EQUIVALENT PHOTONS METHOD and EIKONAL APPROXIMATION in the THEORY of TRANSITION RADIATION

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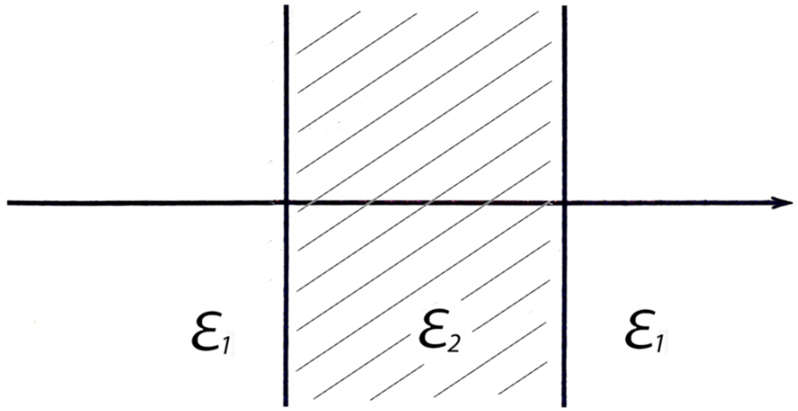
Transition Radiation for Sharp and Smooth Function $\varepsilon(\mathbf{r})$

$\varepsilon(x)$ – sharp function of x



Joining of solutions for ε_1 and ε_2

Problem

$$\frac{\omega_p^2}{\omega} l \geq 1 \quad \text{for } \varepsilon(\mathbf{r})$$

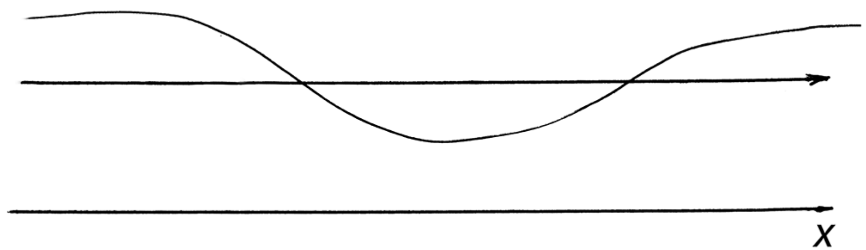
$\varepsilon(\mathbf{r})$ – smooth function of $\mathbf{r}$

1. $|1-\varepsilon(\mathbf{r})| \ll 1$ approximation
2. Semiclassical approximation for $\varepsilon(x)$

M.Ter – Mikaelian (1980)

L.Durand (1975)

J.Lepore, R.Riddell (1976)



Simplification of the Problem for $\varepsilon(\mathbf{r})$

1. Equivalent photons method (the field of relativistic electrons → wave packet of free waves → evolution of this wave packet with time)

2. Eikonal approximation
   (the changing of free waves packet due to $\varepsilon(\mathbf{r}) \neq 1$)

Spectral-Angular Density of Transition Radiation

$$\left(\Delta + \omega^2 \varepsilon_\omega(\mathbf{r})\right)\mathbf{E}_\omega = \operatorname{grad}\operatorname{div}\mathbf{E}_\omega - 4\pi e i\omega \frac{\mathbf{v}}{\mathbf{v}}\,\delta(\boldsymbol{\rho})\, e^{i\omega z/v}$$

$$\frac{dE}{d\omega\, do} = \frac{\omega^2}{\left(8\pi^2\right)^2}\left|\mathbf{k}\times\mathbf{I}\right|^2$$

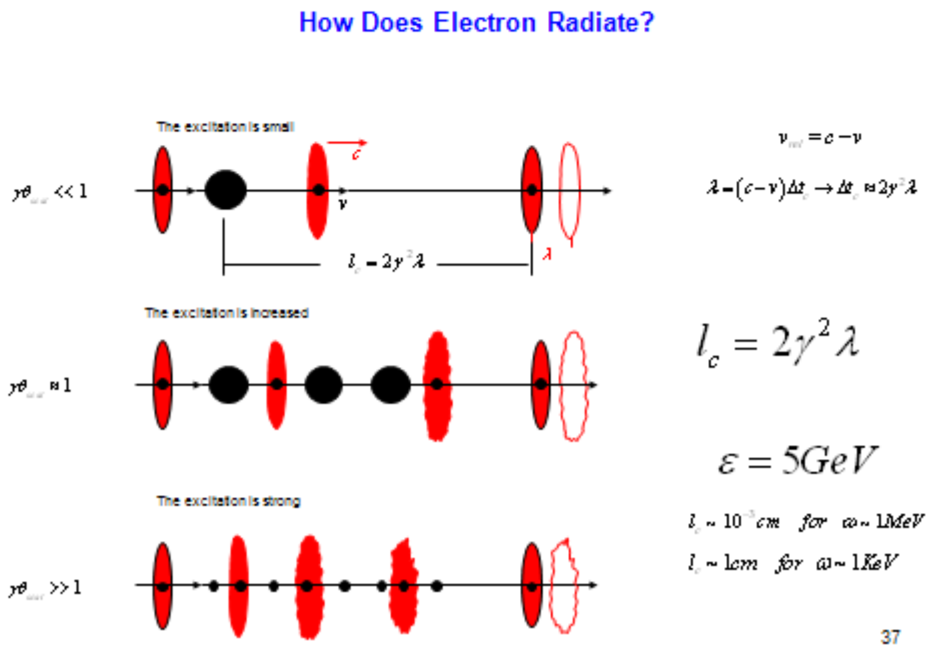
$$\mathbf{I} = \int d^3r\, e^{-i\mathbf{kr}}\left(1-\varepsilon(\mathbf{r})\right)\mathbf{E}_\omega(\mathbf{r})$$

It is necessary to know $E(\mathbf{r})$ in the region where $\varepsilon_\omega(\mathbf{r}) \neq 1$

Proper Field of Relativistic Electrons

$$\mathbf{E}_c(\mathbf{r},t) = -e\left(\nabla_\perp + \frac{n}{\gamma^2}\frac{\partial}{\partial z}\right)\varphi_c(\ ,t)$$

$$\varphi_c(\mathbf{r},t) = \frac{}{\sqrt{(z-vt)}}$$



$$\frac{4\pi}{k_\perp^2 + k_z^2/\gamma^2} e^{-i\mathbf{kv}t}$$

$$c_k = \frac{4\pi\gamma}{k_\perp} e^{-i\mathbf{kv}t} \int_0 dz \cos k_z\, e^{-k_\perp \gamma z} =$$

$$= 4\pi\, e^{-i\mathbf{kv}t} \int_0^\infty \rho\, d\rho\, J_0(k_\perp \rho) K_0(k_z \rho/\gamma)$$

$z_{eff} \sim \lambda$

$\rho_{eff} \sim \gamma\lambda$

Equivalent Photons Method

$$\varphi(\mathbf{r},t)=\frac{1}{\pi^2}\mathrm{Re}\int d^3k\, e^{i(\mathbf{kr}-kt)}\frac{\theta(k_z)}{k_\perp^2+k_z^2/\gamma^2}$$

$c=1$

$for\ t=0\quad \varphi(\mathbf{r},0)=\varphi_c(\mathbf{r},0)$

this packet spreads for $t>0$, but for $\frac{kt}{\gamma^2}\ll 1$ the spreading is small!!!

$$\begin{cases}\varphi(\mathbf{r},t)=\dfrac{e}{\sqrt{(z-t)^2+\rho^2/\gamma^2}}=\displaystyle\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{kr}}c_k, \\ c_k=e^{ik_z t}\dfrac{4\pi}{k_\perp^2+k_z^2/\gamma^2}\end{cases}$$

$$t\ll\frac{\gamma^2}{k}$$

Eikonal Approximation

The changing of free wave packet $\mathbf{E}_\omega(\mathbf{r})$ is determined by

$$\left(\Delta + \varepsilon_\omega(r)\omega^2\right)\mathbf{E}_\omega = \nabla\left(div\,\mathbf{E}_\omega\right)$$

$$\mathbf{E}_\omega = e^{i\omega z}\mathbf{\Phi}(\mathbf{r})$$

The second derivatives of $\mathbf{\Phi}$ are neglected

$$i\frac{\partial}{\partial z}\mathbf{\Phi}_\perp = \frac{\omega}{2}\left(1 - \varepsilon_\omega(\mathbf{r})\right)\mathbf{\Phi}_\perp$$

$$E_\omega^{(eik)}(\mathbf{r}) \approx E_\omega^0(\mathbf{r})\exp\left\{-i\frac{\omega}{2}\int_{-\infty}^{z}dz'\left(1-\varepsilon_\omega(\mathbf{r})\right)\right\}$$

$$E_\omega^0(\mathbf{r}) \approx \frac{2e\omega}{v^2\gamma}e^{i\omega z}\frac{\boldsymbol{\rho}}{\rho}K_1\left(\omega\rho/v\gamma\right)$$

Spectral Angular Density of Transition Radiation

$$\frac{dE}{d\omega\, do} \approx \frac{\omega^4}{(8\pi)^2}\left|\mathbf{I}_{\perp}^{(eik)}\right|^2$$

$$\mathbf{I}_{\perp}^{(eik)} = i\frac{4e}{v^2\gamma}\int d^2\rho\; e^{-i\mathbf{k}_{\perp}\rho}\,\frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right)\left\{e^{-i\frac{\omega}{2}\int_{-\infty}^{\infty}dz\left(1-\varepsilon_{\omega}(\mathbf{r})\right)}-1\right\}$$

Validity conditions

$$\gamma^{-1} \ll \frac{\omega_{\rho}}{\omega} \ll 1, \qquad \theta \ll \frac{\omega_{\rho}}{\omega}$$

$$\omega\theta l \ll 1$$

Conclusions

1. Description method for forward transition radiation for smoth function $\varepsilon_{\omega}(\mathbf{r})$.

2. Spreading of wave photon packet (equivalent photons method for proper field of relativistic electron)

   the wave packet doesn't spread during the time $\Delta t < \gamma^2/\omega$

3. Eikonal approximation for forward transition radiation (validity conditions)

4. Applications
   - thin plate
   - thin dielectric fiber