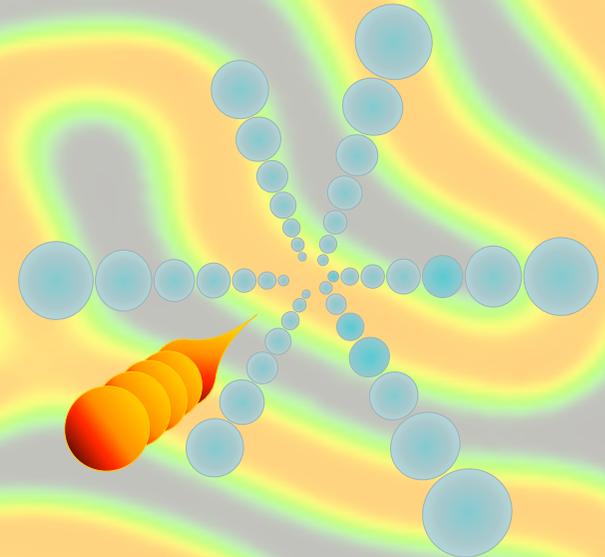


# ENHANCED NEUTRON CONCENTRATION IN URANIUM THIN FILM WAVEGUIDES

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Channeling 2008



# *Plan : "Neutron Number Enhancement in Uranium Thin Film Waveguides"*

- ①. Introductory concepts.
- ②. Neutrons de Broglie wave guiding properties of thin film structures.
- ③. Optimization of uranium thin film waveguides for maximum neutron confinement.
- ④. Conclusion.

# NEUTRON REFLECTION

Incident Neutrons

Reflected Neutrons



neutrons

As in optics, the reflection and refraction of slow neutron de Broglie waves in a material can be described by an index of refraction :

$$n = 1 - \frac{Nb}{2\pi} \lambda^2$$

$\lambda$  – neutrons wavelength

N – number of scatterers per unit volume

b – coherent scattering length

Nb – coherent scattering length density (sometimes)

Transmitted Neutrons

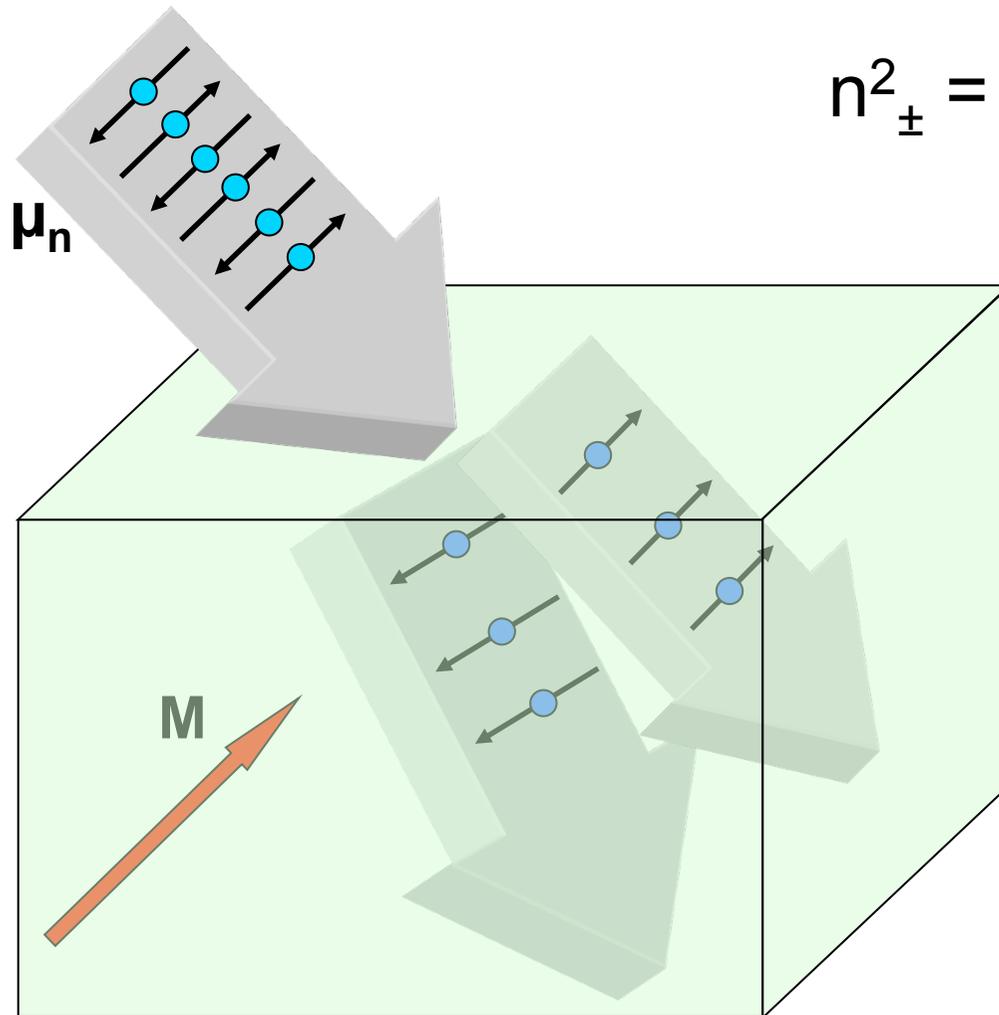
$n_1$

$n_2$

Neutron refractive index for most of materials :  $n < 1$ :  $(Nb/2\pi) \sim 10^{-6} \text{ \AA}^{-2}$  .

# Neutron Magnetic Birefringence

In a ferromagnet film the refractive index for spin up(+) and down (-) neutrons is different due to the neutron magnetic dipole moment interaction with magnetic field.



$$n_{\pm}^2 = 1 - \lambda^2(\delta \pm \gamma M_t)$$

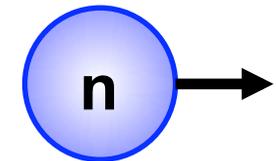
$M_t$  – tangential magnetization component

$$\gamma = 8.44 \cdot 10^{-12} \text{ cm}/\mu_B$$

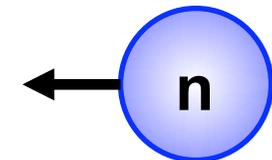
**Magnetization direction**



Low refractive index: spin up (+)

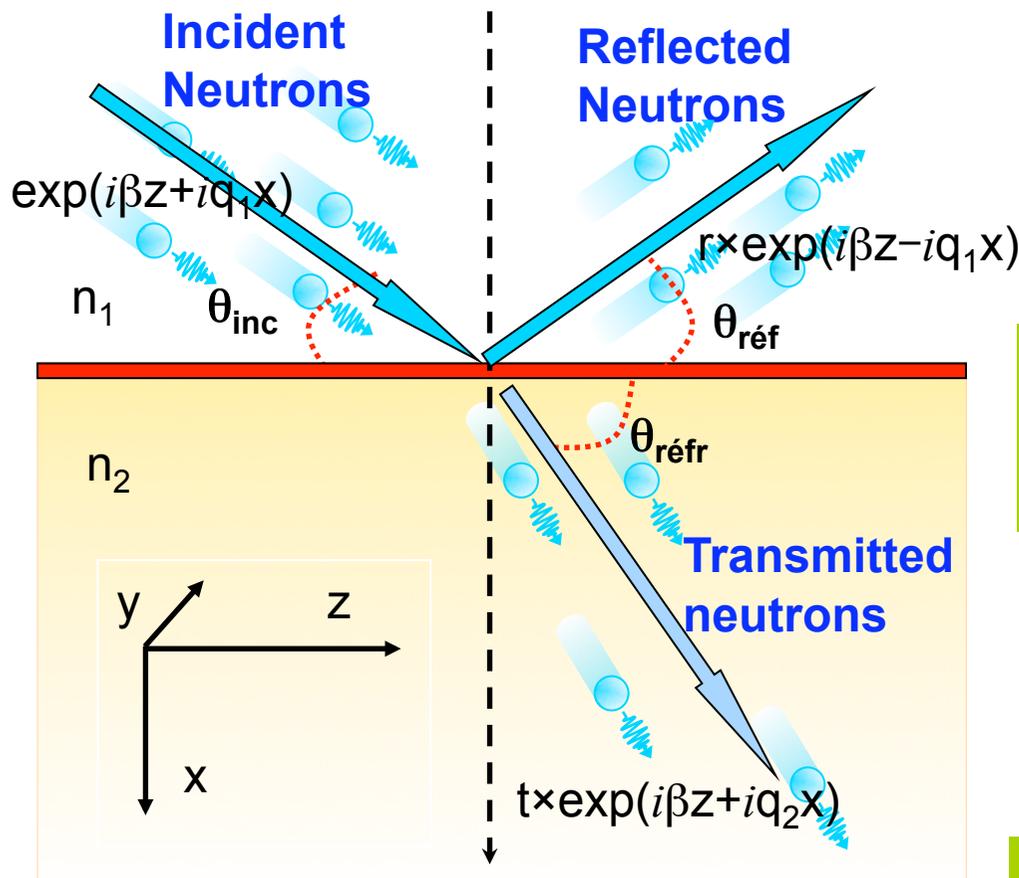


High refractive index: spin down (-)



# NEUTRON REFLECTION

$\beta$  - z component of the wave vector  
 $q$  - x component of the wave vector



neutrons  $\theta_{inc} \sim 0.3^\circ$

## Snell-Décartes law

$$\beta = \frac{2\pi}{\lambda} n_i \cos(\theta_i) = \frac{2\pi}{\lambda} n_j \cos(\theta_j)$$

$$q_i = \frac{2\pi}{\lambda} n_i \sin(\theta_i) = \sqrt{\left(\frac{2\pi}{\lambda} n_i\right)^2 - \beta^2}$$

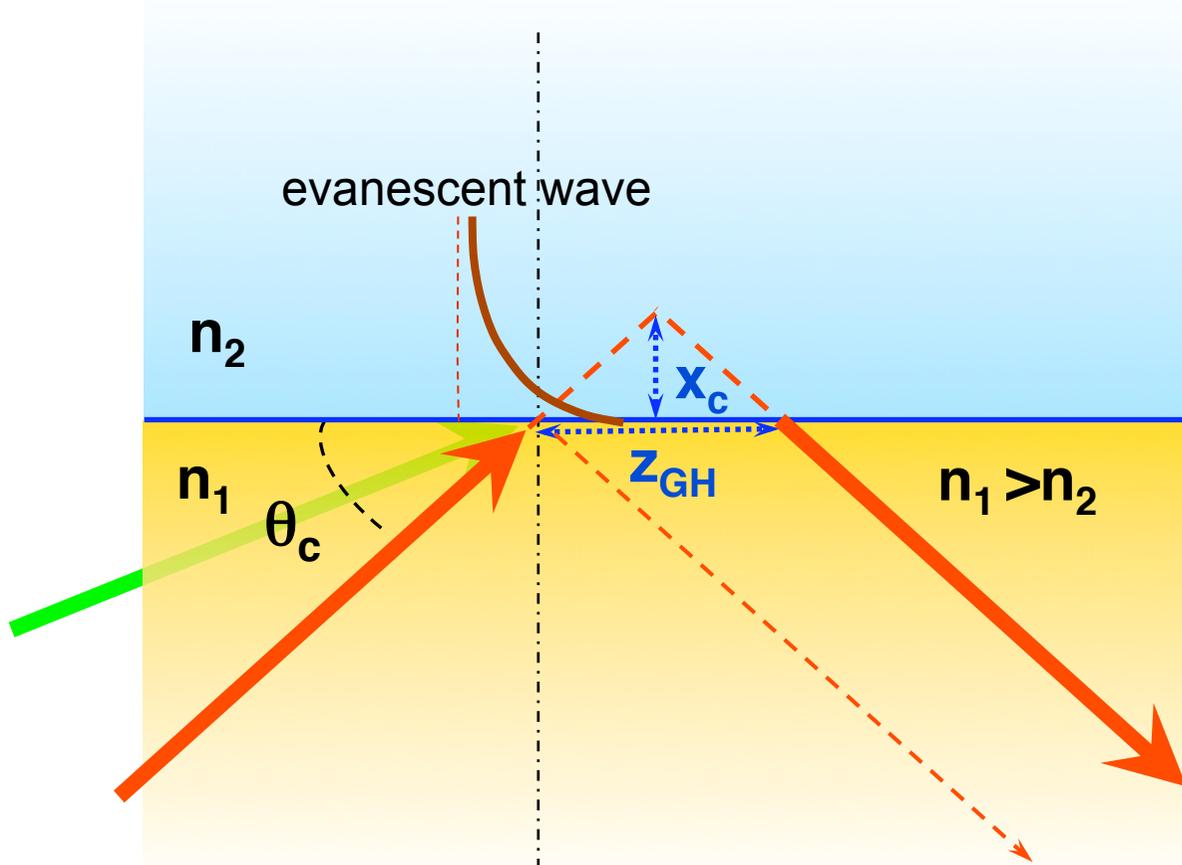
$$k_0 = \frac{2\pi}{\lambda}$$

$$\beta = k_0 n_i \cos(\theta_i)$$

$$q_i = k_0 n_i \sin(\theta_i) = \sqrt{(k_0 n_i)^2 - \beta^2}$$

# Total Reflection

Below some incident critical angle  $\theta_c$  the incident beam is totally reflecting back into the medium 1. When a neutron wave undergoes total internal reflection, an evanescent decaying field appears in the medium with smaller refractive index.



❶ The reflected beam is shifted laterally at a distance  $z_{GH}$  (Goos Hänchen shift):

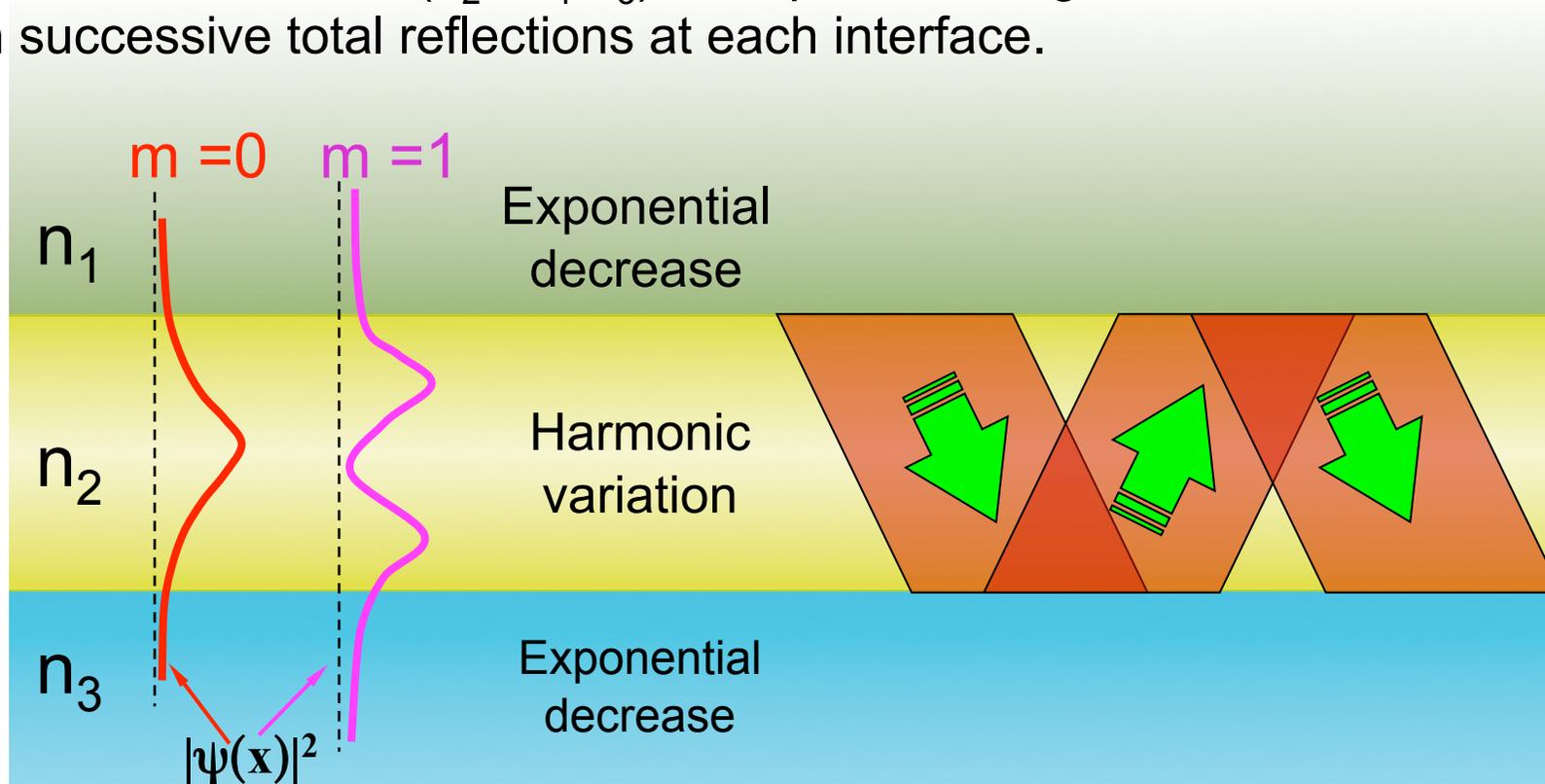
$$z_{GH} = \frac{2\beta}{\sqrt{k_0^2 n_1^2 - \beta^2} \sqrt{\beta^2 - k_0^2 n_2^2}}$$

❷ Evanescent waves penetrate in medium 2 at a typical decaying depth  $x_c$ :

$$x_c = \frac{1}{\sqrt{\beta^2 - k_0^2 n_2^2}}$$

# WAVEGUIDE

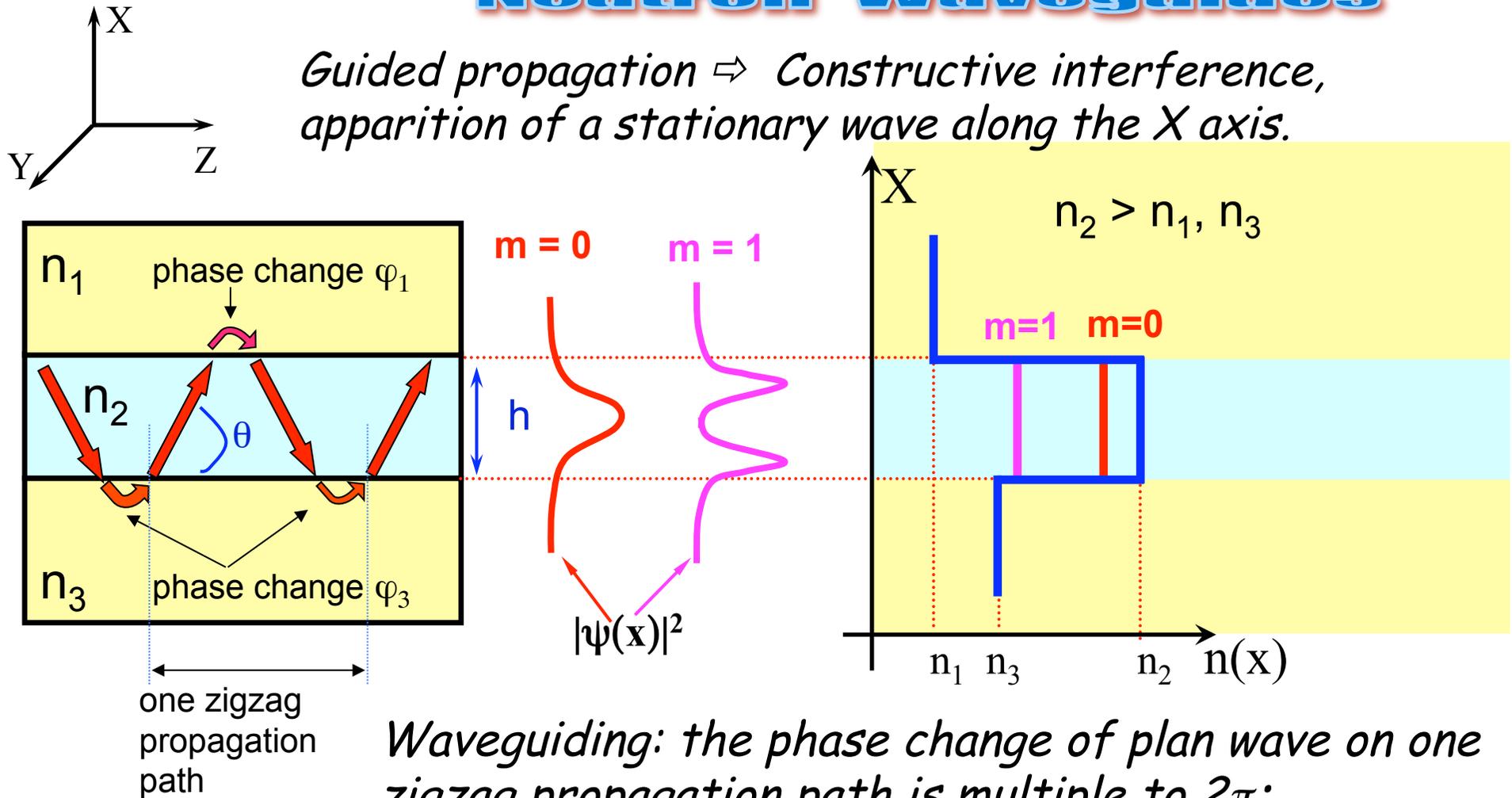
When a film with high refractive index is sandwiched between two media with lower refractive index ( $n_2 > n_1, n_3$ ), it is possible to guide neutrons in thin film with successive total reflections at each interface.



For discrete angles corresponding to the guided modes a stationary resonant wave is formed in the film by constructive interference.

# Neutron Waveguides

Guided propagation  $\Rightarrow$  Constructive interference, apparition of a stationary wave along the X axis.



Waveguiding: the phase change of plan wave on one zigzag propagation path is multiple to  $2\pi$ :

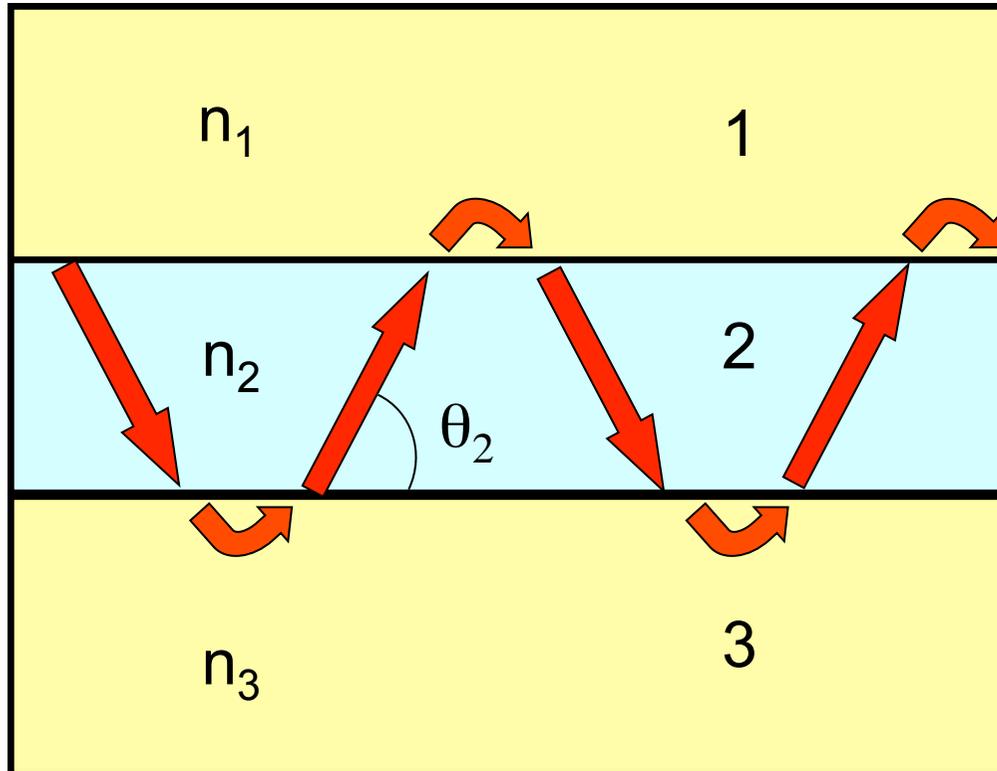
$$2hq_m + \varphi_1 + \varphi_3 = 2\pi m$$

$$q_m = kn_1 \sin\theta, \quad m = 0, 1, 2 \dots,$$

$$k = 2\pi/\lambda, \quad \lambda \text{ is the wavelength}$$

# Total reflections

$$n_2 > n_1, n_3$$



According Snell's Law

$$n_2 \cos \theta_2 = n_1 \cos \theta_1 = n_3 \cos \theta_3$$

For guided propagation

$$\theta_2 < \theta_{c23}, \theta_{c21}$$

$$\cos \theta_1 = \frac{n_2}{n_1} \cos \theta_2 > \frac{n_2}{n_1} \cos \theta_{c21} = 1$$

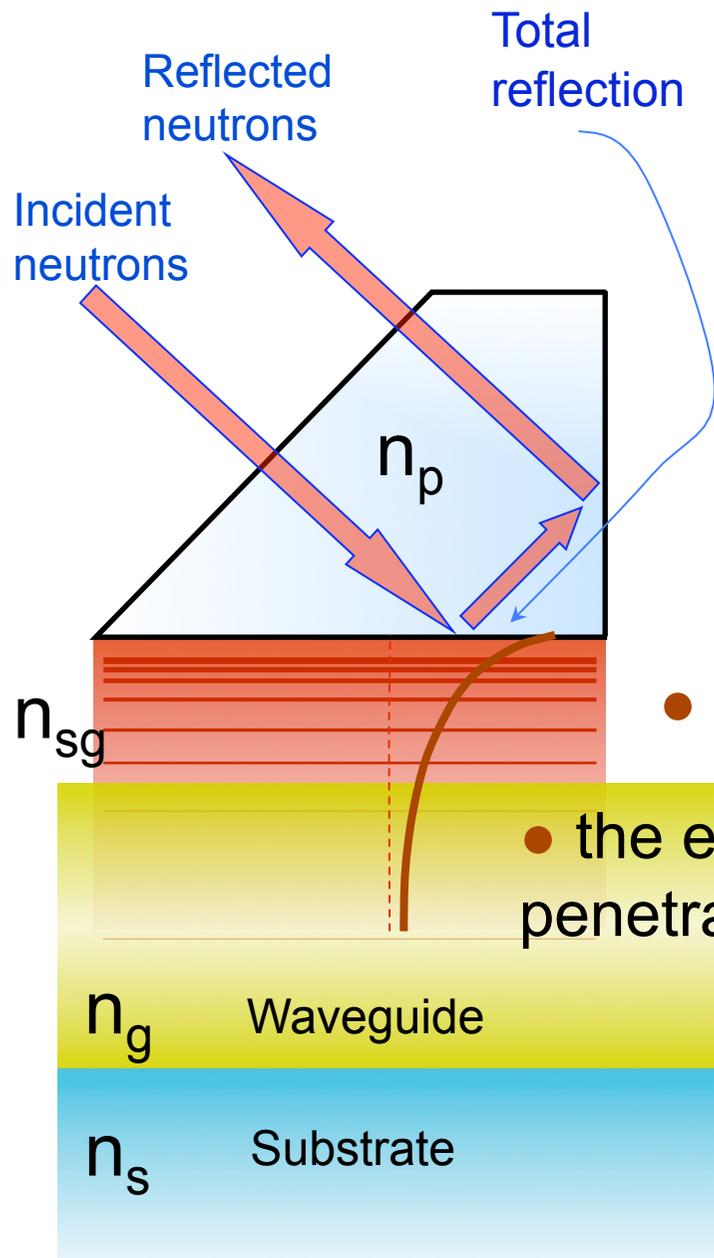
$$\cos \theta_3 = \frac{n_2}{n_3} \cos \theta_2 > \frac{n_2}{n_3} \cos \theta_{c23} = 1$$

**Conclusion:**  $\theta_1$  and  $\theta_3$  are not real angles.

*How to excite the Guided Modes*



# Prism coupler



Beyond some critical incident angles neutrons undergo total internal reflection on the base of the prism if :

$$n_s, n_{sg} < n_g \leq n_p$$

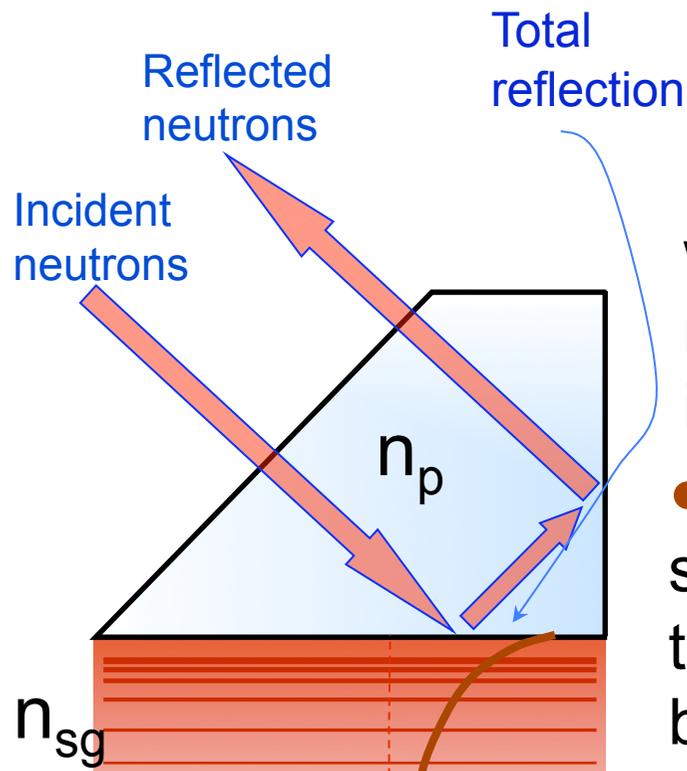
- evanescent wave in the separating gap

- the evanescent wave penetrates into the waveguide

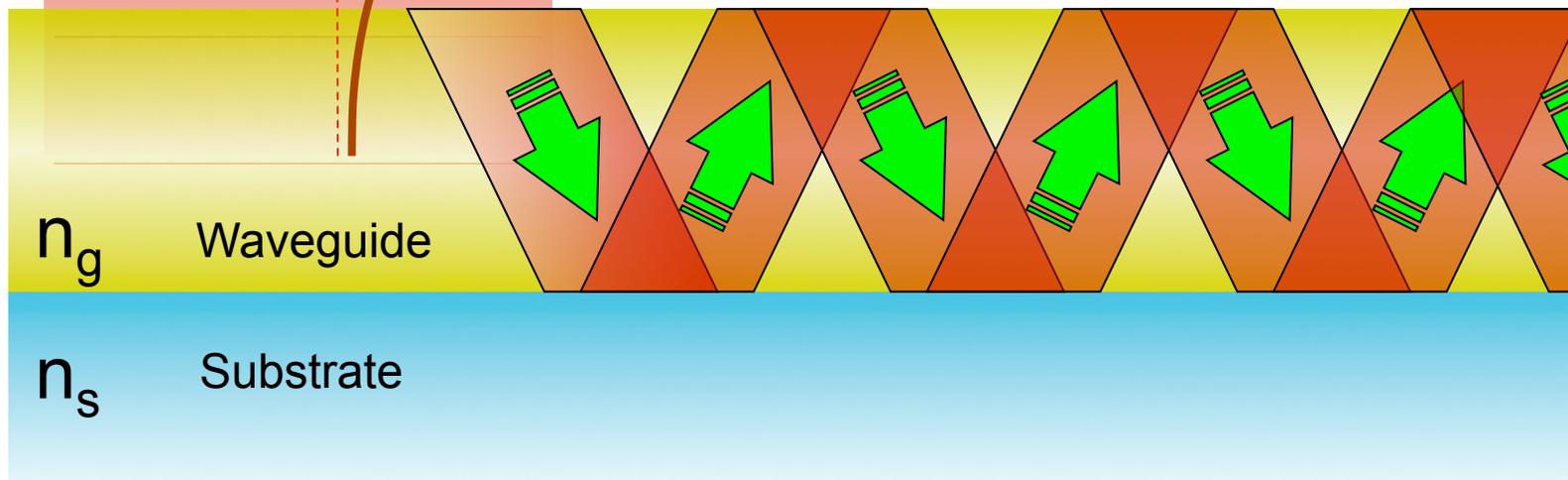
$n_g$  Waveguide

$n_s$  Substrate

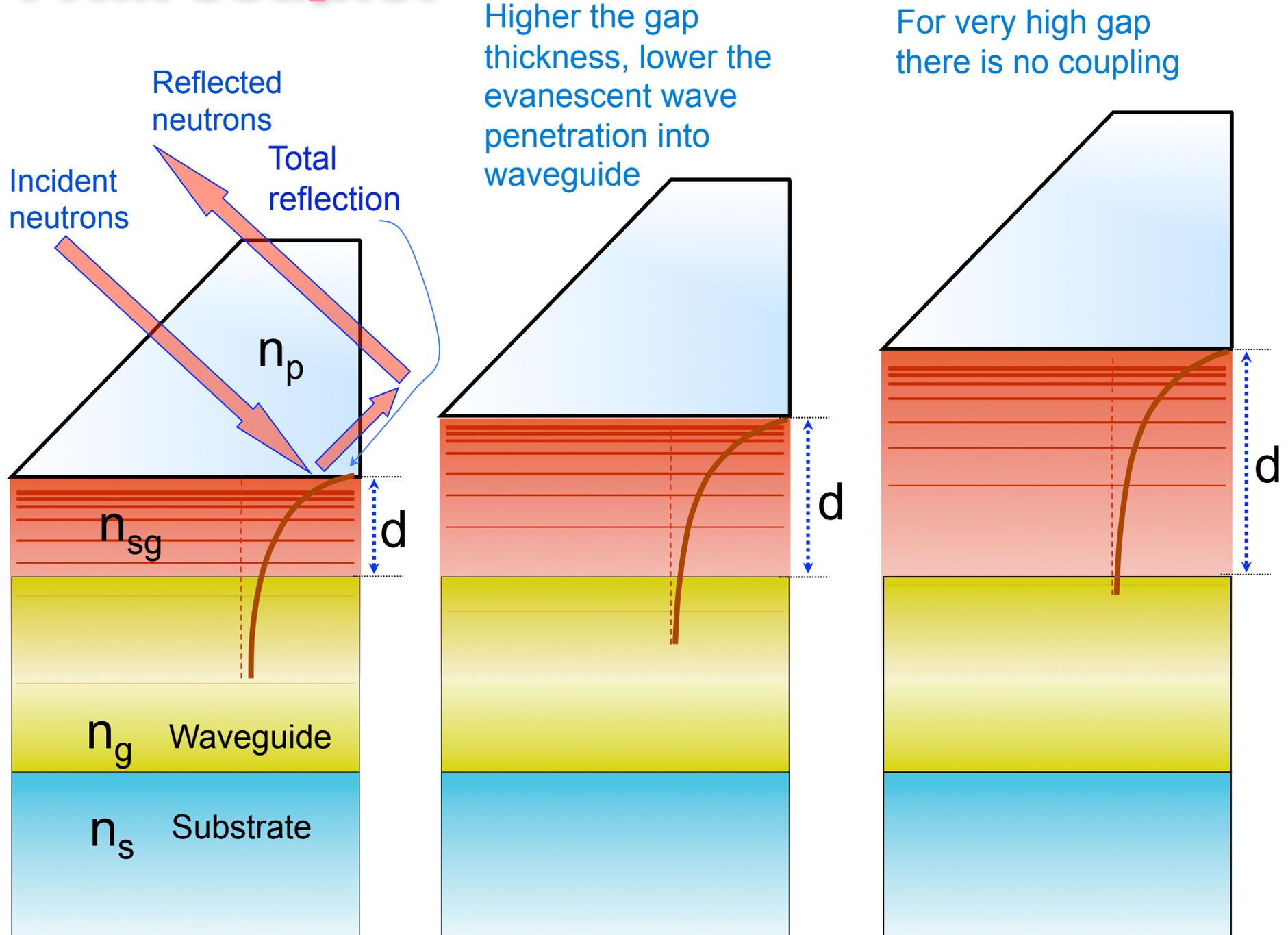
# Prism coupler



- If the z component of the neutron wavevector ( $\beta$ ) matches that of a guided mode then a resonant flow of neutrons into the waveguide occurs.
- In the film these waves are trapped for some resonant angles corresponding to the guided modes and a stationary wave is build up by constructive interference.



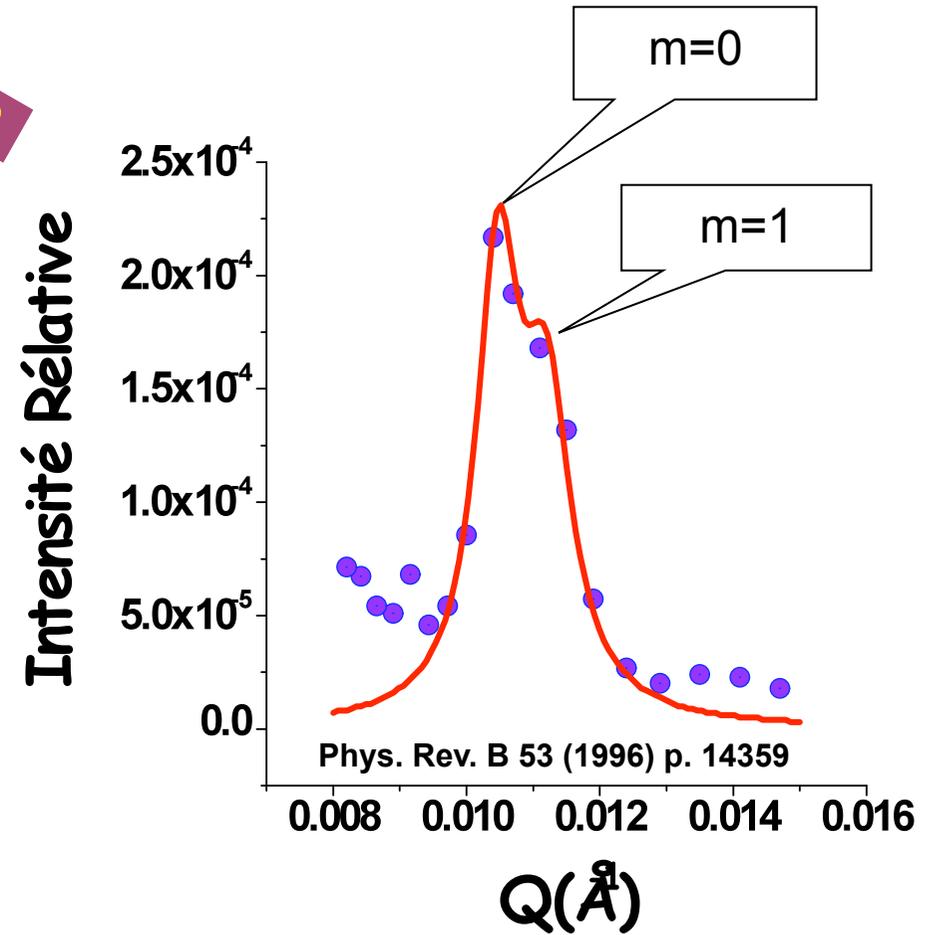
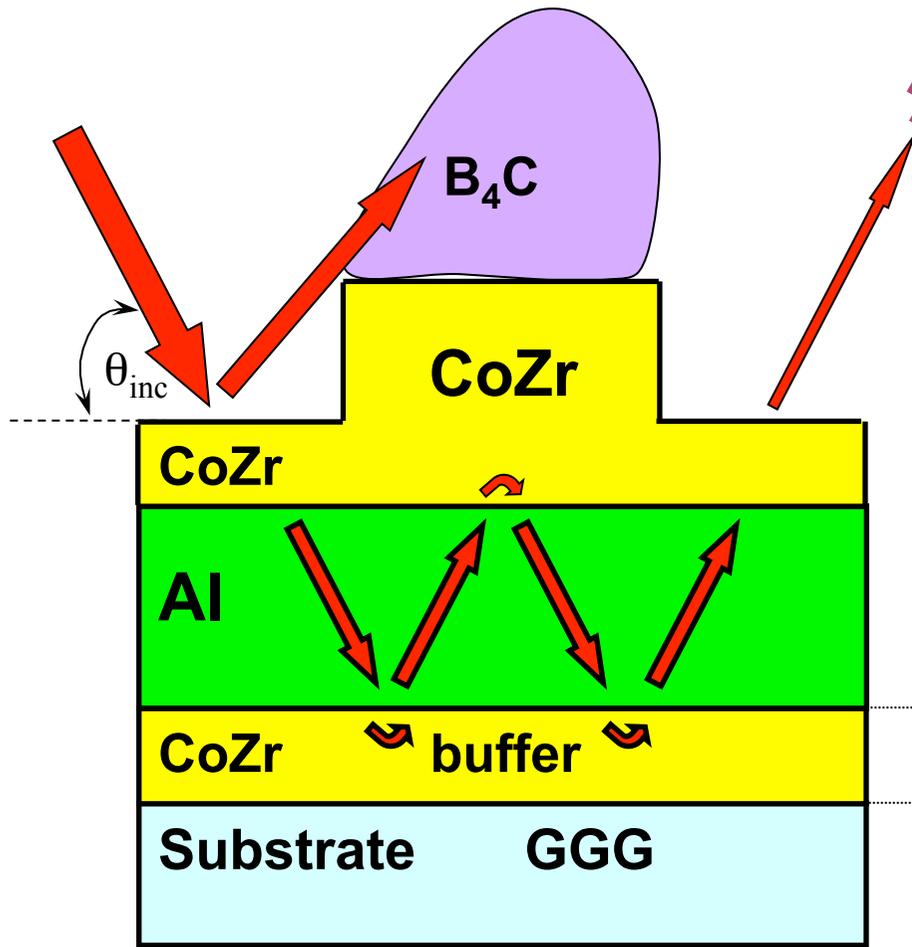
# Prim coupler



Higher the gap thickness, lower the evanescent wave penetration into waveguide

For very high gap there is no coupling

# Experimental demonstration of guided propagation



Scattering vector  $Q = 4\pi \times \sin(\theta_{inc})/\lambda$

Y.P. Feng et al, Phys. Rev. B 49 (1994) p. 10814 (non magnetic).

Our group in collaboration with A. Menelle from LLB de CEA-CNRS (Saclay, France),

S.P. Pogossian et al, Phys. Rev. B 53 (1996) p. 14359 , Phys. Rev. B 56 (1997) p. 4971 , J. Appl. Phys. 81, (1997). p. 4281,

J. Appl. Phys. 83 (1998) p. 1159.

# Neutron Concentration in thin films

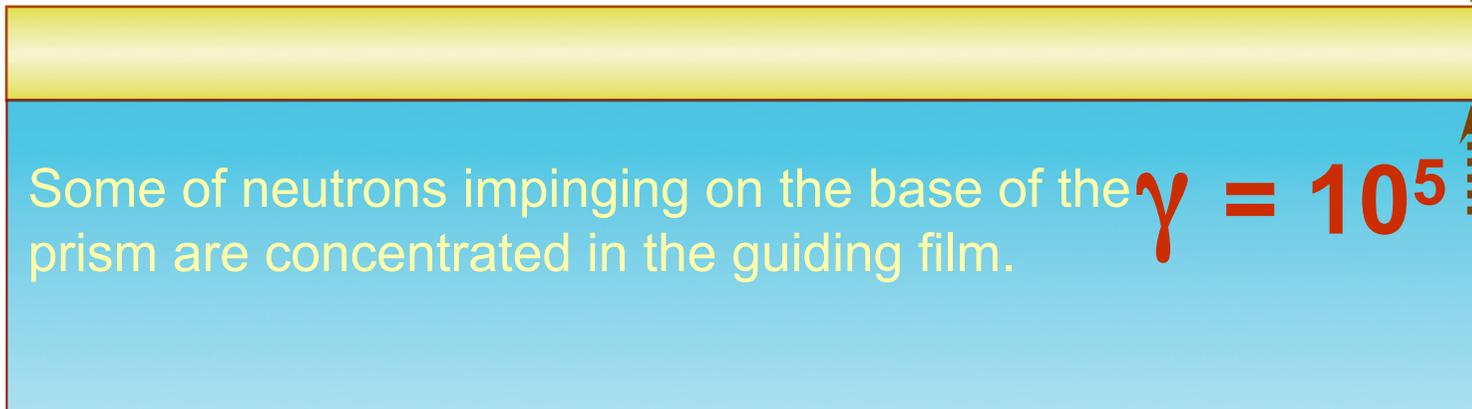
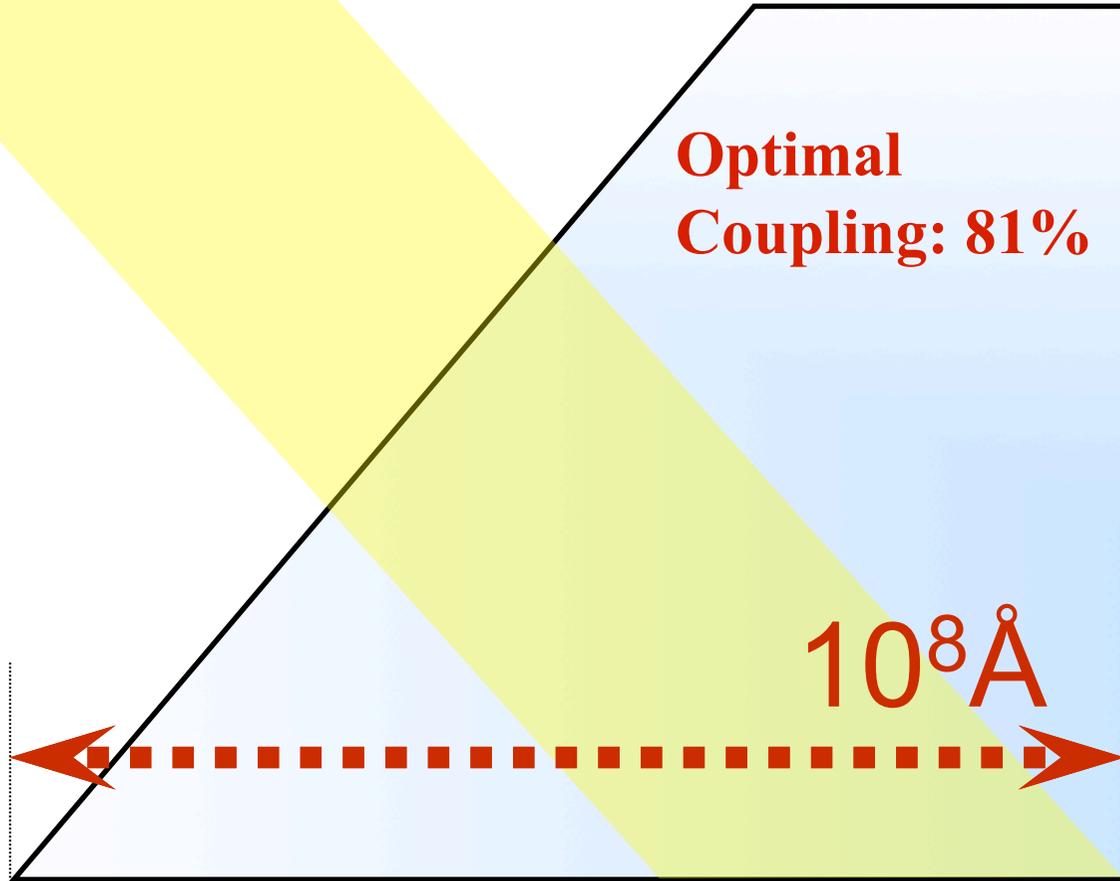
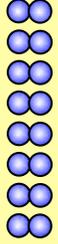
Optimal  
Coupling: 81%

$$\gamma = \frac{\text{---}}{\text{---}}$$

$10^8 \text{ \AA}$

$10^3 \text{ \AA}$

Some of neutrons impinging on the base of the prism are concentrated in the guiding film.  $\gamma = 10^5$

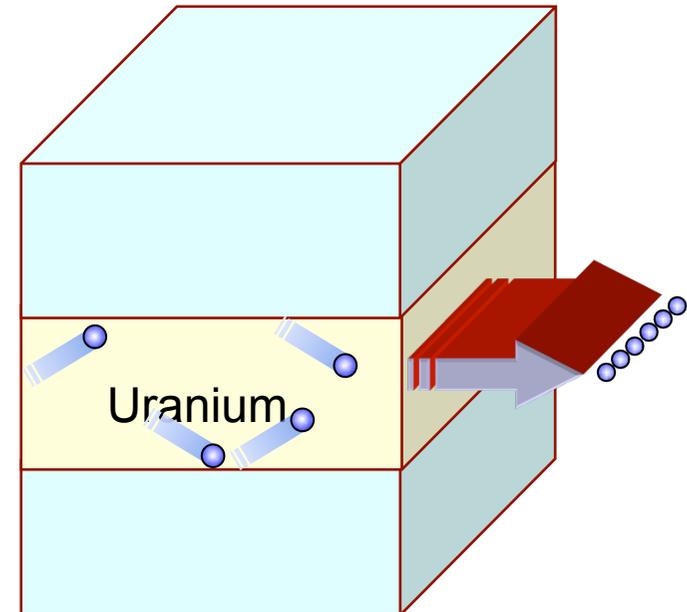
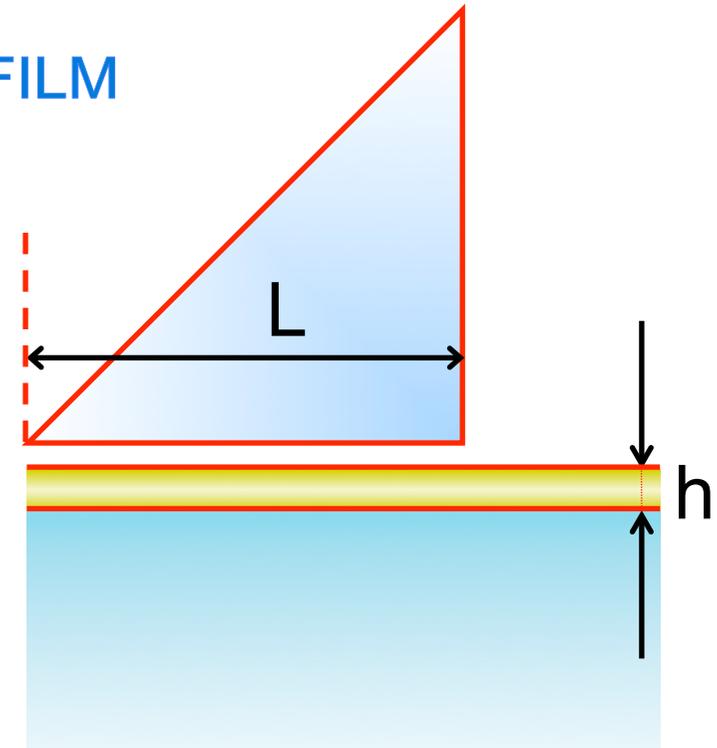


## NEUTRON CONCENTRATION IN U FILM

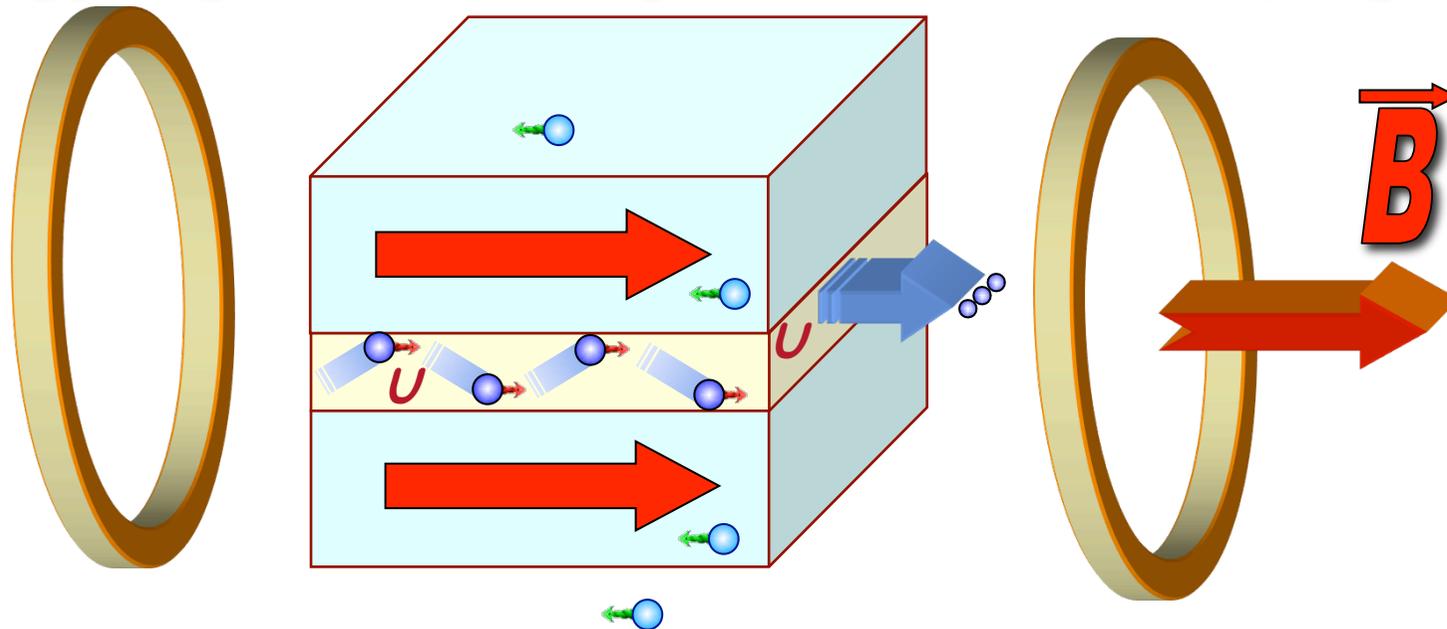
If all incident neutrons on a large surface of the prism coupler ( $\sim\text{cm}$ ) are coupled into the thin guiding film of thickness ( $\sim 10^{-5}\text{cm}$ ). **There should be a strong increase in neutron concentration of about  $\gamma \sim 10^5$ .**

$$\gamma = \frac{L}{h} \approx 10^5$$

*If we use Uranium (235) thin film as a guide, the high concentration of incident additional neutrons will enhance the fission rate of Uranium nuclei. The evolution of the fission will depend on the neutron number in the thin film.*



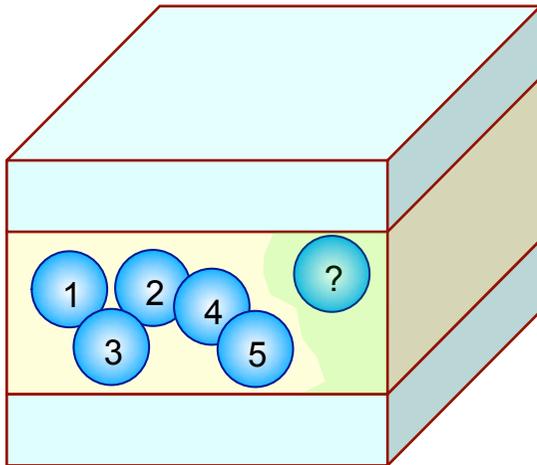
# ENHANCED NEUTRON CONCENTRATION IN U WAVEGUIDES



*If a U film is sandwiched between two ferromagnetic media the neutron number is dependent on the orientation of the neutron magnetic moment with respect to the magnetization vector of the ferromagnetic films.*

*The number of neutrons in U film can be controlled by external magnetic fields, which presents obvious advantages for applications in controlled fission.*

# Optimization of waveguide parameters



**Question:** How to enhance the number of neutrons in a thin film waveguide ?

**Answer:** The concentration of neutrons in U guiding film can be increased by reducing the film thickness.

**Comment:** The smaller the guiding film thickness, the smaller the number of guided modes supported by the waveguide.

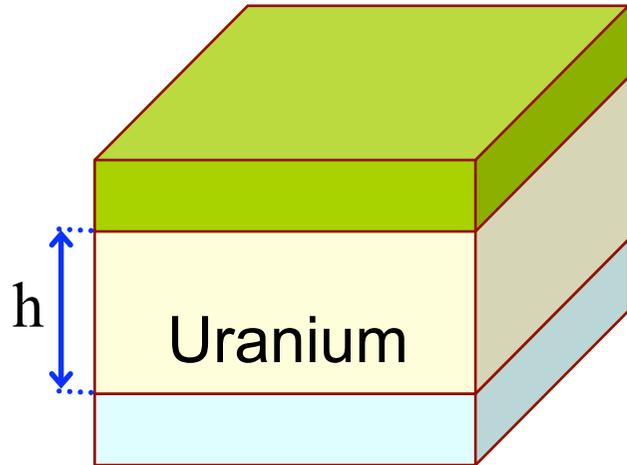
**Conclusion:** The highest neutron concentration is achieved for waveguides supporting only one mode.

**Recommendation :**

**USE SINGLE MODE WAVEGUIDES !**

# Optimization of waveguide parameters

4% enriched U thin film (EU4) is used as an example of a guiding layer



Refractive index :  $n=1- \delta\lambda^2 + i \xi\lambda$

with  $\delta=6.51\times 10^{-7} \text{ \AA}^{-2}$  and  $\xi=1.14\times 10^{-9} \text{ \AA}^{-1}$ .

$\lambda$  is the wavelength of neutron de Broglie waves

S.P. Pogossian, J. Appl. Phys. **102**, (2007) 104501.

$n_m = \beta_m/k = 1 - \lambda^2 a_m$  : effective refractive indices of guided modes

$$2\pi h \sqrt{2(a_m - \delta_2)} - \tan^{-1} \left\{ \frac{\sqrt{2(\delta_1 - a_m)}}{\sqrt{2(a_m - \delta_2)}} \right\} - \tan^{-1} \left\{ \frac{\sqrt{2(\delta_3 - a_m)}}{\sqrt{2(a_m - \delta_2)}} \right\} = \pi m$$

dispersion equation

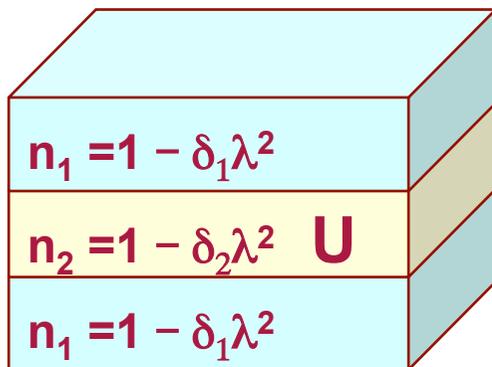
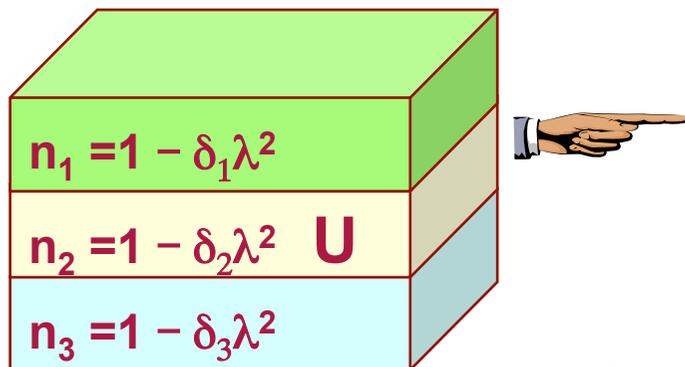
When  $\delta_2 < a_m < \delta_1, \delta_3$

S.P. Pogossian and H. Le Gall, Opt. Comm. **114**, (1995), p. 235 .

# ENHANCED NEUTRON CONCENTRATION IN U WAVEGUIDES

**Thus :** The concentration of neutrons in U guiding film can be increased by reducing the film thickness.

**Objection :** For asymmetrical waveguides ( $\delta_1 \neq \delta_3$ ) there is a limit to it.



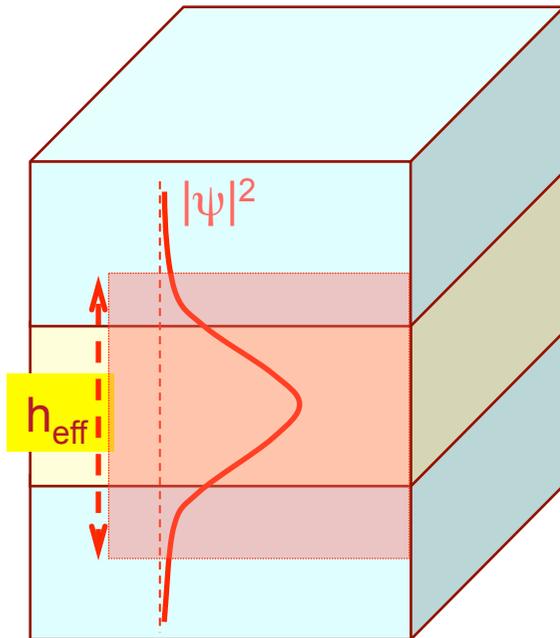
**Solution :**

- ① For a symmetrical guide ( $\delta_1 = \delta_3$ ) there is no cutoff thickness :  $h_{\text{cutoff}} = 0$ .
- ② There is always guided propagation in such a waveguide no matter how thin the guiding film is.

$h_{\text{cutoff}}$

# ENHANCED NEUTRON CONCENTRATION IN U WAVEGUIDES

**Objection :** the neutron concentration in the guiding film cannot be increased infinitely by reducing the waveguide thickness.



For very thin guiding films, the guided waves penetrate deeper into the neighboring regions via their evanescent tail. Guided neutrons are mainly confined to an *effective thickness  $h_{eff}$* :

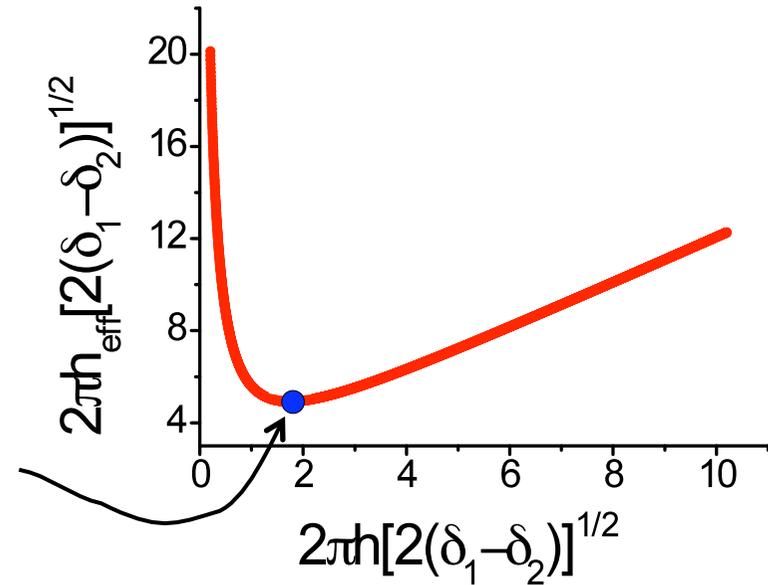


$h_{eff}$

# Neutron guiding in U Thin Films

**Conclusion :** Even though in a symmetrical waveguide the guiding film can be infinitesimally thin, the neutron concentration can not be increased infinitely.

*There is an optimal thickness  $h_{opt}$  which gives the highest neutron concentration.*



$$h_{opt} = 0.277 / \sqrt{2(\delta_1 - \delta_2)}$$

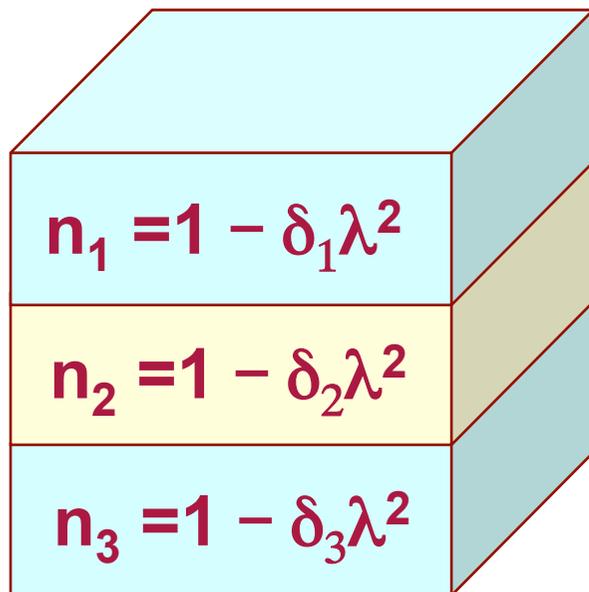
$$a_m = \delta_2 + 0.607 \times (\delta_1 - \delta_2)$$

# NEUTRON CONCENTRATION IN U WAVEGUIDES

**Comment :** optimal waveguide thickness  $h_{opt}=0.277 \times [2(\delta_1 - \delta_2)]^{1/2}$  depends only on the scattering length densities of the guiding film and the neighboring media, and not on the incident neutron wavelength.

**Notice:** *for any neutron wavelength the optimal thickness is the same.*

## Necessary condition for waveguiding



$$n_2 > n_1, n_3$$

$$\delta_2 \text{ (thin film)} < a_m < \delta_1, \delta_3 \text{ (adjacent media)}$$

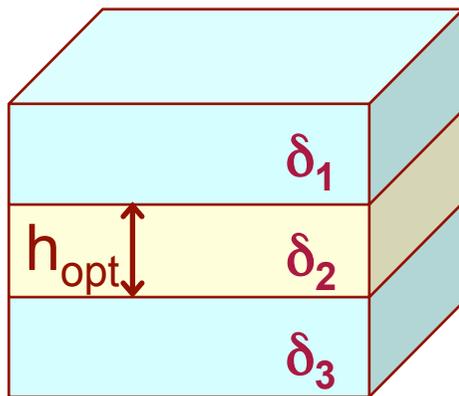


$$\delta_2 \text{ (thin film)} < \delta_1, \delta_3 \text{ (adjacent media)}$$

# NEUTRON CONCENTRATION IN U WAVEGUIDES

**Comment :** in ferromagnetic materials ( Co, Ni, Fe ...) the propagation is spin dependent.

**Conclusion :**  $h_{opt}$  is also spin dependent



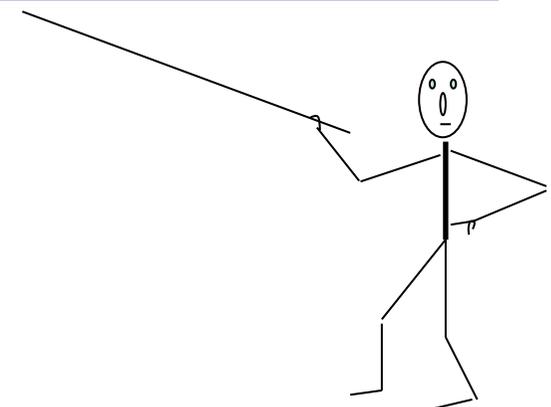
It may happen that in a ferromagnetic material for one polarization (-) there is no guidance ( $\delta_- < \delta_2$ )

while for other polarization (+) the guidance is allowed ( $\delta_+ > \delta_2$ ).

Each polarization has its own optimal thickness. Here  $h_{opt}$  (in Å ) is given for both polarizations when a U film is surrounded by ferromagnetic media of Co, Ni, Fe.

	(+)	(-)
Ni	189	251
Fe	164	-
Co	319	-

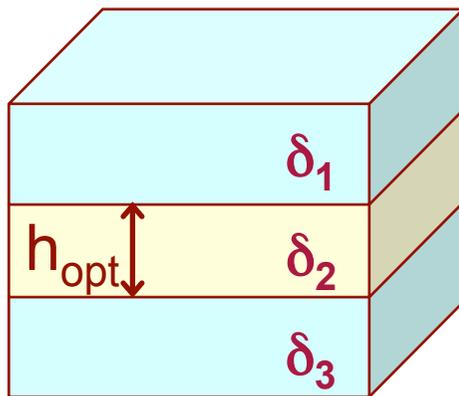
The best choice is : Ni



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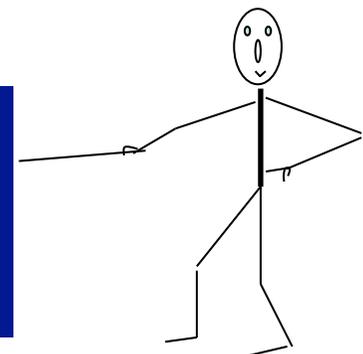
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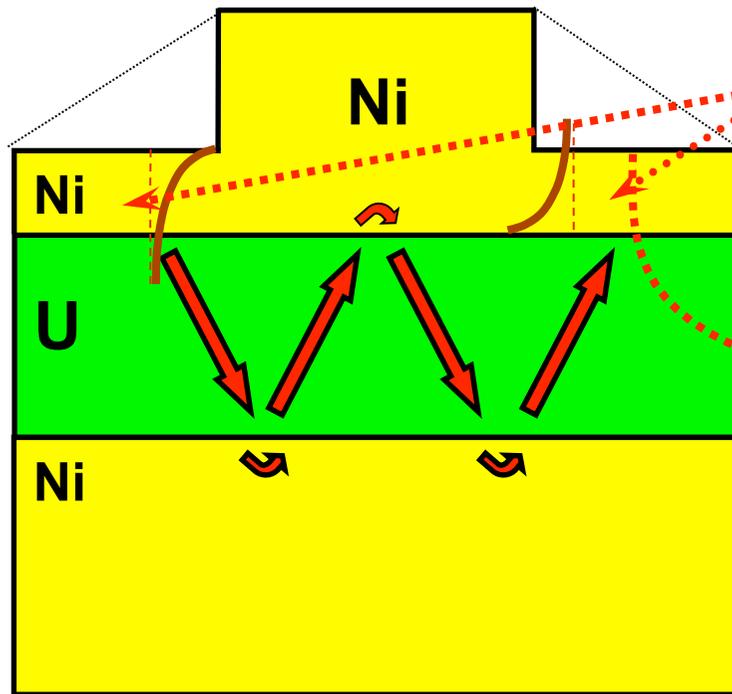
	(+)	(-)
Ni	189	251
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The best choice is : Ni

Both polarizations can be guided



# Waveguiding in the prism region



**Observation :** In the prism region the guided waves will leak partially out of the U film through the Ni gap.

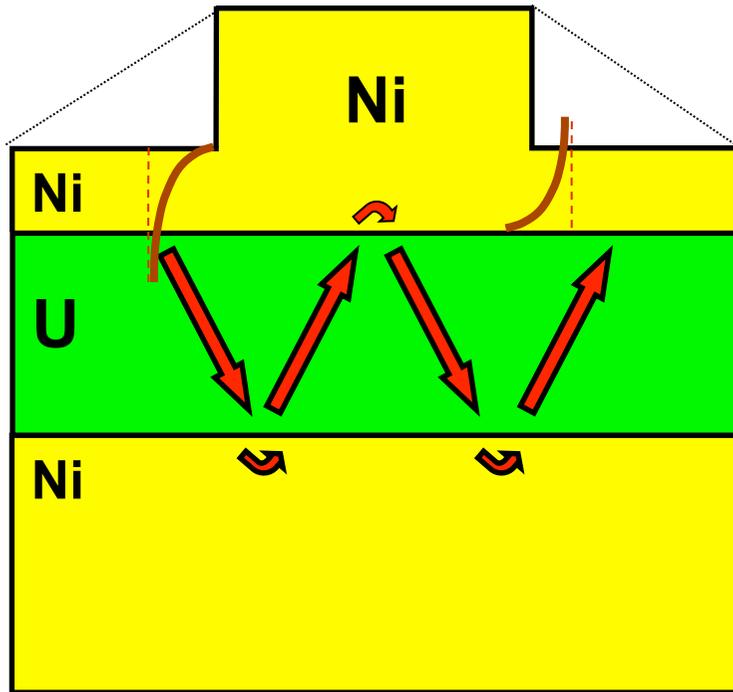
**Result:** alteration of the resonance condition :

$$q_m h - \operatorname{atan}\left(\frac{p_m}{q_m}\right) - \operatorname{atan}(\phi_m) = \pi m$$

$$\phi_m = \frac{p_m}{q_m} \left( \frac{2(1 + \gamma) \times \operatorname{th}(p_m d)}{(\gamma - \tau) + (1 - \gamma\tau) \operatorname{th}^2(p_m d) + \sqrt{[(\gamma - \tau) + (1 - \gamma\tau) \operatorname{th}^2(p_m d)]^2 + 4\tau(1 + \gamma)^2 \operatorname{th}^2(p_m d)}} \right)$$

$$\tau = \left(\frac{p_m}{q_m}\right)^2 \quad \text{and} \quad \gamma = \left(\frac{p_m}{Q_m}\right)^2$$

# Waveguiding in the prism region



Question :

Does the leakage influence the neutron concentration in the waveguide ?

Answer : Yes.

The coupling efficiency should be taken into account. The waveguide thickness optimization should be carried out simultaneously with maximization of neutron coupling efficiency into the guiding film.

# Most efficient coupling condition

It has been established that for the most efficient coupling the following condition should be satisfied [\*]:

$$|T|^2 = \frac{2.513 \times L_{GH}}{L}$$

$|T|^2$  is the transmission coefficient of the Ni gap

$$|T|^2 = \frac{4Q_m q_m}{(q_m + Q_m)^2 + (Q_m^2 + p_m^2)(q_m + p_m^2) \frac{\text{sh}^2(p_m d)}{p_m^2}}$$

$$\begin{cases} q_m = \sqrt{k^2 n_2^2 - \beta_m^2} = 2\pi \sqrt{2(a_m - \delta_2)} \\ p_m = \sqrt{\beta_m^2 - k^2 n_1^2} = 2\pi \sqrt{2(\delta_1 - a_m)} \\ Q_m = \sqrt{k^2 n_p^2 - \beta_m^2} = 2\pi \sqrt{2a_m} \end{cases}$$

\*P.K. Tien, R. Ulrich, J. Opt. Soc. Am. **60** (1970) p.1325.

# Most efficient coupling condition

$$|T|^2 = \frac{2.513 \times L_{GH}}{L}$$

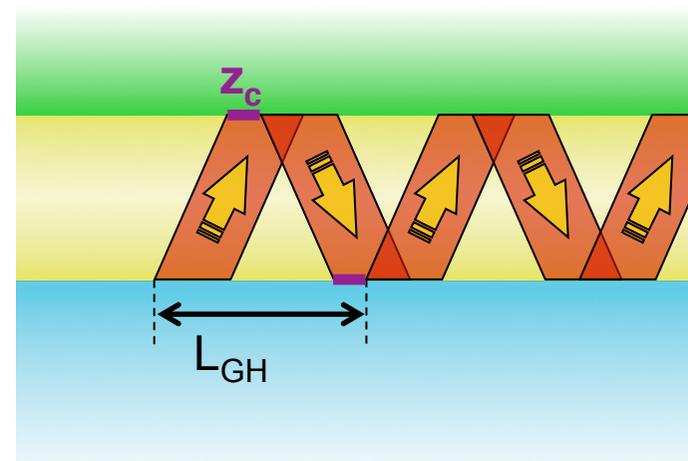
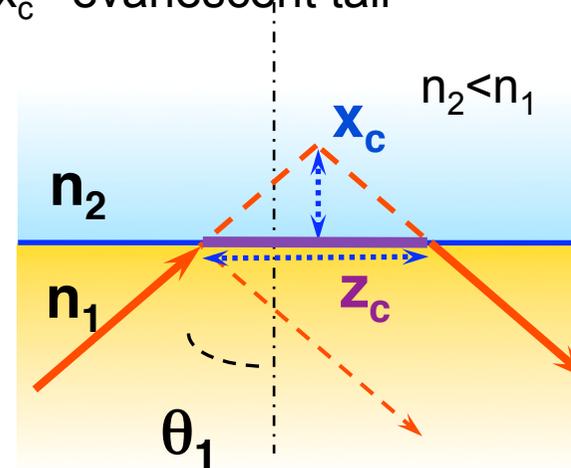
$L_{GH}$  - one zigzag path (taking into account the Goose-Hänchen lateral shift).

In a symmetrical waveguide  $L_{GH}$  is :

$$L_{GH} = \frac{2\beta_m}{q_m} \left( h + \frac{2}{p_m} \right)$$

$L$  - coupling prism length.

$z_c$  - Goose Hänchen lateral shift  
 $x_c$  - evanescent tail



# Optimization Results

**For (-) polarization**

- The mode resonance condition
- The condition for most efficient coupling :

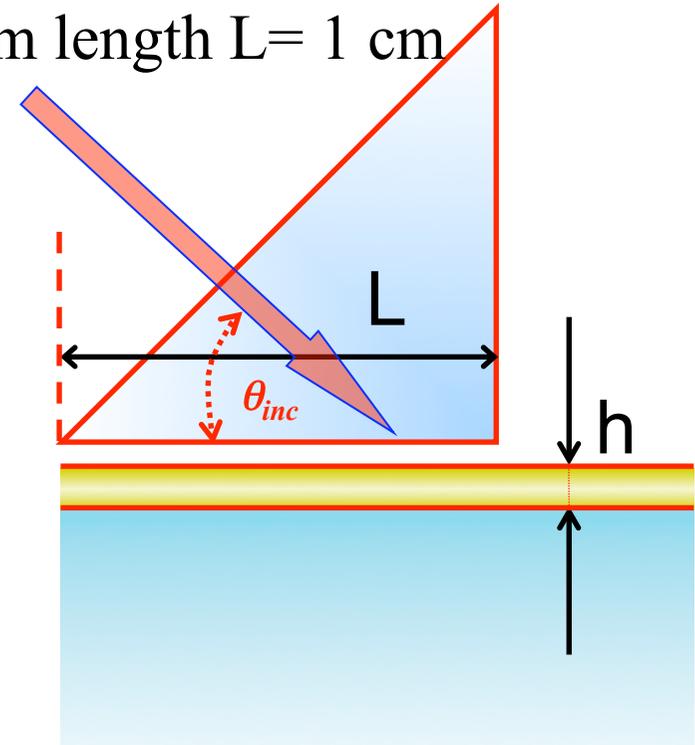
$$h = 244 \text{ \AA} \text{ and}$$
$$d = 653 \text{ \AA}$$

$\frac{I_w}{I_{inc}}$  – the ratio of neutron intensity in the waveguide and in the air

For optimal coupling conditions :  
 $\eta = 0.81$ .  
The enhancement factor is:

$$\frac{I_w}{I_{inc}} = \eta \frac{L}{L_{GH}} \approx 181$$

prism length  $L = 1 \text{ cm}$



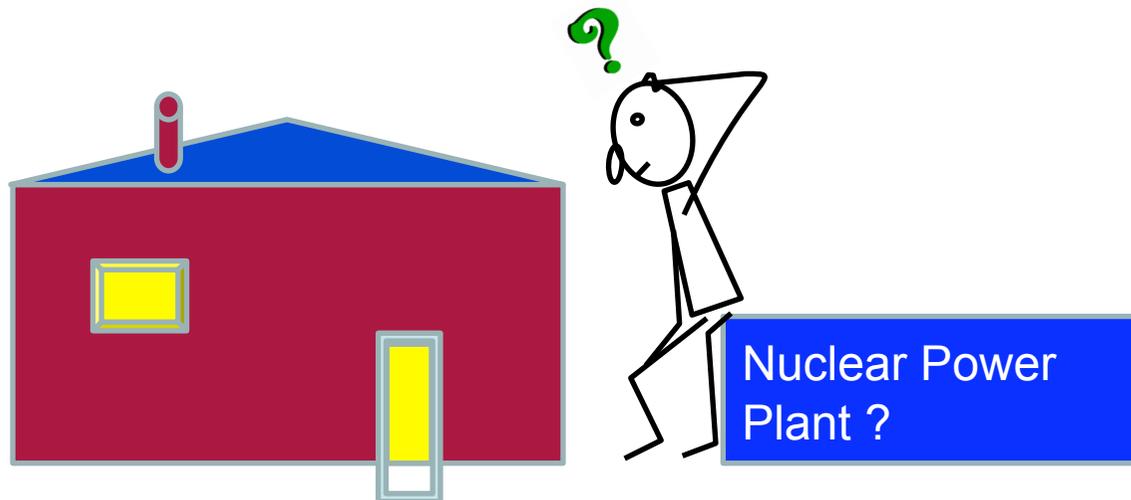
# NEUTRON ENHANCED CONCENTRATION IN U WAVEGUIDES

## Conclusion :

- Optimization of thin film waveguide parameters for enhancement of neutron concentration in 4% enriched U film.
- The U film thickness is : 244 Å
- U film is sandwiched between 3000Å Ni films.
- For optimum coupling the thickness of Ni gap in the coupling region is 653 Å.

## Perspective ?

- miniaturization of nuclear energy production ?



Thank you for your attention

Grazie per la vostra attenzione

Спасибо за ваше внимание

Merci, pour votre attention!

Channeling 2008

