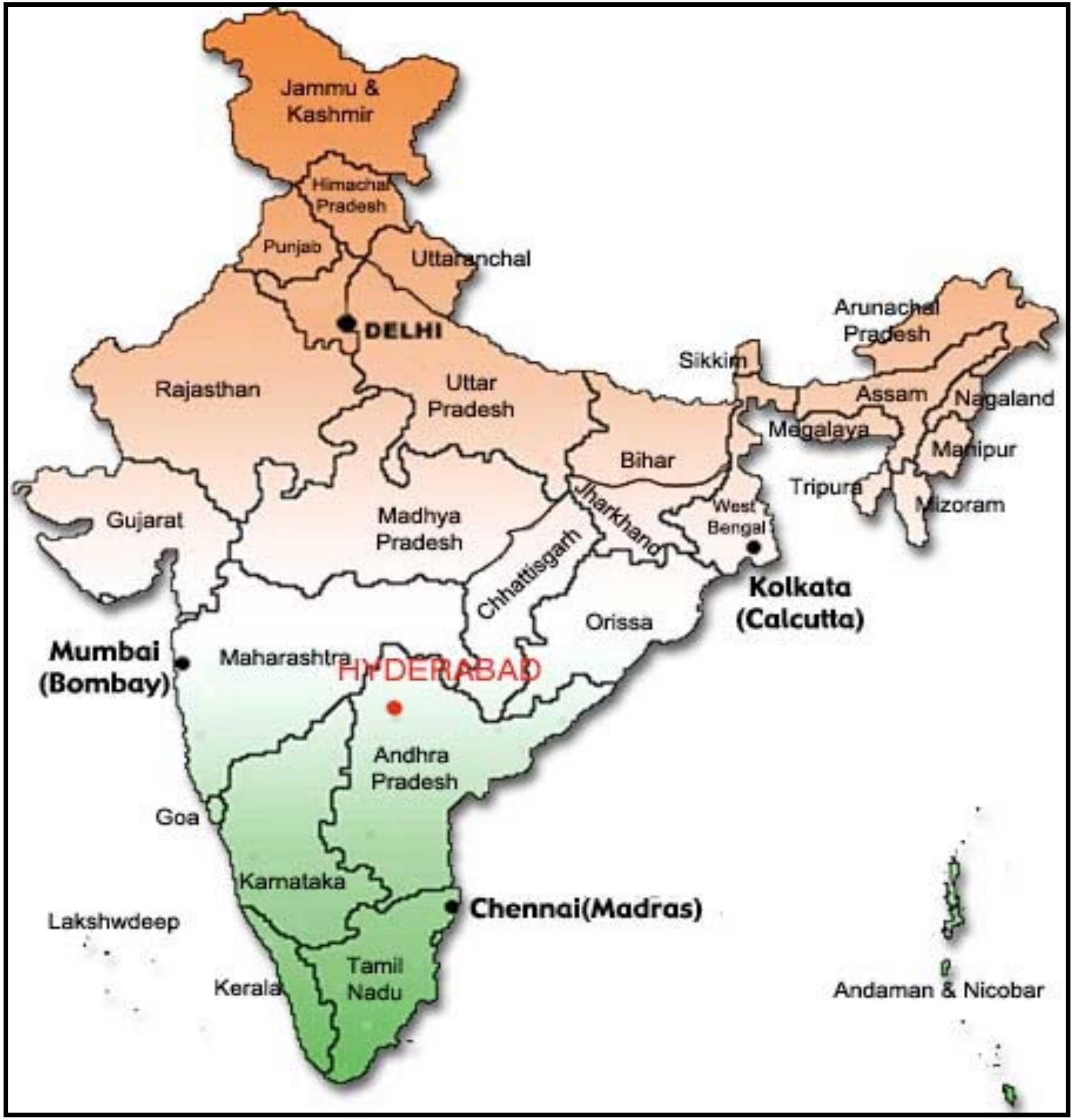
Effects of Dislocations and Periodic Bending on Electron Channeling and Channeling Radiation

Anand P Pathak
University of Hyderabad
India

Collaborators

Dr Juby George (now at IIT Chennai)



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Kolkata (Calcutta)
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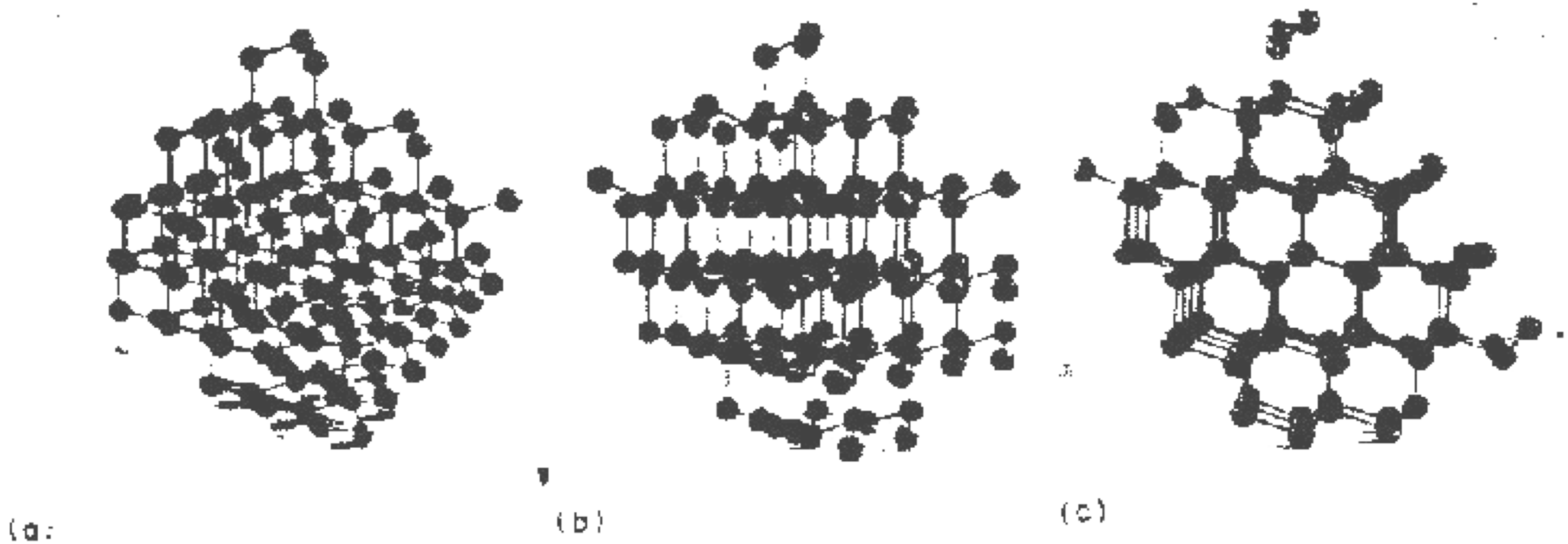
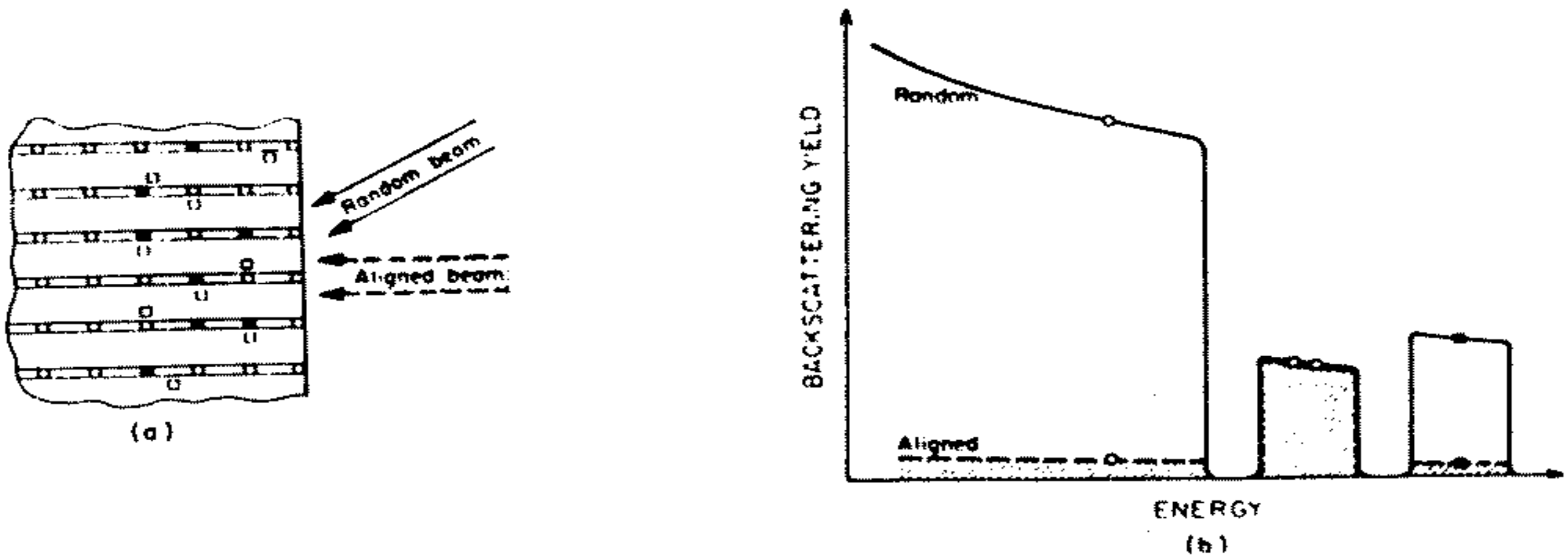
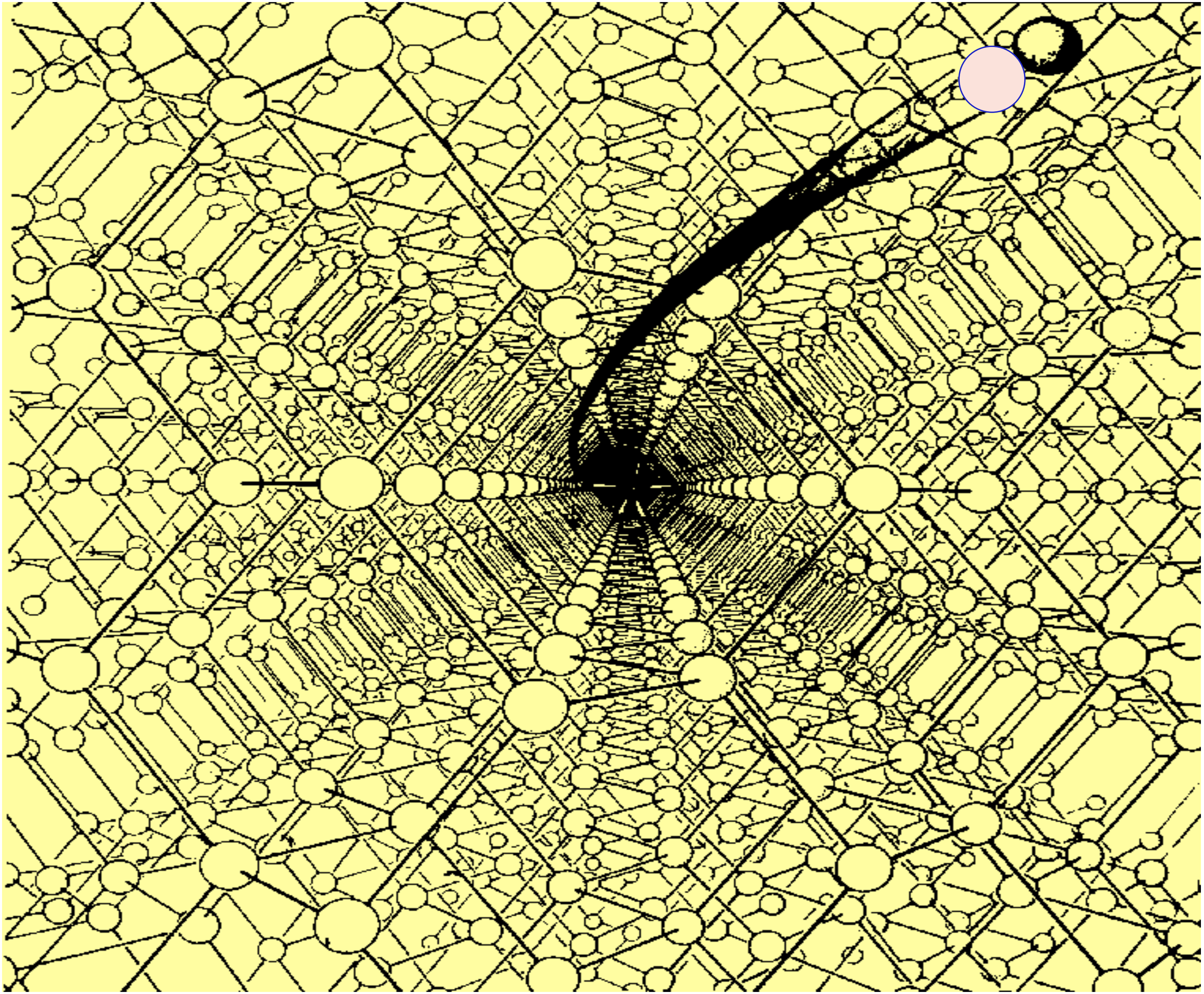
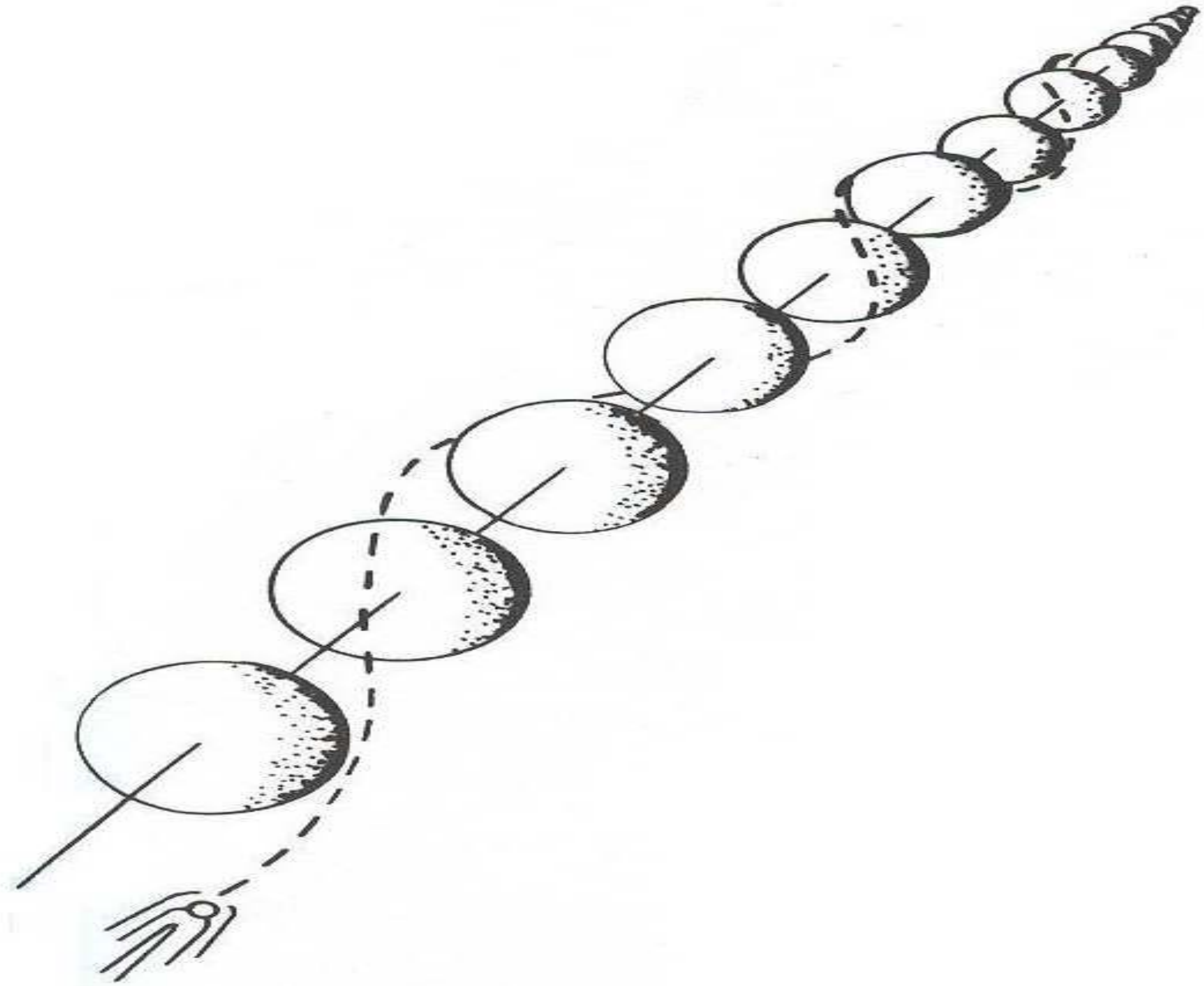


Fig. 8.1 Model of lattice atoms showing the atomic configuration in the diamond-type lattice viewed along (a) random, (b) planar, or (c) axial directions.

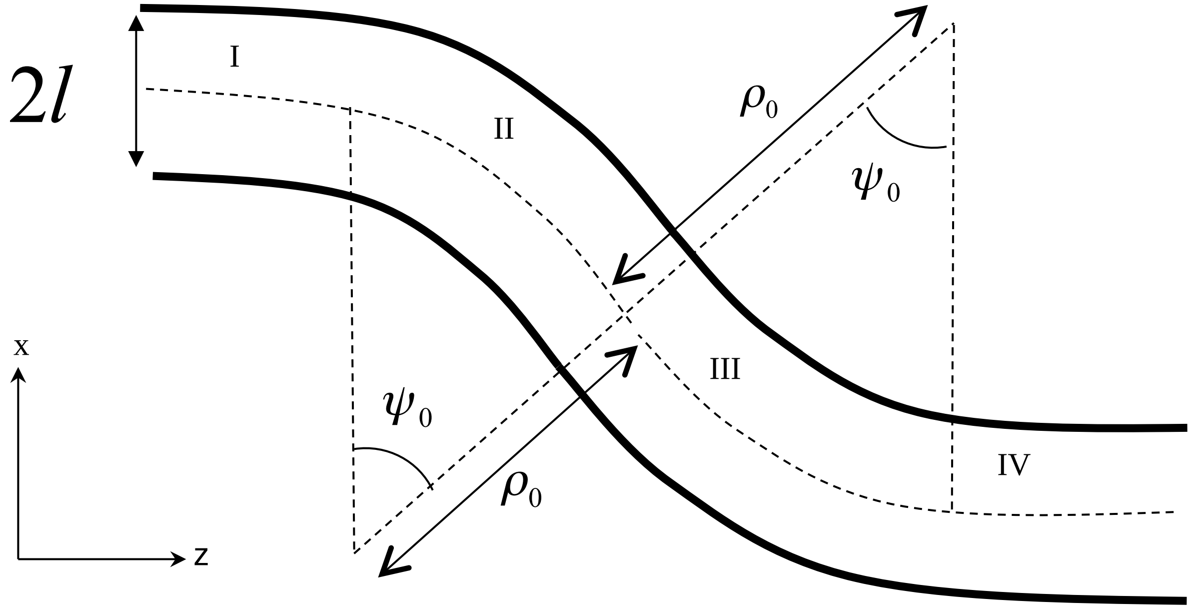






Effects of Dislocations on Positron and Electron Channeling

The Model



➔ For a relativistic particle, the emission process is considered in the rest frame of the particle moving through the crystal.

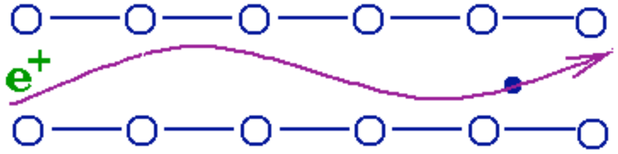
➔ Since the crystal is rushing back at a speed –*v*, it appears Lorentz-contracted

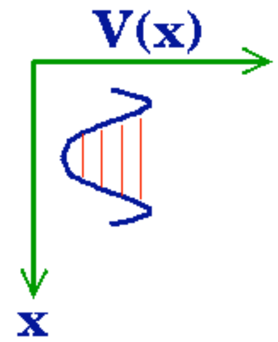
$$d^R = \frac{d}{\gamma}$$

$$E_\perp{}^R = \gamma E_\perp$$

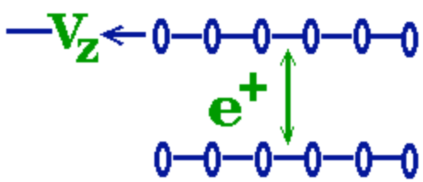
$$V^R = \gamma\, \mathrm{V}$$

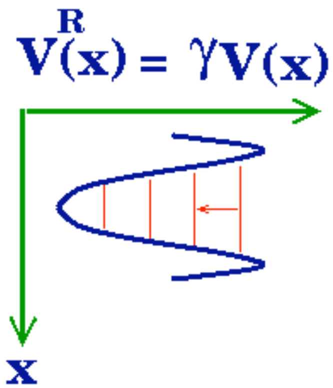
Lab Frame



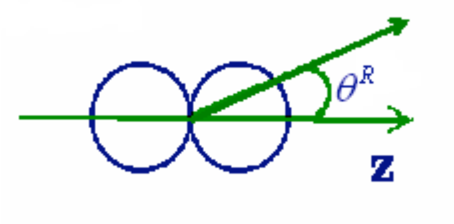


Rest Frame



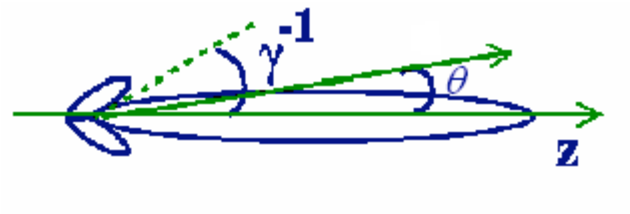


The frequency in the rest frame



$$\omega^R = \gamma\omega_0$$

The emission in the rest frame is observed in the lab frame



$$\omega^L = \frac{\omega_0}{1 - \beta\cos\theta}$$

The maximum frequency is in the forward direction,

i.e., at θ = 0 ($\beta = 1$)

$$\omega_m = 2\gamma^2\omega_0$$

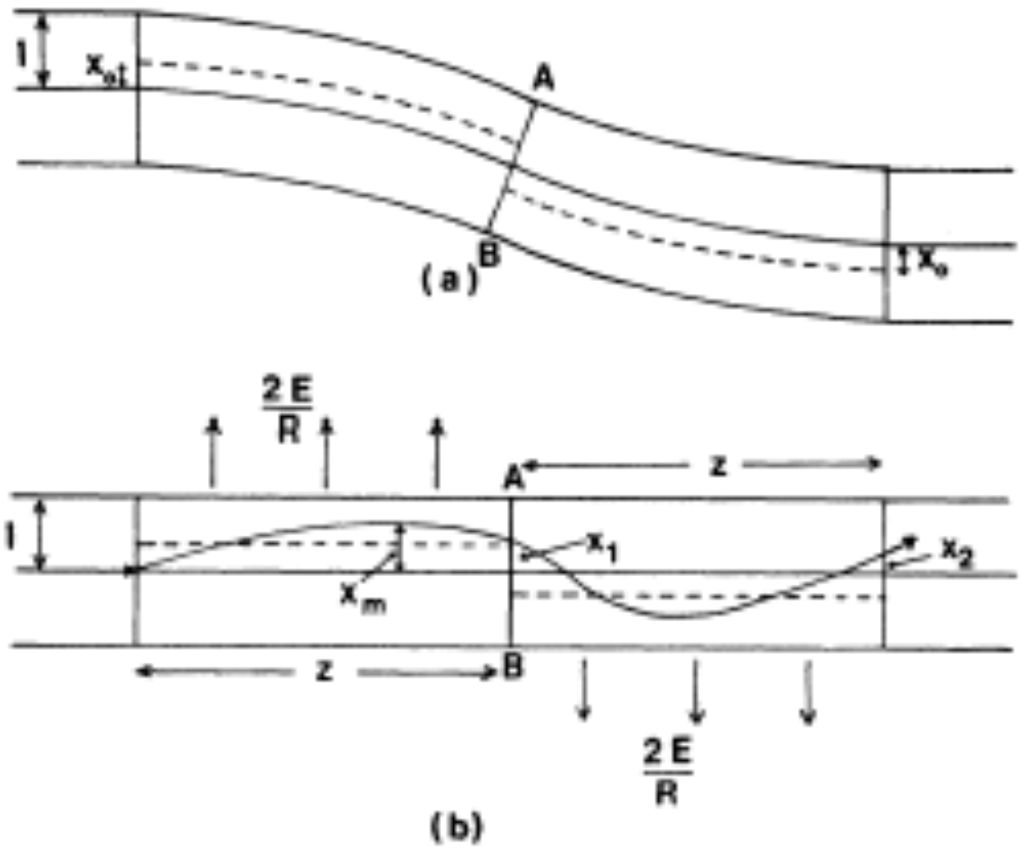


FIG. 3. (a) Typical channel at some finite distance from a dislocation. (b) Straight model channel replacing the channel of part (a) and showing the coordinates used in the text. Here, l is the half-width of the channel, x_m is the amplitude in the first part of the channel, x_0 is the equilibrium position about which the particle will oscillate, and x_1 and x_2 are the positions at which the particle arrives after having traversed the first and second parts of the channel, respectively.

Region I

The Schrödinger Equation for planar channeling

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Psi^I(x,z)+\frac{1}{2}m\omega^2x^2\Psi^I(x,z)=E^I\ \Psi^I(x,z) \qquad E^I=(n+1/2)\hbar\omega+\frac{\hbar^2k^2}{2m}$$

Equations for the transverse and longitudinal motion,

$$-\frac{\hbar^2}{2m}X^{I''}(x)+\frac{1}{2}m\omega^2x^2X^I(x)=E_T^IX^I(x)$$

$$-\frac{\hbar^2}{2m}Z^{I''}(z)=E_L^IZ^I(z)$$

solutions

$$X_n^I(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}(2^n n!)^{-1/2}H_n(ax)e^{-a^2x^2/2}$$

$$Z^I(z)=Ae^{ikz}+Be^{-ikz}$$

where $a=(m\omega/\hbar)^{1/2}$

If x_0 is the initial amplitude of the channelon

$$\Psi^I(x,z)=X^I(x-x_0)Z^I(z)$$

After including the effects of several transverse states

$$\Psi^I(x,z)=A_0X_0^I\,e^{ik_0z}+\sum_{n=0}B_n\,X_n^I\,e^{-ik_nz}$$

Region II

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2_{\rho,\varphi}\Psi^{II}(\rho,\varphi)+V(\rho)\Psi^{II}(\rho,\varphi)=E^{II}\Psi^{II}(\rho,\varphi)$$

The transverse potential due to the curved atomic planes is also assumed as harmonic around the central region

$$V(\rho)=\frac{1}{2}m\omega^2(\rho-\rho_0)^2$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{II}(\rho,\varphi)+\frac{1}{2}m\omega^2(\rho-\rho_0)^2\Psi^{II}(\rho,\varphi)=E^{II}\Psi^{II}(\rho,\varphi)$$

Separation of variables gives the azimuthal equation

$$F^{II''}(\varphi)=-\mu^2F^{II}(\varphi)$$

with solution $$F^{II}(\varphi)=Ce^{i\mu\varphi}+De^{-i\mu\varphi}$$

and radial equation. $$R^{II''}(\rho)+\frac{2m}{\hbar^2}\left[E^{II}-\frac{1}{2}m\omega^2(\rho-\rho_0)^2-\frac{\hbar^2\mu^2}{2m\rho^2}\right]R^{II}(\rho)=0$$

Effective potential

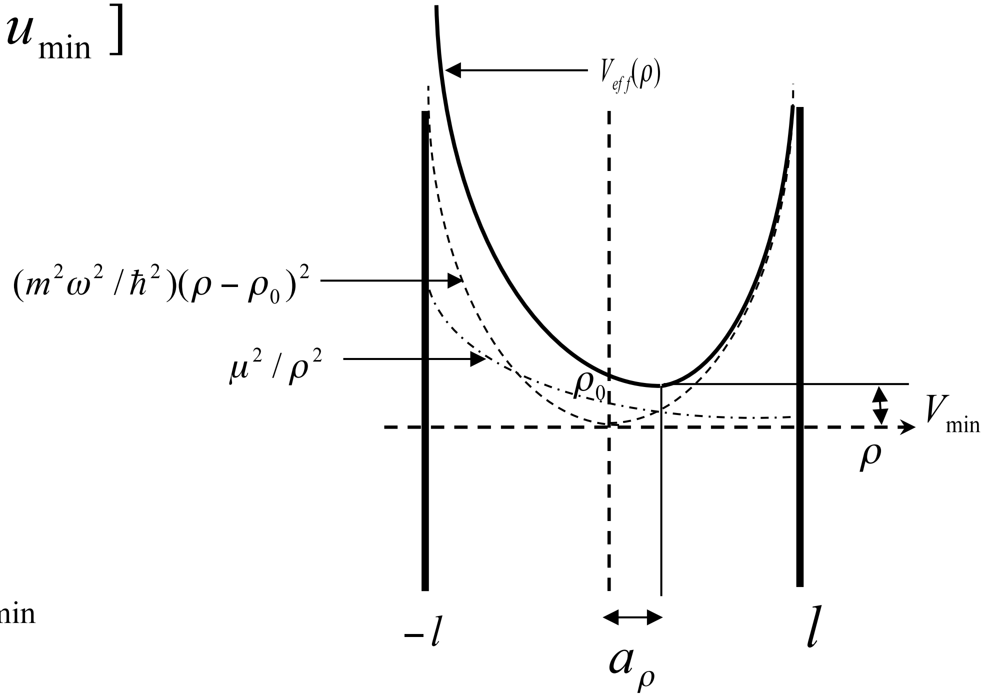
$$V_{eff}(\xi) \approx \frac{\hbar^2}{2m}[\ (\lambda / \rho_0^4)(\xi - a_p)^2 + u_{\min}\ ]$$

with
$$\xi = \rho - \rho_0$$

$$\lambda = a^4 \rho_0^4 + 3\mu^2$$

$$a_p = \mu^2 \rho_0 / \lambda$$

$$u_{\min} = \frac{\mu^2(\lambda - \mu^2)}{\rho_0^2 \lambda} = (2m/\hbar^2)V_{\min}$$



The frequency in the second region

$$\omega' = (\hbar / m)(\lambda / \rho_0^4)^{1/2}$$

After including the effects of several transverse states

$$\Psi^{II}(x,z) = \sum_{m=0} R_m^{II}[C_m e^{i\mu\varphi} + D_m e^{-i\mu\varphi}]$$

Region III

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{III}(\rho,\varphi)+\frac{1}{2}m\omega^2(\rho-\rho_0)^2\Psi^{III}(\rho,\varphi)=E^{III}\Psi^{III}(\rho,\varphi)$$

Separation of variables gives the azimuthal eqn.

$$F^{III''}(\varphi)=-\mu^2F^{III}(\varphi)$$

with solution

$$F^{III}(\varphi)=Ge^{i\mu\varphi}+He^{-i\mu\varphi}$$

and radial eqn.

$$R^{III''}(\rho)+\frac{2m}{\hbar^2}\left[E^{II}-\frac{1}{2}m\omega^2(\rho-\rho_0)^2+\frac{\hbar^2\mu^2}{2m\rho^2}\right]R^{III}(\rho)\cong 0$$

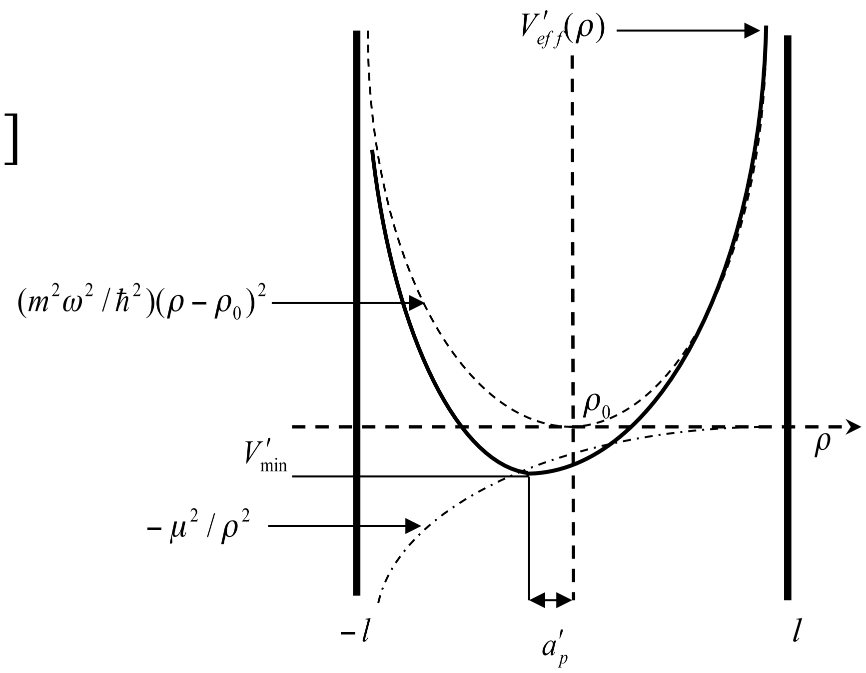
Effective potential

$$V'_{eff}(\xi) \approx \frac{\hbar^2}{2m}[\,(\lambda'/\rho_0^4)(\xi + a'_p)^2 + u'_{\min}\,]$$

with $$\lambda' = a^4\rho_0^4 - 3\mu^2$$

$$a'_p = \mu^2\rho_0/\lambda'$$

$$u'_{\min} = -\frac{\mu^2(\lambda' + \mu^2)}{\rho_0^2\,\lambda'} = -(2m/\hbar^2)V'_{\min}$$



The frequency in the second region

$$\omega'' = (\hbar/m)(\lambda'/\rho_0^4)^{1/2}$$

After including the effects of several transverse states

$$\Psi^{III}(x,z) = \sum_{m=0} R_m^{III}[G_m e^{i\mu\varphi} + H_m e^{-i\mu\varphi}]$$

Region IV

Region 4 is a perfect channel, wavefunction of positron in this region is of the same form as in the 1st region

$$\Psi^{IV}(x,z) = X_n^{IV} I_n e^{ik_n z}$$

Boundary Conditions

Boundary I

$$\Psi^{I}\Big|_{z=0} = \Psi^{II}\Big|_{\varphi=0} \qquad AX^{I} + BX^{I} = R^{II}[C + D]$$

$$\frac{\partial \Psi^{I}}{\partial z}\Bigg|_{z=0} = \frac{1}{\rho_0}\frac{\partial \Psi^{II}}{\partial \varphi}\Bigg|_{\varphi=0} \qquad ikAX^{I} - ikB\,X^{I} = \frac{i\mu}{\rho_0}R^{II}[C - D]$$

Boundary II

$$\Psi^{II}\Big|_{\varphi=\varphi_0} = \Psi^{III}\Big|_{\varphi=0} \qquad R^{II}[Ce^{i\mu\varphi} + De^{-i\mu\varphi}] = R^{II}[G + H]$$

$$\frac{\partial \Psi^{II}}{\partial \varphi}\Bigg|_{\varphi=\varphi_0} = \frac{\partial \Psi^{III}}{\partial \varphi}\Bigg|_{\varphi=0} \qquad R^{II}[Ce^{i\mu\varphi} - De^{-i\mu\varphi}] = R^{II}[G - H]$$

Boundary III

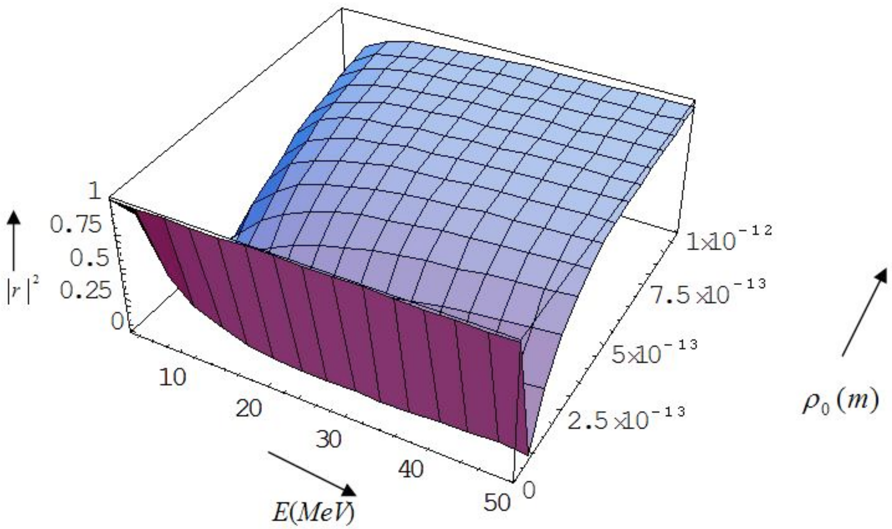
$$\Psi^{III}\Big|_{\varphi=\varphi_0} = \Psi^{IV}\Big|_{z=t} \qquad R^{III}[Ge^{i\mu\varphi} + He^{-i\mu\varphi}] = IX^{IV}e^{ikt}$$

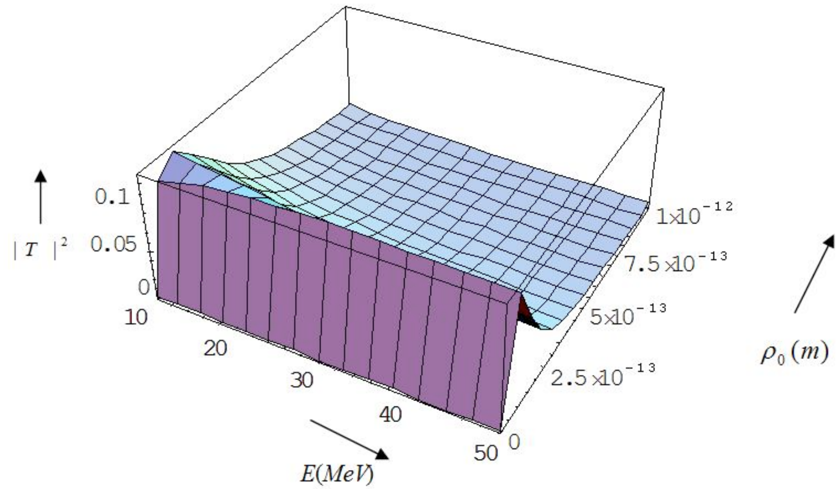
$$\frac{1}{\rho_0}\frac{\partial \Psi^{III}}{\partial \varphi}\Bigg|_{\varphi=\varphi_0} = \frac{\partial \Psi^{IV}}{\partial z}\Bigg|_{z=t} \qquad \frac{i\mu}{\rho_0}R^{III}[Ge^{i\mu\varphi} + He^{-i\mu\varphi}] = ikIX^{IV}e^{ikt}$$

The Reflection and Transmission co-efficient in terms of the various parameters of the dislocation affected channel

$$|r|^2 = \frac{|B|^2}{|A|^2} = \frac{(-\mu^2 + k^2\rho_0^2)^2 Sin^2(2\mu\varphi_0)}{4k^2\mu^2\rho_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + k^2\rho_0^2)^2 Sin^2(2\mu\varphi_0)}$$

$$|T|^2 = 1 - |r|^2 = \frac{4k^2\rho_0^2\mu^2}{4k^2\mu^2\rho_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + k^2\rho_0^2)^2 Sin^2(2\mu\varphi_0)}$$





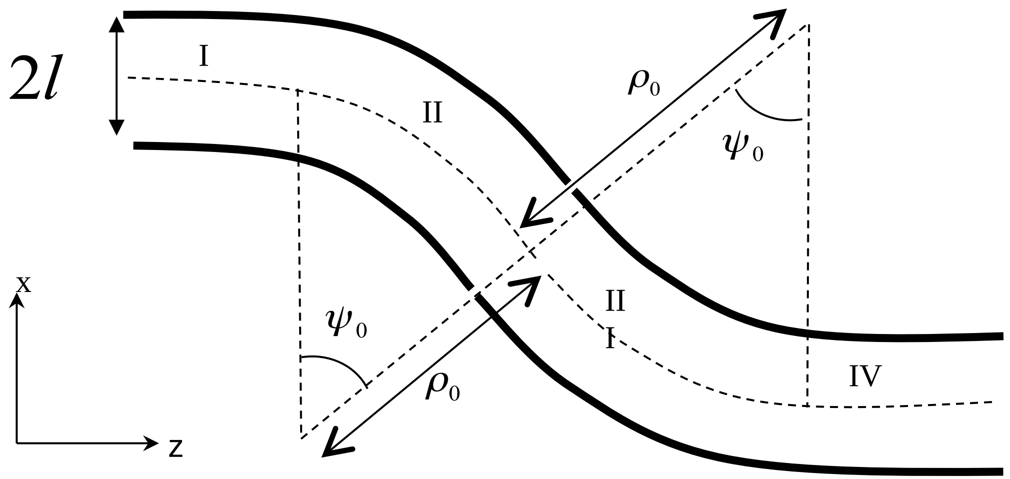
Equation of motion of a crystalline undulator

$$\tilde{x} = x - a\ sin(k_u z)$$

Where a is the amplitude of bending of the channel and

$$k_u = \frac{2\pi}{\lambda_u}$$

The dislocation affected region,



Region I

The Schrödinger Equation for planar channeling

$$-\frac{\hbar^2}{2m}\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta z^2}\right)\,\Psi^I(x,z)+U(x)\,\Psi^I(x,z)=E^I\,\Psi^I(x,z)$$

$$\begin{aligned} U(x) &= V_0\tilde{x}^2 \\ &= V_0(x-a\ sin(k_u z))^2 \end{aligned}$$

Region II

Centrifugal force proportional to $\mu^2/\rho_0{}^2$ is responsible for the curved regions of the channel.

$\mu^2 = l(l+1)$ with l as the orbital angular momentum quantum number and ρ_0 is the radius of curvature of the channel.

Assume a finite number of undulator periods in a length of the dislocation affected region of the channel (low or medium dislocation concentration). If λ_d is the wavelength of the dislocation affected region and x_d is the corresponding amplitude of the waves

The equation of motion of both the waves

$$\lambda_d = n\,\lambda_u$$

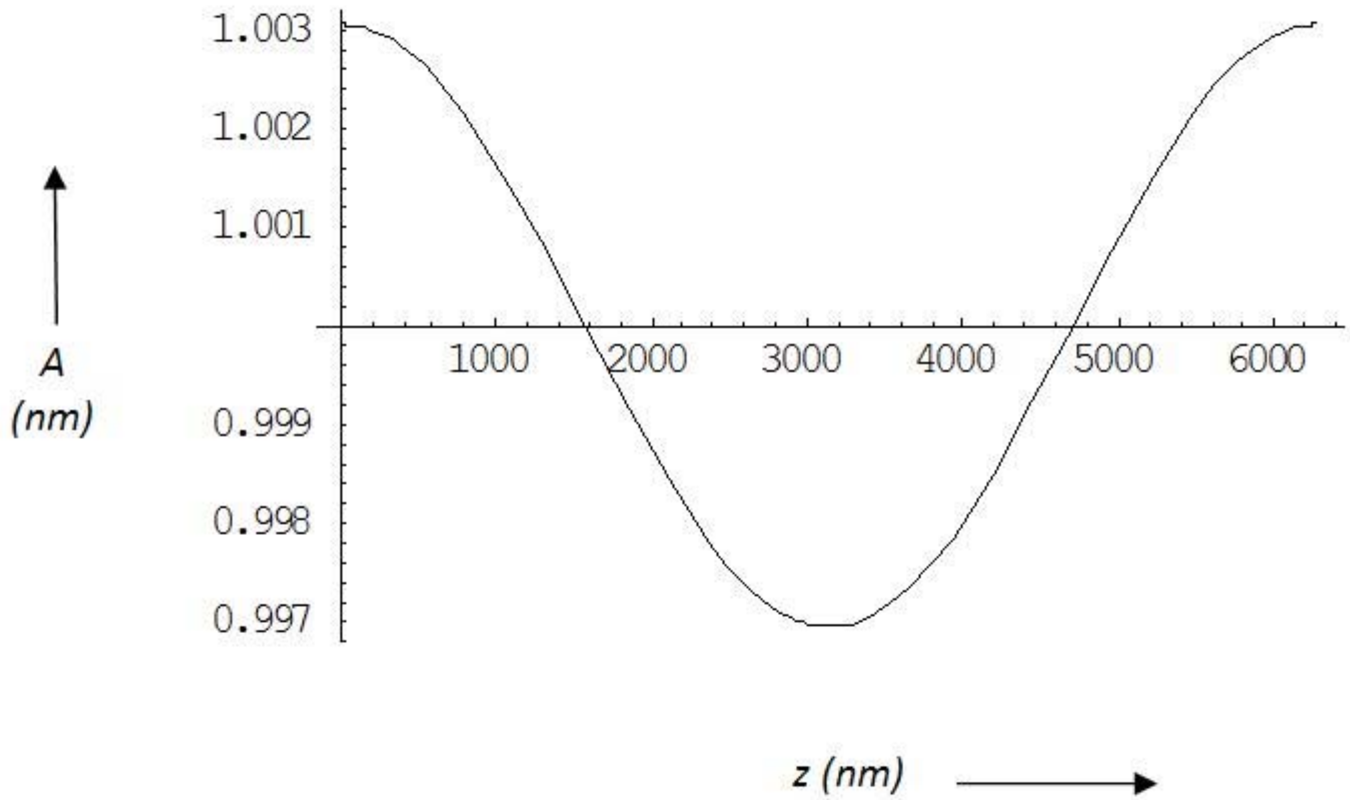
$$\begin{aligned} r_1 &= a\ sin(nk_d z) \\ r_2 &= x_d\ sin(k_d z) \end{aligned}$$

Superposition of the two waves gives

$$r = A\ sin(k_d z + \varepsilon)$$

Where A and ε are the effective amplitude and phase of the final wave.

$$\begin{aligned} A^2 &= a^2 + x_d^2 + 2ax_d\ cos[(n-1)k_d z - \phi] \\ tan\ \varepsilon &= \frac{a\ sin[(n-1)k_d z] + x_d\ sin\phi}{a\ cos[(n-1)k_d z] + x_d\ cos\phi} \end{aligned}$$



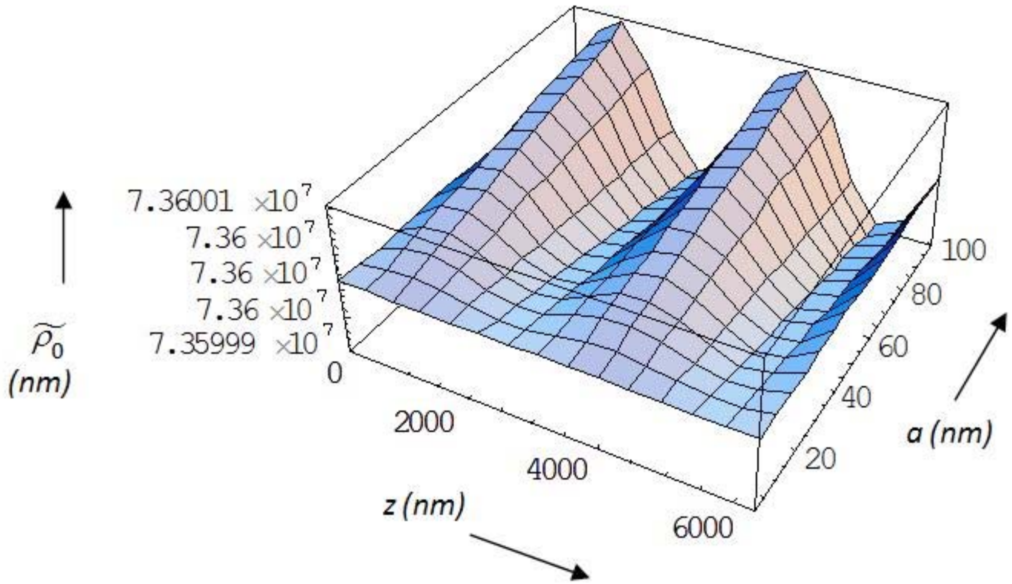
1.003
1.002
1.001
0.999
0.998
0.997
1000
2000
3000
4000
5000
6000
A
(nm)
z (nm)

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{II}(\rho,\varphi)+U(\rho)\Psi^{II}(\rho,\varphi)=E^{II}\;\Psi^{II}(\rho,\varphi)$$

With the channel periodically bent, the radius of curvature of the dislocated affected region,

$$\tilde{\rho_0}=\rho_0-x_d\;sin(k_d z)+A\;sin(k_u z)$$



The variation of parameters of the dislocation affected region with dislocation density,

Dislocation density	r_0	Radius of Curvature	2z (Length of the curved part)
$10^{10}/cm^2$	$0.5\times\ 10^2$ nm	$10.28\ \times\ 10^5$ nm	$6.28\ \times\ 10^2$ nm
$10^9/cm^2$	$1.58\ \times\ 10^2$ nm	$10.26\ 6\times\ 10^6$ nm	$9.92\ \times\ 10^2$ nm
$10^8/cm^2$	$0.5\ \times\ 10^3$ nm	$10.28\ \times\ 10^7$ nm	$6.28\ \times\ 10^3$ nm

Range of various parameters of the periodically bent channel affected with dislocation corresponding to a dislocation density of $10^8/cm^2$,

a (cm)	λ_u (cm)	R_u (cm)	E (Mev)	x_d amplitude of the dislocation wave
$1\ \times\ 10^{-7}$	$3.14\times\ 10^{-4}$	$2.5\ \times\ 10^{-2}$	142.363	$2.198\ \times\ 10^{-3}$
$10\ \times\ 10^{-7}$	$3.14\times\ 10^{-4}$	$2.5\ 6\times\ 10^{-3}$	14.236	$2.198\ \times\ 10^{-4}$
$100\ \times\ 10^{-7}$	$3.14\ \times\ 10^{-4}$	$2.5\times\ 10^{14}$	1.412	$2.198\ \times\ 10^{15}$

The effective potential

$$V_{eff} = V_0(\rho - \tilde{\rho}_0)^2 + \frac{\hbar^2}{2m}\frac{\mu^2}{\rho^2}$$

$$V_{eff}(\xi) = \frac{\hbar^2}{2m}\left[\frac{\lambda}{\tilde{\rho_0}^4}(\xi - \tilde{a_p})^2 + U_{min}\right]$$

$$\xi = \rho - \tilde{\rho}_0$$

$$\lambda = 3\mu^2 + b^4\tilde{\rho_0}^4$$

$$\tilde{a_p} = \frac{\tilde{\rho}_0\mu^2}{\lambda}$$

$$U_{min} = \frac{\mu^2}{\lambda\tilde{\rho_0}^2}(\lambda - \mu^2)$$

$$b = \left(\frac{m\omega}{\hbar}\right)^{1/2}$$

The frequency of oscillation in the region,

$$\omega' = \left(\frac{\hbar}{m}\right)\sqrt{\frac{\lambda}{\tilde{\rho}_0{}^4}}$$

With the effective wavefunction

$$\Psi^{II}(\rho,\varphi) = \sum_{m=0} R_m^{II}\left[C_m e^{i\mu\varphi} + D_m e^{-i\mu\varphi}\right]$$

Region III

The effective potential

$$V_{eff}(\xi) = \frac{\hbar^2}{2m}\left[\frac{\lambda'}{\tilde{\rho_0}^4}(\xi + \tilde{a_p'})^2 + U'_{min}\right]$$

$$\lambda' = -3\mu^2 + b^4\tilde{\rho_0}^4$$

$$\tilde{a_p'} = \frac{\tilde{\rho_0}\mu^2}{\lambda'}$$

$$U'_{min} = -\frac{\mu^2}{\lambda'\tilde{\rho_0}^2}(\lambda' + \mu^2)$$

The frequency of oscillation in the region,

$$\omega'' = \left(\frac{\hbar}{m}\right)\sqrt{\frac{\lambda'}{\tilde{\rho_0}^4}}$$

The wavefunction in region *III*

$$\Psi^{III}(\rho,\varphi) = \sum_{m=0} R_m^{III}\left[G_m e^{i\mu\varphi} + H_m e^{-i\mu\varphi}\right]$$

Region *IV*

$$\Psi^{IV}(x,z) = X_n^{IV} I_n e^{ik_n z}$$

The reflection and transmission coeffcients

$$|R|^2 = \frac{(-\mu^2 + k^2\tilde{\rho_0}^2)^2\ sin^2(2\mu\varphi_0)}{4k^2\mu^2\tilde{\rho_0}^2 cos^2(2\mu\varphi_0) + (\mu^2 + k^2\tilde{\rho_0}^2)^2\ sin^2(2\mu\varphi_0)}$$

$$|T|^2 = \frac{4k^2\mu^2\tilde{\rho_0}^2}{4k^2\mu^2\tilde{\rho_0}^2 cos^2(2\mu\varphi_0) + (\mu^2 + k^2\tilde{\rho_0}^2)^2\ sin^2(2\mu\varphi_0)}$$

When $\lambda_d < \lambda_u$

Range of various parameters of the periodically bent channel affected with dislocation at $\lambda_u = 2\,\lambda_d$

Dislocation density	λ_d (cm)	λ_u (cm)	a (cm)	R_u (cm)	E (MeV)
$1.5\times 10^9/cm^2$	1.66×10^{-4}	3.32×10^{-4}	1×10^{-7}	2.8×10^{-2}	150
			10×10^{-7}	2.8×10^{-3}	15
			100×10^{-7}	2.8×10^{-4}	1.5

Equation of motion,

$$\tilde{x} = x - a\ sin(k_u v t)$$

$$\ddot{\tilde{x}} = \ddot{x} + a k_u^2 v^2\ sin(k_u v t)$$

$$\frac{1}{R} = a k_u^2\ sin(k_u v t)$$

$$\ddot{\tilde{x}} + \frac{qe}{m\gamma}U(\tilde{x}) - \frac{\gamma v^2}{R}\tilde{x} = 0$$

The maximum amplitude of oscillation

$$\tilde{x}_m = \frac{m\gamma^2 v^2}{qeV_0 R}$$

And the equilibrium axis shifts to,

$$\tilde{x}_0 = \frac{m\gamma^2 v^2}{2qeV_0 R}$$

The period of oscillation of the particle in the channel,

$$T = \left(\frac{m\gamma}{2qeV_0}\right)^{1/2} Sin^{-1}\left\{1 - \frac{2qeV_0 R}{m\gamma^2 v^2} cos(k_u z)\right\}$$

Region I

The Schrödinger Equation for electron planar channeling

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\Psi^I(x,z)+U(x)\Psi^I(x,z)=E^I\ \Psi^I(x,z) \qquad E^I=\frac{mV_0^2}{2\hbar^2n^2}+\frac{\hbar^2k^2}{2m}$$

$$U(x)=-\frac{V_0}{x+a_T} \qquad V_0=2Z_1Z_2e^2Nd_pCa^2$$

Equations for the transverse and longitudinal motion,

$$-\frac{\hbar^2}{2m}X^{I''}(x)+\frac{V_0}{x+a_T}X^I(x)=E_T^IX^I(x)$$

$$-\frac{\hbar^2}{2m}Z^{I''}(z)=E_L^IZ^I(z)$$

If x_0 is the initial amplitude of the channelon

$$\Psi^I(x,z)=X^I(x-x_0)Z^I(z)$$

After including the effects of several transverse states, we can write

$$\Psi^I(x,z)=A_0X_0^I\,e^{ik_0z}+\sum_{n=0}B_n\,X_n^I\,e^{-ik_nz}$$

Region II

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2_{\rho,\varphi}\Psi^{II}(\rho,\varphi) + V(\rho)\Psi^{II}(\rho,\varphi) = E^{II}\Psi^{II}(\rho,\varphi)$$

The transverse potential due to the curved atomic planes is assumed to shift with respect to lattice plane, due to curvature;

$$V(\rho) = -\frac{V_0}{(\rho-\rho_0)+a_T}$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{II}(\rho,\varphi) - \frac{V_0}{(\rho-\rho_0)+a_T}\Psi^{II}(\rho,\varphi) = E^{II}\Psi^{II}(\rho,\varphi)$$

Separation of variables gives the azimuthal equation

$$F^{II''}(\varphi) = -\mu^2 F^{II}(\varphi)$$

with solution

$$F^{II}(\varphi) = Ce^{i\mu\varphi} + De^{-i\mu\varphi}$$

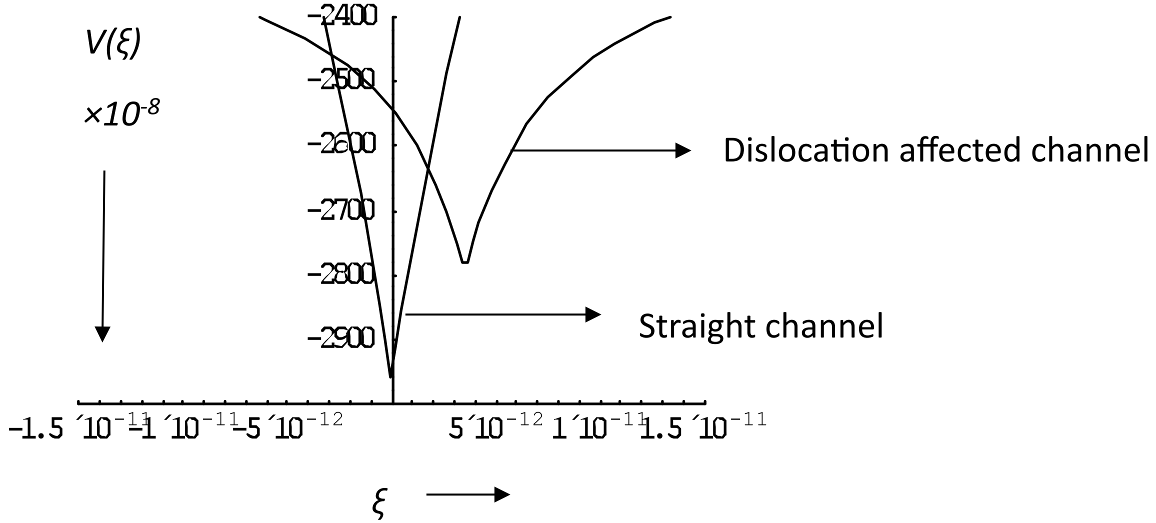
and radial equation

$$R^{II''}(\rho) + \frac{2m}{\hbar^2}\left[E^{II} + \frac{V_0}{(\rho - \rho_0) + a_T} - \frac{\hbar^2\mu^2}{2m\rho^2}\right]R^{II}(\rho) = 0$$

Effective potential for electron case

$$V_{eff}(\xi) = \frac{\hbar}{2m}\left\{\frac{\lambda_1'^3}{\lambda_1^2\rho_0^4 a_{TF}^3[2\xi + \frac{\lambda_1'}{\lambda_1}]} - \frac{\lambda_1'^2}{\lambda_1\rho_0^4 a_{TF}^3} + \frac{\lambda_1''}{\rho_0^4 a_{TF}^3}\right\}$$

$$\begin{aligned}
\lambda_1 &= -2a^4\rho_0^4 + 3\mu^2 a_{TF}^3 \\
\lambda_1' &= -a^4\rho_0^4 a_{TF} + \mu^2 a_{TF}^3\rho_0 \\
\lambda_1'' &= -2a^4\rho_0^4 a_{TF}^2 + \mu^2 a_{TF}^3\rho_0^2
\end{aligned}$$



After including the effects of several transverse states

$$\Psi^{II}(x,z) = \sum_{m=0} R_m^{II}[C_m e^{i\mu\varphi} + D_m e^{-i\mu\varphi}]$$

Region III

Effective potential is given by

$$V_{eff}(\xi) = \frac{\hbar}{2m}\left\{\frac{\lambda_2'^3}{\lambda_2^2\rho_0^4 a_{TF}^3[2\xi + \frac{\lambda_2'}{\lambda_2}]} - \frac{\lambda_2'^2}{\lambda_2\rho_0^4 a_{TF}^3} + \frac{\lambda_2''}{\rho_0^4 a_{TF}^3}\right\}$$

$$\begin{aligned} \lambda_2 &= -2a^4\rho_0^4 - 3\mu^2 a_{TF}^3 \\ \lambda_2' &= -a^4\rho_0^4 a_{TF} - \mu^2 a_{TF}^3 \rho_0 \\ \lambda_2'' &= -2a^4\rho_0^4 a_{TF}^2 - \mu^2 a_{TF}^3 \rho_0^2 \end{aligned}$$

After including the effects of several transverse states

$$\Psi^{III}(x,z) = \sum_{m=0} R_m^{III}[G_m e^{i\mu\varphi} + H_m e^{-i\mu\varphi}]$$

Region IV

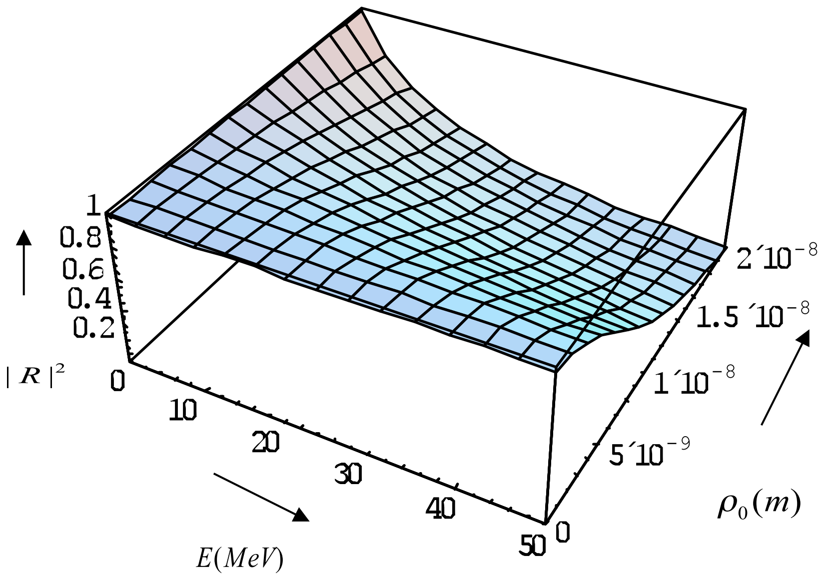
Region 4 is a perfect channel, wavefunction of electron in this region is of the same form as in the 1st region

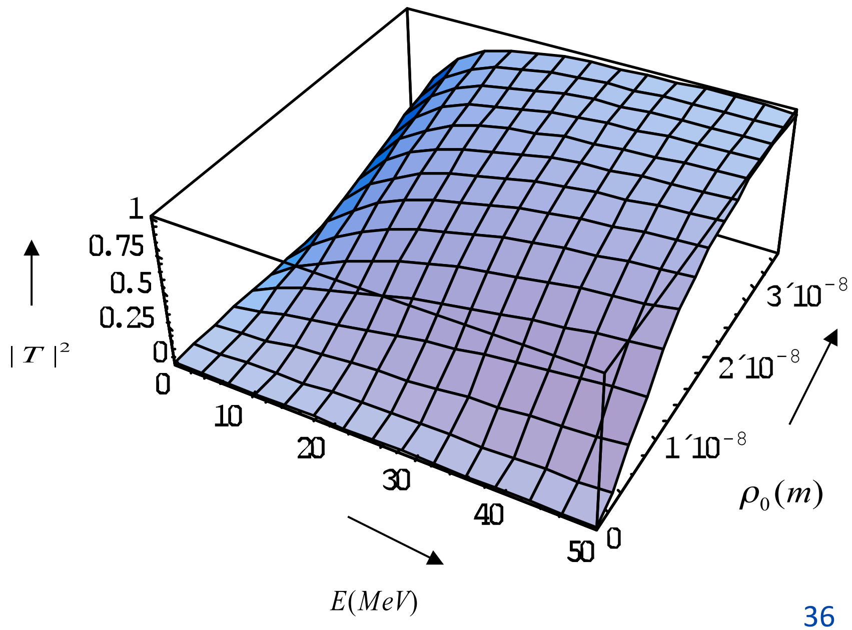
$$\Psi^{IV}(x,z) = X_n^{IV} I_n e^{ik_n z}$$

The <u>Reflection and Transmission co-efficients</u> in terms of the various parameters of the dislocation affected channel

$$|R|^2 = \frac{|B|^2}{|A|^2} = \frac{(-\mu^2 + 2mE\rho_0^2)^2 Sin^2(2\mu\varphi_0)}{8mE\mu^2\rho_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + 2mE\rho_0^2)^2 Sin^2(2\mu\varphi_0)}$$

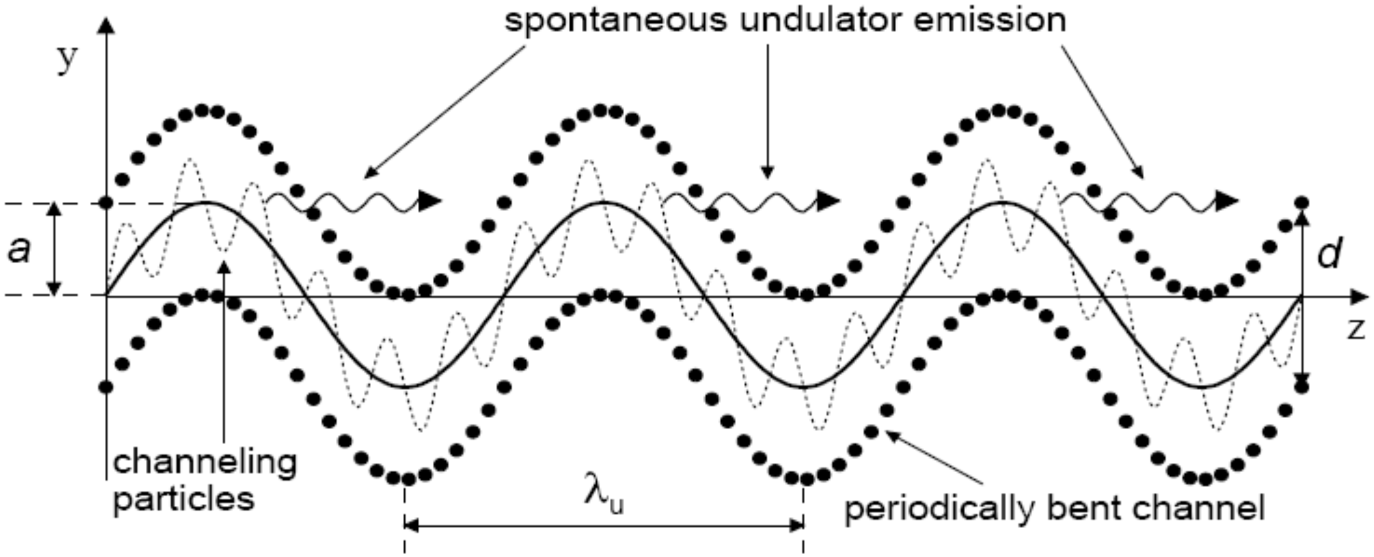
$$|T|^2 = 1 - |R|^2 = \frac{8mE\rho_0^2\mu^2}{8mE\mu^2\rho_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + 2mE\rho_0^2)^2 Sin^2(2\mu\varphi_0)}$$





Effects of Periodic Bending on Channeling

Crystalline Undulator



A crystalline Undulator consist of

- ➔ A channel which is periodically bent
- ➔ Channeling of ultra relativistic positively charged particles

Channeling takes place if the maximum centrifugal force due to the bending is less than the maximal force due to the interplanar field.

We consider a crystal whose planes are periodically bent following a perfect harmonic shape

$$x(z) = a\ sin(k_u z)$$

The transverse and longitudinal coordinates of a channeled particle in such a periodically bent crystal

$$\tilde{x} = x - a\ sin(k_u z)$$

Where a is the amplitude of bending of the channel and

$$k_u = \frac{2\pi}{\lambda_u}$$

Region I & IV

The Schrödinger Equation for planar channeling

$$-\frac{\hbar^2}{2m}\left(\frac{\delta^2}{\delta x^2}+\frac{\delta^2}{\delta z^2}\right)\ \Psi^I(x,z)+U(x)\ \Psi^I(x,z)=E^I\ \Psi^I(x,z)$$

$$U(x) = -\frac{V_0}{\tilde{x}+a_T}$$

Region II & III

Centrifugal force proportional to μ^2/ρ_0^2 is responsible for the curved regions of the channel.

$\mu^2 = l(l+1)$ with l as the orbital angular momentum quantum number and ρ_0 is the radius of curvature of the channel.

Assume that a finite number of undulator periods are there in a length of the dislocation affected region of the channel.

If λ_d is the wavelength of the dislocation affected region and x_d is the corresponding amplitude of the waves

$$\lambda_d = n\,\lambda_u$$

Both these waves can be written in the form

$$r_1 = a\ sin(nk_d z)$$
$$r_2 = x_d\ sin(k_d z)$$

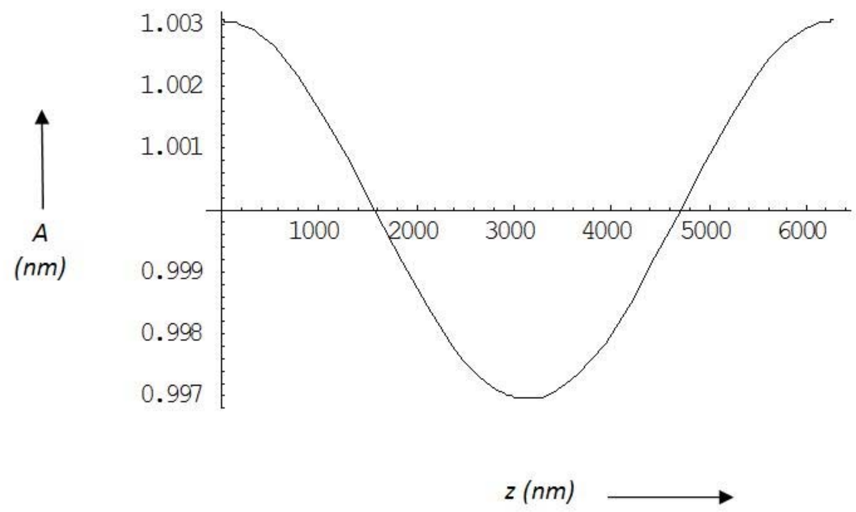
Addition of the waves gives

$$r = A\ sin(k_d z + \Phi)$$

Where A and Φ are the effective amplitude and phase of the final wave.

$$A^2 = a^2 + x_d^2 + 2ax_d\ cos[(n-1)k_d z - \phi]$$
$$tan\ \Phi = \frac{a\ sin[(n-1)k_d z] + x_d\ sin\phi}{a\ cos[(n-1)k_d z] + x_d\ cos\phi}$$

Amplitude is no longer constant but varies periodically with respect to the depth



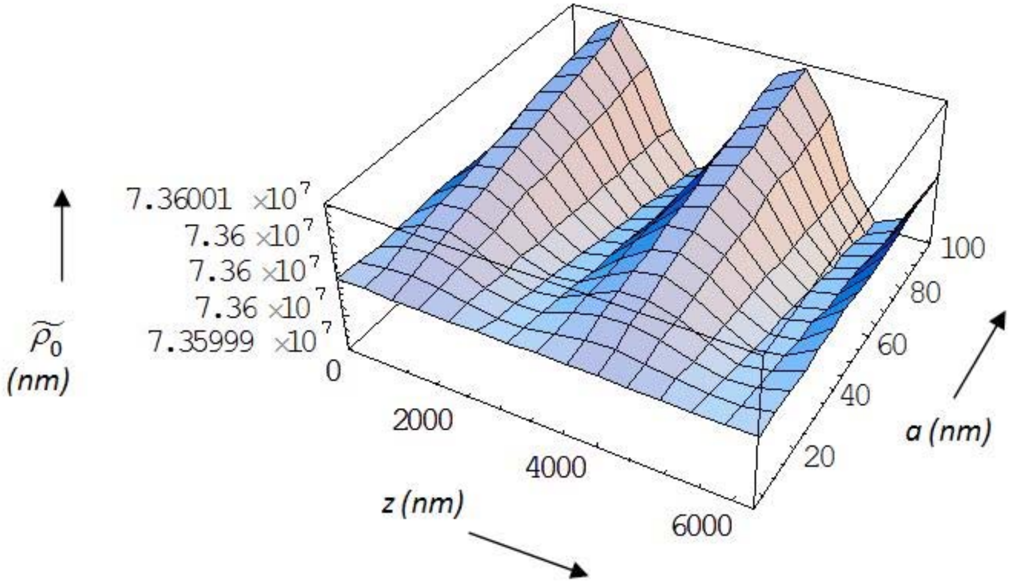
$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{II}(\rho,\varphi)+U(\rho)\Psi^{II}(\rho,\varphi)=E^{II}\ \Psi^{II}(\rho,\varphi)$$

With the channel periodically bent, the radius of curvature of the dislocated affected region,

$$V(\rho)=-\ \frac{V_0}{(\rho-\tilde{\rho}_0)+a_T}$$

$$\tilde{\rho}_0=\rho_0-x_d\ sin(k_d z)+A\ sin(k_u z)$$

Larger the value of a, larger is the variation of $\tilde{\rho}_0$ with *z*.



Assume a finite number of undulator periods in a length of the dislocation affected region of the channel (low or medium dislocation concentration). If λ_d is the wavelength of the dislocation affected region and x_d is the corresponding amplitude of the waves

$$\lambda_d = n \, \lambda_u$$

The equation of motion of both the waves

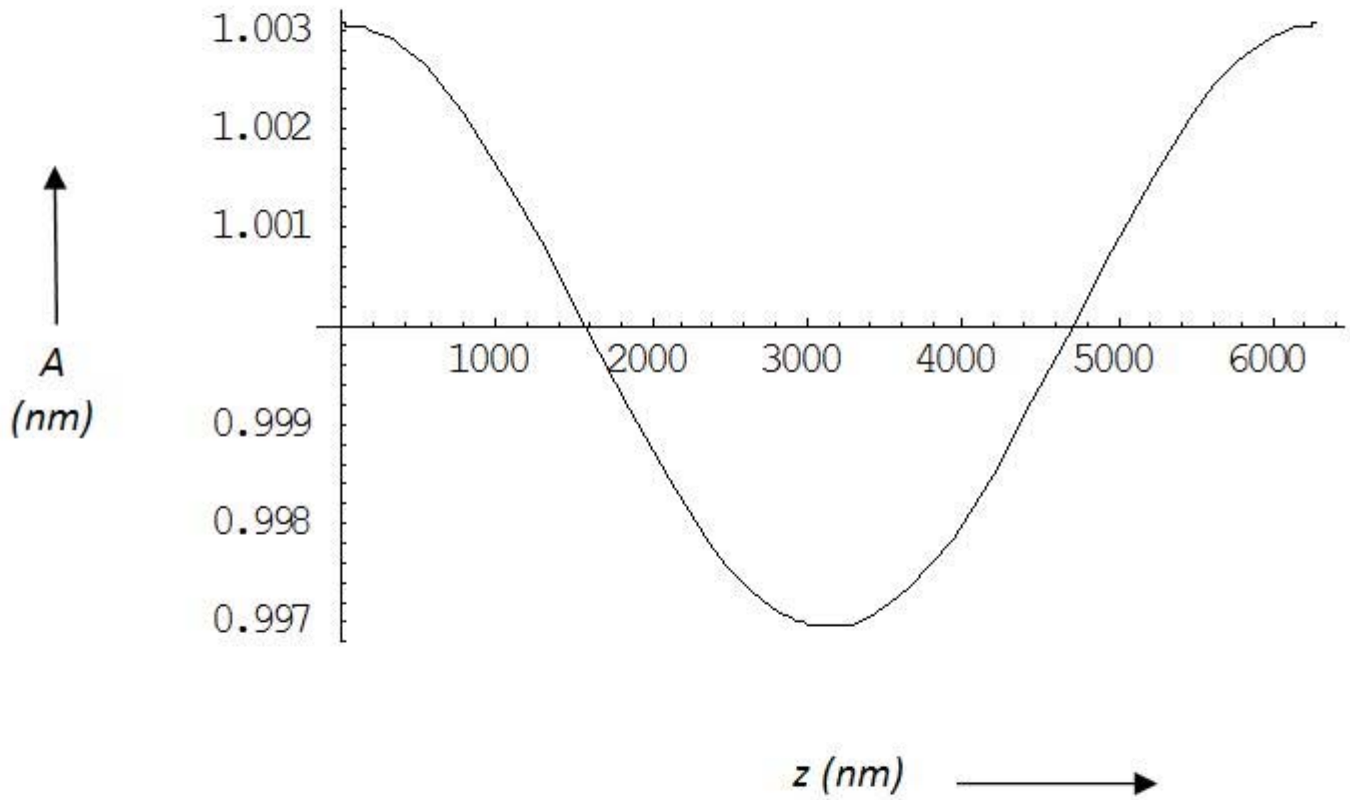
$$\begin{aligned} r_1 &= a \; sin(nk_d z) \\ r_2 &= x_d \; sin(k_d z) \end{aligned}$$

Superposition of the two waves gives

$$r = A \; sin(k_d z + \varepsilon)$$

Where A and ε are the effective amplitude and phase of the final wave.

$$\begin{aligned} A^2 &= a^2 + x_d^2 + 2a x_d \; cos[(n-1)k_d z - \phi] \\ tan \; \varepsilon &= \frac{a \; sin[(n-1)k_d z] + x_d \; sin\phi}{a \; cos[(n-1)k_d z] + x_d \; cos\phi} \end{aligned}$$

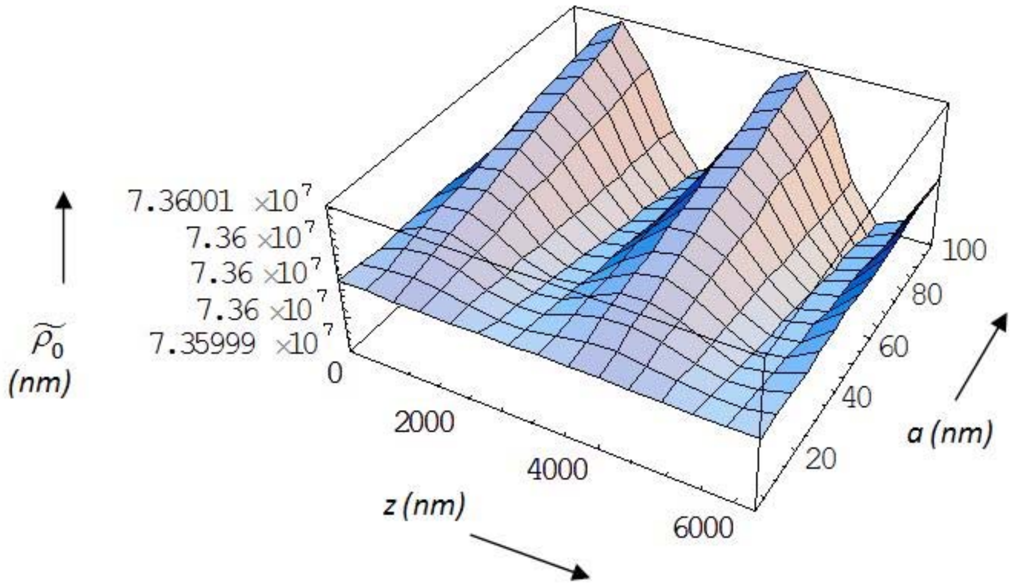

1.003
1.002
1.001
0.999
0.998
0.997
1000
2000
3000
4000
5000
6000
A (nm)
z (nm)

The Schrödinger Equation

$$-\frac{\hbar^2}{2m}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi^{II}(\rho,\varphi)+U(\rho)\Psi^{II}(\rho,\varphi)=E^{II}\ \Psi^{II}(\rho,\varphi)$$

With the channel periodically bent, the radius of curvature of the dislocated affected region,

$$\tilde{\rho_0}=\rho_0-x_d\ sin(k_d z)+A\ sin(k_u z)$$



The variation of parameters of the dislocation affected region with dislocation density,

Dislocation density	r_0	Radius of Curvature	2z (Length of the curved part)
$10^{10}/cm^2$	$0.5\times\ 10^2$ nm	$10.28\ \times\ 10^5$ nm	$6.28\ \times\ 10^2$ nm
$10^9/cm^2$	$1.58\ \times\ 10^2$ nm	$10.26\ 6\times\ 10^6$ nm	$9.92\ \times\ 10^2$ nm
$10^8/cm^2$	$0.5\ \times\ 10^3$ nm	$10.28\ \times\ 10^7$ nm	$6.28\ \times\ 10^3$ nm

Range of various parameters of the periodically bent channel affected with dislocation corresponding to a dislocation density of *$10^8/cm^2$*,

a (cm)	λ_u (cm)	R_u (cm)	E (Mev)	x_d amplitude of the dislocation wave
$1\ \times\ 10^{-7}$	$3.14\times\ 10^{-4}$	$2.5\ \times\ 10^{-2}$	142.363	$2.198\ \times\ 10^{-3}$
$10\ \times\ 10^{-7}$	$3.14\times\ 10^{-4}$	$2.5\ 6\times\ 10^{-3}$	14.236	$2.198\ \times\ 10^{-4}$
$100\ \times\ 10^{-7}$	$3.14\ \times\ 10^{-4}$	$2.5\times\ 10^{14}$	1.412	$2.198\ \times\ 10^{15}$

When $\lambda_d < \lambda_u$

Range of various parameters of the periodically bent channel affected with dislocation at $\lambda_u = 2\lambda_d$

Dislocation density	λ_d (cm)	λ_u (cm)	a (cm)	R_u (cm)	E (MeV)
$1.5\times10^9/cm^2$	1.66×10^{-4}	3.32×10^{-4}	1×10^{-7}	2.8×10^{-2}	150
			10×10^{-7}	2.8×10^{-3}	15
			100×10^{-7}	2.8×10^{-4}	1.5

Equation of motion,

$$\tilde{x} = x - a\ sin(k_u vt)$$

$$\ddot{\tilde{x}} = \ddot{x} + ak_u^2 v^2\ sin(k_u vt)$$

$$\frac{1}{R} = ak_u^2\ sin(k_u vt)$$

$$\ddot{\tilde{x}} + \frac{qe}{m\gamma}U(\tilde{x}) - \frac{\gamma v^2}{R}\tilde{x} = 0$$

The maximum amplitude of oscillation

$$\tilde{x}_m = \frac{m\gamma^2 v^2}{qeV_0R}$$

And the equilibrium axis shifts to,

$$\tilde{x}_0 = \frac{m\gamma^2 v^2}{2qeV_0R}$$

The period of oscillation of the particle in the channel,

$$T = \left(\frac{m\gamma}{2qeV_0}\right)^{1/2} Sin^{-1}\left\{1 - \frac{2qeV_0R}{m\gamma^2 v^2}cos(k_u z)\right\}$$

The reflection and transmission coefficients FOR ELECTRONS case

$$\left|R\right|^2 = \frac{(-\mu^2 + 2mE\tilde{\rho}_0^2)^2 Sin^2(2\mu\varphi_0)}{8mE\mu^2\tilde{\rho}_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + 2mE\tilde{\rho}_0^2)^2 Sin^2(2\mu\varphi_0)}$$

$$\left|T\right|^2 = \frac{8mE\tilde{\rho}_0^2\mu^2}{8mE\mu^2\tilde{\rho}_0^2 Cos^2(2\mu\varphi_0) + (\mu^2 + 2mE\tilde{\rho}_0^2)^2 Sin^2(2\mu\varphi_0)}$$

Dislocations in a periodically bent crystal changes the channeling and dechanneling coefficients by the parameters of the crystalline undulator.

For low dislocation density, $\lambda_d > \lambda_u$, the channelled particle SEES the effects of dislocations because several undulations of crystalline undulator are within one period of dislocation affected channel.

In the opposite case of $\lambda_d < \lambda_u$, (High dislocation density) the undulator effects are largely UNEFFECTED by dislocations, because dislocation affected regions are like point defects on the scale of undulator affected regions


Thank You

Thank You