Parametric Channeling and Collapse of Charged Particles Beams in Crystals

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Introduction.

In aspects of channeling physics considered previously the influence of specific channeled motion on the internal state of the particle is traditionally doesn’t taken into account and the effect of the internal state on channeled motion is being ignored.

In the report a new type of self-organized orientational motion of charged particles with internal energy structure in crystals, i.e., **parametric channeling** is proposed The essence of this motion mode is associated with strong parametric coupling of characteristics of two systems:

- orientational oscillations caused by the interaction of the moving particle charge with averaged fields of crystal axes and planes (oscillations associated with channeling)
- transitions caused by intraparticle processes (resonant electronic transitions in atoms or vibrations caused by relative intramolecular motion).

In general case, this is a nonlinear parametric process whose efficiency is controlled by the evolution of these two coupled systems and the possibility of mutual transformation of their functioning modes.

Depending on the system examined, this process can be described in the classical approximation (molecules) or within the quantum-mechanical approach (atoms, nuclei, electrons).

In the report the following types of parametric motion are examined:

Parametric processes during the channeling of

- an ionized molecule

1. oriented across the channel axis;
2. oriented at an acute angle to the channel axis;

- an atomic ion or nuclei;

- an electron during axial channeling.

Parametric processes during channeling of an ionized molecule.

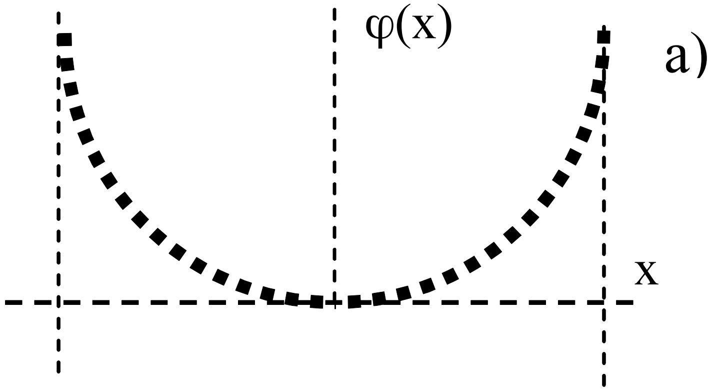
We consider the features of the effect of intramolecular vibrations on orientational motion of an ionized molecule in the planar channel. There is a wide variety of molecules and charge distributions in them. The analysis is restricted to the consideration of the motion of a diatomic molecule AB^+ in which one atom is ionized and another is neutral. The problem is analyzed on the basis of the classical consideration in the coordinate system of molecule center of mass.

Two cases with different relative orientations of molecule and planar channel axes :

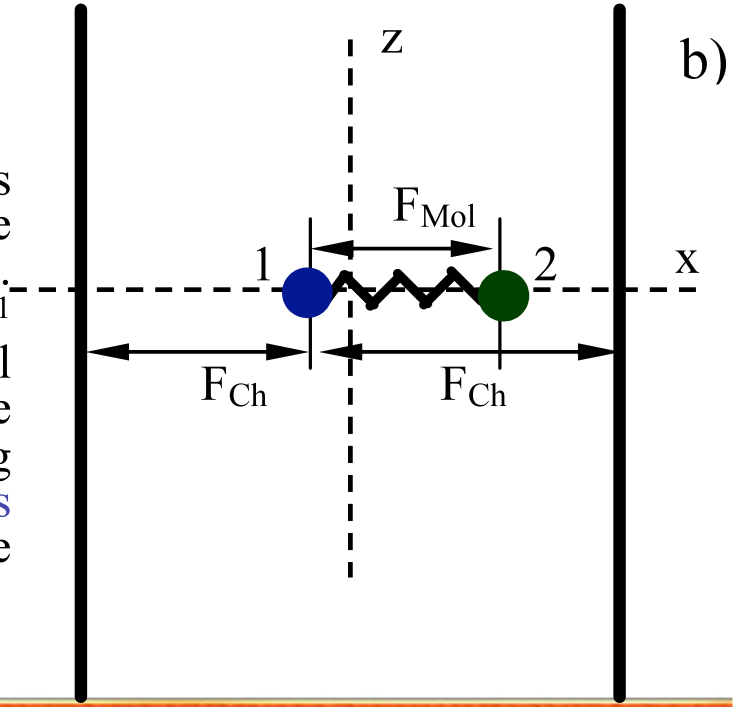
1. parallel
2. perpendicular

The time of quantized spatial rotation of the moving molecule axis at a small orbital quantum number $l \leq 1$ in many cases much exceeds the time of flight of this molecule through a thin crystal in the channeling mode; therefore, the rotational motion of the molecule appears “frozen” in the channeling region.

The motion of a diatomic gas ionized molecule focused across the x axis of the planar channel beamline. The relative orientation of an ion (*1*) and neutral atom (*2*) in structure of a molecule in the channel beamline is shown in Fig.



The range of potential applicability of the channeling effect for such a molecule orientation:



such a motion mode can be feasible, e.g., for mesomolecules such as $pp\mu^-$, $pp\pi^-$, and ppK^- formed by substituting the electron in the molecule with a negative muon or a hadron. The size of such mesomolecules doesn't exceed (2–4).10^{-11} cm, which is smaller than the channel width by several orders of magnitude. Such orientational motion can also be observed for "ordinary" molecules such as HeH^+ during channeling in intercalated crystals or zeolite-type crystals with microchannels whose width is much larger than the molecule size.

The Lagrangian of this system is written as $L=\frac{m_1\dot{x}_1^2}{2}+\frac{m_2\dot{x}_2^2}{2}-\frac{m_1\omega_1^{\,2}x_1^{\,2}}{2}-\frac{m_2\omega_2^{\,2}(x_1-x_2)^2}{2}$

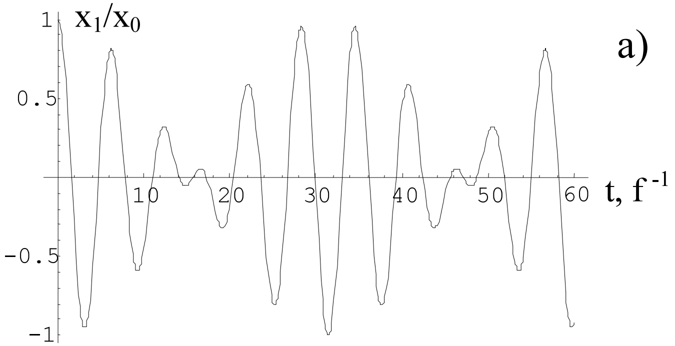
Using the Lagrange equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}-\partial L/\partial x_i=0$

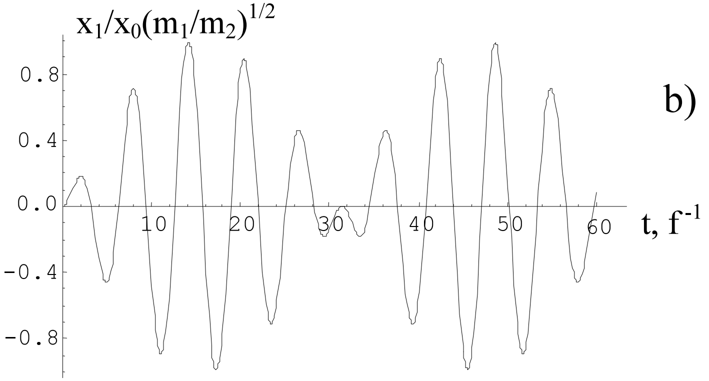
Equations of motion

$$m_1\ddot{x}_1+k_1x_1+k_2(x_1-x_2)=0 \qquad m_2\ddot{x}_2+k_2(x_2-x_1)=0$$

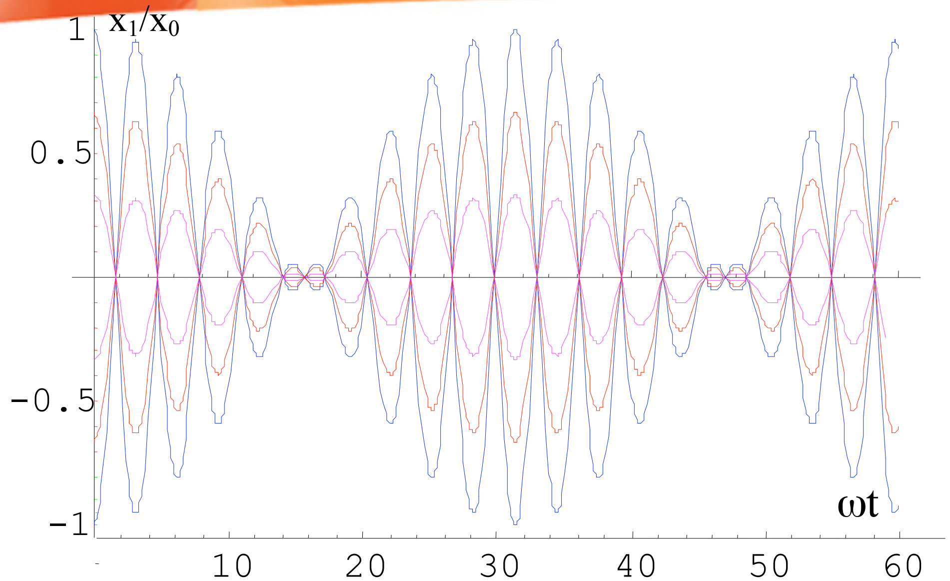
The solutions of these equations

$$x_1=x_0\cos(\sqrt{k_2/4m_1}\,t)\cos(\sqrt{k_2/m_2}\,t) \qquad x_2=x_0\sqrt{m_1/m_2}\sin(\sqrt{k_2/4m_1}\,t)\sin(\sqrt{k_2/m_2}\,t)$$





The energy transfer from oscillatory motion of the molecule in the channel to internal vibrations of the same molecule is a periodic process. In the channel regions along the *z* axis, where the molecule vibration amplitude decreases to zero, $x_1 = 0$, local quasi-cooling of the beam occurs.



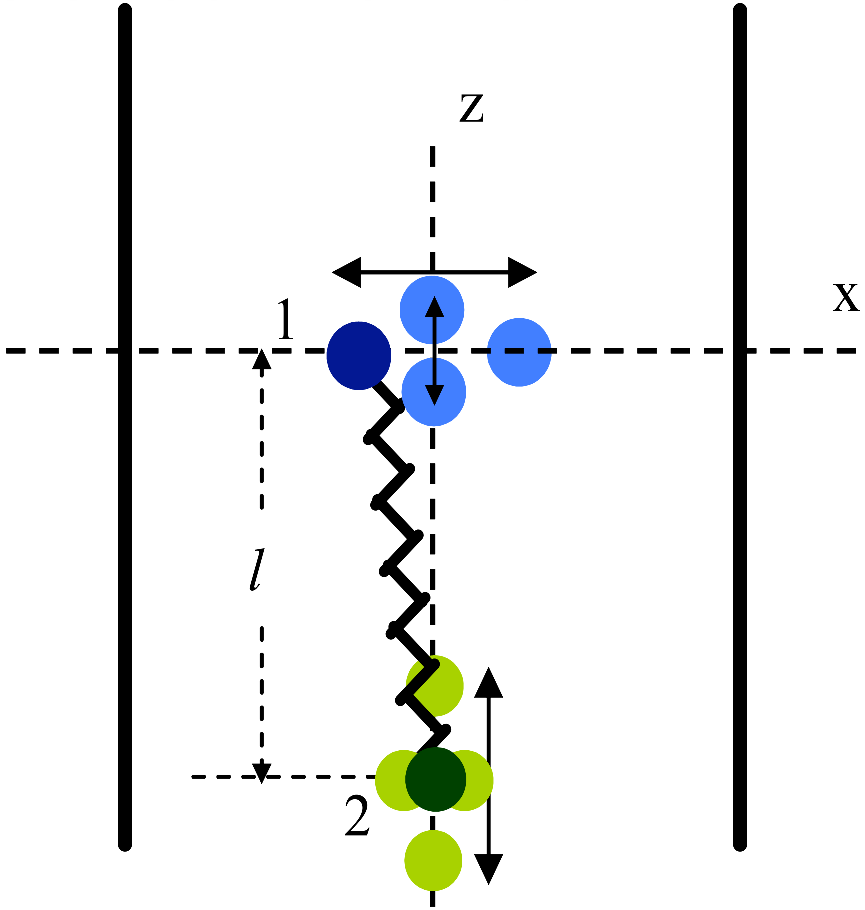
Motion dynamics are independent of the initial vibration amplitude. For all molecules similarly oriented at the channel input, the time taken for maximum beam compression will be the same.

Under resonant conditions between vibrations, the major factor providing transverse cooling of the beam is the ratio of masses of charged and neutral particles. For maximum possible cooling, the charged atom mass should be several times larger than the neutral atom masses: the larger the mass ratio, the larger the efficiency of the energy transfer.

Quasi-cooling of the ionic state of the molecule achieved in such a way will be accompanied by an increase in the amplitude of intramolecular vibrations. The maximum amplitude of internal vibrations can significantly exceed the initial vibration amplitude of the molecule as a single unit in the channel. On the one hand, this result limits the application of this effect to achieve parametric collapse of the beam; on the other hand, it allows the prediction of a number of additional effects, in particular, the possibility of molecule dissociation during the parametric interaction.

Parametric processes during channeling of the ionized molecule oriented at an small angle to the channel axis.

The features of orientational motion of a similar diatomic ionized molecule whose axis is oriented at a small angle to the channel axis. At such an orientation of the molecule axis, ionized molecules with longitudinal sizes greatly exceeding the channel width can be involved in channeling. The relative orientation of particles composing such a molecule is shown in Fig.



Relative orientation of the (*1*) ion and (*2*) neutral atom in the diatomic molecule oriented at an acute angle to the channel axis.

The motion of both atoms in the molecule is described by the system of Lagrange equations

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}_1}-\frac{\partial L}{\partial x_1}=m_1\ddot{x}_1+k_1x_1+\frac{k_2(x_1-x_2)(z_2-z_1)}{l}=0 \qquad \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}_2}-\frac{\partial L}{\partial x_2}=m_2\ddot{x}_2+\frac{k_2(x_2-x_1)(z_2-z_1)}{l}=0$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{z}_1}-\frac{\partial L}{\partial z_1}=m_1\ddot{z}_1-k_2(z_2-z_1)-\frac{k_2(x_2-x_1)^2}{2l}=0 \qquad \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{z}_2}-\frac{\partial L}{\partial z_2}=m_2\ddot{z}_2+k_2(z_2-z_1)+\frac{k_2(x_2-x_1)^2}{2l}=0$$

After complex simplifications the final solutions can be found

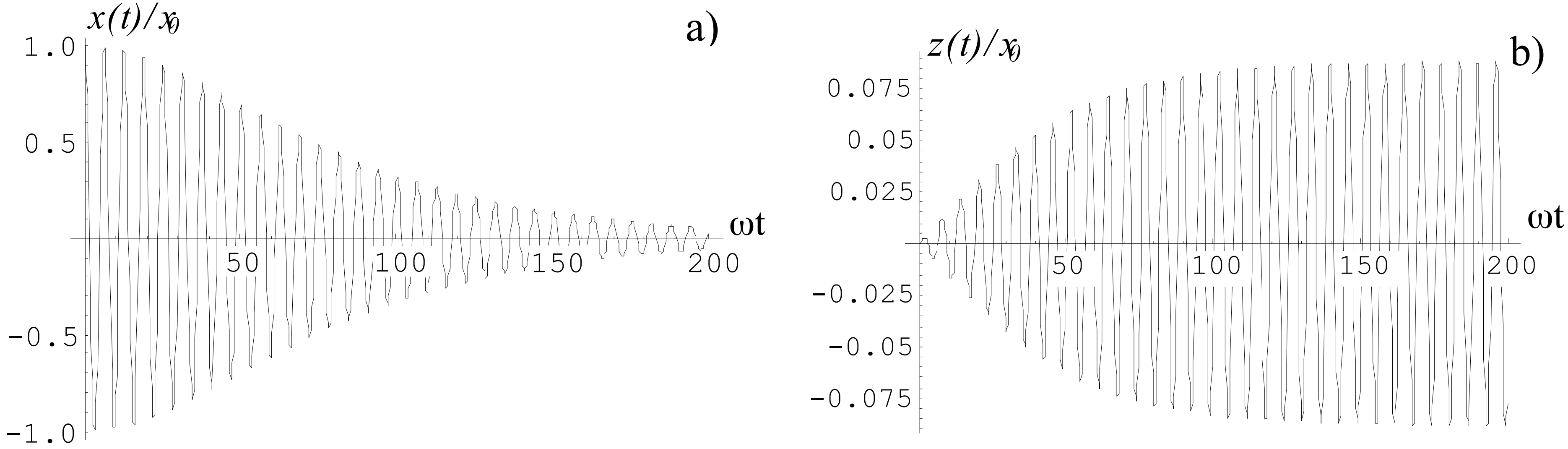
$$x=X_0\sec h\left(\frac{\sqrt{k_1}}{2\sqrt{m_2+m_1}}\sqrt{\frac{m_2}{m_1}}\frac{X_0}{l}t\right)\cos\left(\sqrt{\frac{k_1}{m_1}}t\right)$$

$$z=Z_0\,\mathrm{th}\left(\frac{\sqrt{k_1}}{2\sqrt{m_2+m_1}}\sqrt{\frac{m_2}{m_1}}\frac{X_0}{l}t\right)\sin\left(2\sqrt{\frac{k_1}{m_1}}t\right)$$

$$Z_0=X_0\sqrt{\frac{m_2}{4(m_2+m_1)}}$$

If the molecule consists of two identical atoms ($m_1=m_2=m$), one of which is charged and another is neutral, the resonant condition $\omega_z=2\omega_x$ takes the form $k_2=2k_1$.

A concrete example of these solutions at the ratio $X_0/l = 0.2$ of the initial transverse vibration amplitude (a) and the length of the molecule (b) is shown in Fig.



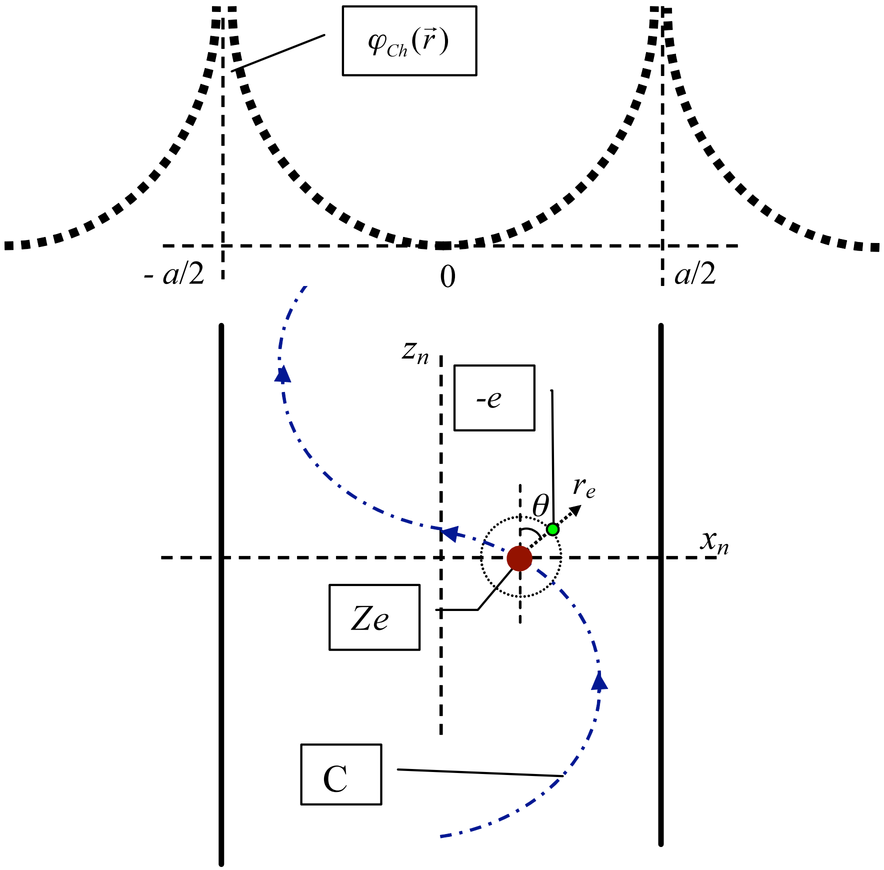
Peculiarities of this process in compare with previous one:

- irreversible energy transfer from the transverse oscillatory (channeled) motion of the ion in the channel to the internal longitudinal vibration of the molecule is observed
- no stringent conditions are imposed on masses of atoms composing the molecule, masses can be widely varied providing the resonance condition required for the efficient process.

Features Of The Implementation Of Parametric Processes During Channeling Of Atomic Ions And Nuclei In Crystals

Consideration of the features of the coupling between the electronic states in the atomic ion volume and the state of the same ion within the channel can be used to reveal the strong parametric coupling between these processes.

We consider the features of orientational motion of the atomic ion with nucleus charge *Ze* and a single atomic electron in the planar channel. The relative orientation of the nucleus and atomic electron in the channel is shown in Fig.



The Hamiltonian of the atomic ion in the channel

$$\hat{H}(\vec{r}_n,\vec{r}_e) = \hat{T}_n(\vec{r}_n) + Ze\varphi_{Cn}(\vec{r}_n) + \hat{T}_e(\vec{r}_e) - e\varphi_e(\vec{r}_e) - e\varphi_{Cn}(\vec{r}_n + \vec{r}_e)$$

The evolution of the electronic and nuclear subsystems is defined by the nonstationary Schrodinger equation

$$i\hbar \frac{\partial \Psi(x_n,\vec{r}_e,t)}{\partial t} = \hat{H}(x_n,\vec{r}_e)\Psi(x_n,\vec{r}_e,t)$$

According to the superposition principle, the wave function of the channeled atomic ion in the general case is given by

$$\Psi(x_n,\vec{r}_e,t) = \sum_k \sum_\beta c_{k\beta}(t)\psi_k(x_n)\psi_\beta(\vec{r}_e)e^{-i(E_k+E_\beta)t/\hbar}$$

Efficient parametric channeling based on the interference of the vibrational state of the nucleus in the channel field and the electronic transition in the nucleus field can occur only in the case of close transition frequencies in either subsystem. We restrict the analysis to the case when such a condition is satisfied only for a single transition from a lower state to one of excited states in either subsystem.

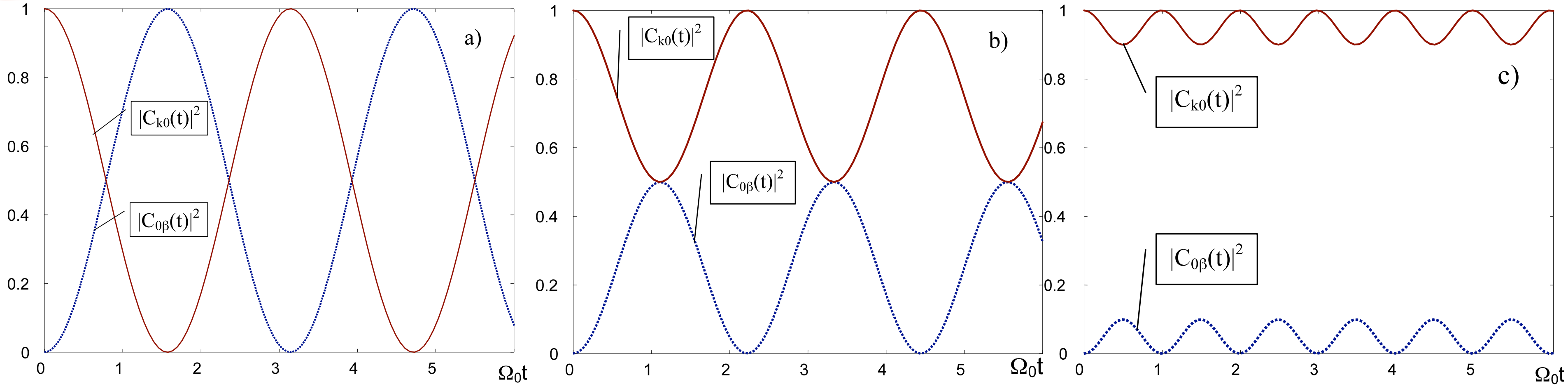
$$\Psi(x_n,\vec{r}_e,t) = c_{k_0\beta}(t)\,\psi_{k_0}(x_n)\psi_{\beta}(\vec{r}_e)e^{-i(E_{k_0}+E_{\beta})t/\hbar} + c_{k\beta_0}(t)\,\psi_{k}(x_n)\psi_{\beta_0}(\vec{r}_e)e^{-i(E_{k}+E_{\beta_0})t/\hbar}$$

The expressions for the probabilities of two alternative states could be found

$$|c_{k\beta_0}(t)|^2 = 1-\frac{\Omega_0^2}{\Omega^2}\sin^2\Omega t$$

$$|c_{k_0\beta}(t)|^2 = \frac{\Omega_0^2}{\Omega^2}\sin^2\Omega t$$

These equations characterize the process of parametric coupling between channeled and intra-atomic states and allow us to consider the effect of parametric cooling of channeled particles with simultaneous excitation of intrinsic electronic states of the ion.



Variation in the efficiency of parametric coupling between the excited channeled state of the atomic ion and the excited intra-atomic state of the same ion as a function of the difference between the energies of the quantized transition of the ion in the channel and the electronic transition within the ion.
a) $\delta\omega = 0$; b) $\delta\omega = \pm 2|V_{k0,0\beta}|/\hbar$ c) $\delta\omega = \pm 6|V_{k0,0\beta}|/\hbar$.

This process for the various relations between Ω_0 and $\delta\omega$ is shown in Fig. The energy exchange efficiency depends on the frequency detuning $\delta\omega$ defined by the difference of total energies of both states. In the case of $\delta\omega = 0$, the maximum parametric coupling and periodic exchange of the total energy between interacting systems are observed.

The significant difference between the frequencies of interlevel transitions of valence electrons in atoms and the frequencies of ion transitions between channeling levels does not make it possible to satisfy this condition in the case of consideration of only transitions corresponding to unperturbed atoms with degenerate levels.

Such a condition can be satisfied if we take into account the fact that atoms moving within the channel are in a strong nonuniform electric field of crystal planes.
Due to the interaction of the atomic quadrupole moment *Qxx* with the electric field gradient, removal of the degeneracy of the orbital quantum number and level splitting in moving atomic ions occur. The energy of this interaction:

$$W_Q = Q_{xx}\frac{\partial^2\varphi(x_n)}{\partial x_n^2} = -Q_{xx}\frac{\partial E(x_n)}{\partial x_n} \qquad \Delta W_Q = W_Q(j_2) - W_Q(j_1)$$

The frequency of degenerate state splitting of the atom is close to the frequency ω_{Ch} of the transition between channeling levels and it confirms the possibility of satisfying the resonant condition to achieve the efficient parametric interaction.

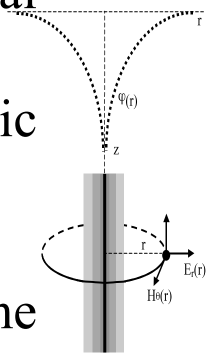
The results obtained are also applicable to the case of channeled motion of nuclei whose volume is characterized by the resonance transition with an energy difference which satisfies the resonant condition. The frequency of the intranuclear transition far exceeds the transition frequency during channeling, although certain exceptions are known (e.g., the transition with an energy of several electronvolts in uranium nuclei), where frequencies of these transitions are comparable.

Features Of The Parametric Interaction And Cooling Of The Electron Beam During Axial Channeling

Some parametric processes have to be observed during the channeling of nonrelativistic and relativistic elementary particles without internal structure but with spin and magnetic moment. The interaction of this moment with the effective magnetic field induced in the rest frame of the moving particle channeled in the field of crystal axes and planes forms an energy structure which is similar in a certain sense to the internal structure of atoms and molecules.

$\varphi(\mathbf{r})$ - averaged potential of the crystal axis

$H_\theta(r)$ - effective azimuthal magnetic field



When the electron moves along the crystal axis, its transverse motion is quantized. The rest frame of the particle is characterized by

- $H_\theta(r)$
- the quantized electron spin direction.

It is a two-level system whose energy levels correspond to different spin orientations, and the transition energy is defined by the effective magnetic field strength. In this case the effect of coupled oscillations can be achieved.

The effective magnetic field of the crystal axis.

$$H(r) = \frac{2J(r)}{rc} \equiv \frac{2\gamma v}{rac} \int_0^r (< g_n(r') >_{z,T} + < g_e(r') >_{z,T}) 2\pi r' dr'$$

Or, using approximations, $< g_n(r) >_{z,T} = \frac{Ze}{\pi u^2} e^{-r^2/u^2}$ $\quad < g_e(r) >_{z,T} = -\frac{Ze}{2\pi a_{TF}^2} e^{-r/a_{TF}}$

$$H(r) = \frac{2\gamma v Ze}{rac} \{(1 - e^{-r^2/u^2}) - \frac{1}{a_{TF}^2} \int_0^r e^{-r'/a_{TF}} r' dr'\}$$

The maximum strength of this field corresponds to the region close to the crystal axis. For relativistic particles with $v \approx c$, at the typical values $u \approx 10^{-9}$ cm, $a \approx 1.5 \,.\, 10^{-8}$ cm, and $Z \approx$ 10–30, the strength can reach a very high value

$$H_{\max} \approx \frac{\gamma v Ze}{uac} \approx 10^{12} \gamma \, Oe$$

In the associated coordinate system the two-component Pauli equation

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = \hat{H}_0 \Psi(\vec{r},t) - \frac{e\hbar \hat{\vec{\sigma}} \vec{H}(r)}{2m_e c} \Psi(\vec{r},t)$$

The total wave function of a channeled particle with spin represents the superposition of two-component wave functions for particles with different spin orientations and is given by

$$\Psi(\vec{r},t) = \sum_{\alpha} \psi_{\alpha}(\vec{r}) \; e^{-iE_{\alpha}t/\hbar} \left| \begin{matrix} c_{\alpha,s_z=\hbar/2}(t) \\ c_{\alpha,s_z=-\hbar/2}(t) \end{matrix} \right|, \quad \sum_{\alpha} \sum_{s_z=-\hbar/2}^{\hbar/2} | c_{\alpha,s_z}(t) |^2 = 1$$

The probability of the parametric transition between channeling levels caused by synchronous transitions between spin states:

$$|\,c_{g,s_z=\hbar/2}(t)\,|^2 = \frac{\Omega^2_{eg(0)}}{\Omega^2_{eg}}\sin^2(\Omega_{eg}t)$$

$$|\,c_{e,s_z=-\hbar/2}(t)\,|^2 = \cos^2(\Omega_{eg}t) + \frac{(\delta\omega_{eg}/2)^2}{\Omega^2_{eg}}\sin^2(\Omega_{eg}t) \equiv 1 - \frac{\Omega^2_{eg(0)}}{\Omega^2_{eg}}\sin^2(\Omega_{eg}t)$$

These solutions confirm the possibility of efficient energy exchange between channeling states and the states controlled by the interaction of the spin magnetic moment.

These solutions confirm the possibility of efficient energy exchange between channeling states and the states controlled by the interaction of the spin magnetic moment. From the formal point of view, these relations are identical to previous results. But there is a fundamental difference between these processes.

The maximum efficiency formally corresponds to the difference of only energies of channeled states.

$$\delta\omega_{eg} = (E_e - E_g)/\hbar \to 0$$

Fulfillment of this condition is inconsistent with the problem statement, i.e., the transition between states with different energies of channeled states.

Rather high efficiency of parametric processes can also be provided under the condition $\delta\omega_{eg} \neq 0$, but only if the requirement $|\Omega_{eg(0)}| >> |\delta\omega_{eg}/2|$ is satisfied. Based on the above expressions for $|\Omega_{eg(0)}| \sim \gamma$, we can easily ensure that such a condition will be satisfied for relativistic particles with large γ. Indeed, as shown above, the effective magnetic field at real parameters can reach a very large value, $H_{max} \approx 10^9\gamma$ Oe

	The ionized molecule oriented across the channel axis	The ionized molecule oriented at an acute angle to the channel axis	Atomic ions in crystals	Atomic nuclei in crystals	The electron beam
The periodicity	Periodical	Irreversible	Periodical	Periodical	Periodical
Types of channeling can be applied to	Planar or axial	Planar or axial	Planar or axial	Planar or axial	Axial only
Accompanied effects	Great increase in the amplitude of intramolecular vibrations	Increase in the amplitude of intramolecular vibrations	Ion excitation	Nuclei excitation	Spin transition
Peculiarities	Applicable to mesomolecules such as ppμ–, ppπ– and ppK–; ordinary molecules such as HeH+ during channeling in intercalated crystals or zeolite-type crystals with wide microchannels	No stringent conditions are imposed on masses of atoms	Impossible to provide in the case of consideration of only unperturbed atoms with degenerate levels transitions. Quadrupole interaction of the atom with the electric field gradient is needed	Applicable only to special nuclei with low level transition (e.g., the transition with an energy of several electronvolts in uranium nuclei)	Great γ is needed to provide qusiresonance condition