How to make coherent bremsstrahlung circularly polarized

Vincenzo Guidi, Andrea Mazzolari

Department of Physics and INFN - University of Ferrara

Victor Tikhomirov

Institute for Nuclear Problems Minsk, Republic of Belarus

Outline

• Coherent bremmstrahlung and its linear polarization

•String-of-strings crystal orientation

• Polarization of crystal field harmonics

• Circular polarization of radiation of positrons channeled in bent crystals with string-of-strings orientation

• Polarization asymmetry of channeled positron production

• Other manifestations of circular polarization of the crystal field harmonic

• Conclusions

Coherent bremsstrahlung

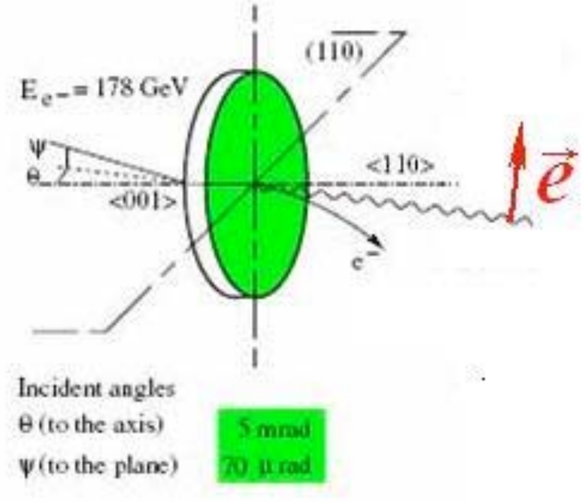
Predicted: Ferretti, Ter-Mikaelian, Dyson and Überall,..

Observed: first - Diambrini-Palazzi et al. (1960) in Frascati

at present - Arends et al., Mainz (MAMI, 855 MeV),
Klein et al., Bonn (ELSA, 3 GeV),
Avakian et al., CERN (20-170 GeV),
Klein et al., Jeff. Lab. (6 GeV).

Polarization: linear

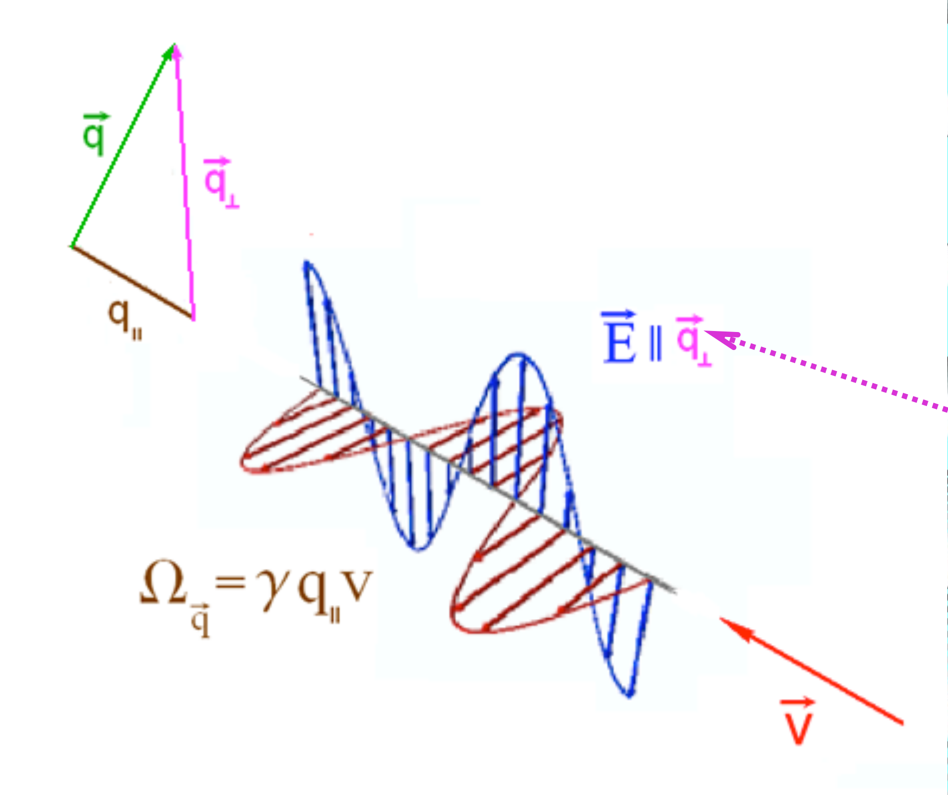
Jeff. Lab. $P_\gamma = 84\%$
for the production
of ρ and ω mesons



Crystal field can be represented by preudophotons

with frequency $\Omega_{\vec{q}} = \gamma q_{\parallel} v$

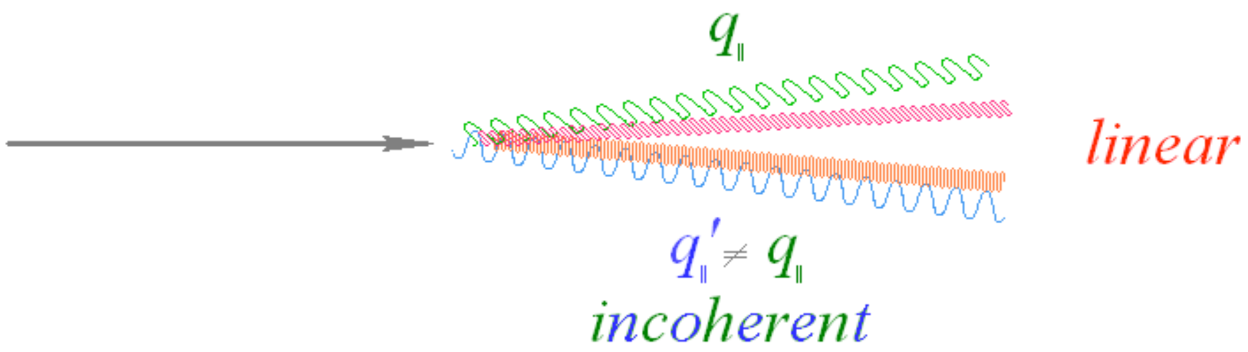
and **linear** polarization $E \parallel \vec{q}_{\perp}$



$$V(\vec{r}) = \sum_{\vec{q}} V_{\vec{q}} e^{i\vec{q}\vec{r}}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r}) = -i \sum_{\vec{q}} \vec{q} V_{\vec{q}} e^{i\vec{q}\vec{r}}$$

CB is (has been!) linearly polarized

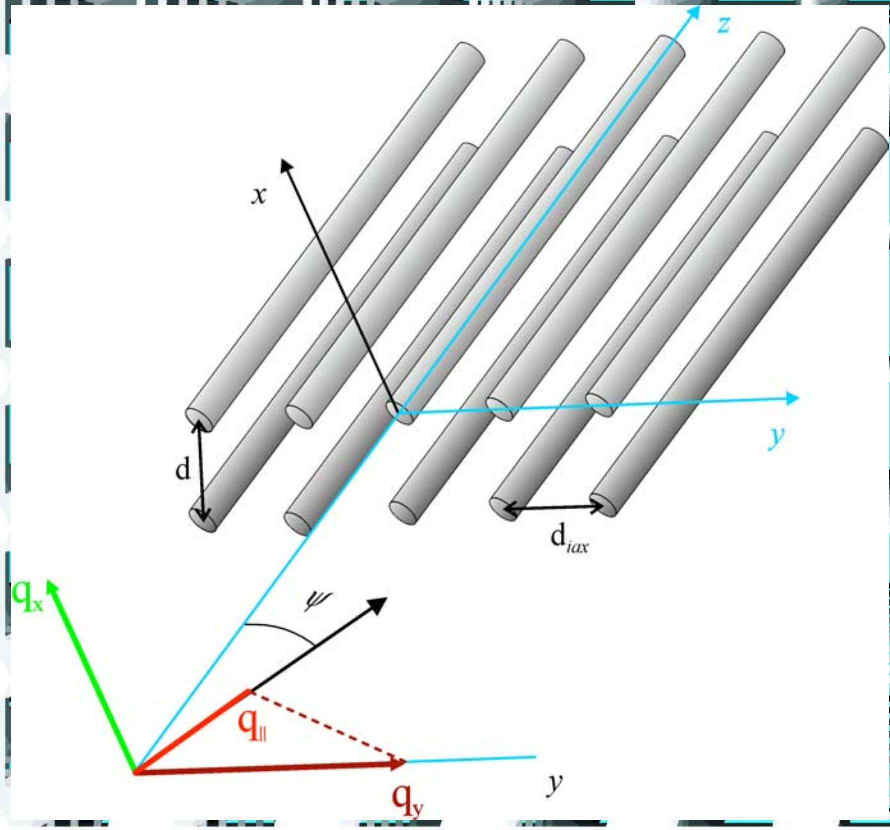


Coherent Bremsstrahlung ~ backward Compton scattering of linearly polarized photons with energies $\Omega_{\vec{q}} = \gamma q_{\parallel} \upsilon$

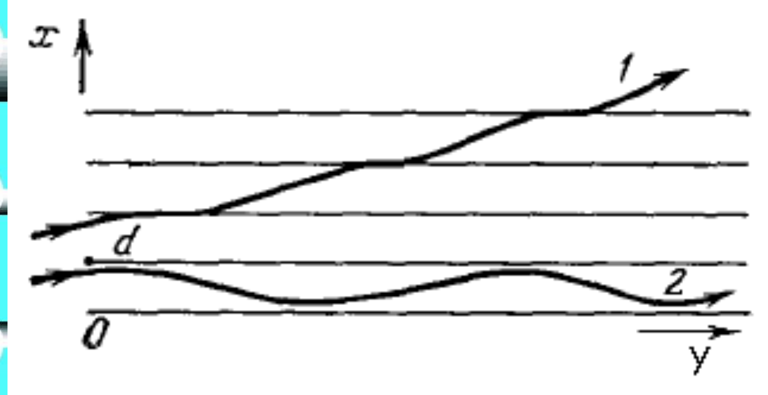
$\Omega_{\vec{q}} \neq \Omega_{\vec{q}'} \Rightarrow$ scattering is ***incoherent***

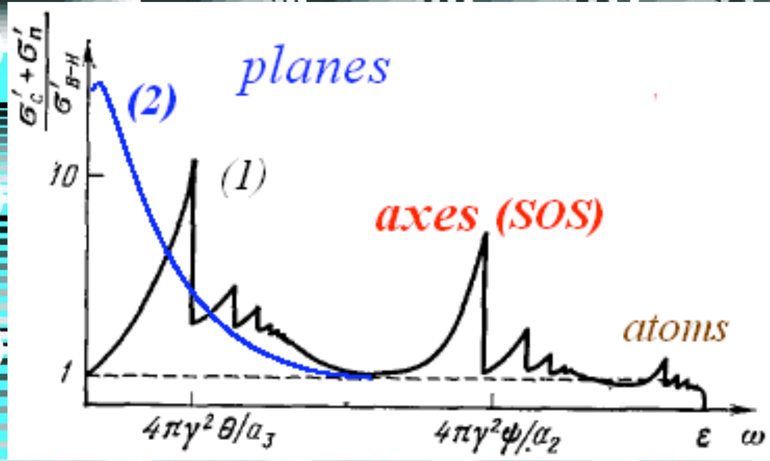
$\Rightarrow$ polarization remains ***linear***

String-of-strings geometry (Lindhard; Akhiezer&Shul'ga)



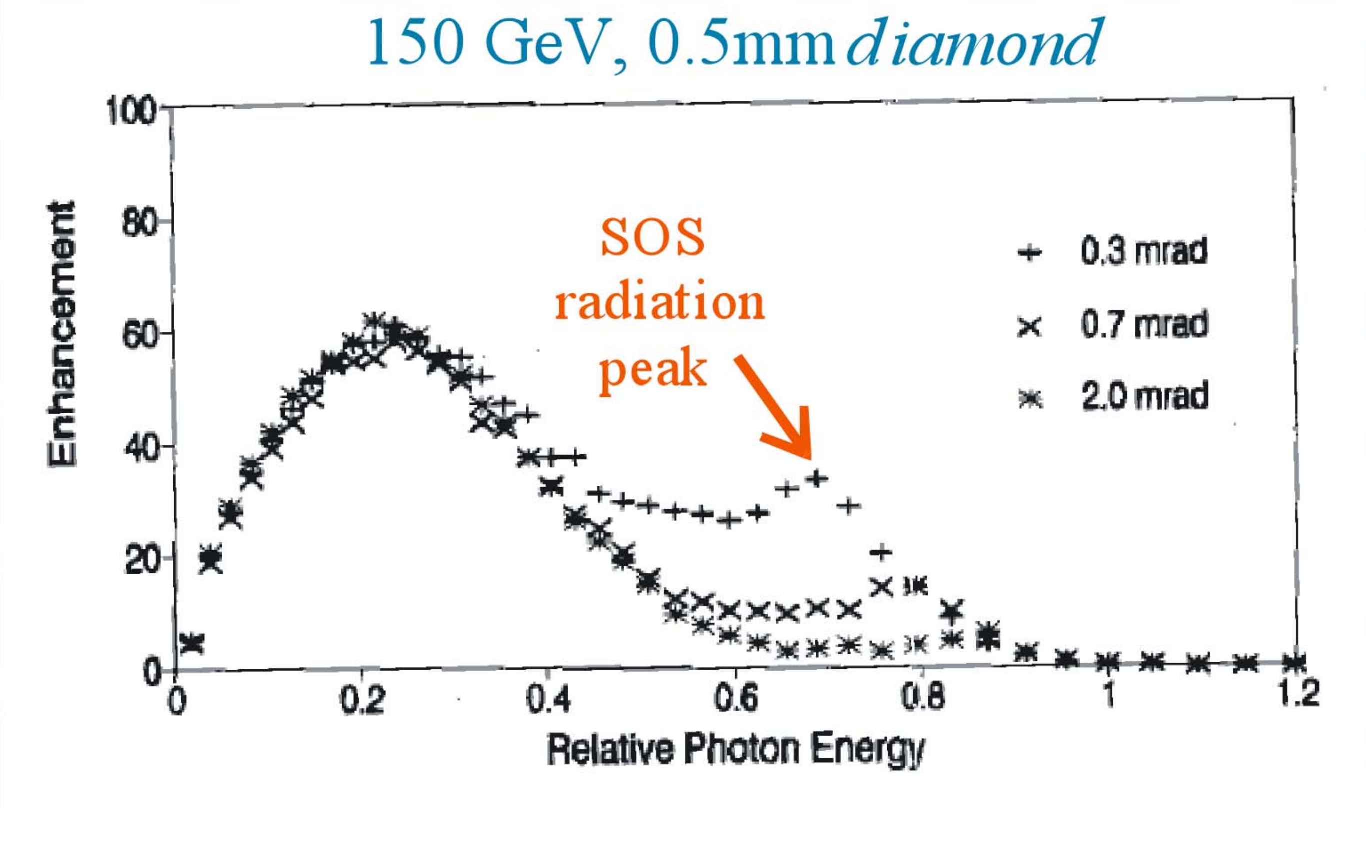
motion in transverse plane





Experimental observation of hard radiation peak in string-of-strings geometry

R. Medenwaldt et al. PLB 281(1992)153



PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS 11, 041001 (2008)

Coherent bremsstrahlung, coherent pair production, birefringence, and polarimetry in the 20–170 GeV energy range using aligned crystals

A. Apyan,[1,*] R. O. Avakian,[1] B. Badelek,[2] S. Ballestrero,[3,¶] C. Biino,[4,5] I. Birol,[6] P. Cenci,[7] S. H. Connell,[8] S. Eichblatt,[6] T. Fonseca,[6] A. Freund,[9] B. Gorini,[5] R. Groess,[10] K. Ispirian,[1] T. J. Ketel,[11] Yu. V. Kononets,[12] A. Lopez,[13] A. Mangiarotti,[14] B. van Rens,[11] J. P. F. Sellschop,[10,†] M. Shieh,[6] P. Sona,[14] V. Strakhovenko,[15] E. Uggerhøj,[16,‡] U. I. Uggerhøj,[17] G. Unel,[18] M. Velasco,[5,§] Z. Z. Vilakazi,[10,∥] and O. Wessely[2]

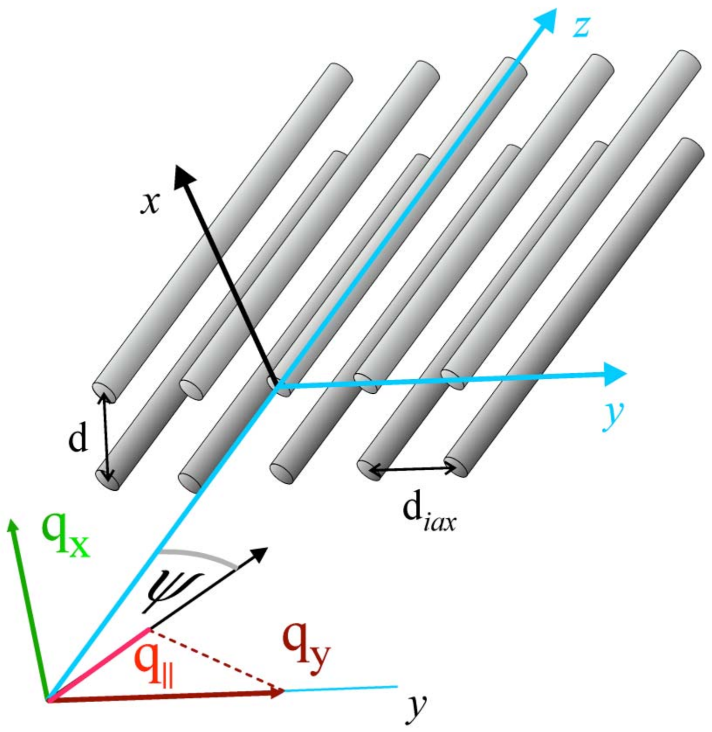
(NA59 Collaboration)

…using crystals, *only linear polarization* may be produced.

… Our measurements and our calculations indicate *low photon polarization* for the high-energy SOS photons.

Nevertheless, We'll show that **circular polarization** of radiation of positrons channeled in *bent crystals* with string-of-strings orientation **can be high!**

SOS radiation can be circularly polarized!



$$q_{1x} = \frac{2\pi}{d}, \quad q_{1y} = \frac{2\pi}{d_{inax}},$$

$$q_x = q_{1x} n_x, \quad q_y = q_{1y} n_y,$$

$$\vec{q} = q_x \vec{e}_x + q_y \vec{e}_y,$$

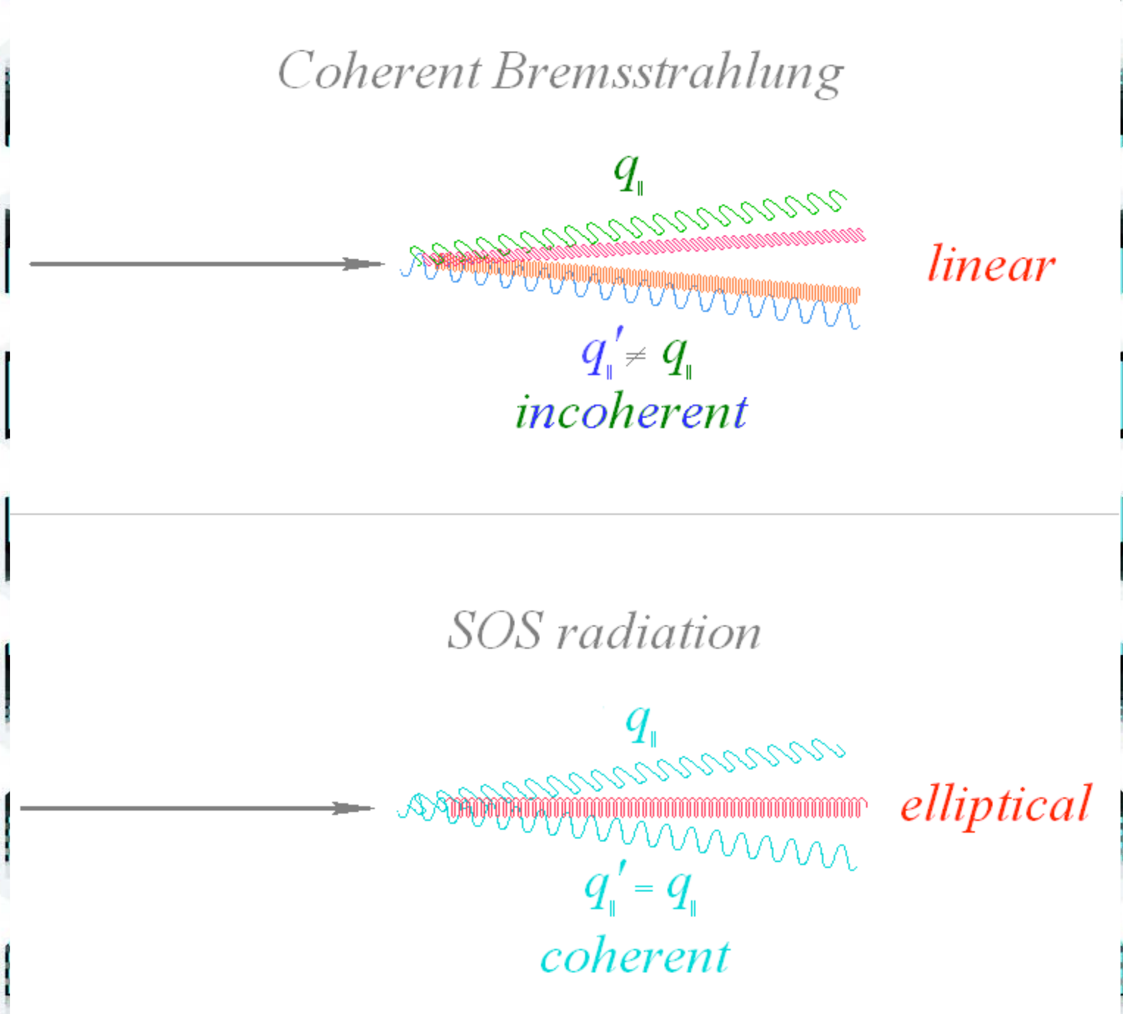
$$q_{\parallel} = \vec{q}\vec{\upsilon}/\upsilon = \psi q_y \Rightarrow$$

$$if \; q_x = q_{1x} n_x \neq q'_x = q_{1x} n_x$$

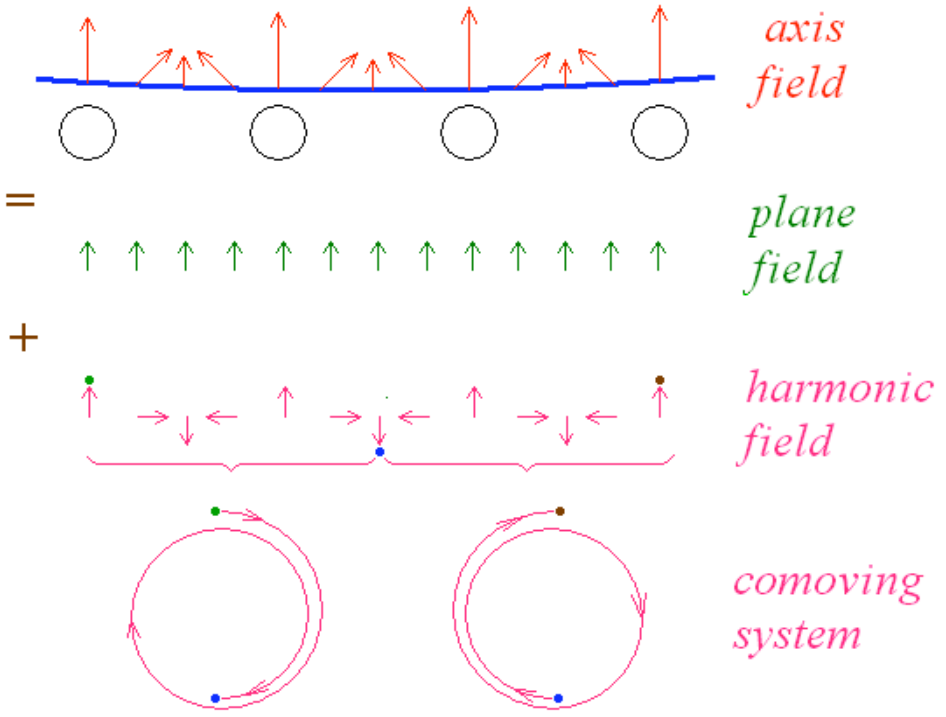
$$q_{\parallel} = q'_{\parallel}$$

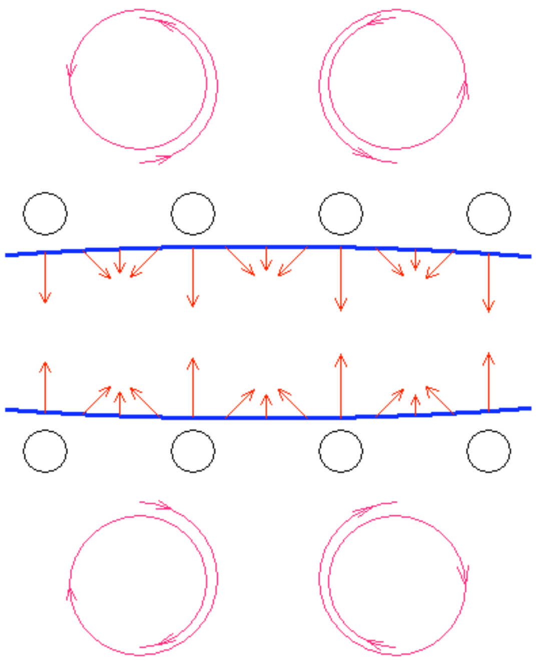
different q_x*, same* $q_{\parallel}$*!*

Coherence of "crystal photons" at SOS geometry allows to obtain **circular polarization**



Intuitive prove of crystal field harmonic circular polarization





CP is opposite on opposite channel sides!

"Mathematical prove" of crystal field harmonic circular polarization

V.V.Tikhomirov , JETP 109(1996)1188

Fourier decomposition of the axial potential

$$V(x,y) = \sum_{q_x, q_y} V(q_x, q_y) e^{iq_x x} e^{iq_y y},$$

where $q_x = 2\pi n_x/d$, $q_y = 2\pi n_y/d_{inax}$, $n_{x,y} = 0, \pm 1, \pm 2, ..$
d is the inter-planar distance,
d_{inax} is the inter-axis distances inside the plane.

Planar potential

$$V(x) = \sum_{n_x = 0, \pm 1, \pm 2 ..} V(q_x) e^{iq_x x}.$$

Slowly varying amplitudes

$$E_{xn}(x) = 2 \sum_{n_x = 0,1,...} V(q_x, q_y) q_x sin(q_x x),$$

$$E_{yn}(x) = 2q_y \sum_{n_x = 0,1,...} V(q_x, q_y) cos(q_x x)$$

of the averaged crystal electric field

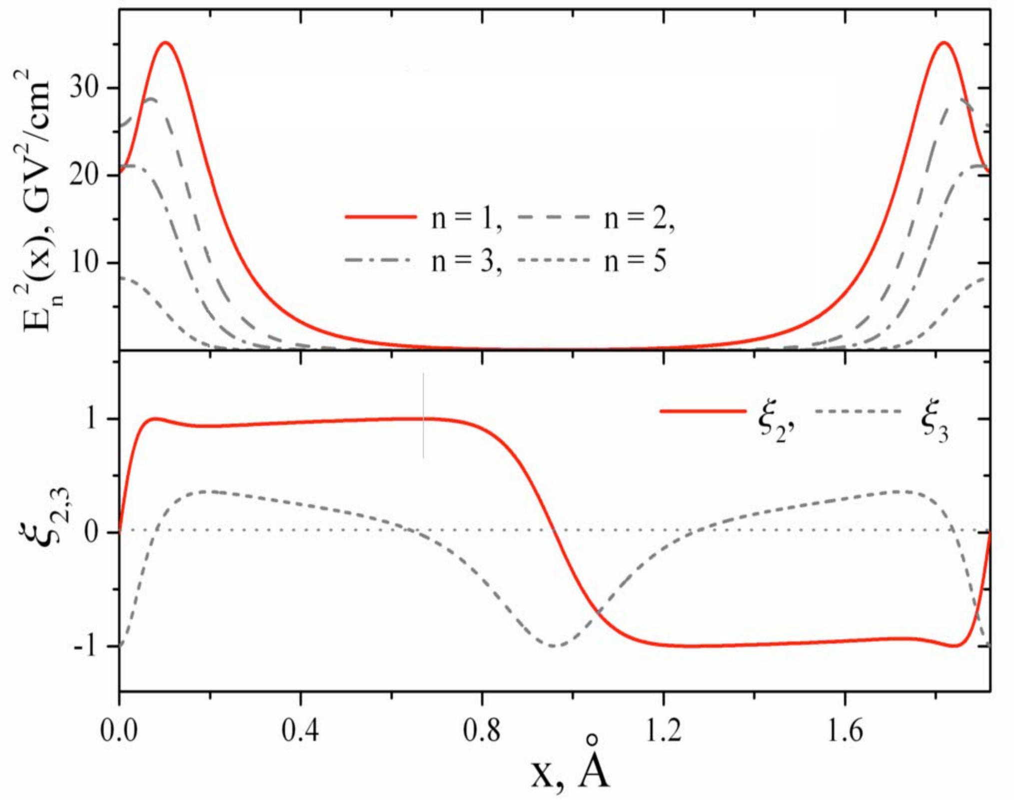
$$\mathbf{E} = -\vec{\nabla} V(x,y) = -\mathbf{n}_x V'(x) + \sum_{n=\pm 1, \pm 2 ...} [\mathrm{n}_x E_{xn}(x) - i\mathrm{n}_y E_{yn}(x)] e^{iq_y y}.$$

Stokes parameters of the crystal field harmonic

$$\xi_{2n} = \frac{2E_{xn}E_{yn}}{E_n^2}, \quad \xi_{3n} = \frac{E_{xn}^2 - E_{yn}^2}{E_n^2}, \quad E_n = \sqrt{E_{xn}^2 + E_{yn}^2}.$$

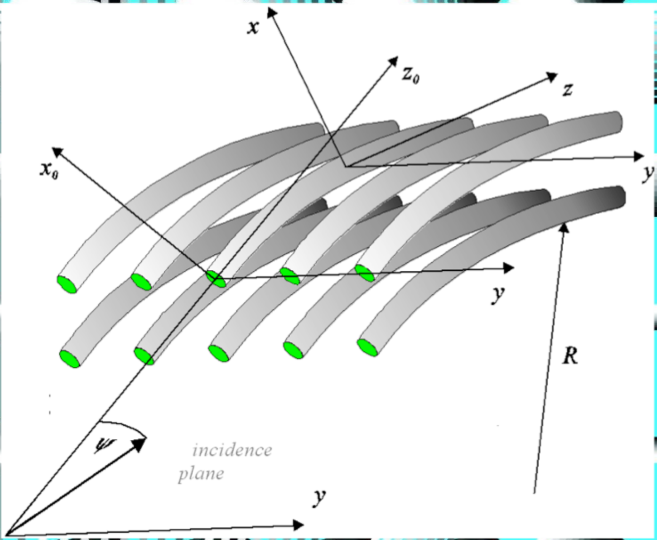
Space symmetry properties of the amplitudes

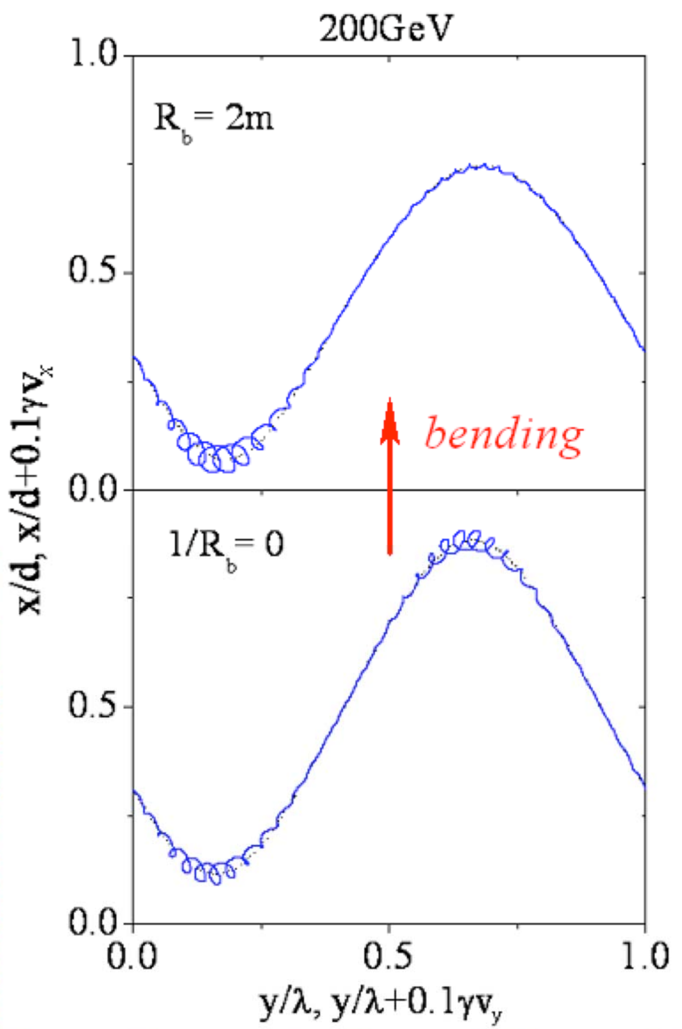
$$E_{xn}(d - x) = -E_{xn}(x), \qquad E_{yn}(d - x) = E_{yn}(x)$$



CP is opposite on opposite channel sides!

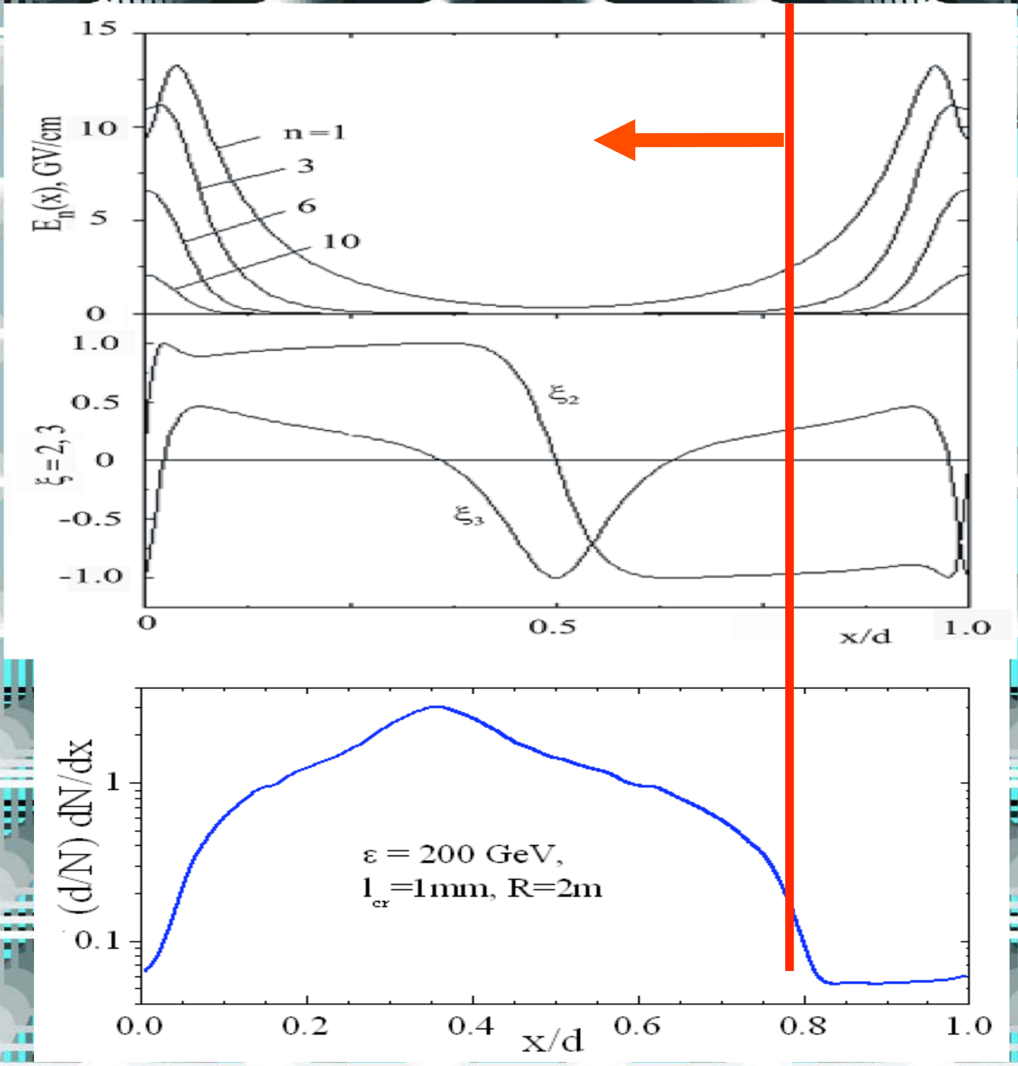
Crystal **bending** gives rise to a **preferred spirality** of positron trajectories



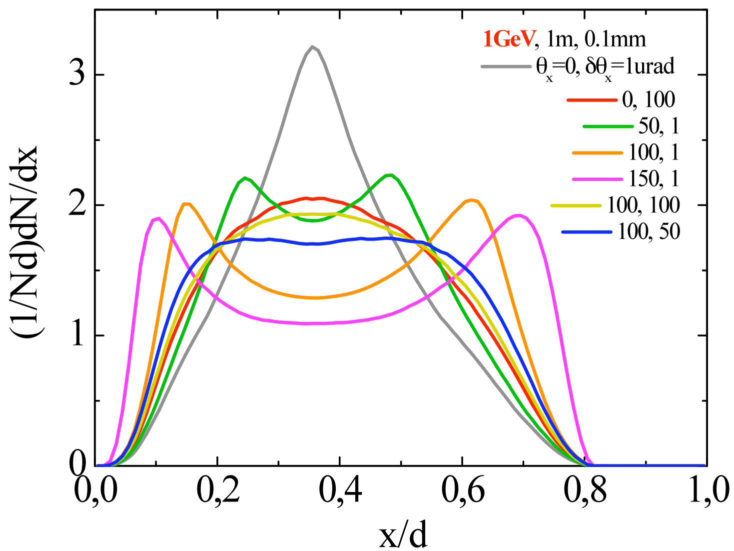


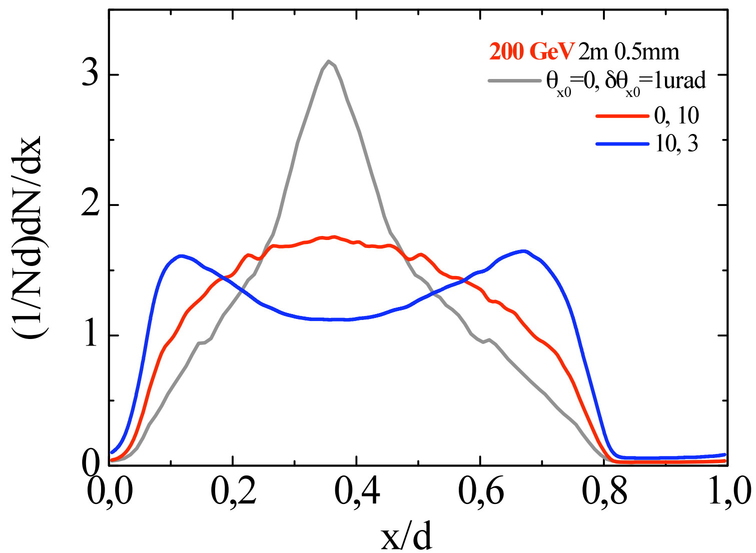
Projections on xy plane of positron trajectories (dots) and velocities (solid spirals) in Si crystal with (top) and without bending (bottom). Velocity components, measured in units of $0.1/\gamma$ are plotted from the corresponding positron coordinates. Velocity projections rotate in opposite directions near the opposite crystal planes (at x ~ 0.15d and x ~ 0.85d). Crystal bending violates trajectory symmetry amplifying velocity oscillations at x ~ 0.1d and diminishing them at x ~ 0.7d.

Channeled positrons move in the regions of specific circular polarization in bent crystals



Channeled positron distributions in transverse coordinate

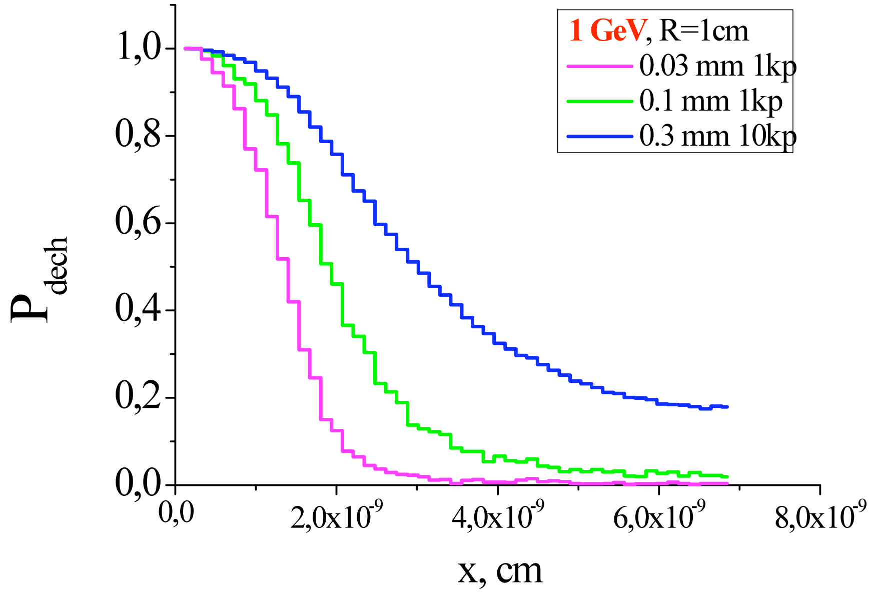


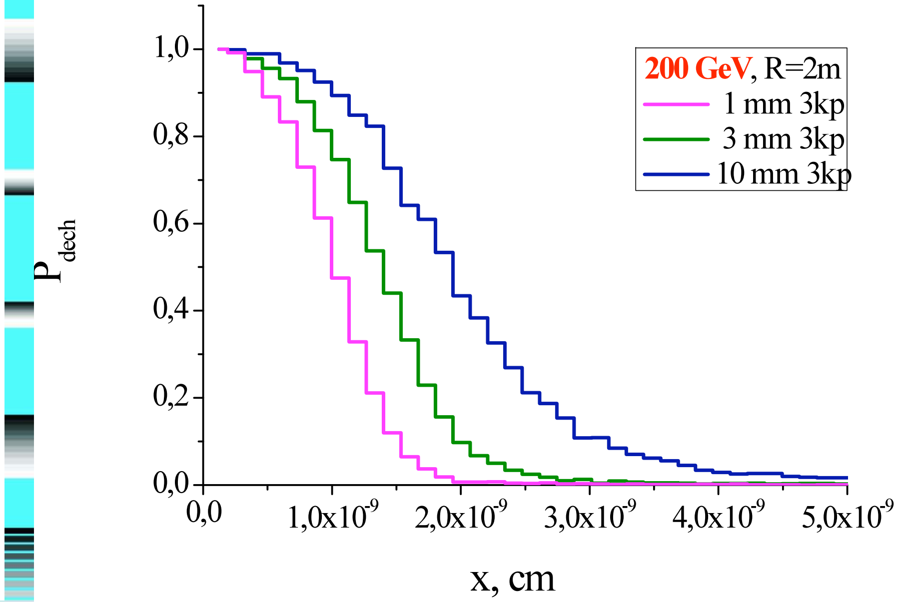


It is really possible:

1. to depopulate the region of $0.8 < x/d < 1$
2. to populate the region of $x/d \sim 0.1$

Positron dechanneling probability *vs* initial transverse coordinate at $\theta = 0$





Positrons spend considerable time at x ~ 0.2Å

A simplest "local coherent bremsstrahlung" theory allowing to evaluate circular polarization of emitted gamma quanta

Gamma-quantum polarization matrix
[V.M. Strakhovenko, Phys. Rev. **A68**:042901, 2003]:

$$\frac{dw_{ij}}{d\omega d^2\theta} = \frac{\alpha\omega^2}{16\pi^2}\frac{\varepsilon}{\varepsilon'}\int\int dt_1 dt_2 L_{ij}\exp\left\{i\omega\frac{\varepsilon}{\varepsilon'}[t_1 - t_2 - \mathbf{n}(\mathbf{r}_1 - \mathbf{r}_2)]\right\}:$$

$$L_{ij} = \varphi(\varepsilon)[(\mathbf{e}_i\mathbf{v}_1)(\mathbf{e}_j\mathbf{v}_2) - (\mathbf{e}_j\mathbf{v}_2)(\mathbf{e}_i\mathbf{v}_1)]$$

$$+2[(\mathbf{e}_i\mathbf{v}_1)(\mathbf{e}_j\mathbf{v}_2) + (\mathbf{e}_j\mathbf{v}_2)(\mathbf{e}_i\mathbf{v}_1)] + \delta_{ij}(\mathbf{v}_1\mathbf{v}_2 - 1 + \gamma^{-2}),$$

$$c = \hbar = 1, \quad \alpha = 1/137, \quad i,j = x,y$$

$$\varepsilon' = \varepsilon - \omega, \varphi = \varepsilon/\varepsilon' + \varepsilon'/\varepsilon,$$

$$\mathbf{v}_{1,2} = \mathbf{v}(t_{1,2}), \quad \mathbf{r}_{1,2} = \mathbf{r}(t_{1,2}).$$

Its expansion up to $O((\gamma v)^2)$ terms

$$dw_{ij} = \frac{i\alpha d\omega}{4\pi\gamma^2}\int_{-\infty}^{\infty} dt\int_{-\infty}^{\infty}\frac{d\tau}{\tau - i0}$$

$$\times\left\{\delta_{ij}\left[\frac{1}{4}(\varphi-2)(\mathbf{g}_2 - \mathbf{g}_1)^2 + \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} d\tau'\mathbf{g}^2(t+\tau')\right]\right.$$

$$\left.+\frac{\varphi}{2}(g_{2i}g_{1j} - g_{2j}g_{1i}) - (g_{2i}g_{1j} + g_{2j}g_{1i})\right\}\exp\left(-i\frac{\omega m^2}{2\varepsilon\varepsilon'}\right),$$

$$\mathbf{g}_{1,2} = \gamma\left[\mathbf{v}_\perp(t_{1,2}) - \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} d\tau'\mathbf{v}_\perp(t+\tau')\right]$$

$$= -i\sum_{n=\pm1,\pm2,\ldots}\frac{1}{mq_\parallel}[\mathbf{n}_x E_{xn}(x) - i\mathbf{n}_y E_{yn}(x)]$$

$$\times[\exp(\mp iq_\parallel\tau) - \sin(q_\parallel\tau/2)/(q_\parallel\tau/2)]\exp(iq_\parallel t + iq_y y_0), \qquad q_\parallel = q_y\psi.$$

The "index" symmetry properties

$$E_{x-n}(x) = E_{xn}(x), \qquad E_{y-n}(x) = -E_{yn}(x)$$

allow to obtain

$$dw_{ij} = \frac{i\alpha d\omega}{4\pi\gamma^3}\int_{-\infty}^{\infty} dt\int_{-\infty}^{\infty}\frac{d\tau}{\tau - i0}\sum_{n=\pm1,\pm3,\ldots}\frac{1}{m^3 q_\parallel^3}$$

$$\times\{\delta_{ij}E_n^2[1 - f^2(\zeta) + (\varphi(\varepsilon) - 2)\sin^2\zeta]$$

$$-\varphi(\varepsilon)E_{in}E_{jn}\epsilon_{ijz}[\sin 2\zeta - 2f(\zeta)\sin\zeta]$$

$$-2E_{in}E_{jn}\delta_{ij}[\cos\zeta + f^2(\zeta) - 2f(\zeta)\cos\zeta]\}\exp\left(-i\frac{\omega m^2}{2\varepsilon\varepsilon'}\right),$$

$$\zeta = q_\parallel\tau/2, \qquad f(\zeta) = \sin\zeta/\zeta.$$

Integration over τ leads to the "local" probability of gamma quantum emission having the "Compton scattering form":

$$dw_{ij} = (A + \mathbf{B}\vec{\sigma})_{ij}/2,$$

$$(A, \mathbf{B}) = \frac{\alpha}{\gamma^3}\sum_{n=1,3,\ldots}\left(\frac{E_n}{mq_\parallel}\right)^3(a, \mathbf{b})\theta(1-\nu),$$

$$a = \frac{\varphi(\varepsilon)}{2} + 2\nu(1-\nu), \qquad \nu = \frac{\omega m^3}{2\varepsilon\varepsilon'|q_\parallel|},$$

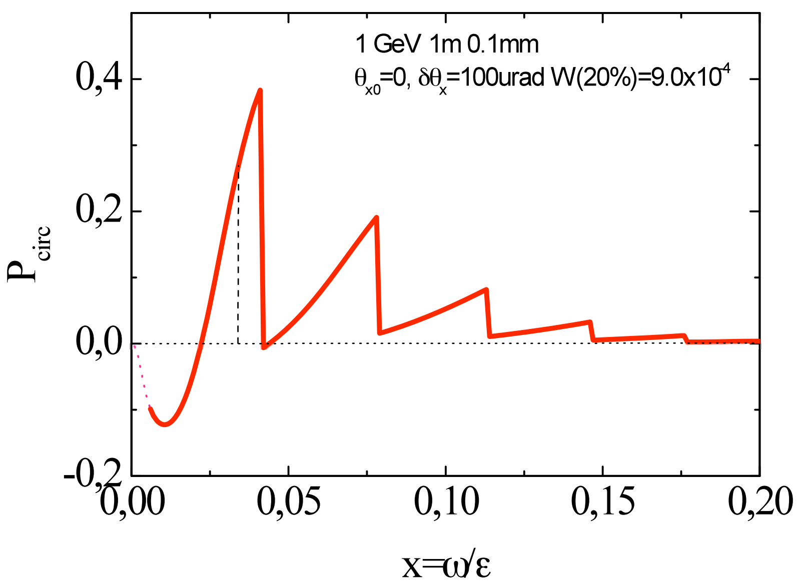
$$b_1 = 0, \quad b_2 = \xi_{3n}\varphi(\nu - 1/2), \quad b_3 = \xi_{3n}\nu^2,$$

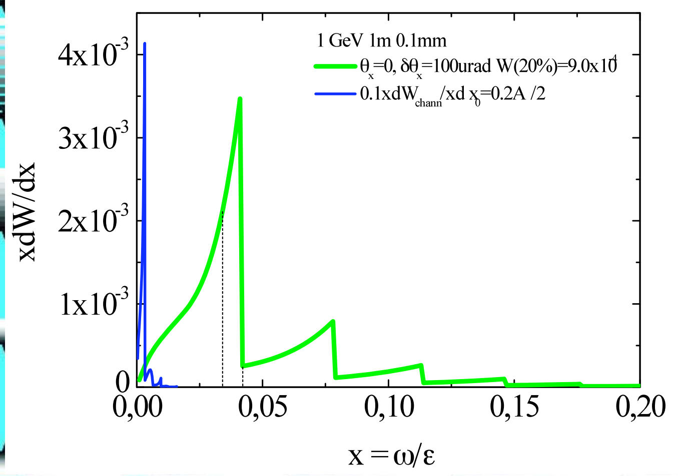
$b_2 \propto \xi_{3n}$ TERM DESCRIBES GAMMA CIRCULAR POLARIZATION!

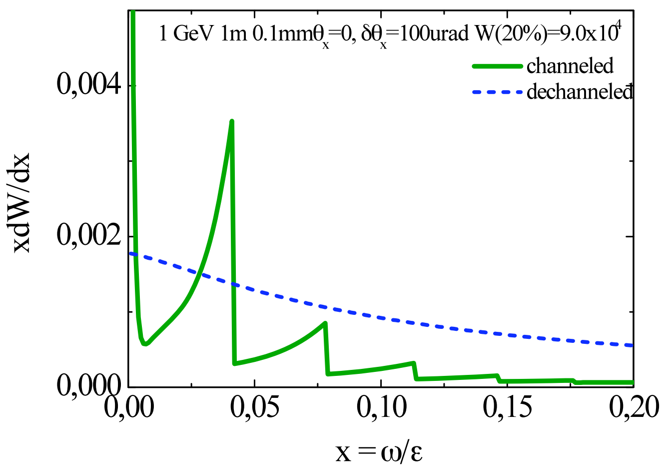
Strakhovenko:

$$b_1 = \xi_{1n}\nu^2, \qquad b_2 = 0, \qquad b_3 = \xi_{3n}\nu^2.$$

Polarization and spectrum of 1 GeV channeled e^+



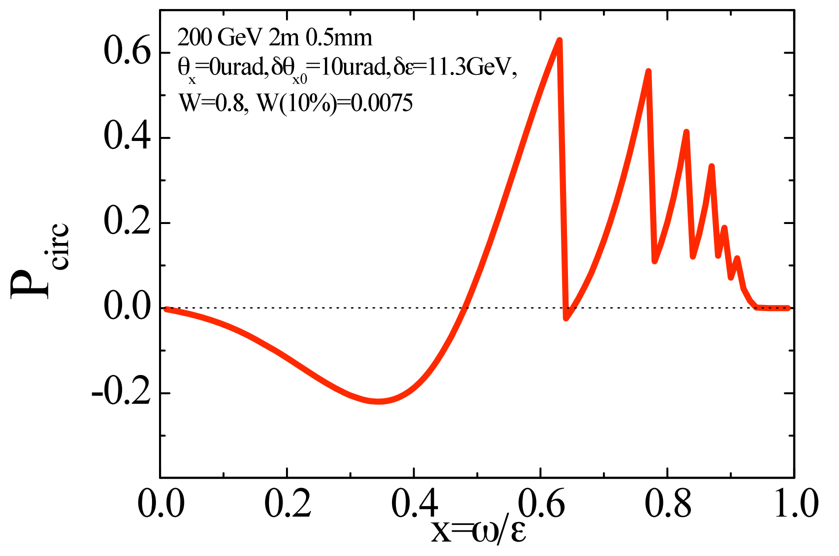


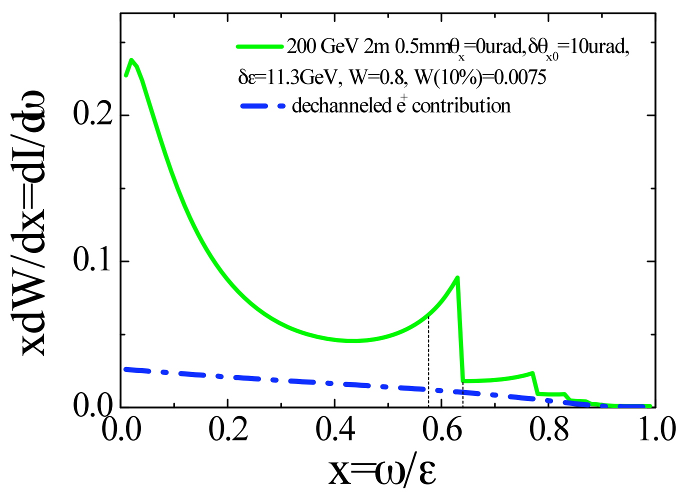


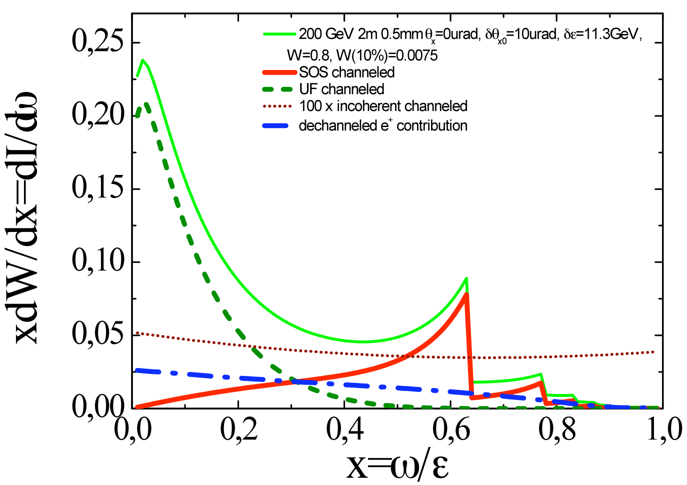
l_{cr} = 0.1mm, R = 1cm! W ~ 10^{-3}

Compton scattering in magnetized iron can be used to measure gamma polarization!

Polarization and spectrum of 200 GeV channeled e^+



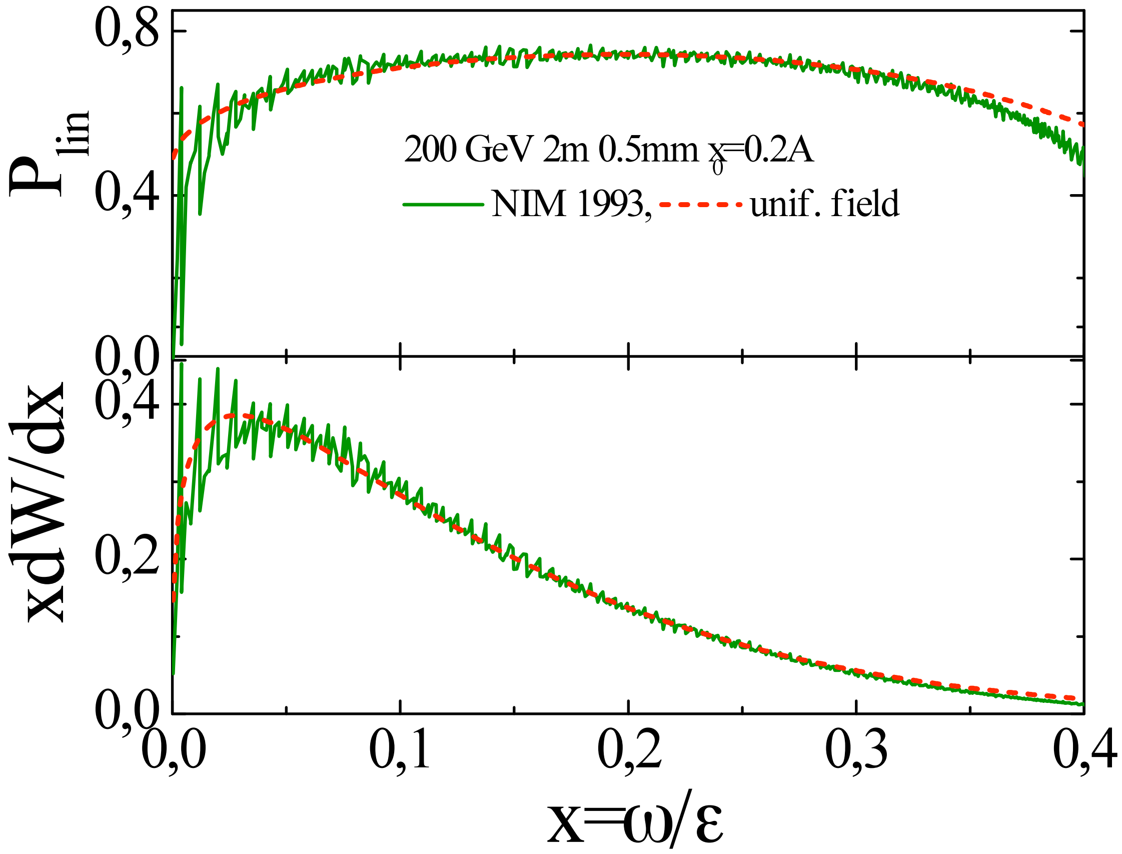




l_{cr} = 0.2mm, R = 2m – easily available!

W ~ 10^{-2} !

Synchrotron-like approximation at 200 GeV e^+



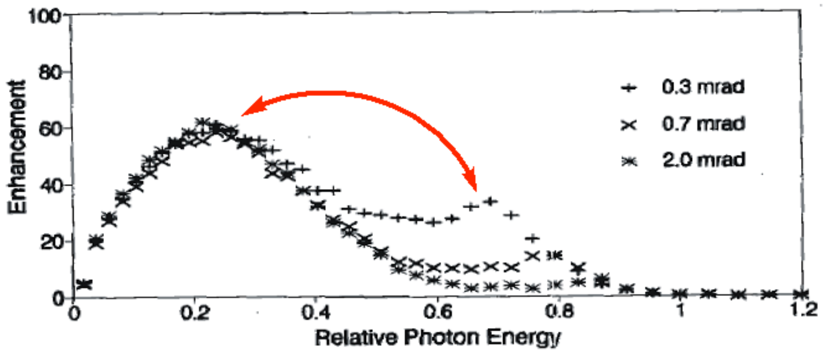
Tikhomirov V.V. *A new method to calculate the characteristics of radiation and pair production under high energies and arbitrary angles of particle incidence relative to the crystal planes. Nucl. Instrum. and Methods.* 1993. V. B82. P. 409-416. ***Based on Baier-Katkov method.***

The more sophisticated theory:

adding circular polarization to the approach of
V.N. Baier, V.M. Katkov, V.M. Strakhovenko, NIM B69(1992)258

1. + channeling radiation
2. + circular polarization
3. planar potential => SOS radiation
4. SOS oscillations => channeling radiation

150 GeV, 0.5mm, diamond



$$dw = \frac{2\alpha d\omega}{\gamma^2} \sum_{n=1,2,\dots} \frac{e^2 E_n^2}{m^2 q_\parallel^2}$$

$$\times \left\{ (g_1+\nu)\int_{b_+}^{\infty} Ai(y)dy - 2g_1 \int_{b}^{\infty} Ai(y)dy + (g_1-\nu)\int_{b_-}^{\infty} Ai(y)dy \right.$$

$$+ \mu\nu\left(\frac{\chi}{u}\right)^{1/3}\left[(g_4+4\nu g_2)Ai(b_+) - 2\varphi Ai(b)(1-\Lambda) + (g_4-4\nu g_2)Ai(b_-)\right]$$

$$\left. + \nu^2\left(\frac{\chi}{u}\right)^{2/3}\left[(g_3-g_2)Ai'(b_+) - 2(g_3+g_2)Ai'(b) + (g_3-g_2)Ai'(b_-)\right] \right\};$$

$$dw_{12} = -dw_{21} = -\frac{4i\alpha d\omega}{\gamma^2} \sum_{n=1,2,\dots} \frac{e^2 E_{xn} E_{yn}}{m^2 q_\parallel^2}$$

$$\times \frac{\varphi}{4} \left\{ (g_5+\frac{1}{2})\int_{b_+}^{\infty} Ai(y)dy - 2g_5 \int_{b}^{\infty} Ai(y)dy + (g_5-\frac{1}{2})\int_{b_-}^{\infty} Ai(y) \right.$$

$$\left. + \mu\nu\left(\frac{\chi}{u}\right)^{1/3}\left[Ai(b_+) - Ai(b_-)\right] + \frac{1}{4}\nu\left(\frac{\chi}{u}\right)^{2/3}\left[3Ai'(b_+) - 10Ai'(b) + 3Ai'(b_-)\right] \right\},$$

$$b = (\omega\chi/\varepsilon)^{2/3}(1+\rho/2), \qquad b_\pm = (\omega\chi/\varepsilon)^{2/3}[(1+\rho/2) \pm 1/\nu],$$

$$g_1 = \frac{\varphi}{4} + \nu^2(1+\rho/2) - 4\nu^2\mu^2\Lambda, \qquad g_2 = [\varphi(1+\rho/2) - 1]\Lambda,$$

$$g_3 = 1 - 4\varphi\mu^2\Lambda, \qquad g_4 = \varphi(1+\Lambda), \qquad g_5 = \nu(1+\rho/2) + \nu\mu^2,$$

$$\mu = \frac{\chi}{s} = \frac{eE}{2m|q_\parallel|}, \qquad \Lambda = \frac{2E_{xn}^2}{E_n^2}.$$

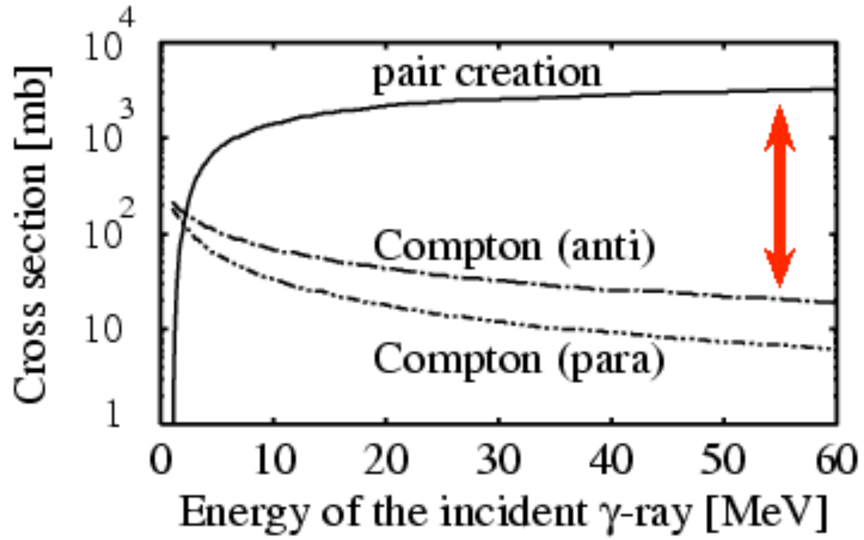
Incoherent scattering contribution to the local radiation probability

$$dw_{inc} = \frac{d\omega}{3L_{rad}\varepsilon^2\omega}\left\{2\left[\varepsilon^2 + \varepsilon'^2\right] + \omega^2\right\}$$

absolutely necessary at **ε ~ 1 TeV**

Pair production probability dependence on gamma-quantum circular polarization

Essential for
measurement
of gamma
circular polarization
at 100 MeV and above



Probability and polarization asymmetry of channeled positron production

$$\vartheta_x = \theta_x - z/R$$

$$\frac{dw}{d\vartheta_x} = \frac{dw(x)}{d\vartheta_x} = \int\int \frac{d\sigma(\theta)}{d\theta_x d\theta_y} d\theta_y dz = R\int\int \frac{d\sigma(\theta)}{d\theta_x d\theta_y} d\theta_x d\theta_y,$$

$$w(x) = \frac{1}{2}\left[w_{xy}(x) + dw_{yx}(x)\right] =$$

$$= \frac{\alpha^2 R}{2\omega}\int_0^\omega \frac{d\varepsilon}{\omega}\sum_{n=1,2,..} E_n^2(x) q_\parallel^{-2}[\varphi + 4\nu(1-\nu)]\theta(1-\nu)\int_{-\vartheta_x max}^{\vartheta_x max}[1 - P_{dech}(\vartheta_x)]d\vartheta_x$$

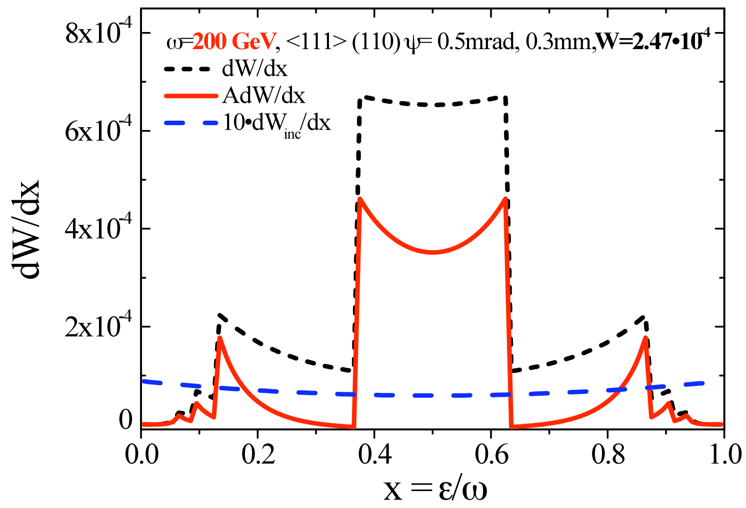
$$Aw(x) = \frac{i}{2}\left[dw_{xy}(x) - dw_{yx}(x)\right] =$$

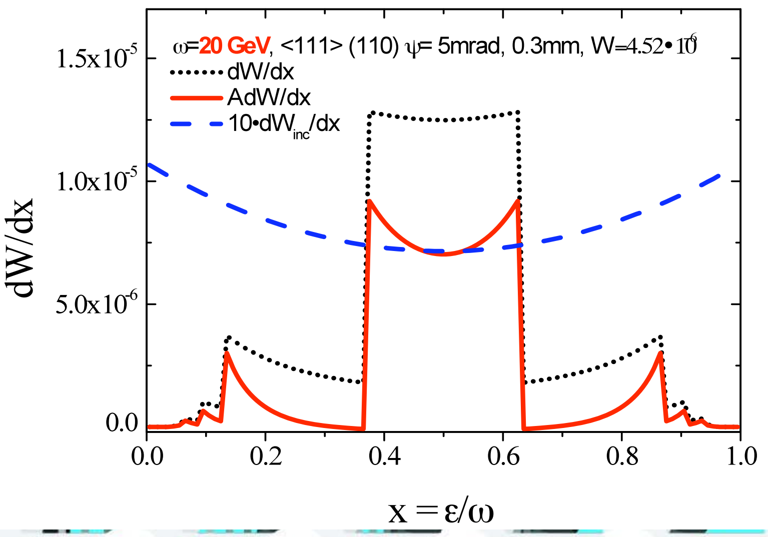
$$= \frac{\alpha^2 R}{2\omega}\int_0^\omega \frac{d\varepsilon}{\omega}\sum_{n=1,2,..} E_n^2(x) q_\parallel^{-2}\xi_2\varphi(2\nu-1)\theta(1-\nu)\int_{-\vartheta_x max}^{\vartheta_x max}[1 - P_{dech}(\vartheta_x)]d\vartheta_x,$$

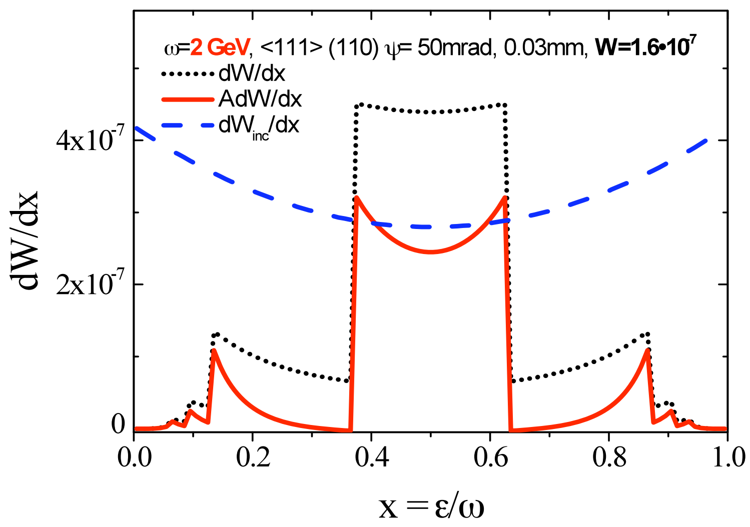
$$\varepsilon' = \varepsilon - \omega, \qquad \varphi = \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}, \qquad \nu = \frac{\omega m^2}{2\varepsilon\varepsilon' q_\parallel}, \qquad q_\parallel = n q_{1y}\psi v$$

are expressed through values integrated over both transverse angles

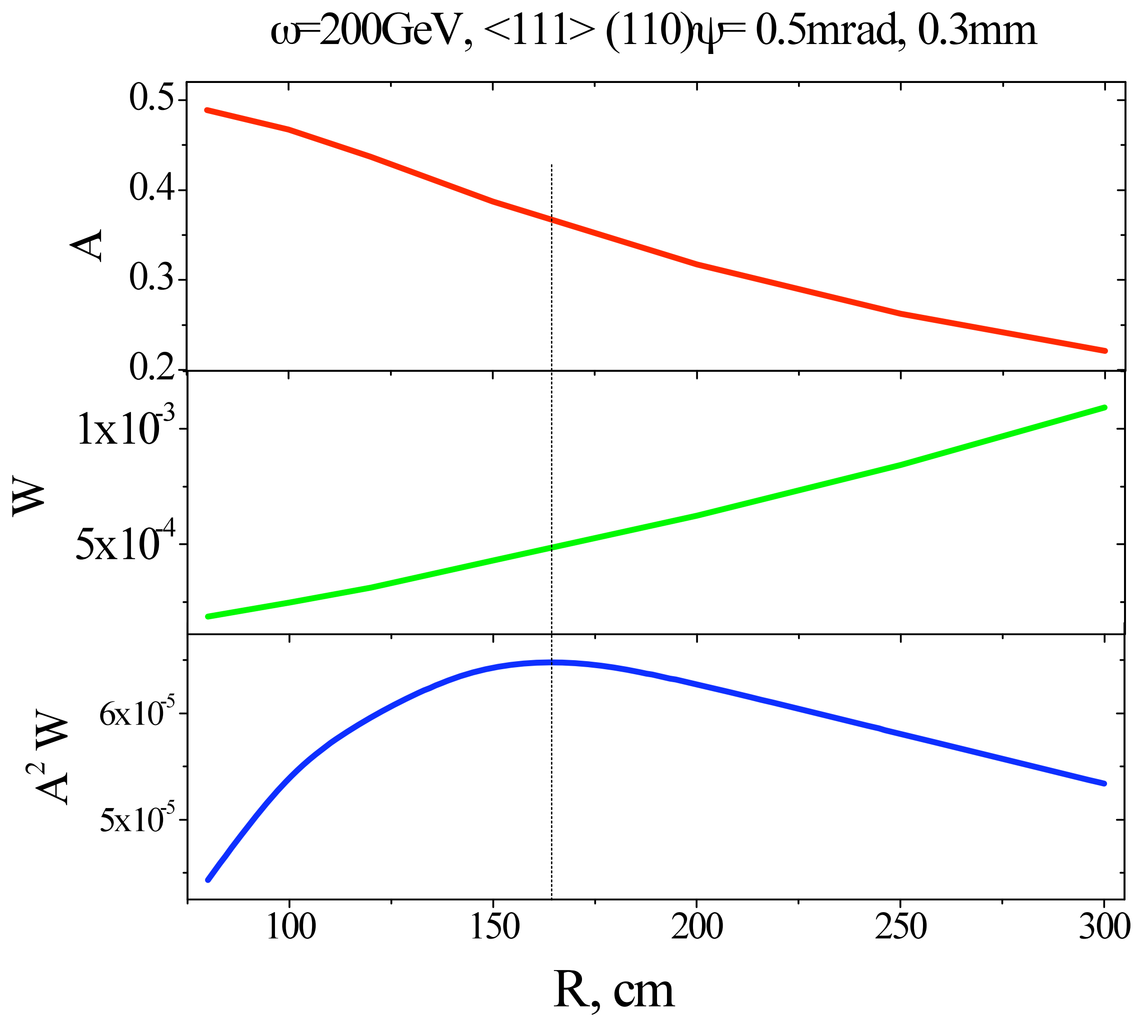
Differential pair production **probability** and polarization **asymmetry** at 2, 20 and 200 GeV



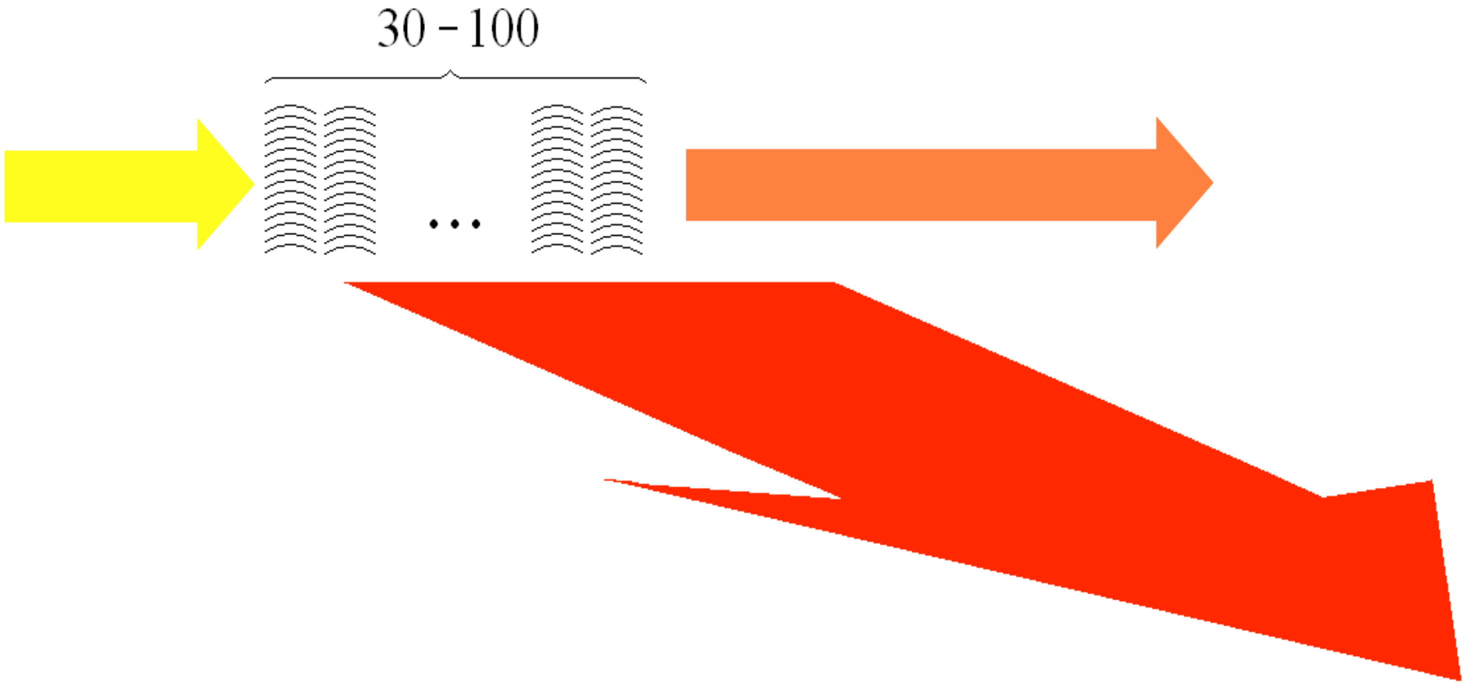




Bending radius optimization at 200 GeV



Tens of crystal can be used for gamma CP measurements

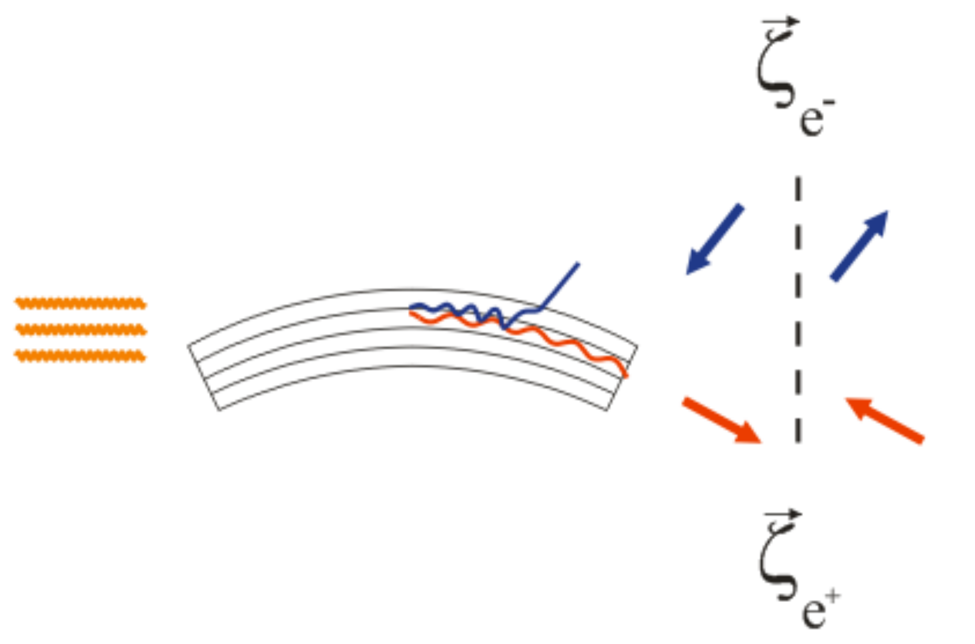


$W(\hbar\omega \sim 100\,GeV) \Rightarrow 10^{-2}, \quad W(\hbar\omega \sim 10\,GeV) \Rightarrow 10^{-4}, \quad W(\hbar\omega \sim 1\,GeV) \Rightarrow 10^{-5}$

Positrons become longitudinally polarized after circularly polarized gamma-quanta emission

Longitudinal positron polarization can be measured

Longitudinally polarized ***electrons*** can be produced



Polarization effect predicted earlier

Vacuum dichroism and birefringence

V. G. Baryshevsky and V. V. Tikhomirov, *Usp. Fiz. Nauk* **159**, 529 (1989) [*Sov. Phys. Usp* **32**, 1013 (1989)].

V. G. Baryshevsky and V. V. Tikhomirov, *Yad. Fiz.* **36**, 697 (1982) [*Sov. J. Nucl. Phys.* **36**, 408 (1982)];*Phys. Lett. A* **90**, 153 (1982).

V. G. Baryshevsky and V. V. Tikhomirov, *Nucl. Instr. Meth. A* **234**, 430 (1985).

V. V. Tikhomirov, *Sov. J. Nucl. Phys.* **53**, 338 (1991).

Electron radiative self-polarization in crystals

V. G. Baryshevsky, A.O. Grubich *Pis'ma Zh. Tekh. Fiz.* **5**, 1529 (1979).

V. V. Tikhomirov, *Pis'ma v Zh. Eksp. Teor. Fiz.* **58**, 168 (1993).

Polarized electron-positron pair production by gamma-quanta

V. G. Baryshevsky and V. V. Tikhomirov, *Zh. Eksp. Teor. Fiz.* **85**, 232 (1983) [*Sov. Phys. JETP* **58**, 135 (1983)];*Phys. Lett. A* **96**, 215 (1983).

V. G. Baryshevsky and V. V. Tikhomirov, *Yad. Fiz.* **48**, 670 (1988) [*Sov. J. Nucl. Phys.* **48**, 429 (1988)].

Positron (electron) anomalous magnetic moment modification influencing spin rotation

V. G. Baryshevsky, *Pis'ma Zh. Tekh. Fiz.* **5**, 182 (1979) [*Sov. Tech. Phys. Lett* **5**, 73 (1979)].

V. G. Baryshevsky and A. O. Grubich, *Yad. Fiz.* **44**, 1114 (1986) [*Sov. J. Nucl. Phys.* **44**, 721 (1986)].

V. V. Tikhomirov, *Sov. J. Nucl. Phys.* **61**, 1188 (1996).

Electron spin rotation in a circularly polarized crystal field harmonics

V. V. Tikhomirov, *Pis'ma v Zh. Eksp. Teor. Fiz.* **61**, 177 (1995).

V. V. Tikhomirov, *Phys. Rev. D* **53**, 7213 (1996).

V. V. Tikhomirov, *Sov. J. Nucl. Phys.* **62**, 664 (1999).

V. V. Tikhomirov, *Zh. Eksp. Teor. Fiz.* **109**, 1188 (1996).

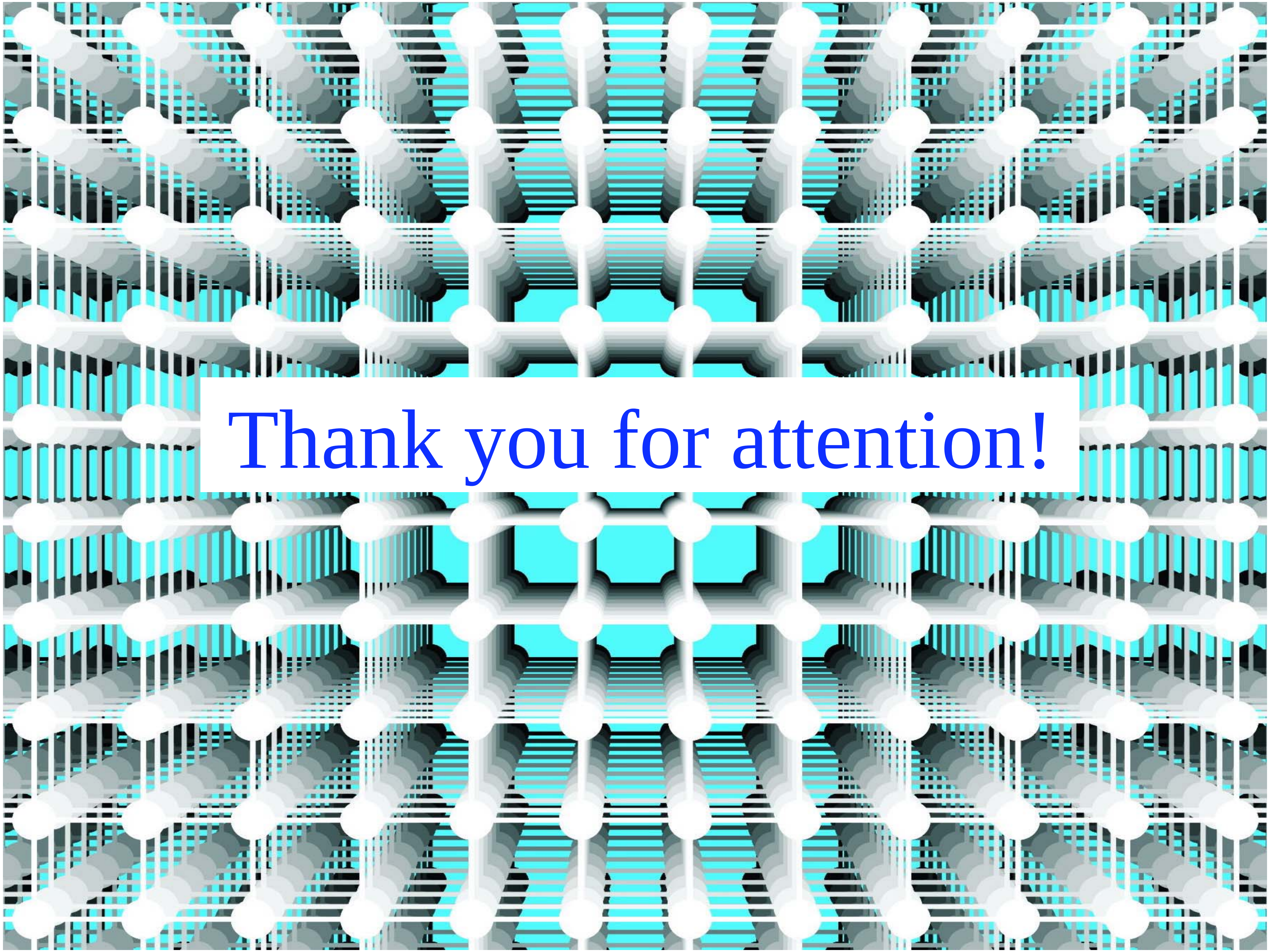
all of them:

– are described by Baier-Katkov method

– observable mostly at the LHC energies

Conclusions

- Hard string-of-string radiation peak is highly circularly polarized
- Circular gamma-quantum polarization can be measured in a new way
- Both effects are observable at 1 GeV and above


Thank you for attention!

The Mir castle