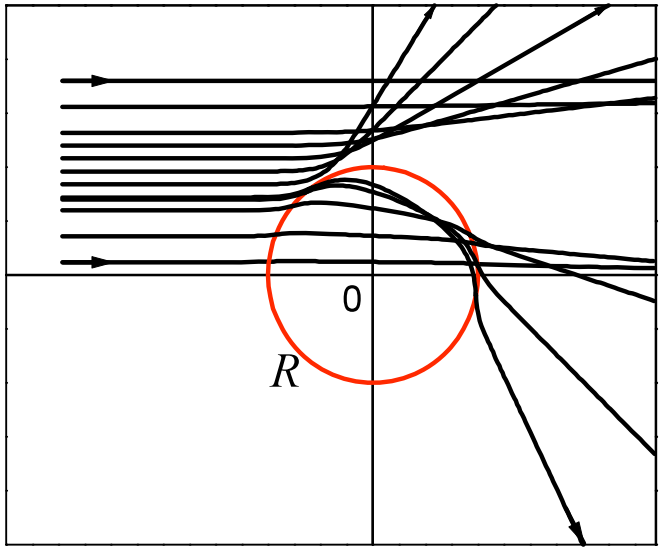


$$R_{nanotube} \approx 10^{-7}\, cm$$

e^+

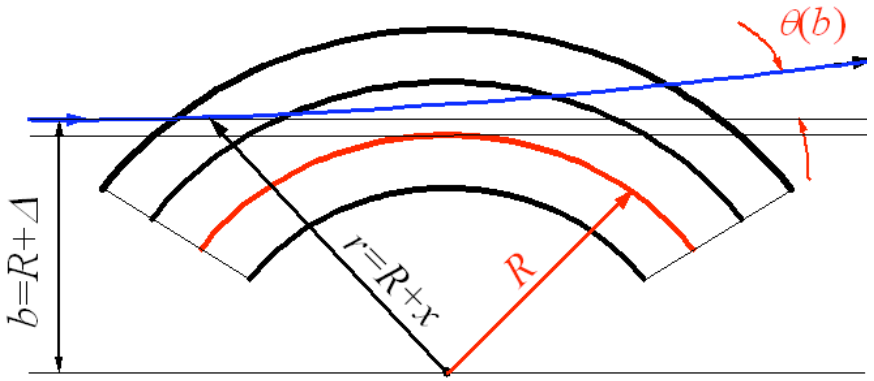


e^-



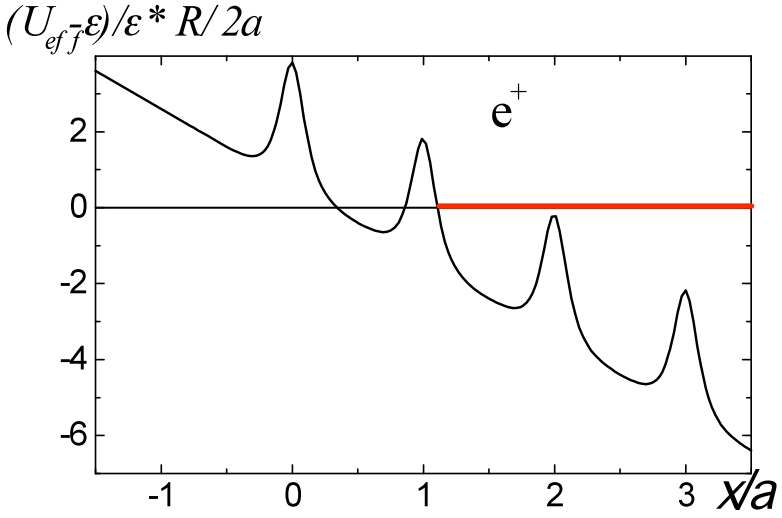
X. Artru et al., Phys. Reports, 2005

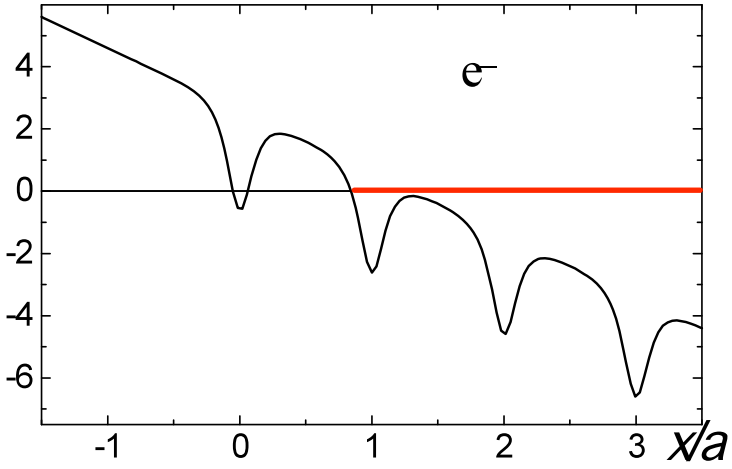
Potential for Beam Reflection by Bent Crystal Planes



$$\vartheta(b) = \pi - 2bv\sqrt{E}\int_{r_0}^{\infty} \frac{dr/r^2}{\sqrt{E - U_{eff}(r,b)}}$$

$$U_{eff}(r,\Delta) = E + 2U(x) + 2E(\Delta - x)/R$$

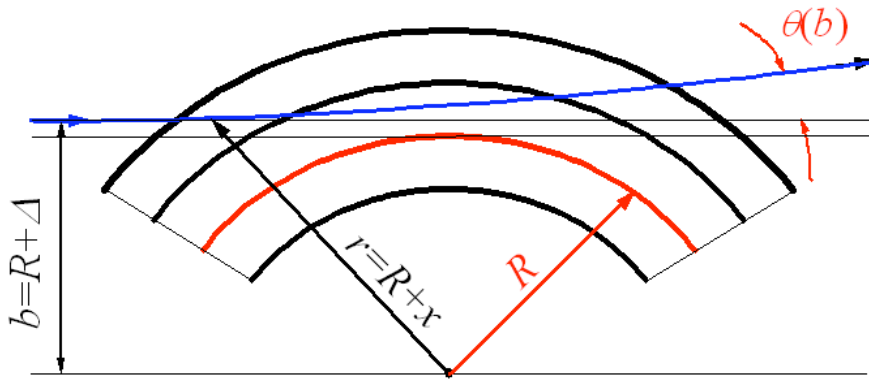


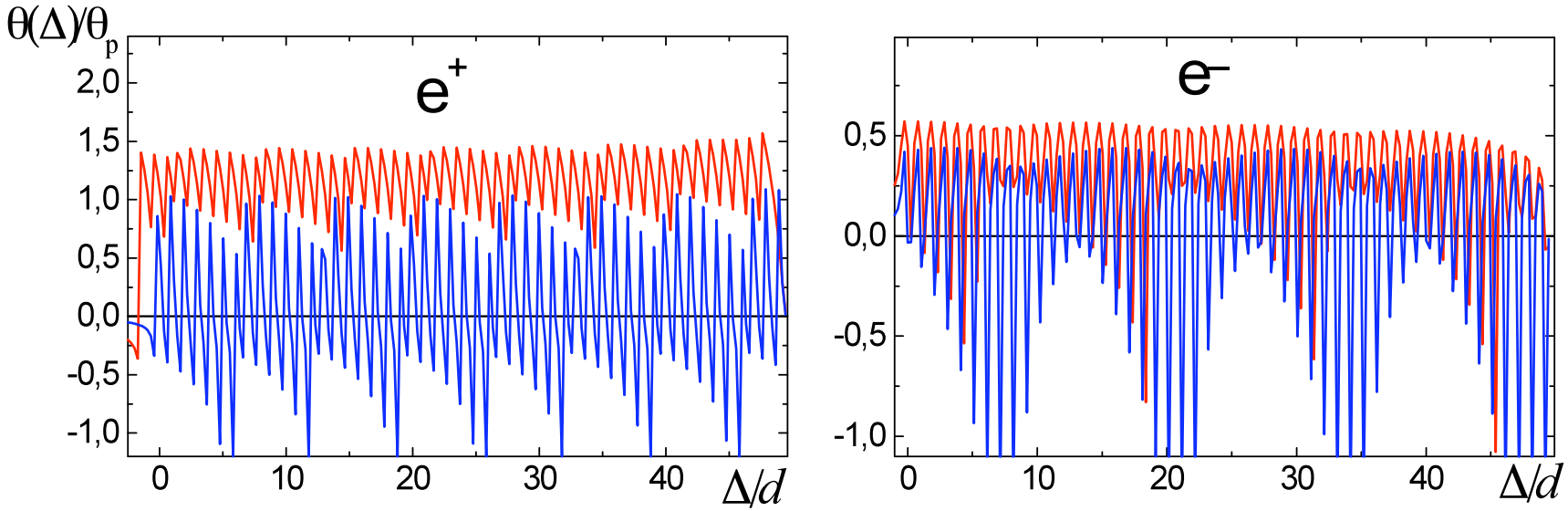


Condition for bending: $\alpha = \frac{4U_0}{E}\frac{R}{d} >> 1$

Deflection Functions for Beam Reflection in Crystal

N.F. Shul'ga, V.I. Truten', V.V. Boyko, 2009





E=300 GeV α=2.

E=3000 GeV α=0.2

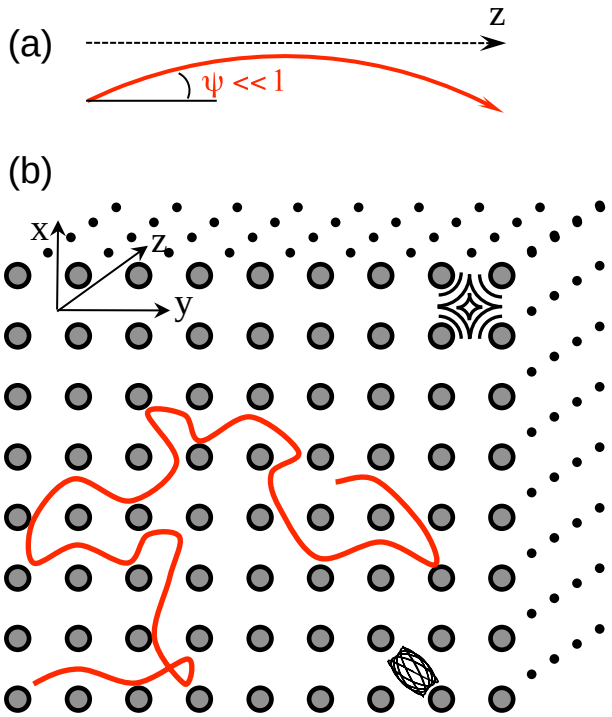
Axial Channeling

$$\frac{d\vec{p}}{dt} = -\vec{\nabla}U(\vec{r})$$

$$U(\vec{r}) \rightarrow U(x,y) = \frac{1}{L_z}\int_{-\infty}^{\infty} dz \sum_n u(\vec{r} - \vec{r}_n)$$

J. Lindhard (1965)

$$\begin{cases} p_z = const \\ \ddot{\vec{\rho}} = -\dfrac{1}{E}\vec{\nabla}U(x,y) \\ p_z >> p_\perp \end{cases}$$

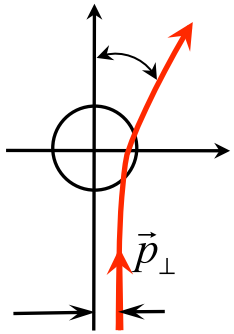


motion
- finite
- infinite
  - scattering by single atomic string
  - multiple scattering by atomic strings

Motion of a fast positively charged particle (a) in the field of a single atomic string and (b) in the periodic field of atomic strings of a diamond crystal in the plane orthogonal to the <100> axis

Scattering by Atomic String

N.Shul'ga, S.Fomin, V.Truten', (1984)

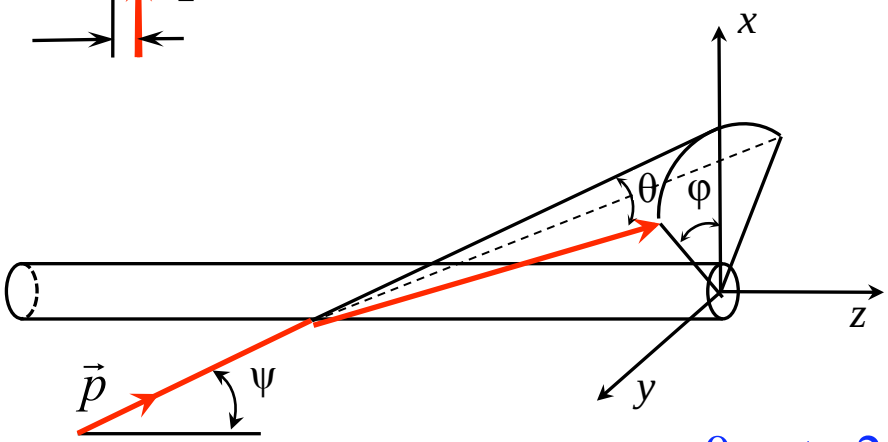


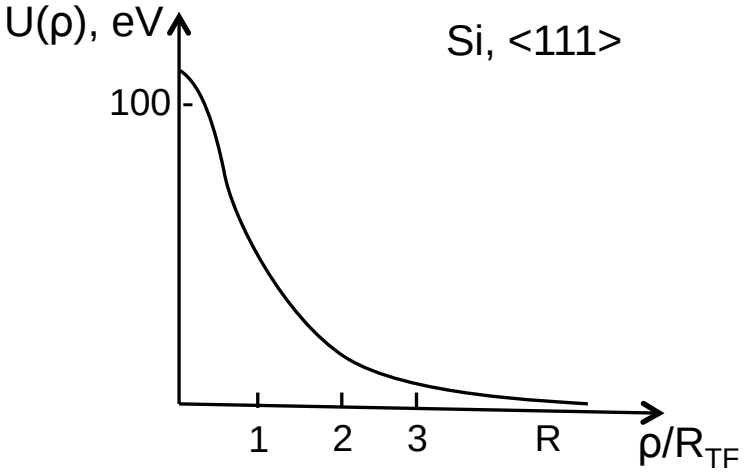
$$p_z = const$$

$$\ddot{\vec{\rho}} = -\frac{1}{E}\vec{\nabla}U(x,y)$$

$$p_z >> p_\perp$$

$$\vec{\rho} = (x,y)$$





$$\theta = \psi \cdot 2\sin\left(\frac{\phi(b)}{2}\right)$$

$$\phi(b) = \pi - 2b\int_{\rho_0}^{\infty} \frac{d\rho}{\rho^2\sqrt{1-\frac{U(\rho)}{\varepsilon_\perp}-\frac{b^2}{\rho^2}}}, \qquad \varepsilon_\perp = \frac{E\psi^2}{2}$$

Multiple Scattering on Atomic strings

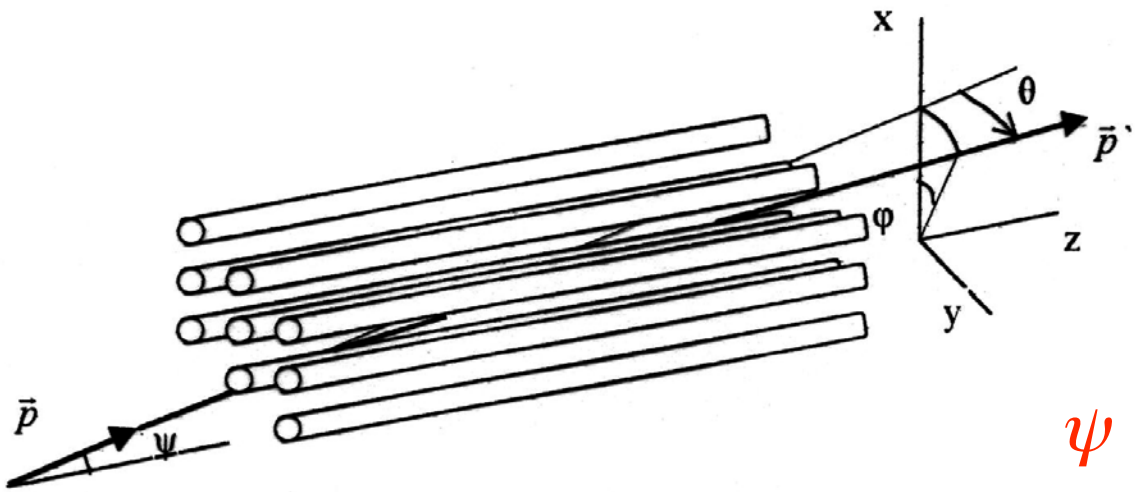


Fig. 3.11.

$$\psi \sim \psi_c$$

$$\psi_c = \sqrt{2U_0/E}$$

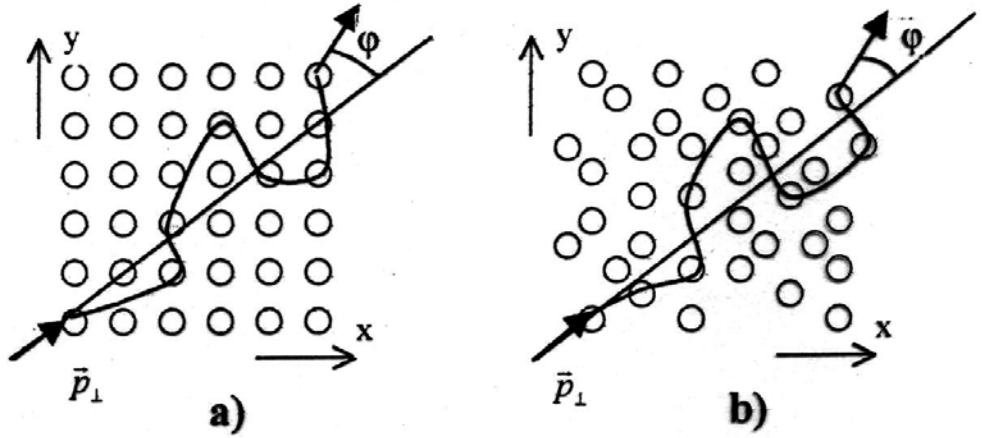


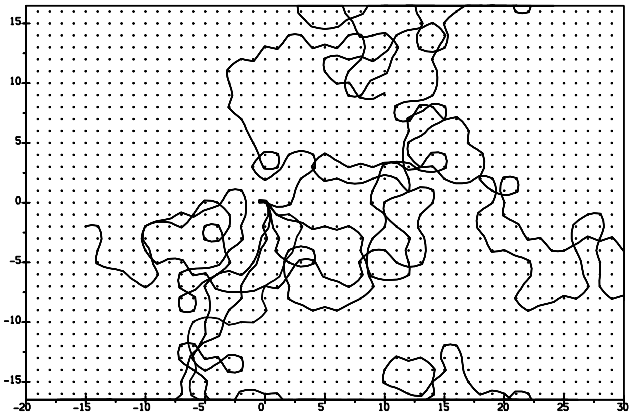
Fig. 3.12.

V. Beloshitskii, M. Kumakhov (1973), $\psi < \psi_c$

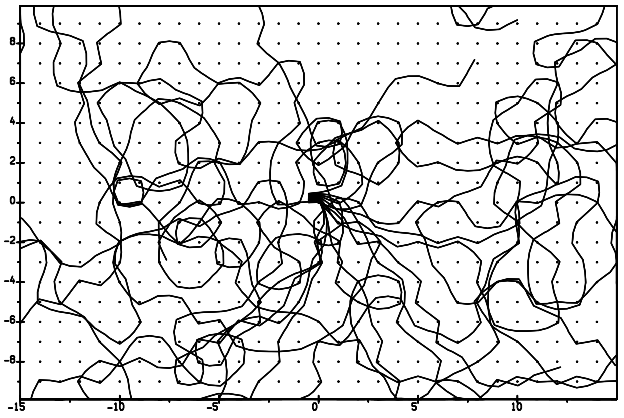
N. Shul'ga, V. Truten', S. Fomin (1982), $\psi > \psi_c$

Dynamical Chaos at Multiple Scattering for e

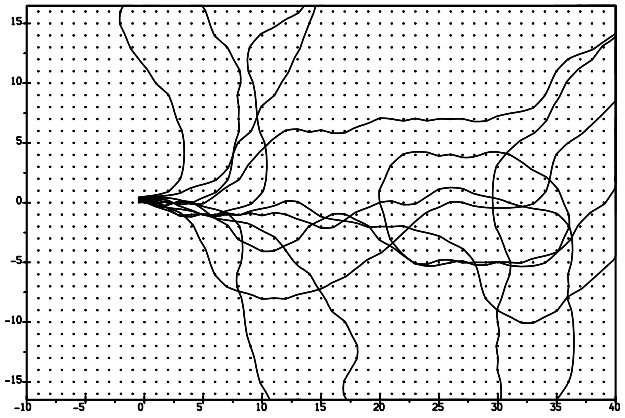
$$z = \psi / \psi_c$$



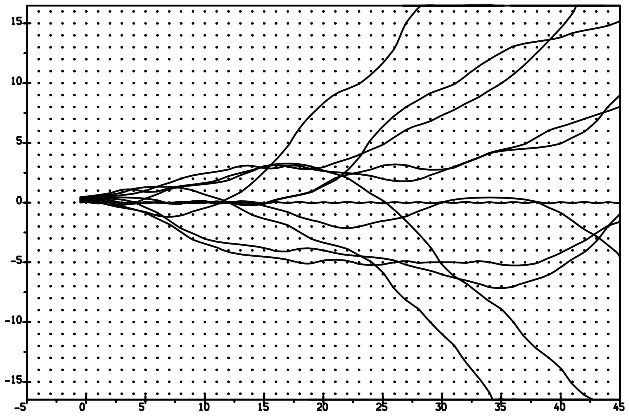
Z=0.5



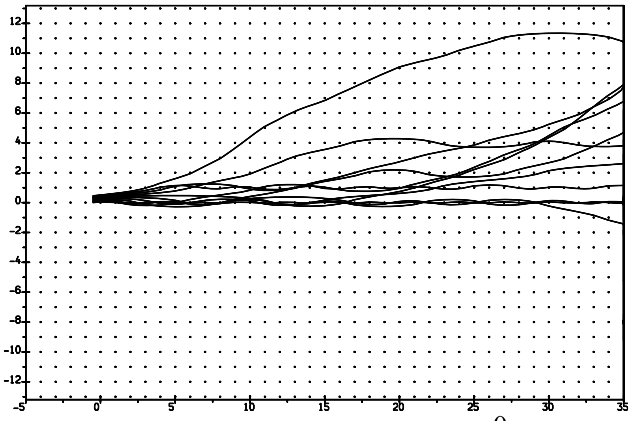
Z=0.7



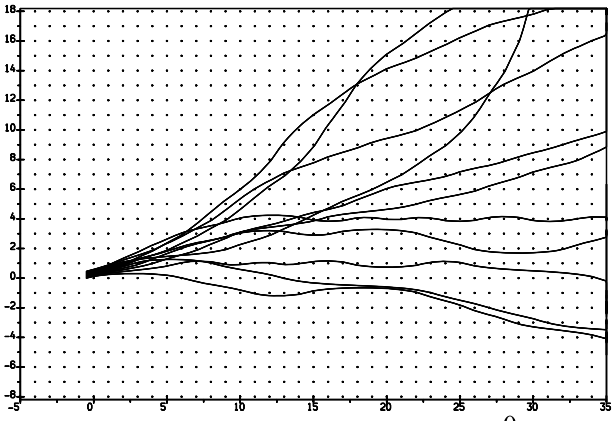
Z=1.5



Z=1



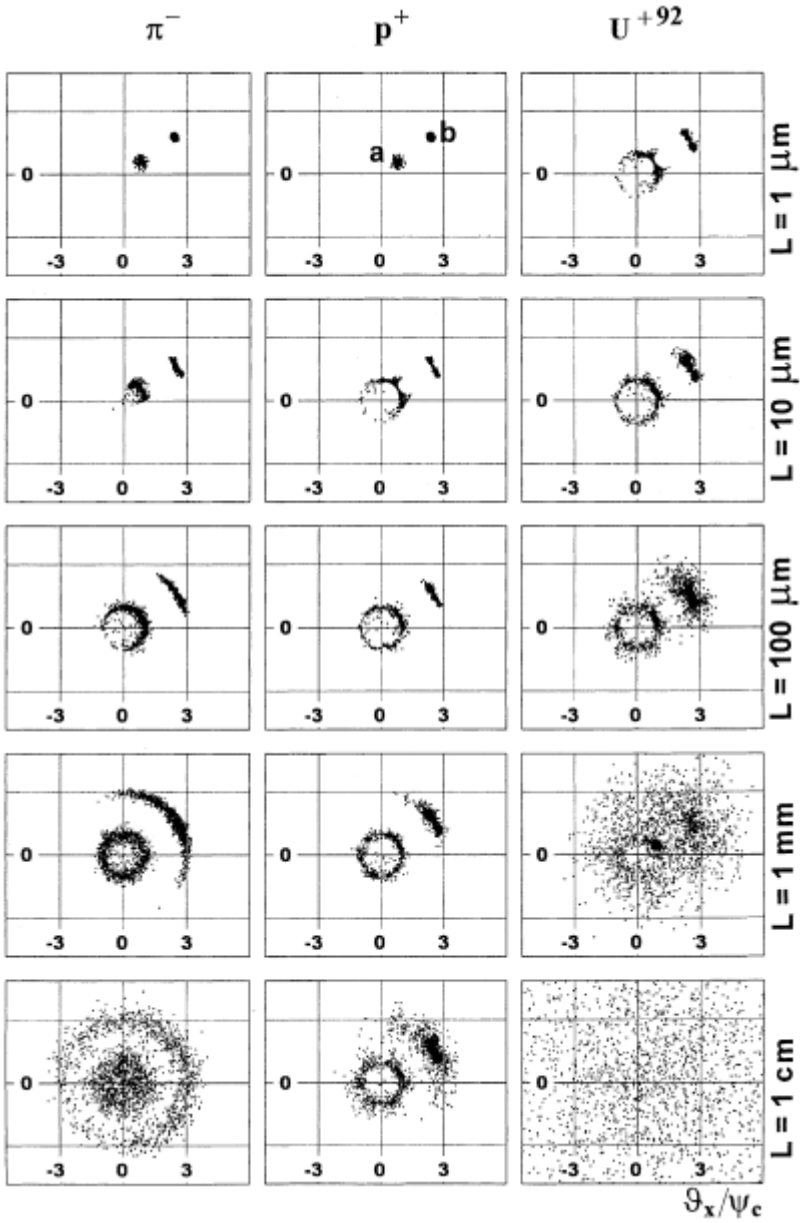
Z=2, $\alpha = 6^0$



Z=2, $\alpha = 15^0$

A. Akhiezer, N Shul'ga, V. Truten', Physics Reports, 1991

Multiple Scattering in Oriented Crystal (simulation)

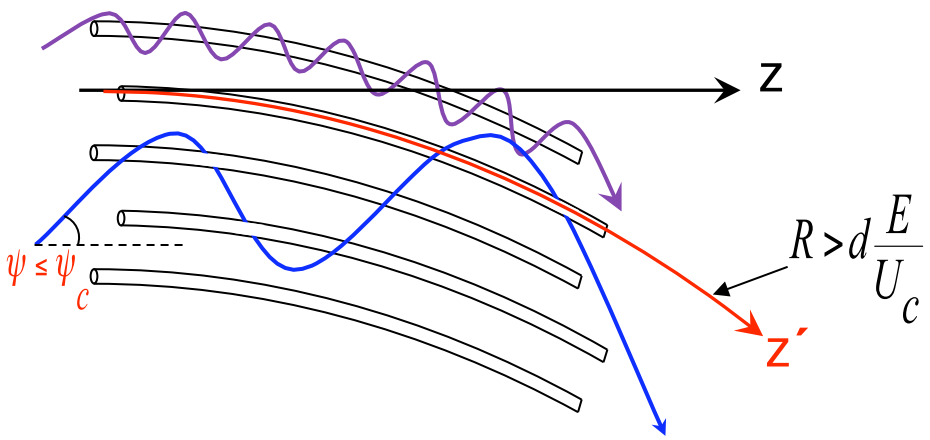


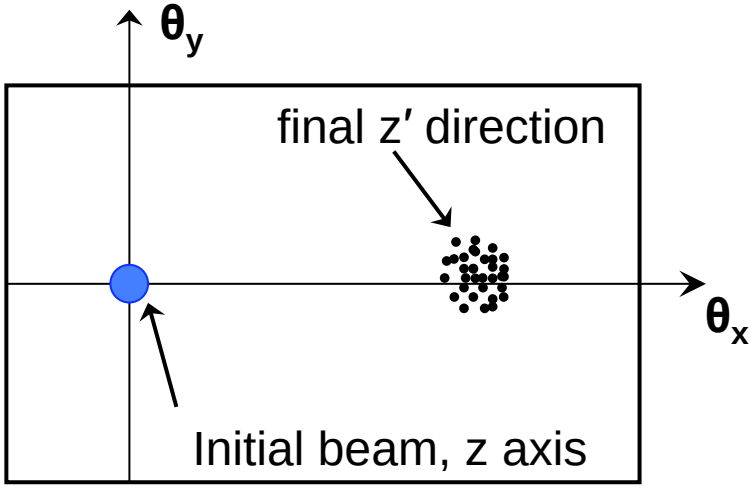
Si, <111>, E=450 GeV

A. Greenenko, N. Shul'ga,
NIM B193(2002) 133

Stochastic Mechanism of Beam Deflection

A. Greenenko, N. Shul'ga (1991)





Condition for bending: $\overline{\Delta\psi^2} < \psi_c^2$

$$\alpha = \frac{l_\perp}{R\psi_c}\frac{L}{R\psi_c} < 1$$

$$l_\perp \sim \frac{1}{\psi_c n d R_{TF}}$$

A.A. Greenenko, N.F. Shul'ga, NIM B **173** (2001) 178

Beam Deflection in Bent Crystal (simulation)

Si, <111>

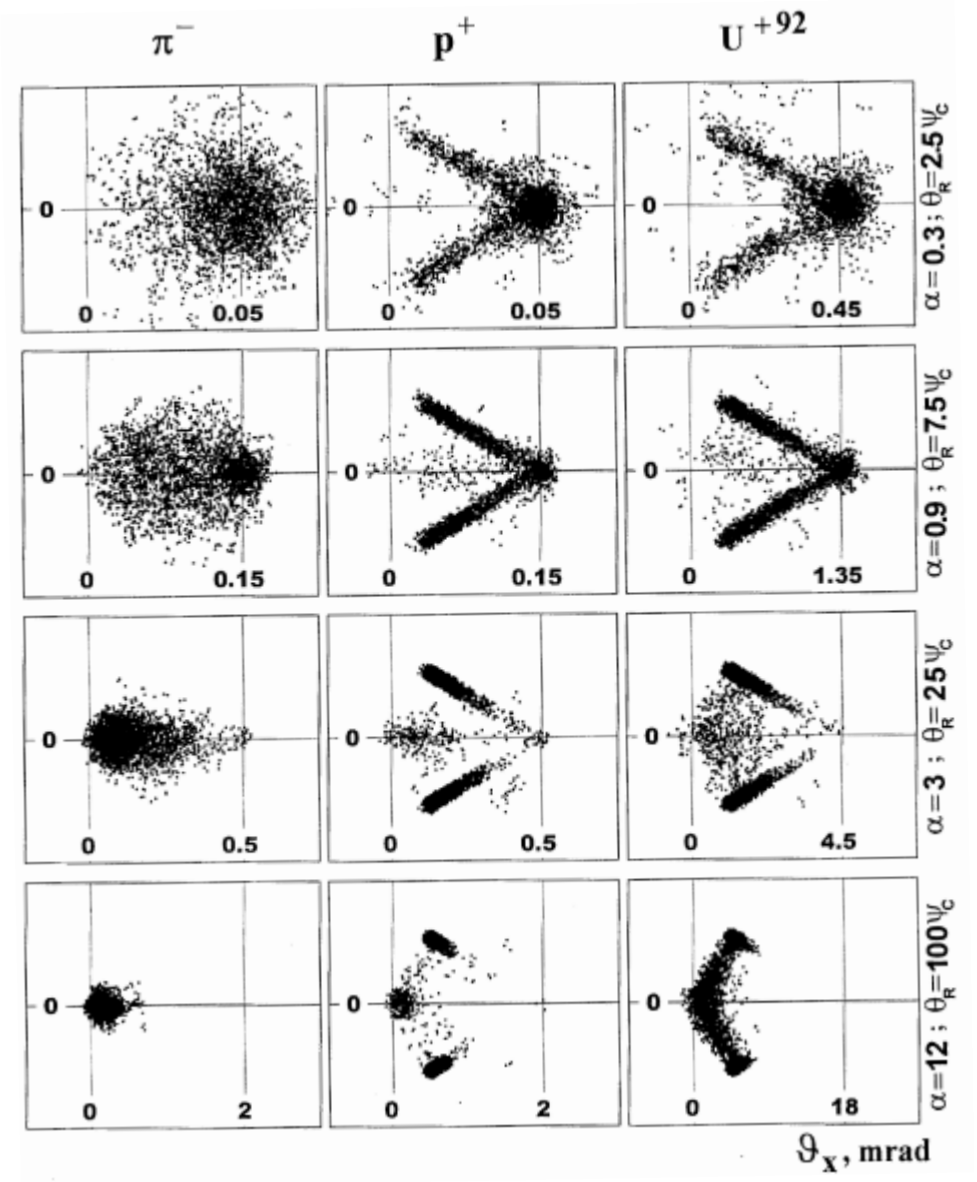
E=450 GeV

$$L_{\max}^{\pi^-,p^+} = 10\,cm$$

$$L_{\max}^{U^{+92}} = 9\,mm$$

$$\alpha = \frac{l_\perp}{R\psi_c}\frac{L}{R\psi_c} < 1$$

$$l_\perp \approx \frac{1}{\psi_c n\, d\, a_{TF}}$$



α = 0.3

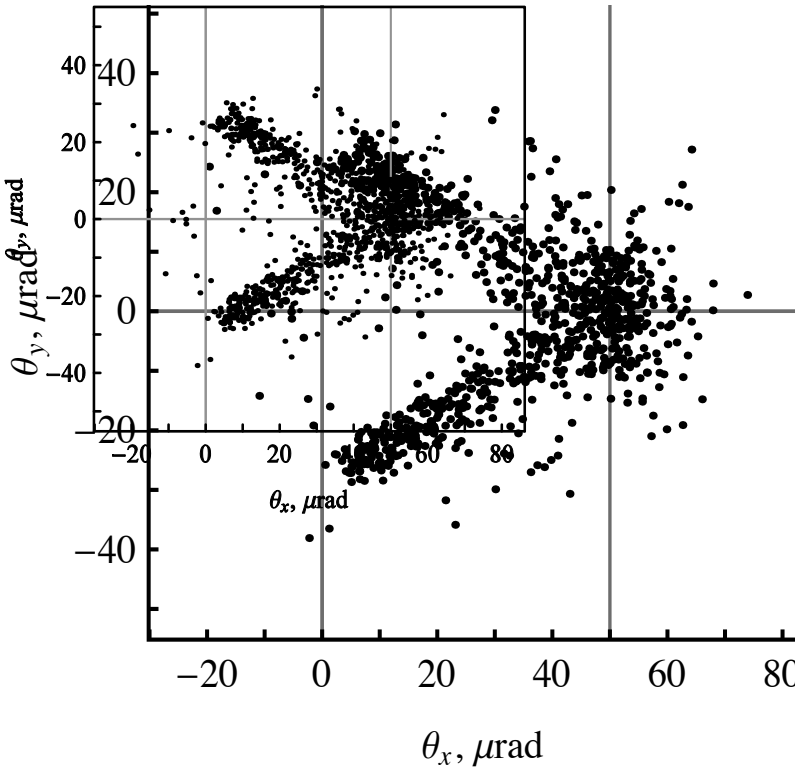
α = 0.9

α = 3

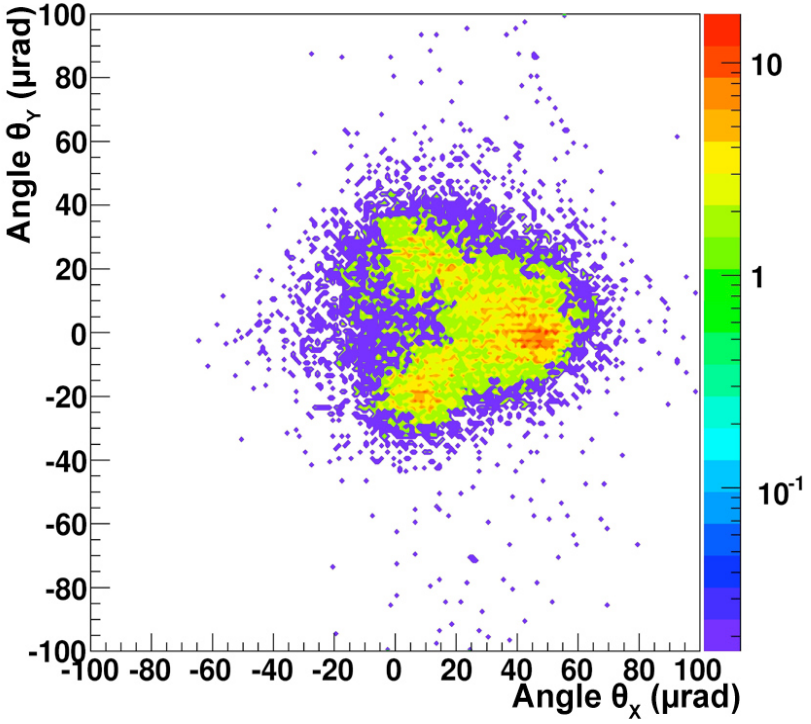
α = 12

R=50 m R=50 m R=50 cm

Angular Distribution of 400 GeV Protons after Passing 2 mm of Bent Si Crystal with R=40 m



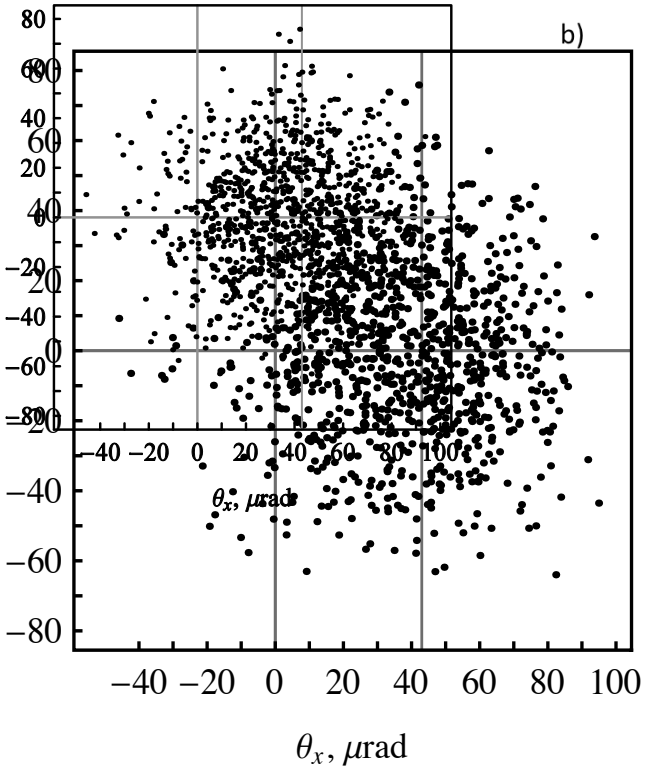
Simulation results



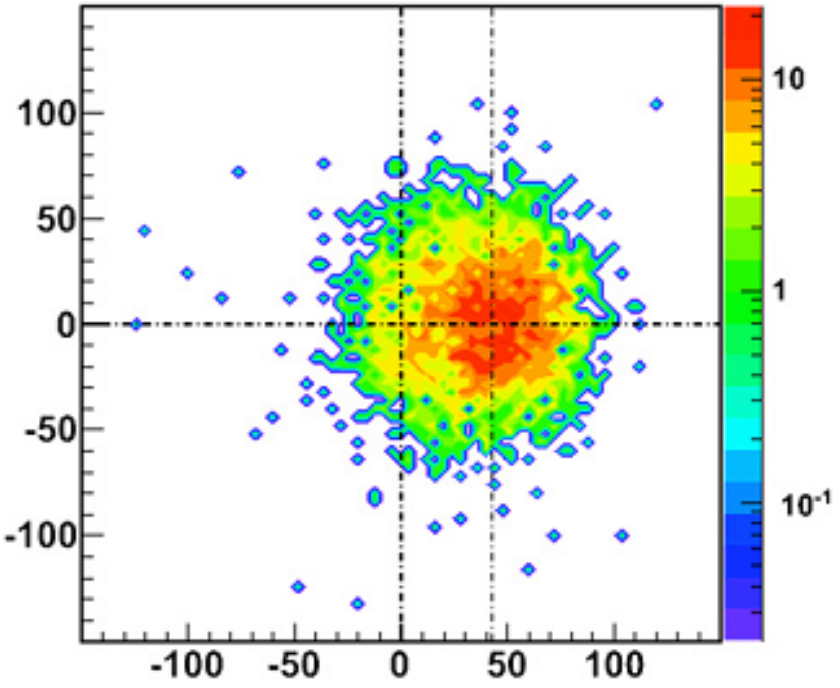
CERN experiment

W. Scandale et al. Phys. Rev. Lett. 101 (2008), 164801

Angular Distribution of 150 GeV π^--mesons after Passing 1.172 mm of Bent Si Crystal with R=40 m



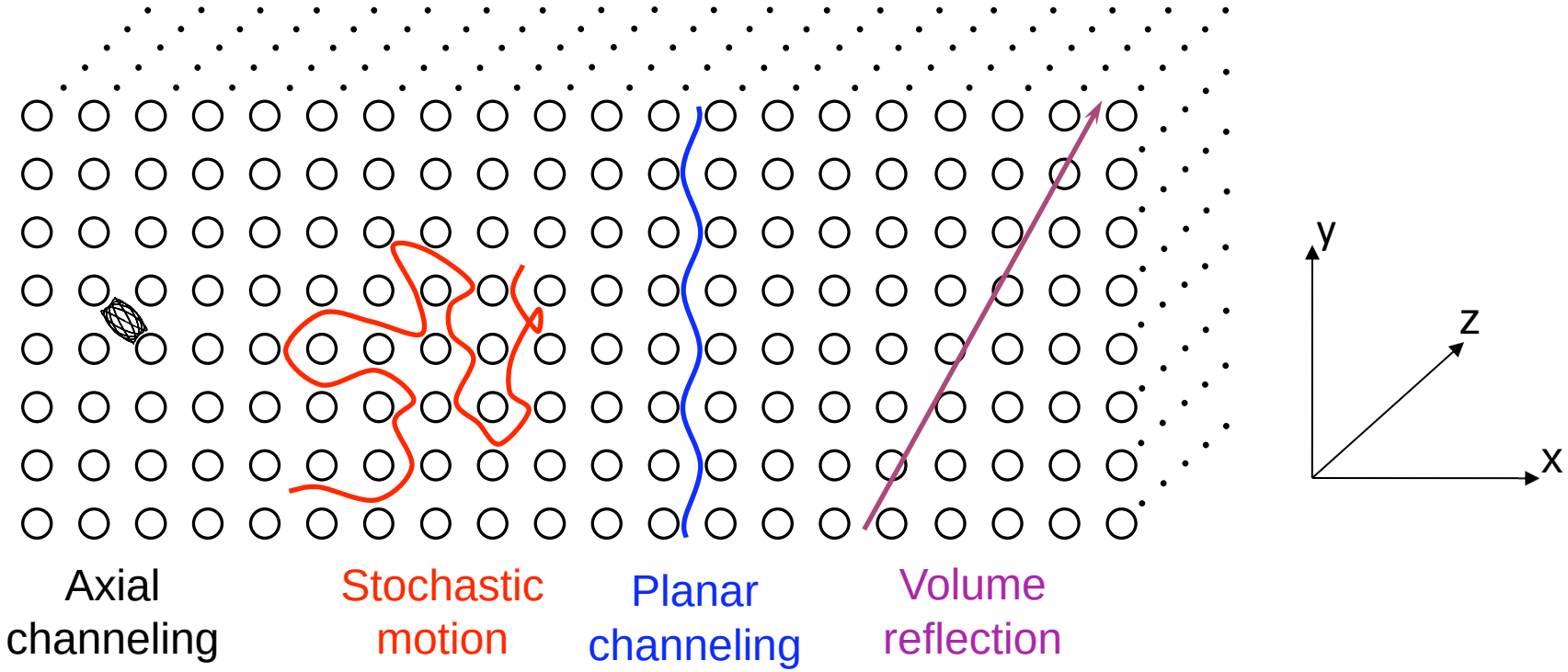
Simulation results



CERN experiment

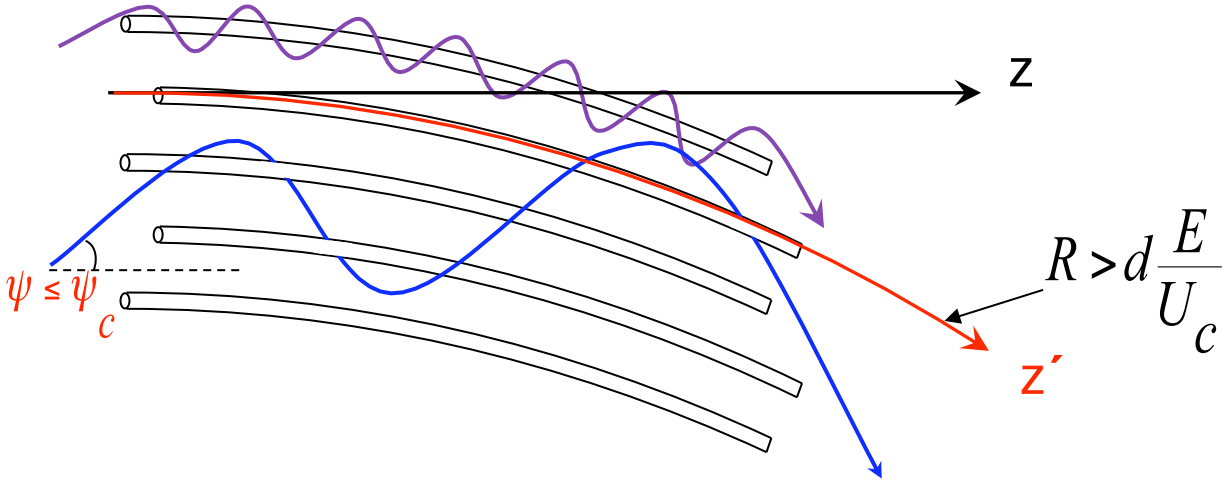
W. Scandale et al. Physics Letters B 680 (2009) 301-304

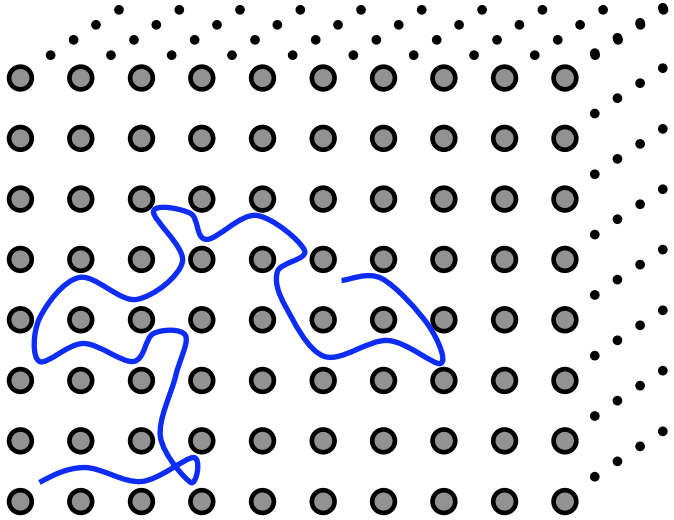
Initial Conditions for Beam Deflection by Bent Crystals

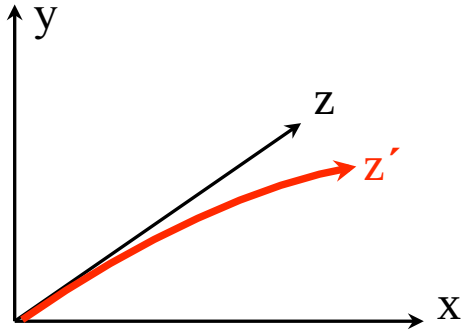


Stochastic Mechanism of Beam Deflection

$$\psi_x \approx \psi_y < \psi_c$$

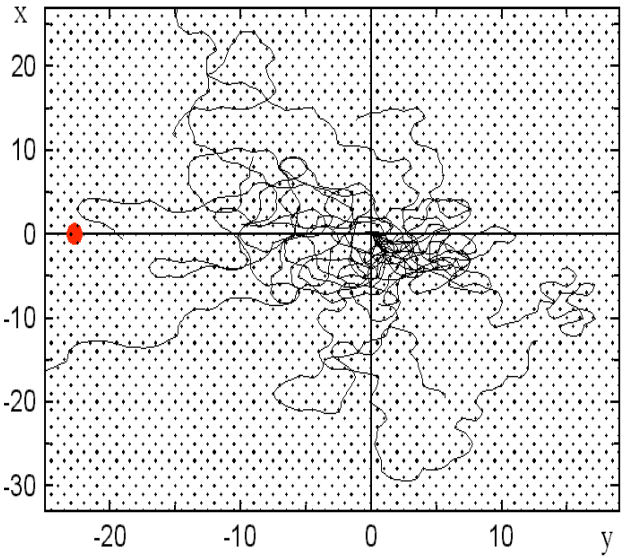


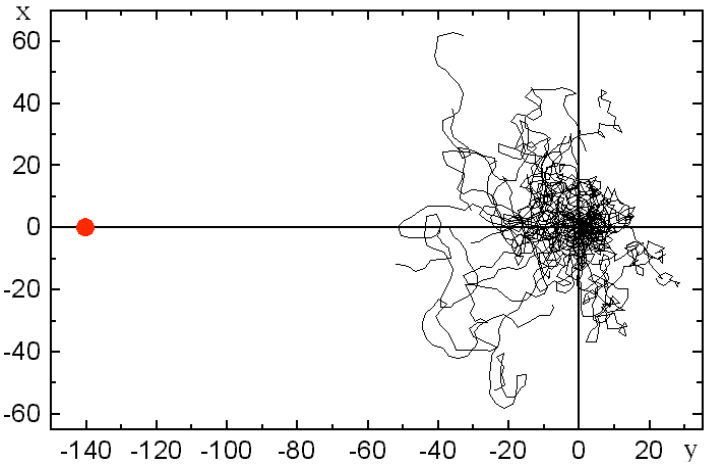




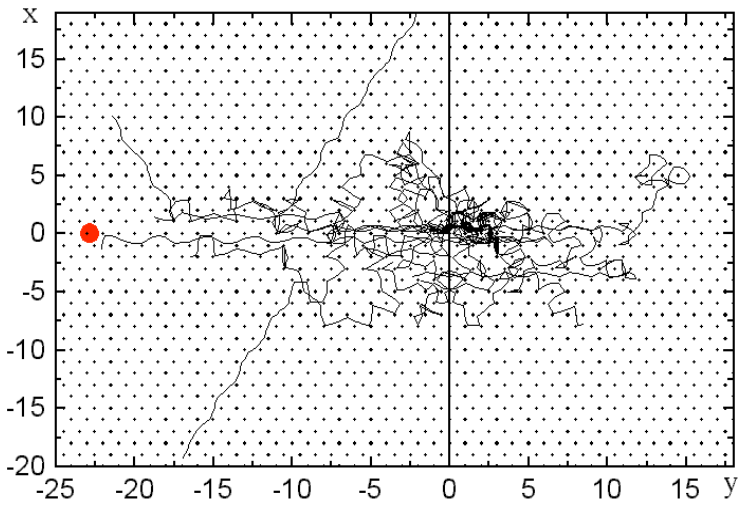
Trajectories in Stochastic Mechanism of Particles Deflection in a Bent Crystal for E=300 GeV, R=100 m

e^-

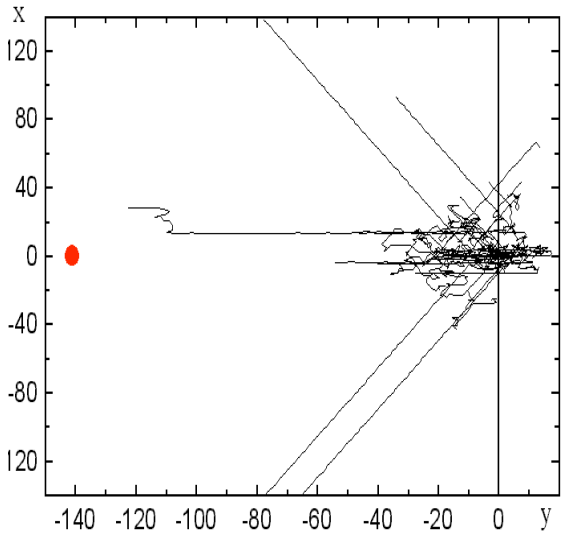




e^+



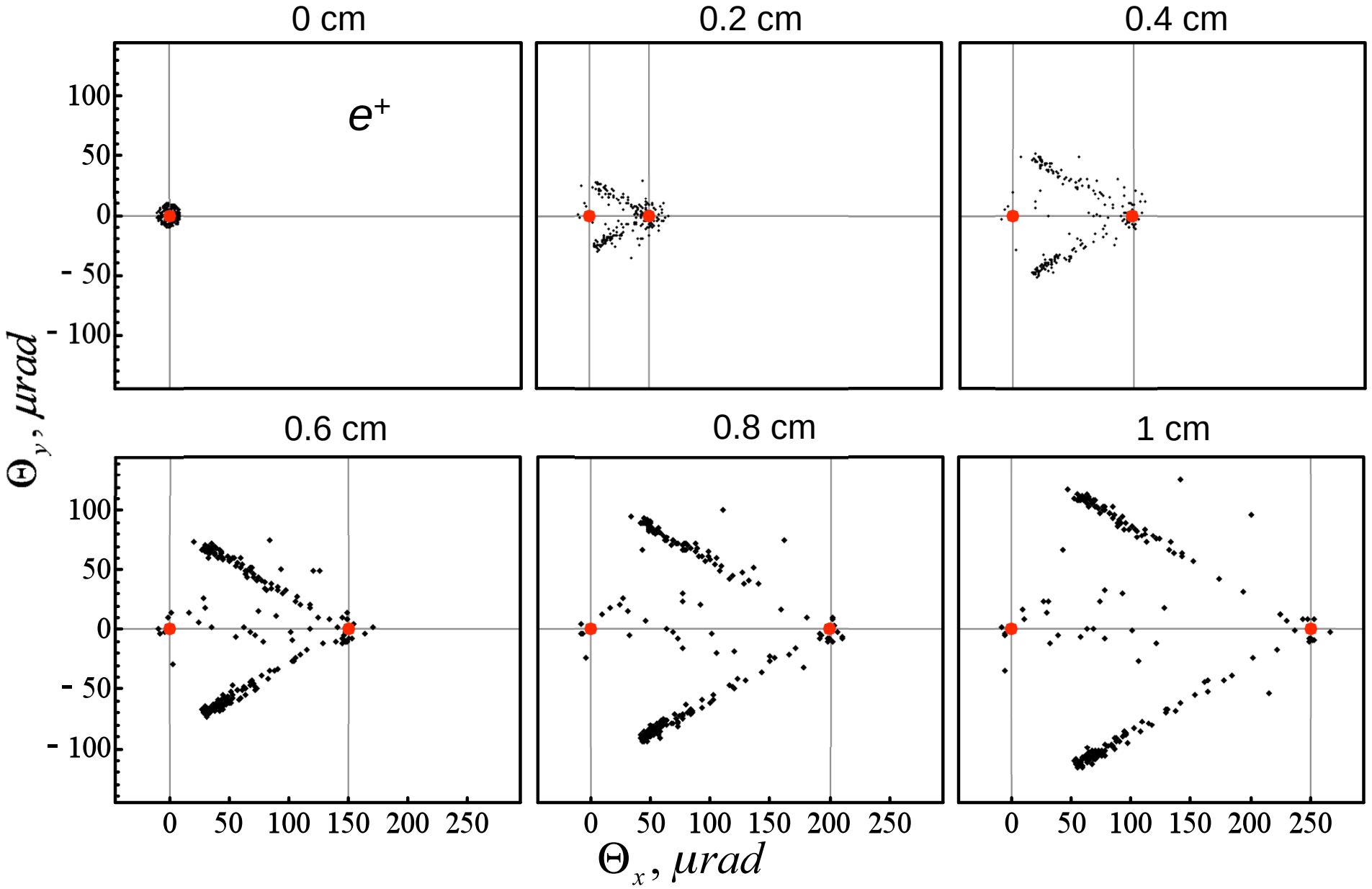
L=0.1 cm



L=0.25 cm

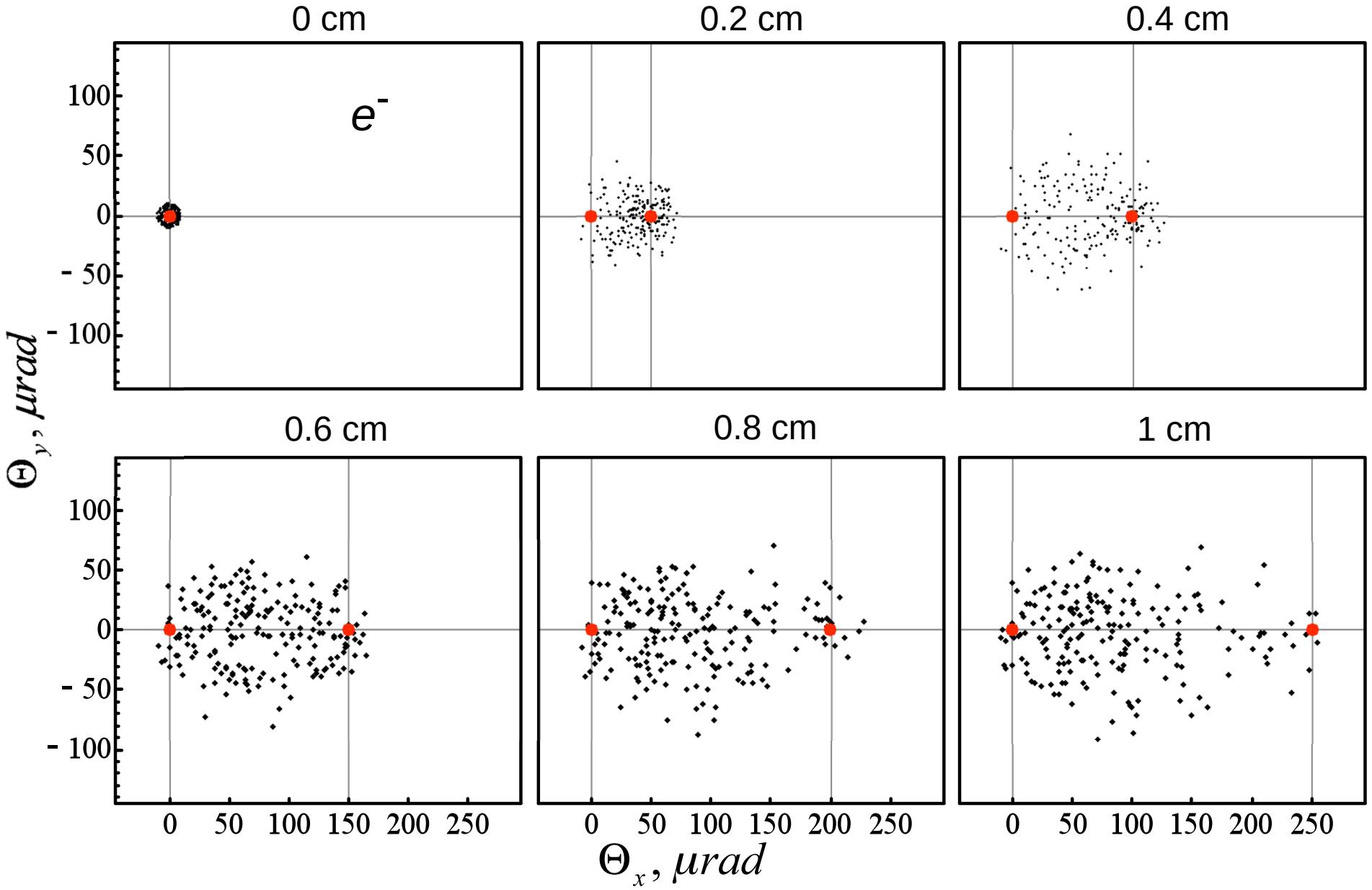
Beam Deflection in a Bent Crystal (stochastic mechanism)

e^+, E=400 GeV, R=40 m, 200 particles, $\sqrt{\psi_x^2 + \psi_y^2} < \frac{1}{3}\psi_c$



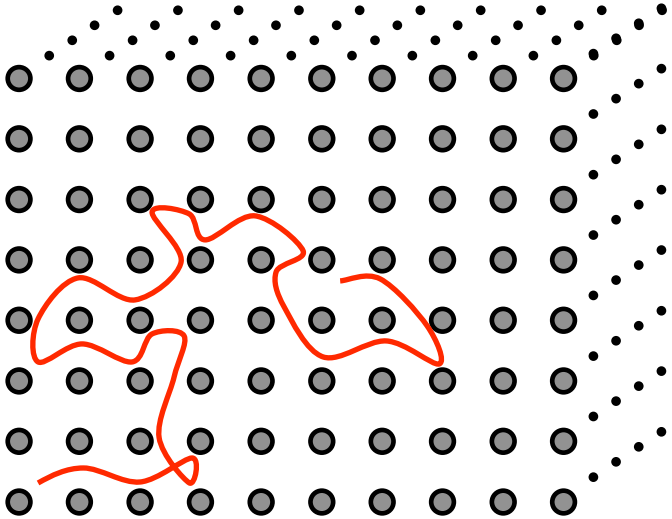
Beam Deflection in a Bent Crystal (stochastic mechanism)

e^-, E=400 GeV, R=40 m, 200 particles, $\sqrt{\psi_x^2 + \psi_y^2} < \frac{1}{3}\psi_c$



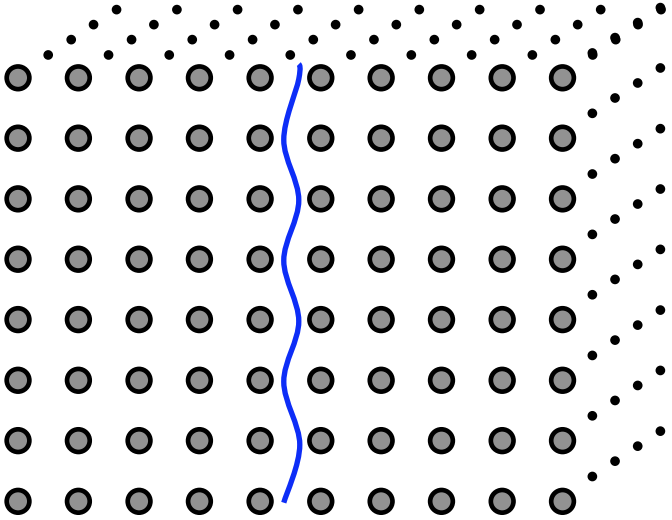
Beams Initial Conditions

Stochastic Mechanism

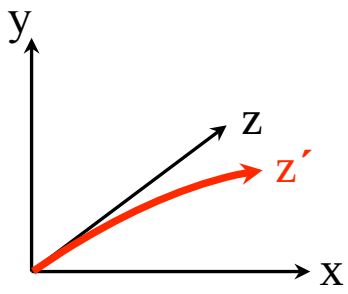


$\psi_x \approx \psi_y < \psi_c$

Planar Channeling

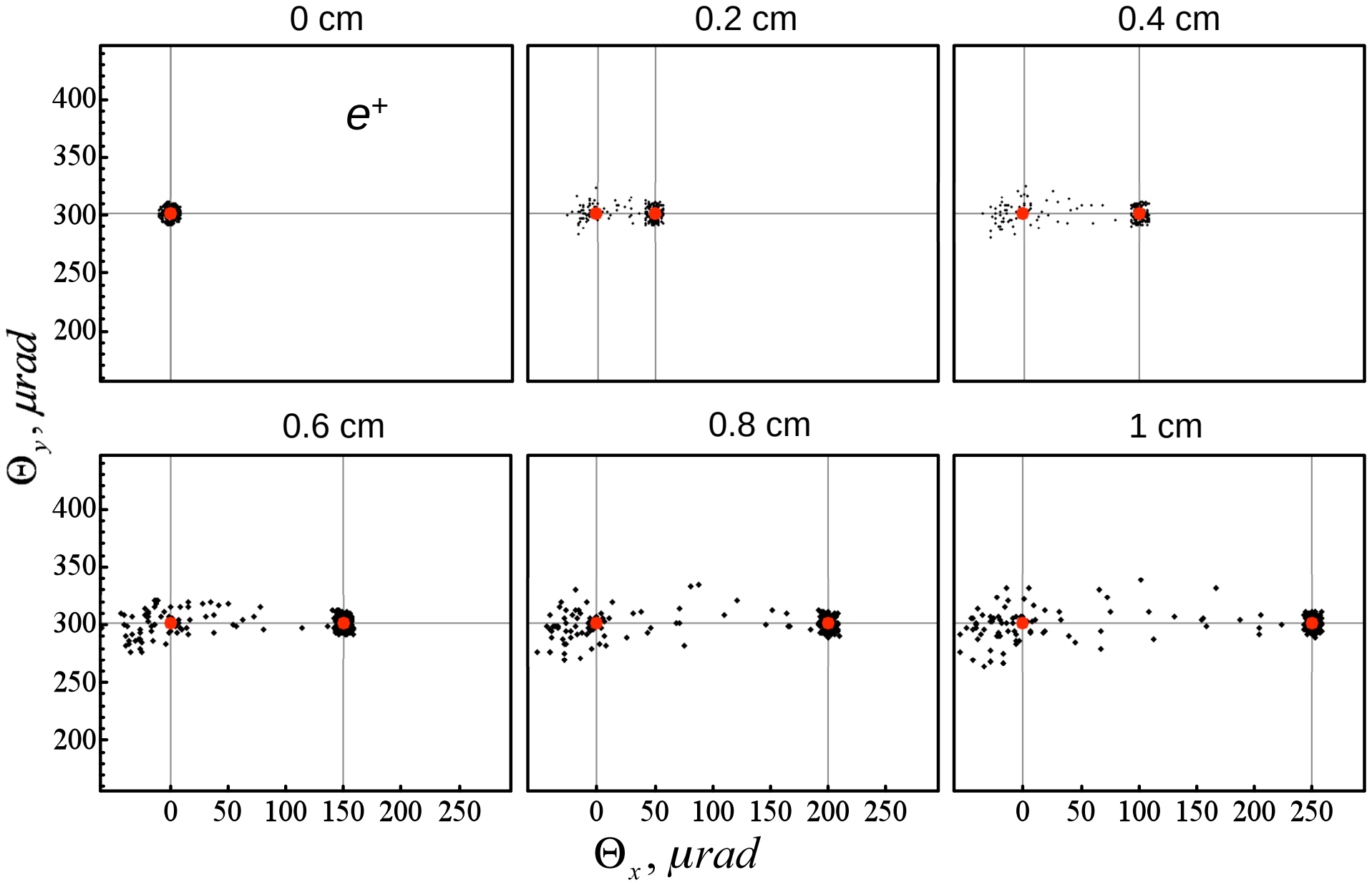


$\psi_x < \theta_p, \quad \psi_y >> \psi_c$



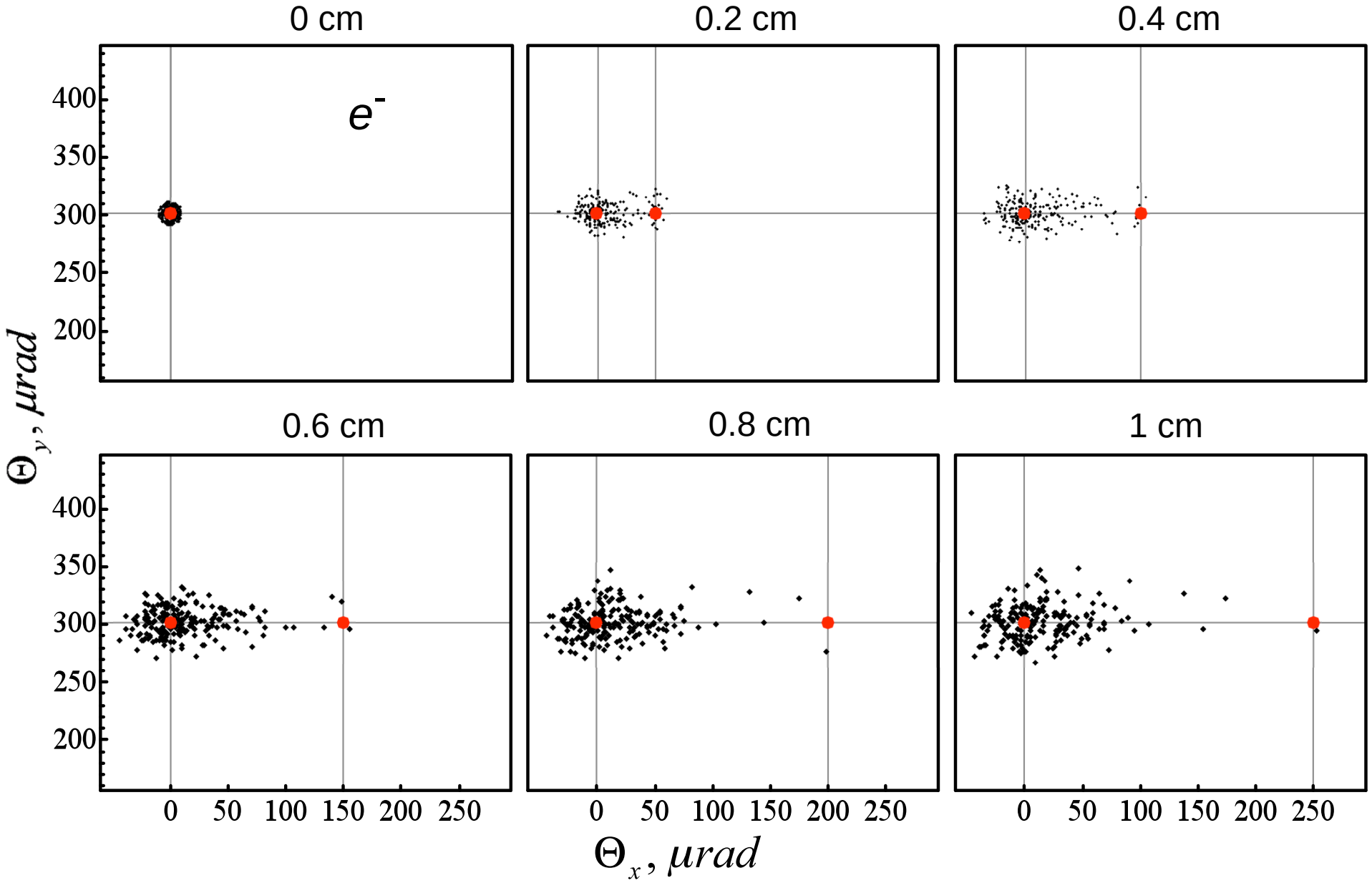
Beam Deflection in a Bent Crystal (plane channeling)

e+, E=400 GeV, R=40 m, 200 particles, $\psi_x \approx 0, \quad \psi_y \approx 10\psi_c$



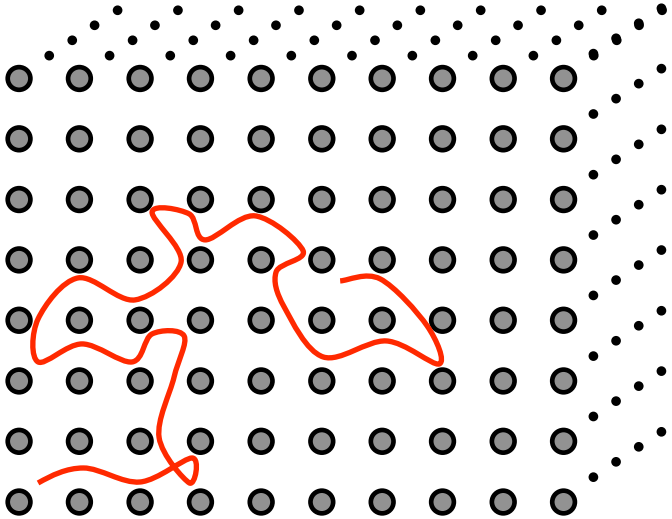
Beam Deflection in a Bent Crystal (plane channeling)

e-, E=400 GeV, R=40 m, 200 particles, $\psi_x \approx 0, \quad \psi_y \approx 10\psi_c$



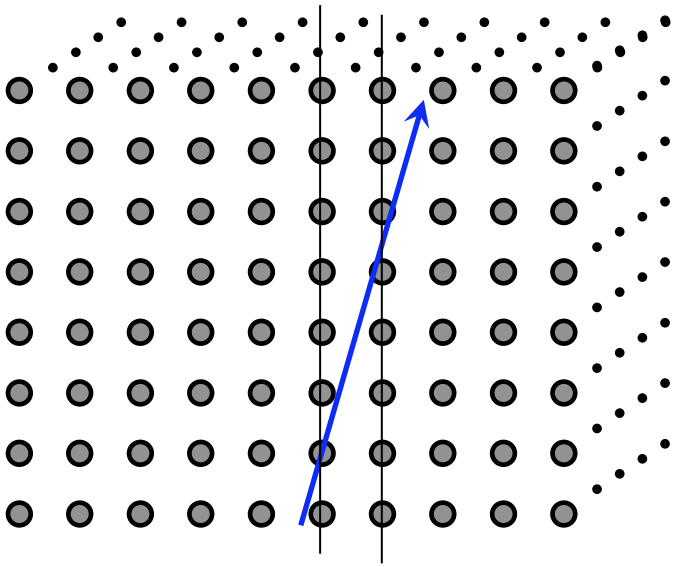
Beams Initial Conditions

Stochastic Mechanism

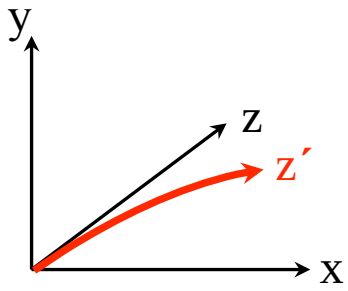


$\psi_x \approx \psi_y < \psi_c$

Volume Reflection

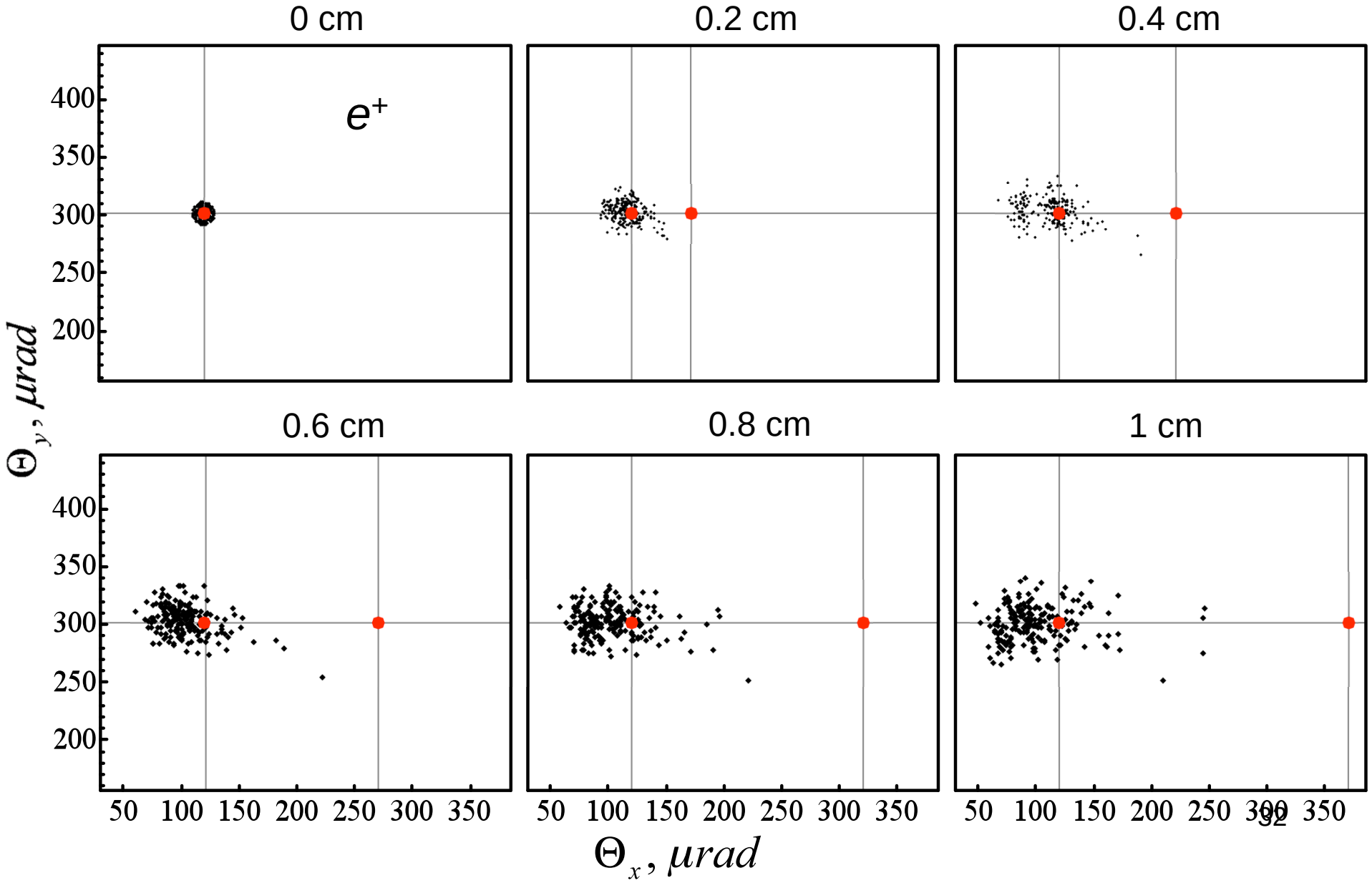


$\psi_x > \theta_p, \quad \psi_y >> \psi_c$



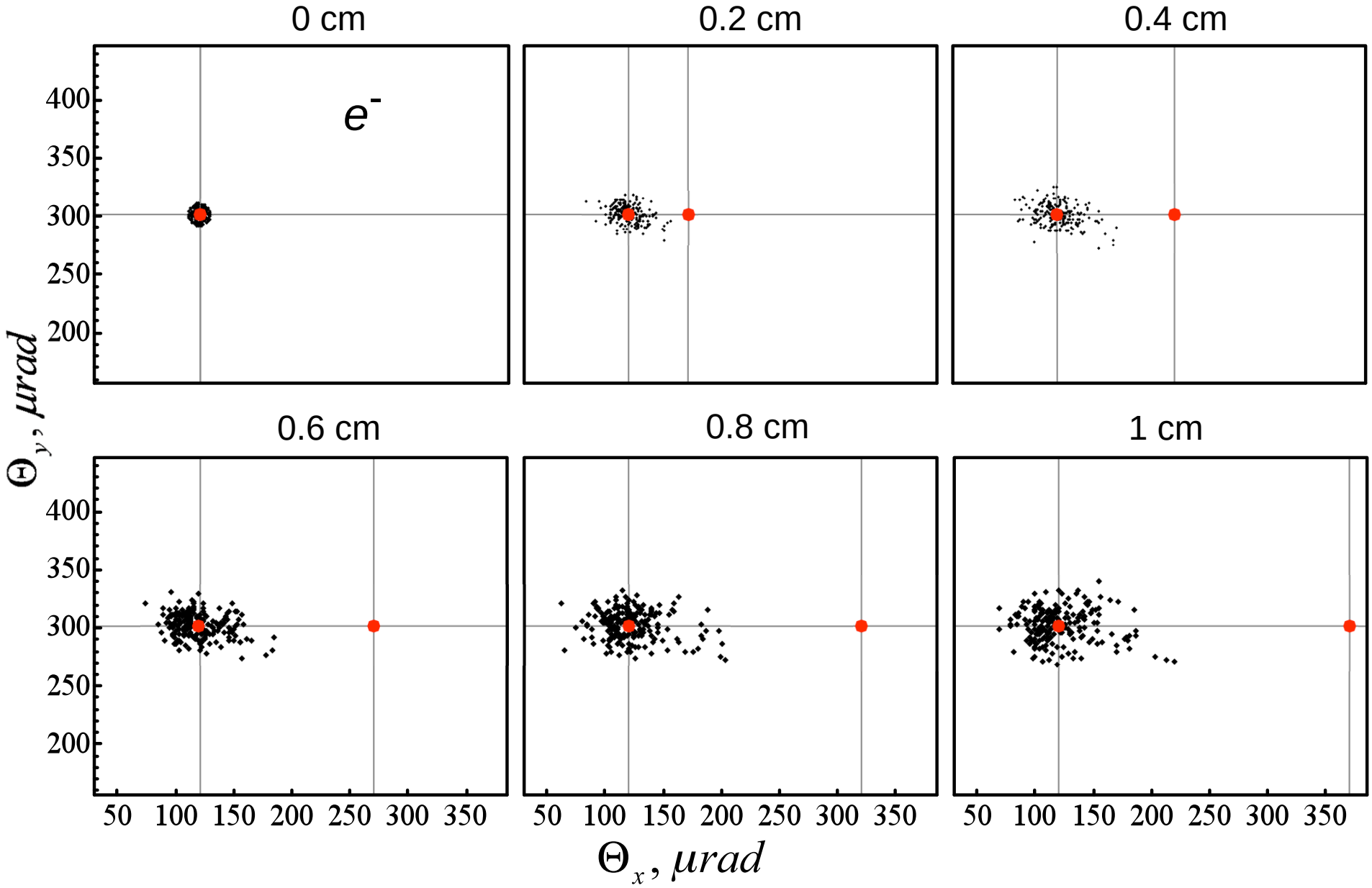
Beam Deflection in a Bent Crystal (volume reflection)

e^+, E=400 GeV, R=40 m, 200 particles, $\psi_x \approx 4\psi_c$, $\psi_y \approx 10\psi_c$



Beam Deflection in a Bent Crystal (volume reflection)

e⁻, E=400 GeV, R=40 m, 200 particles, $\psi_x \approx 0, \quad \psi_y \approx 10\psi_c$



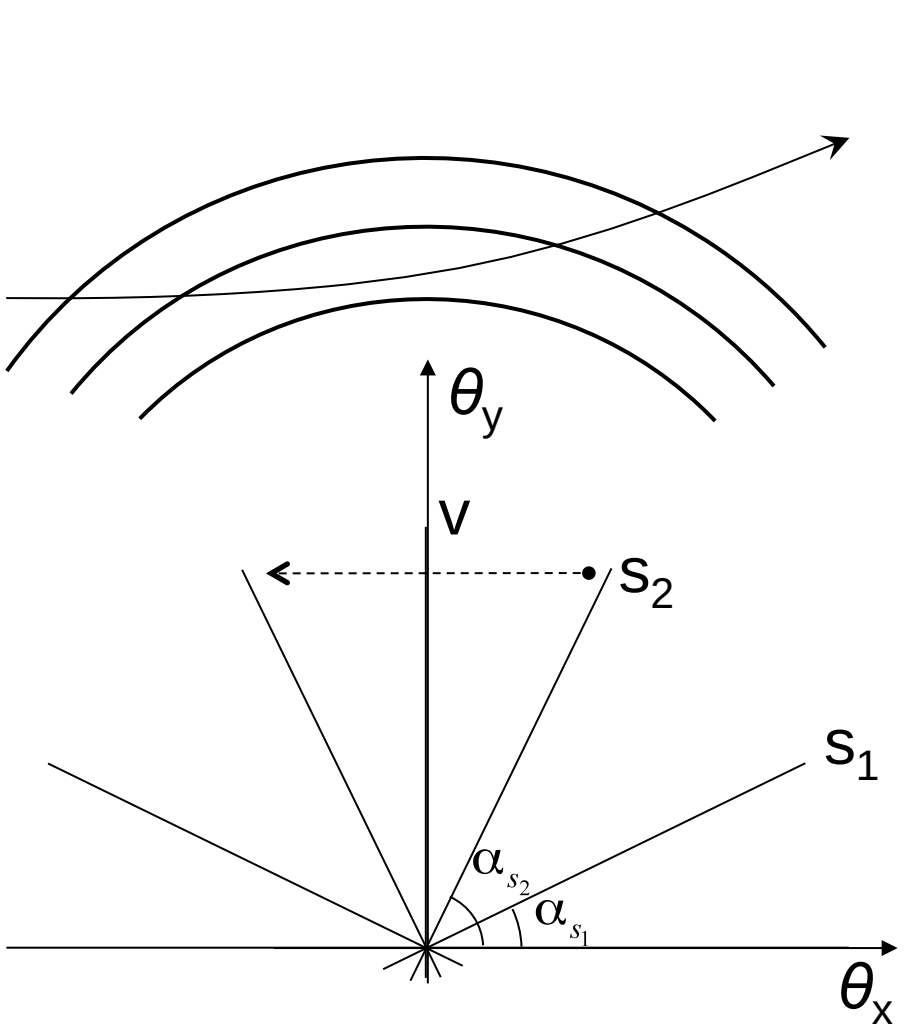
CONCLUSIONS
(analogies)

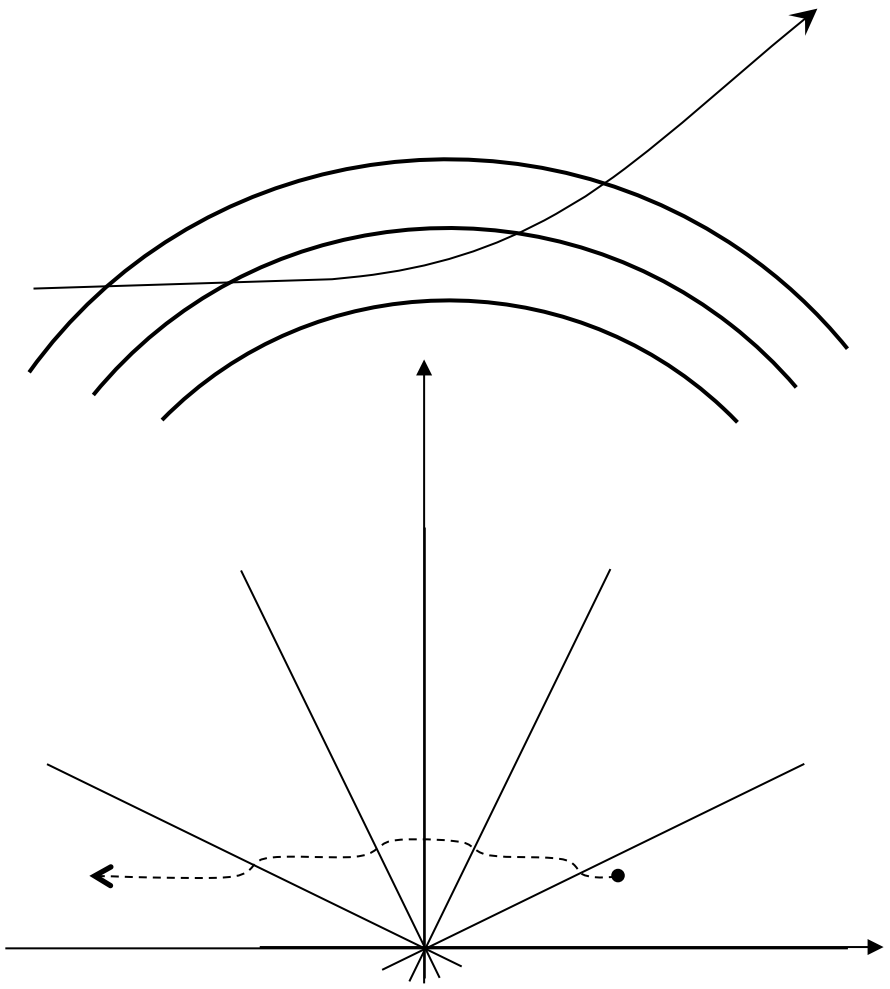
Bent crystal	Central field
Plane channelling	Finite motion in the central field, precession, ….
Volume reflection	Scattering in the central field, orbiting, ….
Stochastic	Dynamical chaos Multiple scattering

CONCLUSIONS
(efficiency)

Plane channelling	e^+ e^- — ?	$R > a\frac{\varepsilon}{U_0}$	$\theta >> \theta_p$
Volume reflection	e^+ e^- — ?	$R >> a\frac{\varepsilon}{U_0}$	$\theta \le 2\theta_p$
Stochastic	e^+ e^- — !!!	$\frac{l_\perp}{R\psi_c}\frac{L}{R\psi_c} < 1$	$\theta \sim 10\psi_c$

Thank You for Your Attention





$$\theta_{out} \approx 2\theta_{p-v}$$

$$\theta_{\text{out}} \approx 2\left(\theta_{p-s_1}\sin\left(\alpha_{s_1}\right)+\theta_{p-s_2}\sin\left(\alpha_{s_2}\right)+\theta_{p-v}\right)$$