Applicability Condition of Macroscopic Transition Radiation Theory

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Introduction.

Transition radiation is emitted when a charge particle moves across the interface between two different media. The usual way to describe the transition radiation is to make use macroscopic electrodynamics. Then transition radiation arises as a result of the fast particle self field reflection from the interface between two media, when the particle field reduce to the electromagnetic plane wave. Macroscopic theory can be applicable when this theory applicable for the both of participant field, fast particle self field and radiation field. Hence there are two different conditions to describing transition radiation with help of macroscopic electrodynamics [1]. This fact lead to restriction of the macroscopic electrodynamics application to the transient radiation. It is interesting to estimate the applicability area of such consideration.

2.Macroscopic theory applicability conditions.

Let us consider the transient radiation as a result of the fast particle self field reflection from the interface between two media, when the particle field Fourier component $\boldsymbol{E}_0(\boldsymbol{q})exp(i\boldsymbol{qr} - i\boldsymbol{qv}t)$ reduce to the electromagnetic plane wave $\boldsymbol{E}exp(i\boldsymbol{kr} - i\omega t)$. The conservation of the field frequency and of the tangential to the plane of the interface wave-vektor components q_t and k_t leads to expression to the normal component of $\boldsymbol{q}$:

$$q_n = (\omega - \boldsymbol{k}_t\boldsymbol{v}_t)/v_n \qquad (1)$$

The macroscopic theory can be applicable to the radiation field description if k is small in comparison with the inverse interatomic spacing $1/b$:

$$kb << 1 \qquad (2)$$

The macroscopic electrodynamics can be applicable to the particle field description when q_n is small in comparison with the inverse interatomic spacing $1/b$:

$$q_n b = (\omega - \boldsymbol{k}_t\boldsymbol{v}_t)b/v_n << 1 \qquad (3)$$

In the case of the normal incidence (3) has the form

$$kb << v/c \qquad (4)$$

and for the ultrarelativistic particles inequality (4) coincide with (2) and there is only one condition of the macroscopic theory applicability – inequality (2) [2].

When the condition (2) is satisfied but condition (3) is violated the macroscopic description of transition radiation can be erroneous and it is necessary to make use of microscopic theory.

Let the interface coincide with plane $x = 0$, and the particle velocity disposed in the plane $y = 0$. In that case $v_x = v\, sin\alpha$, $v_y = 0$, $v_z = v\, cos\alpha$,

$k_x = k\, sin\vartheta\, cos\varphi$, $k_y = k\, sin\, \vartheta\, sin\varphi$, $k_z = k\, cos\vartheta$. The inequality (2) can be written as ($\lambda \equiv c/\omega$)

$$1 << (v/c)\{(\lambda/b)sin\, \alpha + cos\alpha\, sin\vartheta\, cos\varphi\} \qquad (5)$$

In the case of nonrelativistic particle, when $v << c$, instead of (4) we can write

$$1 << (c/v) << (\lambda/b)sin\alpha, \qquad (6)$$

Then for nonrelativistic particles at the normal incidence the transient radiation macroscopic description it is applicable practically only in radio wave region.

In the case of the fast particle grazing incidence on the interface between two media $\alpha << 1$ and if $(\lambda/b)\alpha >> 1$ instead of (5) it is possible to write

$$1 << (v/c)(\lambda/b)\alpha << (v/c)(\lambda/b) \qquad (7)$$

and macroscopic description it is applicable only in radio wave region.

3.Microscopic approach.

We consider the case, when the inequality (2) is satisfied and (3) is violated. In this case propagation process of emitted quantum can be described with the use of the macroscopic approach, whereas the field induced by a fast particle and the process of quantum emission should be treated microscopically. By that reason it is sufficiently to change macroscopic expression for polarizing current upon its microscopic expression. The microscopic electron number density $n(\mathbf{r}) = n_0 + \delta n(\mathbf{r})$ depend on co-ordinate and $\delta n(\mathbf{r}) << n_0$. We can to take into account co-ordinate dependence of electron number density as small fluctuations in macroscopic consideration. Dielectric constant of such matter can be written in form

$$\varepsilon(\mathbf{r},\omega)=\varepsilon_0(\omega)+\delta\varepsilon(\mathbf{r},\omega) = \varepsilon_0(\omega) + \delta n(\mathbf{r})\ \partial\varepsilon_0(\omega)/\partial\, n_0 \tag{8}$$

The small value of $\delta\varepsilon(\mathbf{r},\omega)$ allow taking into account only linear terms $\delta\varepsilon((\mathbf{r},\omega)$ in the Maxwell equations solution. Let us $\boldsymbol{j}_0(\mathbf{r},\omega)$ is the polarization current density without fluctuation. In the equation

$$rot\,\boldsymbol{H}(\mathbf{r},\omega) = (4\pi/c)\boldsymbol{j}_0(\mathbf{r},\omega) - (i\omega/c)\varepsilon_0(\omega)\boldsymbol{E}(\mathbf{r},\omega) - (i\omega/c)\delta\varepsilon(\mathbf{r},\omega)\boldsymbol{E}(\mathbf{r},\omega) \tag{9}$$

we can in the last term of (9) to change $\boldsymbol{E}(\mathbf{r},\omega)$ in the last term of (9) by the its first approximation – solution of (9) in the limit $\delta\varepsilon(\mathbf{r},\omega) \rightarrow 0$, $\boldsymbol{E}_0(\mathbf{r},\omega)$, which obey to equation

$$rot\,\boldsymbol{H}_0(\mathbf{r},\omega) = (4\pi/c)\boldsymbol{j}_0(\mathbf{r},\omega) - (i\omega/c)\varepsilon_0(\omega)\boldsymbol{E}_0(\mathbf{r},\omega) \tag{10}$$

In such approximation last term in (9) is known and play the role of external current density

$$\boldsymbol{j}_1(\mathbf{r},\omega) = -(i\omega/4\pi)\delta\varepsilon(\mathbf{r},\omega)\boldsymbol{E}_0(\mathbf{r},\omega) \tag{11}$$

From (9) and (11) we can write

$$rot\,\boldsymbol{H}_1(\mathbf{r},\omega) = (4\pi/c)\boldsymbol{j}_1(\mathbf{r},\omega) - (i\omega/c)\varepsilon_0(\omega)\boldsymbol{E}_1(\mathbf{r},\omega) \tag{12}$$

At the long distance solutions of equations (11) and (13) has a form

$$\boldsymbol{E}_0(\boldsymbol{r},\omega) = (4\pi i/\omega)\int d^3q\, exp(i\boldsymbol{qr})\{\boldsymbol{j}_0(\boldsymbol{q},\omega)+[\boldsymbol{q}[\boldsymbol{qj}_0(\boldsymbol{q},\omega)]]/(q^2-(\omega/c)^2)\} \quad (13)$$

$$\boldsymbol{E}_1(\boldsymbol{r},\omega) = (4\pi i/\omega)\int d^3q\, exp(i\boldsymbol{qr})\{\boldsymbol{j}_1(\boldsymbol{q},\omega)+[\boldsymbol{q}[\boldsymbol{qj}_1(\boldsymbol{q},\omega)]]/(q^2-(\omega/c)^2)\} \quad (14)$$

It useful to emphasize, $\boldsymbol{E}_0$ is the macroscopic solution, but $\boldsymbol{E}_1$ is the correction because of the electron density fluctuations .

We take into account the electron density fluctuation. But in the infinite fluctuated media exists the radiation of uniformly moving charge. This radiation represent the incoherent process with summation radiation intensity of each atom.

The transition radiation is the coherent process. For the transition radiation it is enough to know only value of the polarization current density.

For the transition radiation the decisive role play the near-surface fluctuation and it is possible to consider only near-surface layer. In this case it is possible to limit oneself to take into consideration the electron density fluctuations in this layer. In the thin surface layer can be important the effect of the natural variation of the polarization due to difference between the local field acted on the surface and the inner molecules . Also it is possible the intensified influence of adsorbed atoms and surface defects.

4.Transient radiation from the particle system on the surface.

The intensification of surface region influence make more attractive all problems with the surface particles. Let us consider the transition radiation from the planar system of particle on the surface. The radiation field from one particle determine by local field acted on this particle. The connection between local field $\boldsymbol{E}^{loc}$ and the field of external sources $\boldsymbol{E}^{0}$ for the particles on plane surface was found in [3]:

$$E_i^{loc}(\mathbf{r},\omega) = \left\{\frac{\delta_{ij} - e_i e_j}{1 - \pi n \alpha_l(\omega)\xi} + \frac{e_i e_j}{1 + 2\pi\alpha_t(\omega)\xi}\right\} E^0(\mathbf{r},\omega) \tag{15}$$

where $\boldsymbol{e}$ is the normal to the plane $z = 0$ where the particles are found, $\alpha_l(\omega)$ and $\alpha_t(\omega)$ are the main values of polarizability tensor $\alpha_{ij}(\omega)$ and

$$\xi = \int d^2q\, f(\boldsymbol{q})q \tag{16}$$

is the coefficient describing the in-plain distribution of particles; function $f(\boldsymbol{q})$ is the Fourier transform of distribution function

$$w(\boldsymbol{R}_{ba}) = (1/S)\eta\,(Z_a+Z_{ba})[1-f(X_{ba}+Y_{ba})] \tag{17}$$

The function $\eta(z)$ describes the particle distribution along the direction $\boldsymbol{e}$ and in this work it can be taken as delta-function $\delta(z)$ and

$$\xi = \int q d^2q f(\boldsymbol{q}) \tag{18}$$

The local field Fourier transform can be found from (15) with help of fast particle self field:

$$E_\iota^{loc}(q_x,q_y,Z=0,\omega) = \frac{ie}{2\pi^2 v_z[K^2-(\omega/c)^2+(\omega-\boldsymbol{Kv})^2/v_z^2]} \times \left\{\frac{(\delta_{ij}-e_ie_j)(\omega v/c^2-K)}{1-an_1\pi\alpha(\omega)} + \frac{e_ie_j[\omega v/c^2-(\omega-\boldsymbol{Kv})/v_z}{1+2an_1\pi\alpha(\omega)}\right\} \tag{19}$$

The transition radiation energy in the frequency range $d\omega$ in the element of solid angle $d\Omega$ (oriented along the unit vector $\boldsymbol{n}$) is given by

$$\frac{d^2W(\boldsymbol{n},\omega)}{d\omega d\Omega} = \frac{4e^2 n_1^{2}\;{}^{4}}{c^3 v_z^{2}[K^2-(\omega/c)^2+(\omega-\boldsymbol{K}\boldsymbol{v})/v_z^{2}]^2}\times$$

$$\times\left|\frac{[\boldsymbol{n}\{(\boldsymbol{v}-v_z\boldsymbol{e})\omega/c^2-\boldsymbol{K}\}]}{\alpha^{-1}(\omega)-\pi a n_1} + \frac{[\boldsymbol{ne}]\{\omega v_z/c^2-(\omega-\boldsymbol{K}\boldsymbol{v})/v_z\}}{\alpha^{-1}(\omega)+2\pi a n_1}\right|^2 \qquad (20)$$

References

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