Bremsstrahlung from relativistic bare heavy ions in single crystals

Allan H Sørensen

Channeling 2010

Channeling Radiation

^{208}Pb penetrating Si target at $\gamma = 170$:

$$\psi_1 \ll 1/\gamma \qquad (\gamma\psi_1 = 3.5\times10^{-3} \text{ for } \langle 111\rangle)$$

Hence

- non-relativistic transverse motion for channeled ions in "rest frame"
- far from "constant field" (synchrotron) approximation

Typical lab-frequency in transverse motion corresponding to periodicity *d*

$$\omega_\mathrm{d} \sim 2\pi c\psi/d$$

For $d \sim 2$Å and $\psi \sim \psi_1$ this gives

$$\hbar\omega_\mathrm{d} \sim 2\pi \text{ keV} \times \psi_1$$

$\gamma = E/Mc^2$
Lorentz factor
ψ_1 critical/
Lindhard angle

Channeling Radiation

^{208}Pb penetrating Si target at $\gamma = 170$:

$$\psi_1 \ll 1/\gamma \qquad (\gamma\psi_1 = 3.5\times 10^{-3} \text{ for } <111>)$$

Hence

- non-relativistic transverse motion for channeled ions in "rest frame"
- far from "constant field" (synchrotron) approximation

Typical lab-frequency in transverse motion corresponding to periodicity *d*

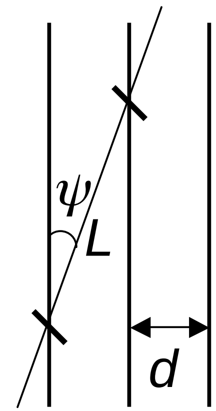
$$\omega_d \sim 2\pi c\psi/d$$

For $d \sim 2$Å and $\psi \sim \psi_1$ this gives

$$\hbar\omega_d \sim 2\pi \text{ keV} \times \psi_1$$

$L = d/\psi$

$\Delta t = L/c$



Transformation to rest frame (time dilation):

$$\omega_d^R = \gamma\omega_d$$

Radiation in rest frame typically at ω_d^R
Transformation to lab gives radiation at:

$$\omega \sim 2\gamma^2\omega_d$$

For ^{208}Pb in Si <111> at $\gamma = 170$:

$\psi_1 = 21\ \mu\text{rad}$

$\hbar\omega_d \sim 2\pi\ \text{keV} \times \psi_1 = 0.13\ \text{eV}$

$$\therefore \qquad \hbar\omega \sim 2\gamma^2\omega_d \sim 7.5\ \text{keV}$$

Way below characteristic energies for incoherent BS
- typically MeV-GeV: leave channeling radiation here!

Procedure of BS calculation

Require projectile to stay intact, that is, restrict to non-contact collisions

$$b > R_1 + R_2 \equiv R_\Sigma$$

Impact parameter $b > R_\Sigma$ not complete guarantee for no break-up - but nearly *

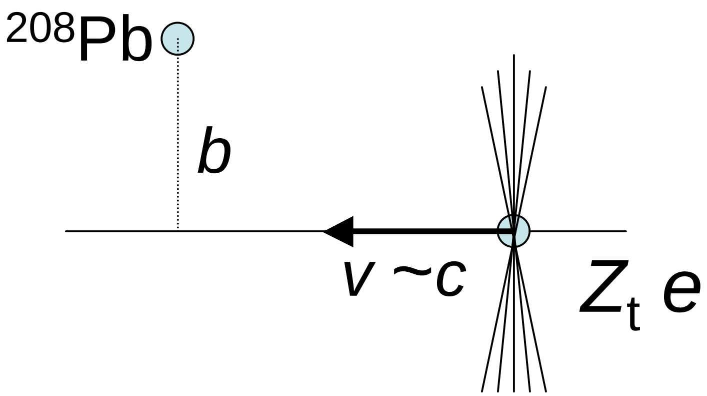
1. EM field of moving object (nucleus, neutral atom, electron) nearly transverse at high γ ; shape as EM radiation pulse
2. Xsections for photon scattering extractable from literature

$\longrightarrow$ use Weizsäcker-Williams method of virtual quanta

* electromagnetic dissociation important near R_Σ

Four contributions to BS

The main contribution:



Scattering of the virtual photons of the *screened* target nucleus on the projectile in the rest frame R of the latter

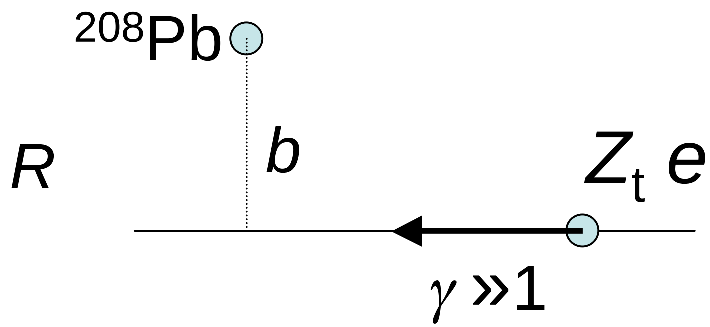
The other three:

1. scattering of the virtual photons of the projectile on target nuclei
2. scattering of the virtual photons of the projectile on target electrons
3. scattering of the virtual photons of target electrons on the projectile

WW - main contribution - step 1

Virtual photon intensity in *R* :

$$\frac{dI}{d\omega d^2b} = \hbar\alpha Z_{\mathrm{t}}^2 \frac{1}{\pi^2 b^2}(xK_1(x))^2$$



divide by $\hbar\omega$ to get # virtual photons per bin (OBS: ω in *R*)

Bare target nucleus:

$$x = \frac{\omega b}{\gamma c}$$

Neutral target atom [Yukawa potential with screening length a_{TF}]:

$$x = \left[\left(\frac{\omega b}{\gamma c}\right)^2 + \left(\frac{b}{a_{\mathrm{TF}}}\right)^2\right]^{1/2}$$

Depletion at small *b* - EM dissociation?

Effective b_{max} at given ω determined by $x \approx 1$ - screening important at high γ

WW - main contribution - step 2

Multiply scattering cross section on virtual photon intensity.

Since observed photon energy and exit angle in lab depend on scattered photon energy and angle i *R* we need differential cross section.

Projectile intact: require coherent action of constituents

a) below $\omega_1 \approx 8$ MeV (typical binding) scattering on point nucleus : *Thomson cross section for point nucleus*
b) beyond ω_1 and up to $\omega_2 = c/R$ coherent scattering on Z quasifree protons : *Z^2 times Thomson cross section for p*
c) beyond ω_2 incoherent scattering on individual protons possible; *restrict to coherent part* to prevent break-up

To b) add *resonance*

WW - main contribution - step 2

A useful fit to experimental data for elastic photon scattering on ^{208}Pb:

$$\frac{d\sigma}{d\Omega'} = Z^2 r_p^2 \, \frac{1}{2}(1 + \cos^2 \psi')$$

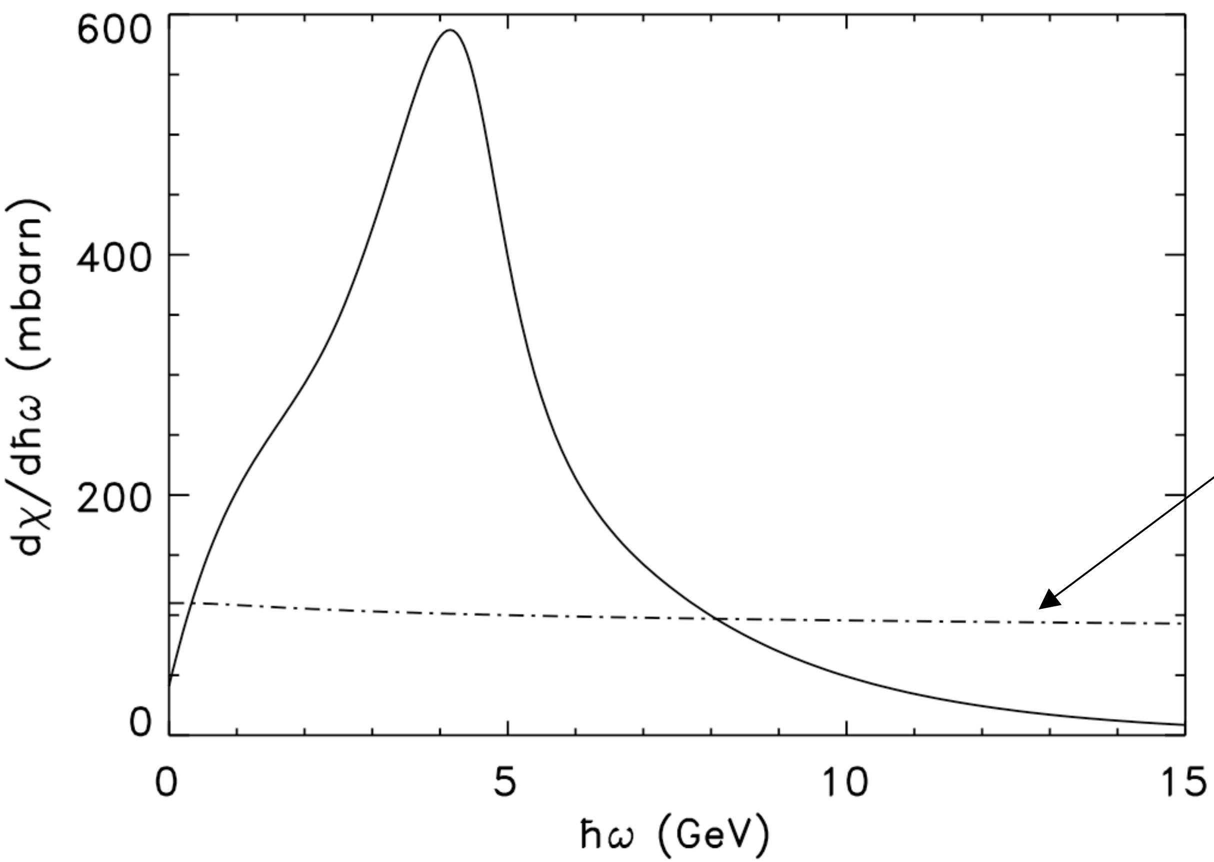
$$\times \left\{ \begin{array}{ll} \left(\frac{ZM_p}{M}\right)^2; & \hbar\omega' < \hbar\omega_1 \\ 0.793 \, \frac{(\hbar\omega')^4}{((\hbar\omega')^2 - (E_m)^2)^2 + (\Gamma\hbar\omega')^2}; & \hbar\omega_1 < \hbar\omega' < \hbar\tilde{\omega}_2 \\ 1.93 \, \exp(-\epsilon(\hbar\omega' - \hbar\tilde{\omega}_2)\sin^2 \frac{\psi'}{2}); & \hbar\tilde{\omega}_2 < \hbar\omega' \end{array} \right\}$$

primes in *R*
r_p classical radius of proton
ψ' scattering angle

resonance: E_m = 13.7 MeV
Γ = 4.15 MeV
depletion: ε = 0.11 MeV^{-1}

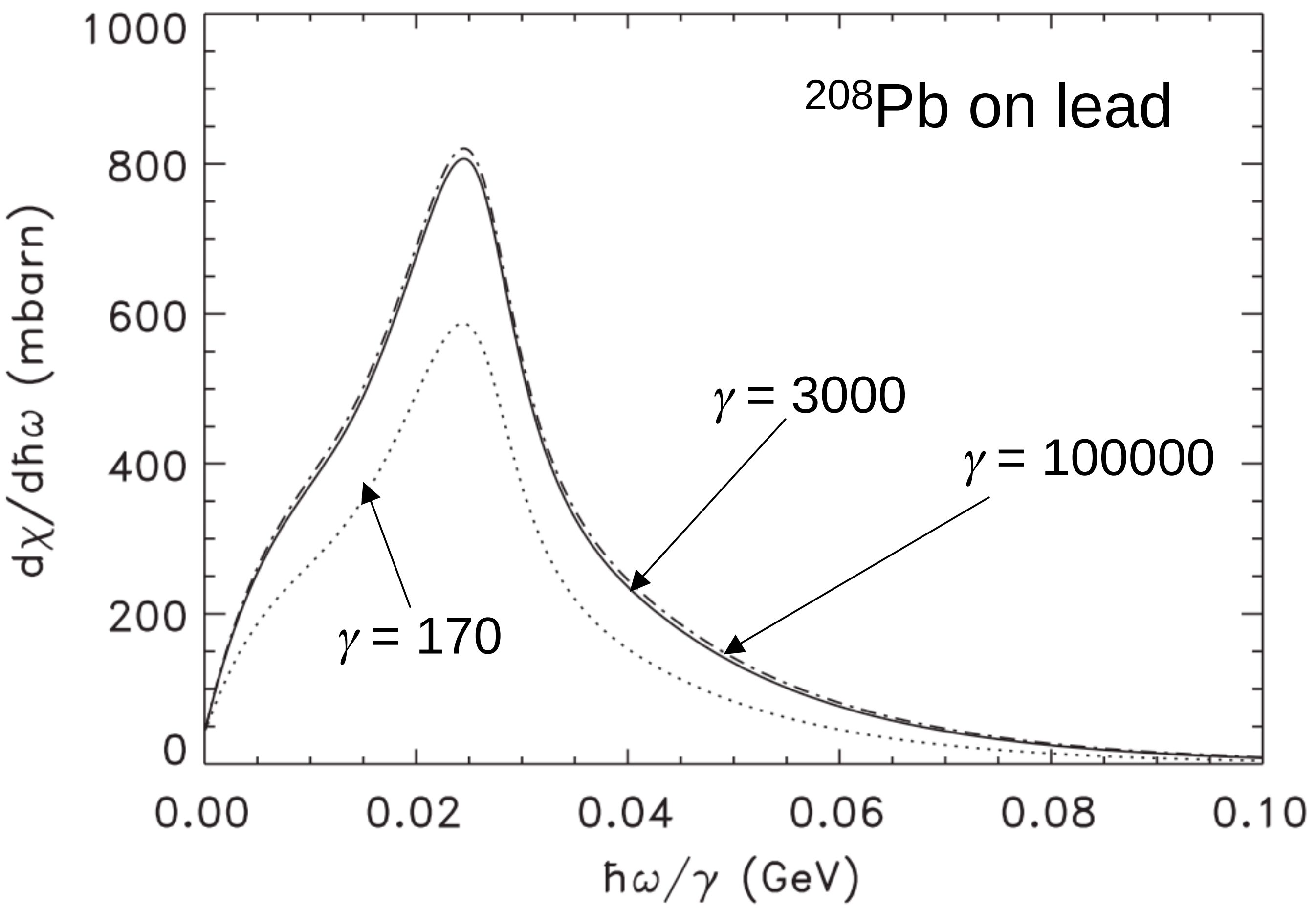
WW - main contribution - step 3

Transform back to lab!
Power spectrum for bare ^{208}Pb on lead target at $\gamma = 170$:



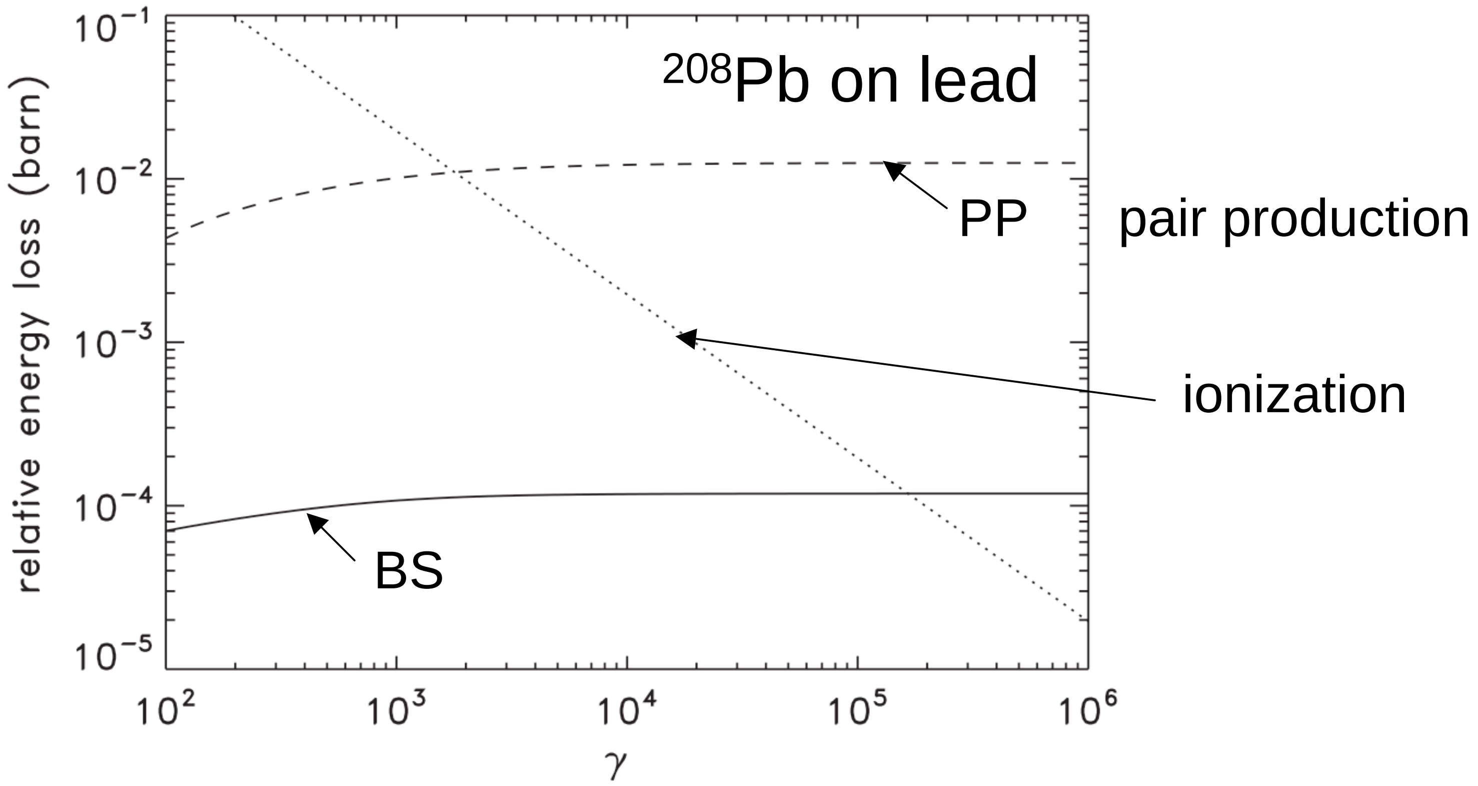
result
for point
nucleus

Scaling with γ :
peak position proportional to γ (ca. $2\gamma E_m$)
peak height saturates due to screening at high γ



Energy loss

Bremsstrahlung can not compete!



For fractional energy loss $-E^{-1}dE/dx$ per cm multiply by 3.30×10^{-2}

Channeling

b-range never beyond screening length in target atom:

BS essentially close-encounter process

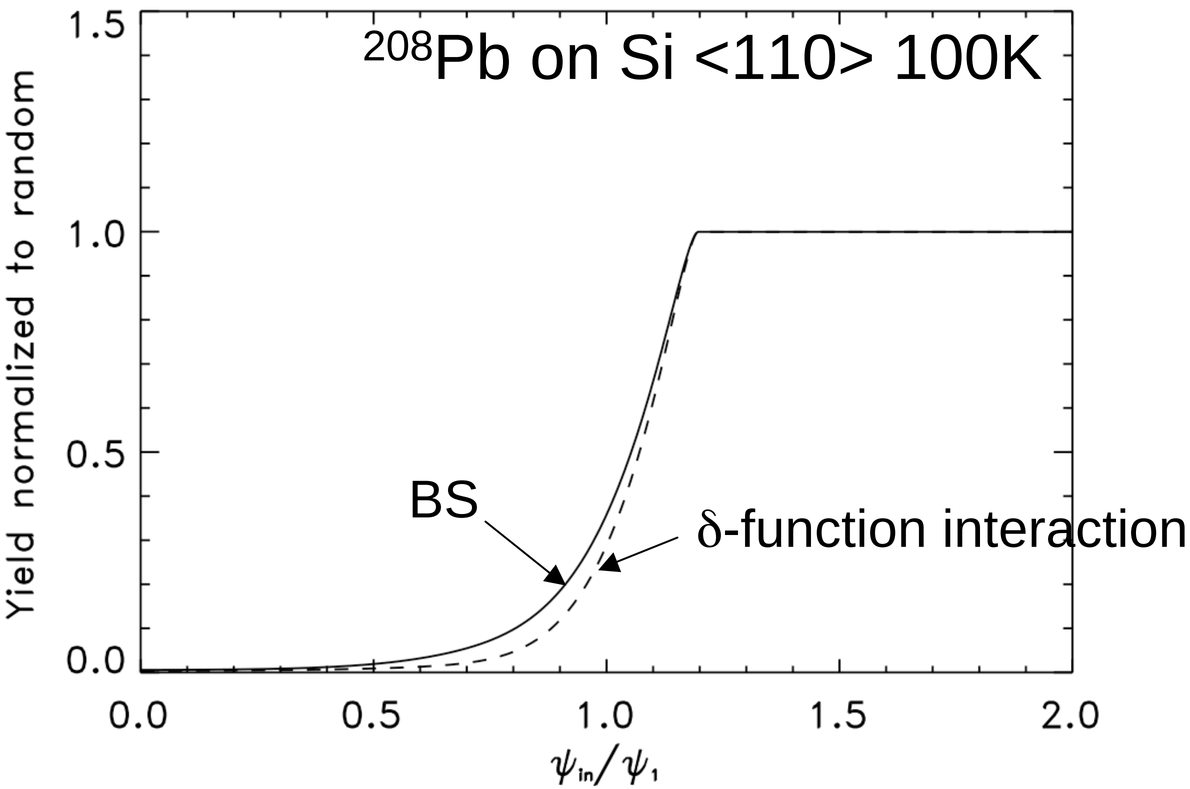
When screening defines range at all energies where photon scattering cross section has support, dependence on ω and *b* factorizes:

$$\frac{d\chi}{d\hbar\omega d^2b} = \frac{d\chi}{d\hbar\omega} \times \frac{1}{2\pi \ln(Ca_{\mathrm{TF}}/R_\Sigma)} \frac{1}{b^2} \left[\frac{b}{a_{\mathrm{TF}}} K_1 \left(\frac{b}{a_{\mathrm{TF}}} \right) \right]^2$$

radiation cross section (power spectrum)

“Complete screening” - longest range!

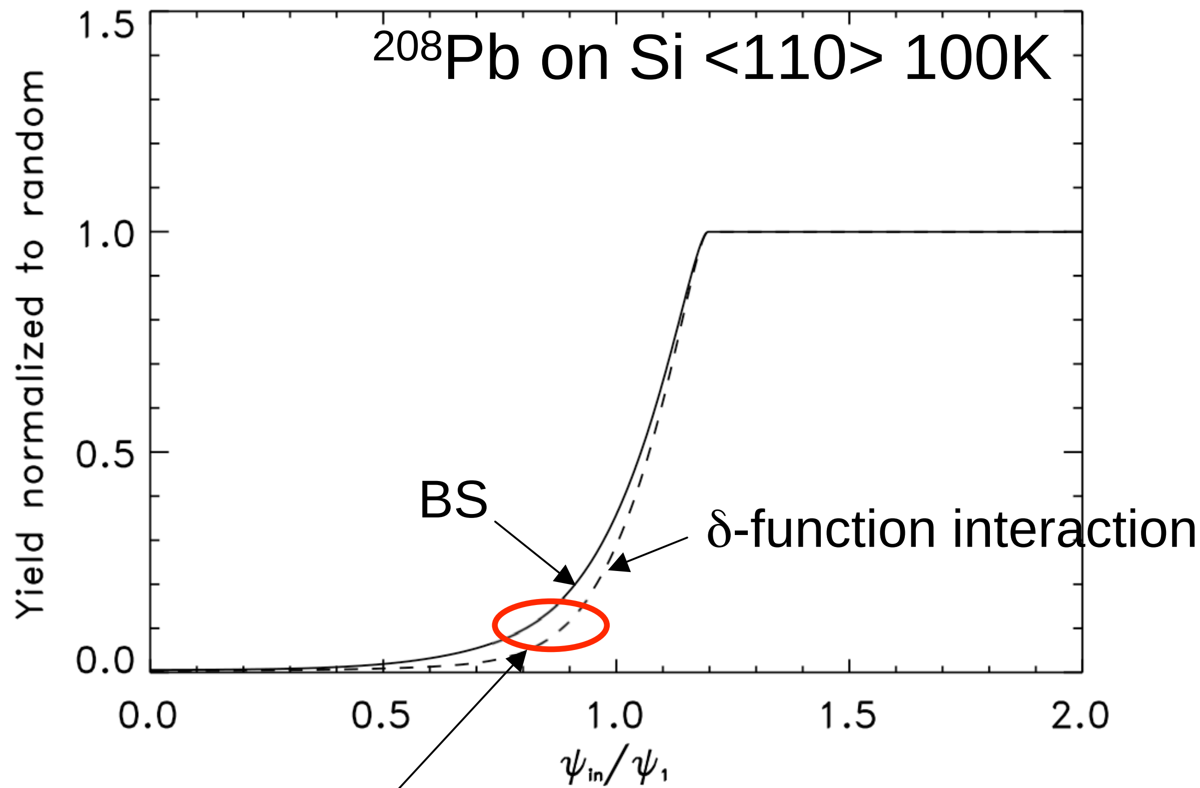
Dip in BS in complete screening limit



$a_{TF}/R_{\Sigma} = 2.4{\times}10^3$

$a_{TF}/\rho = 2.5$

Dip in BS in complete screening limit



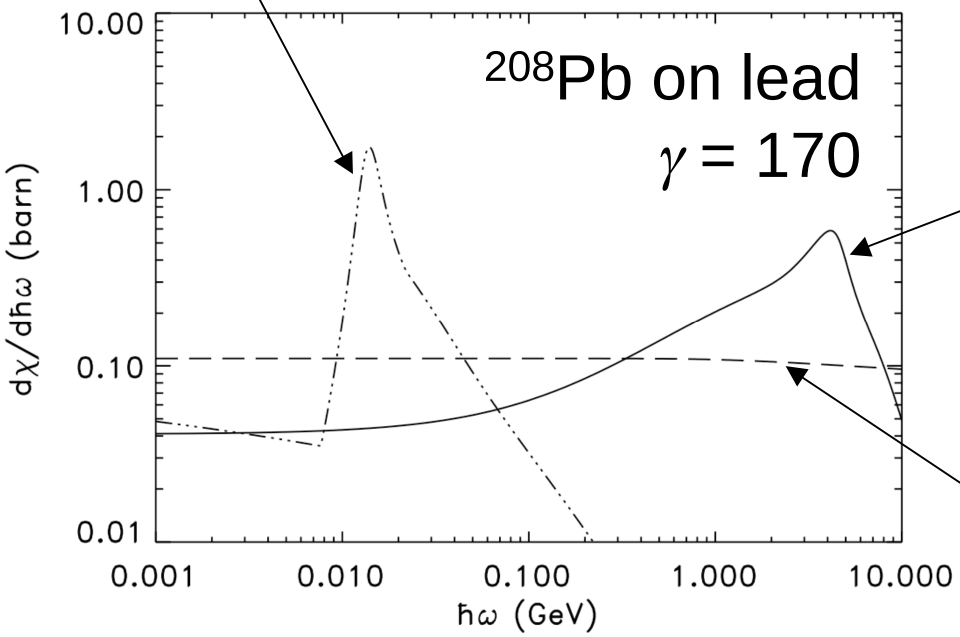
variation with γ
to stay between lines!

$a_{TF}/R_\Sigma = 2.4\times10^3$

$a_{TF}/\rho = 2.5$

The other three contributions to BS

1.
Scattering of the virtual photons of the projectile on target nuclei



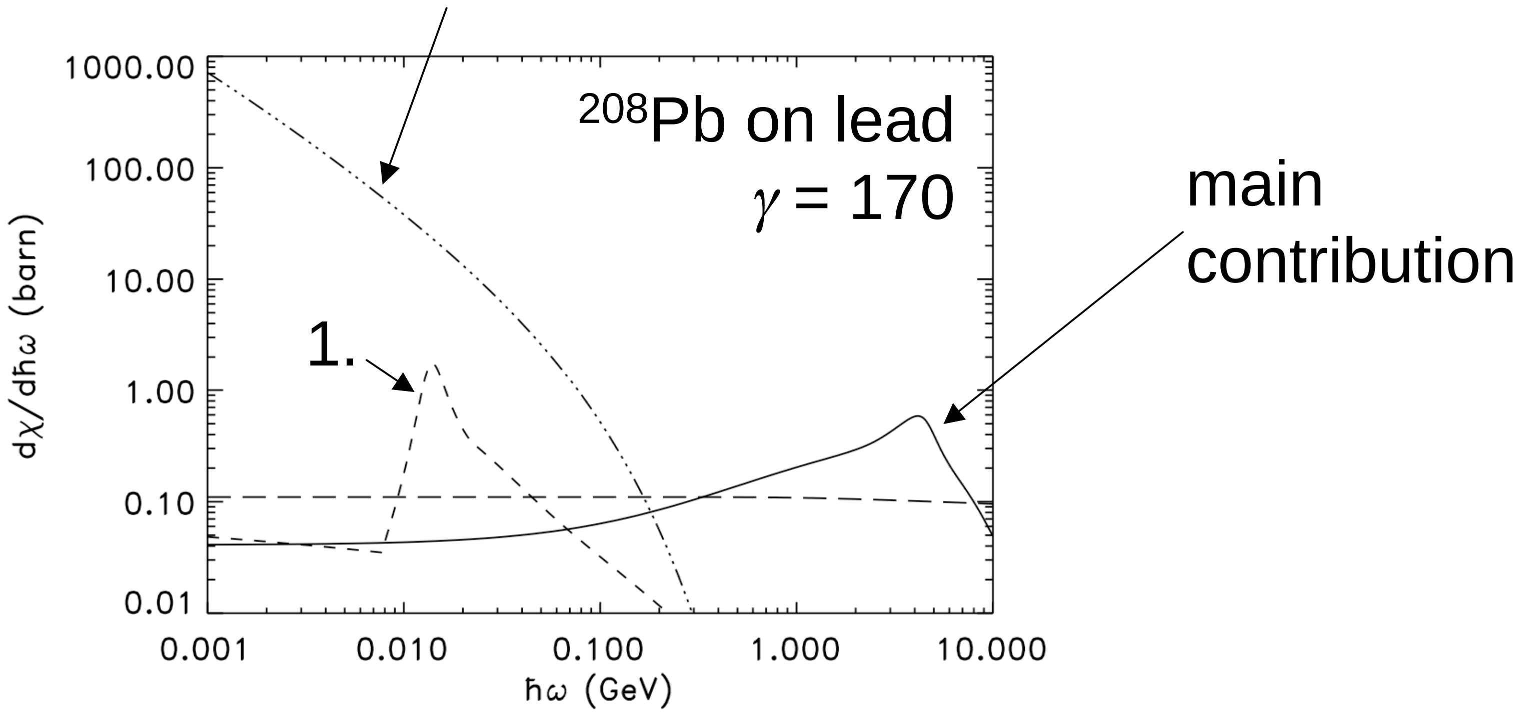
main contribution

hypothetical point nucleus

Confined to MeV energies (GDR)
Essentially no change with γ

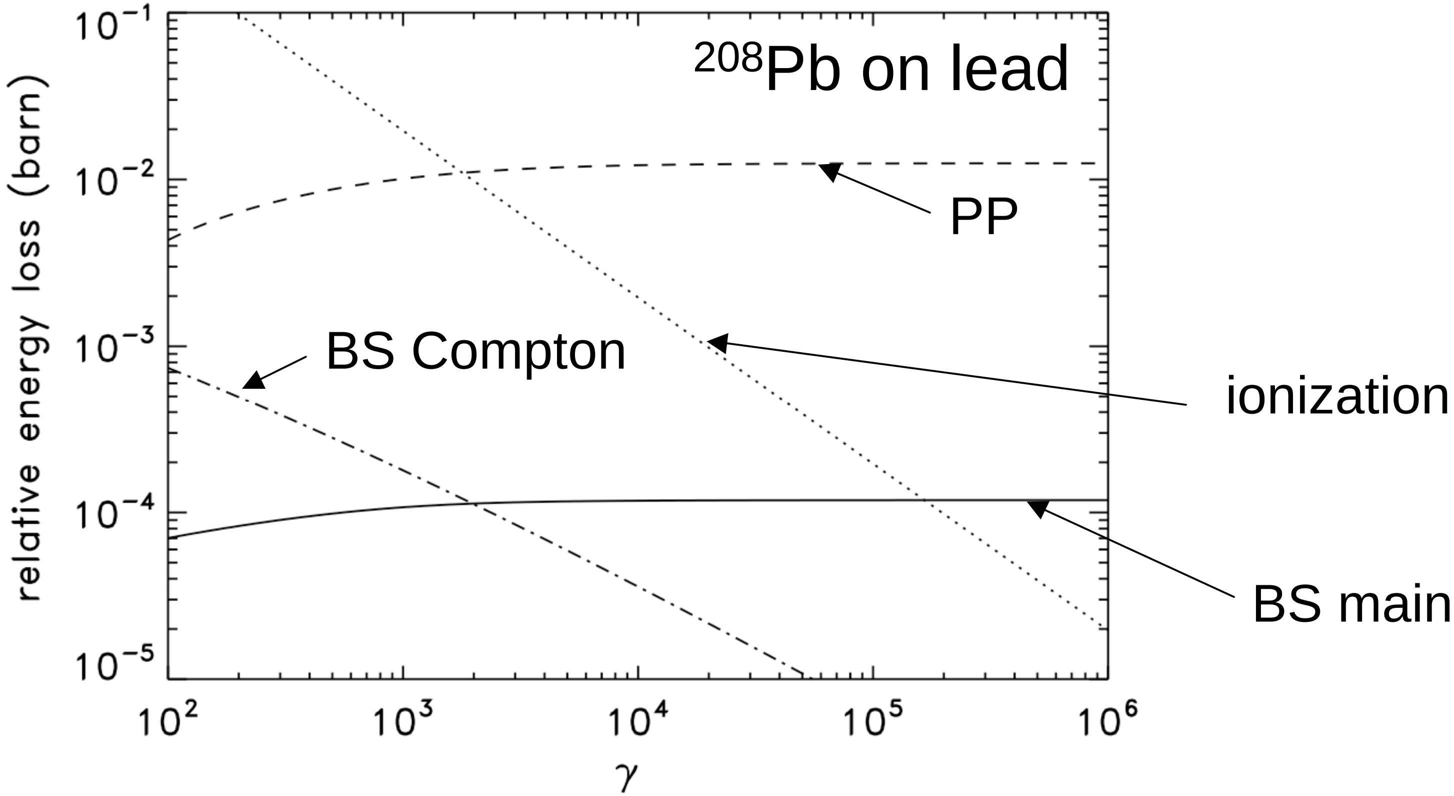
The other three contributions to BS

2. Scattering of the virtual photons of the projectile on target electrons - Compton scattering



Well below peak position in main component - but high yield
Moves up in energy with γ but less fast than main contribution

Compton processes add significantly to energy loss by BS at “moderate” γ :



Despite the extra contribution, BS never dominates energy loss of bare heavy ion penetrating matter

The other three contributions to BS

3. Scattering of the virtual photons of target electrons on the projectile:
Compared to main contribution the change of incoming particle from screened target nucleus to electron implies

1. change of Z_t^2 to $Z_t \times 1^2$ in intensity (major)
2. adjustment of minimum impact parameter (minor)
3. off-set of scattering center (important in channeling)

Essentially, in amorhous medium the contribution from scattering virtual photons of target electrons may be obtained by multiplying main contribution by $1/Z_t$,
that is, the sum of the two is:

$$(1 + 1/Z_t) \times \text{main contribution}$$

The other three contributions to BS

3. Scattering of the virtual photons of target electrons on the projectile:
Compared to main contribution the change of incoming particle from screened target nucleus to electron implies

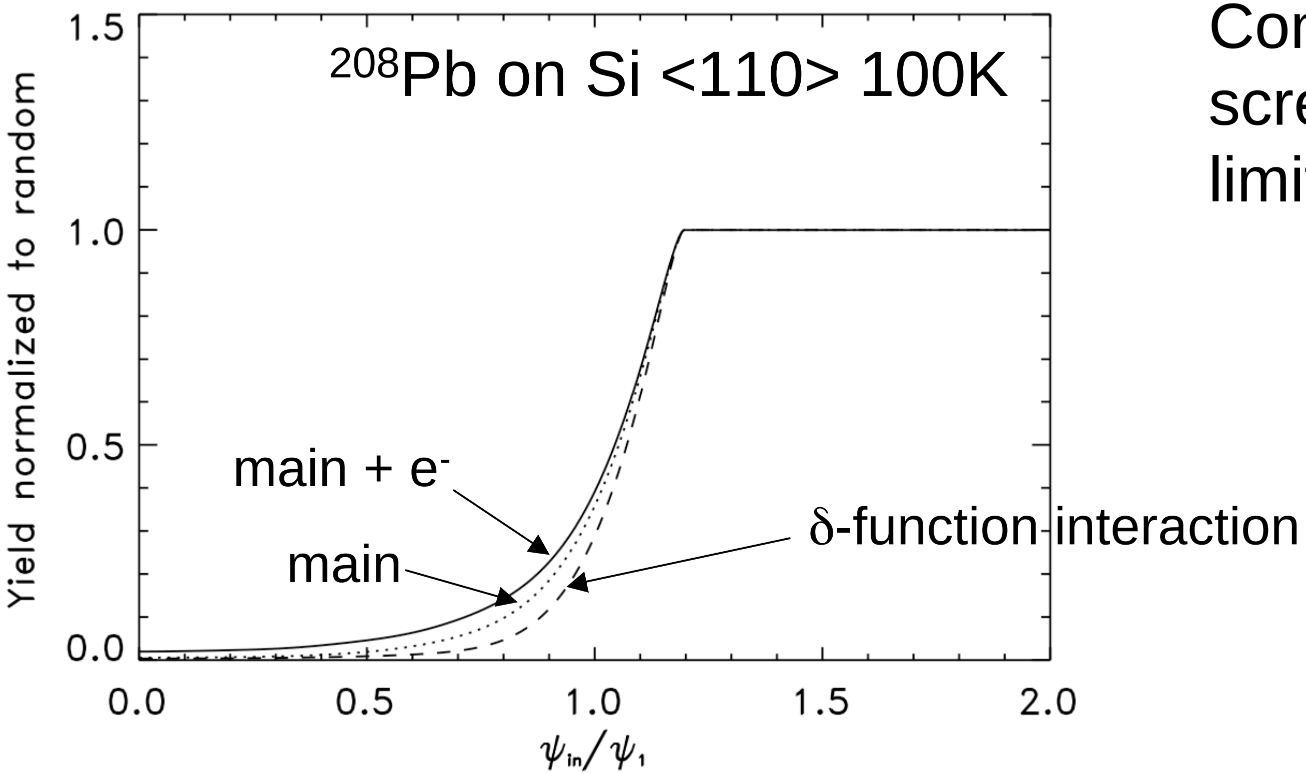
1. change of Z_t^2 to $Z_t \times 1^2$ in intensity (major)
2. adjustment of minimum impact parameter (minor)
3. off-set of scattering center (important in channeling)

Essentially, in amorhous medium the contribution from scattering virtual photons of target electrons may be obtained by multiplying main contribution by $1/Z_t$,
that is, the sum of the two is:

$$(1 + 1/Z_t) \times \text{main contribution} \quad \longrightarrow \quad \text{Si } +7\%$$

Channeling

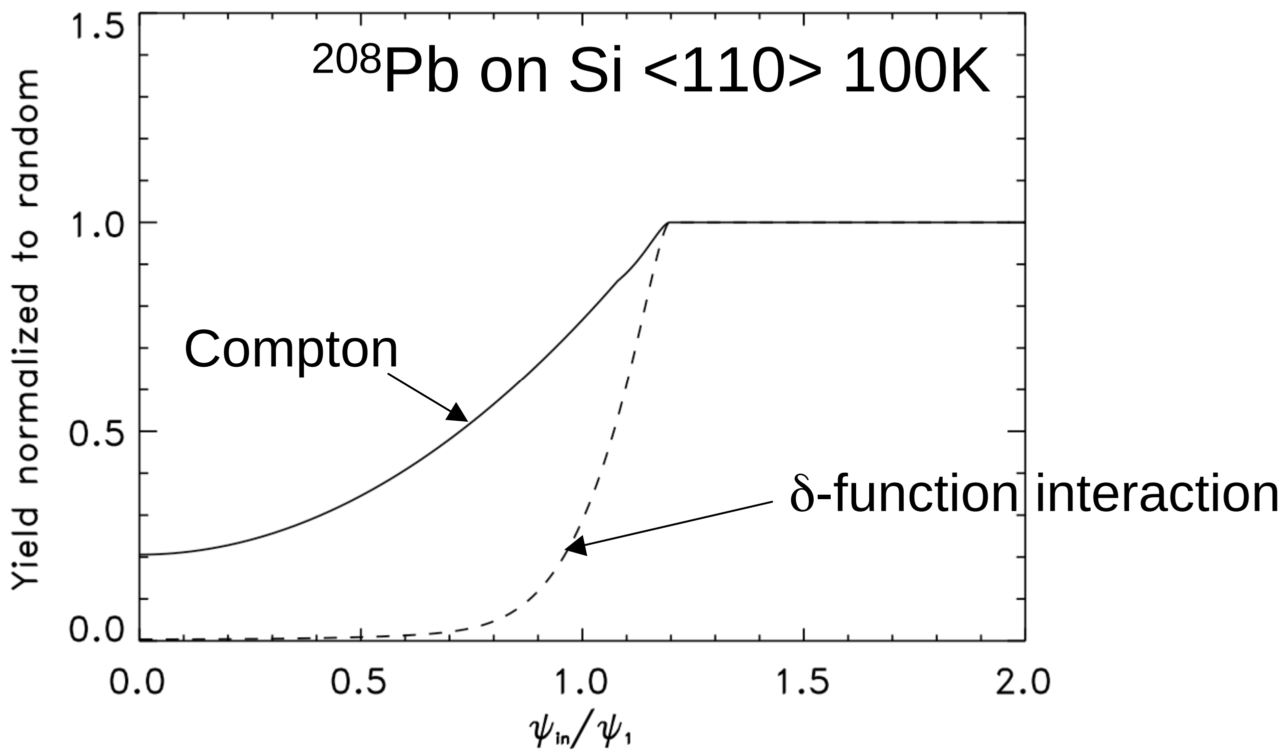
Scattering of the virtual photons of target electrons narrows dips somewhat due to wider range of positions in channel than nuclei + increase in minimum yield



Complete screening limit

Channeling

The Compton component; tests electron distribution - dip fills in



Pair Production

Electron-positron PP is also a close-collision process. But since

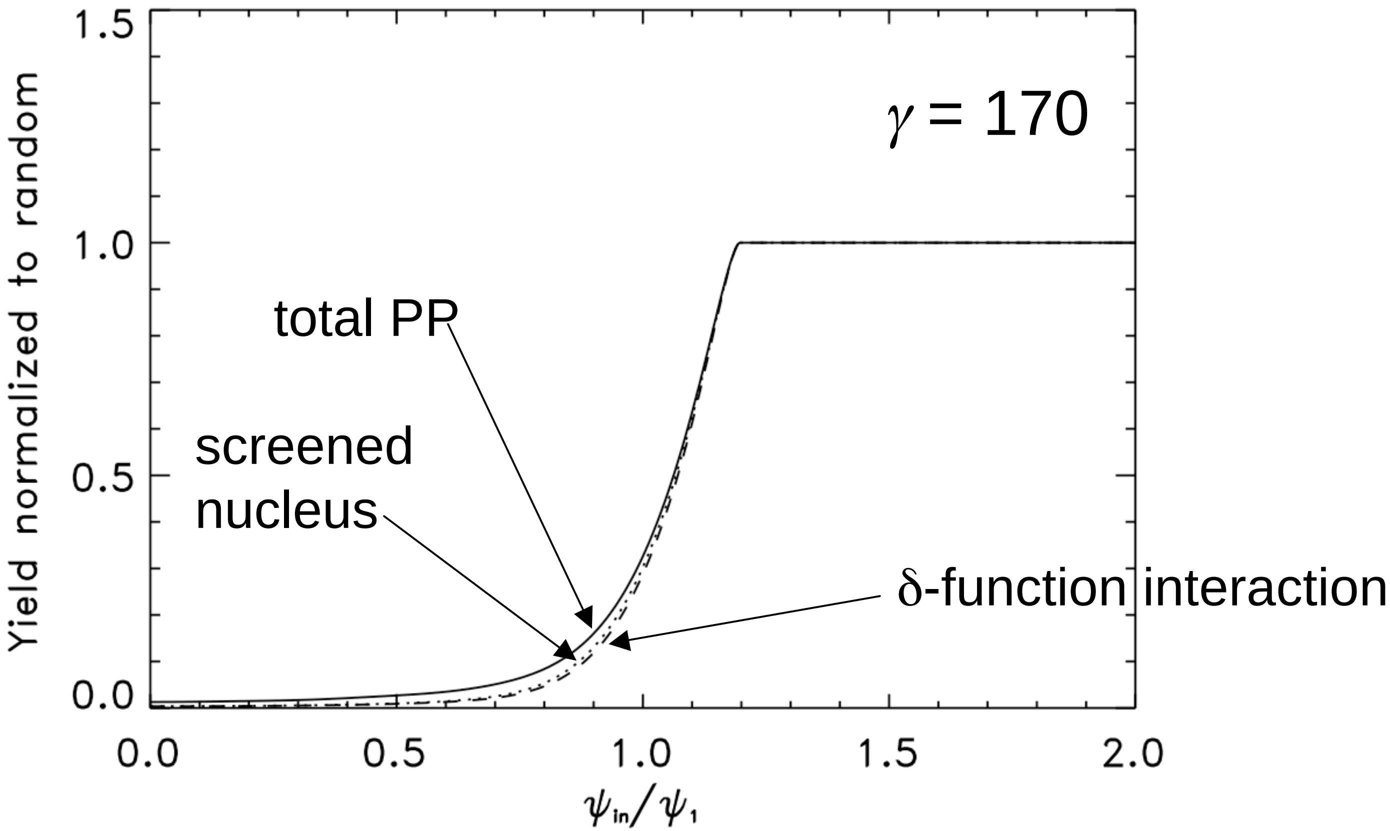
$$L_r = \log(a_{TF}/\rho)/\log(a_{TF}/\ b_{min})$$

is larger in PP than in BS due to larger b_{min}, PP has the potential of showing narrower dips with higher minimum yield.

Ex - ^{208}Pb on Si cooled to 100K:

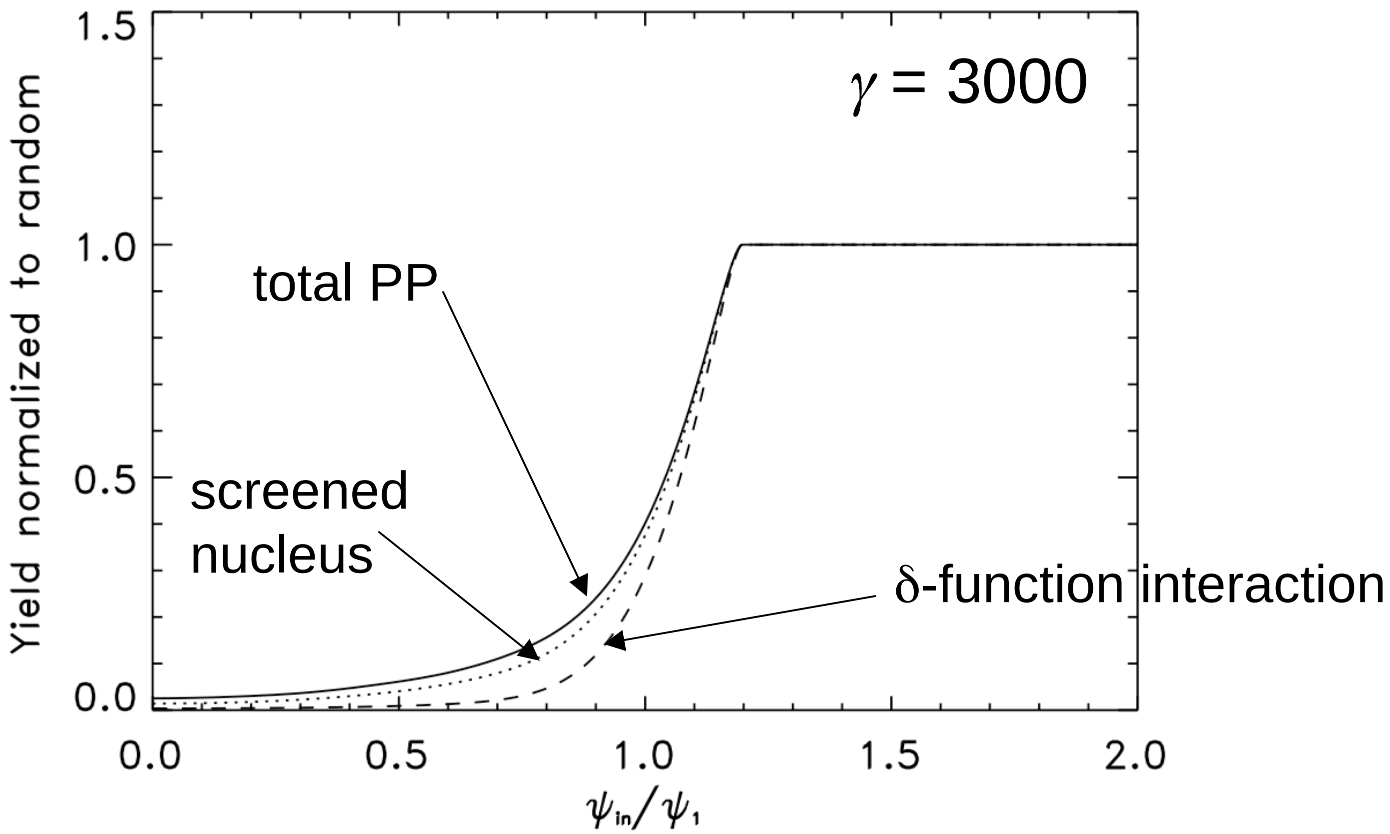
BS:	$L_r = 0.12$
PP:	$L_r = 0.20$

PP: ^{208}Pb on Si <110> 100K



Only the action of target electrons brings a slight deviation from result for δ-function interaction

PP: ^{208}Pb on Si <110> 100K



Not much different from BS in complete screening limit!
Nuclear contribution slightly higher, total about the same.

Credit:

Tue V. Jensen