Channeling 2008
Erice, Italy

Diffracted Channeling Radiation and Other Compound Radiation Processes

Hideo Nitta

Department of Physics

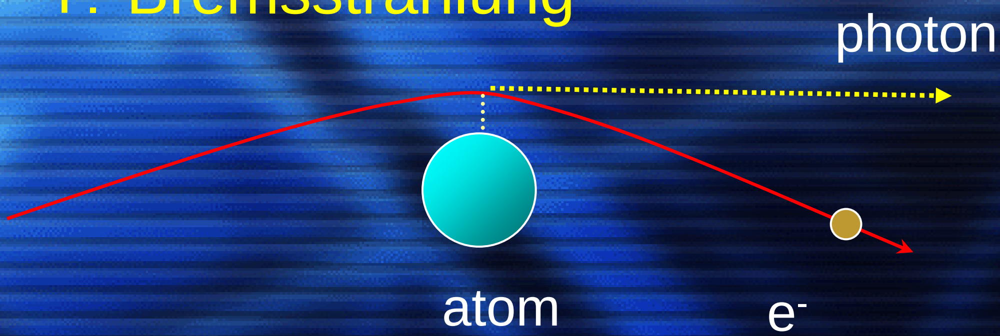
Tokyo Gakugei University

Contents

- Diffracted Channeling Radiation (DCR)
- Diffracted Coherent Bremsstrahlung (DCB)
- CPR (PXR) by Neutral Particles

Fundamental processes of radiation

1. Bremsstrahlung



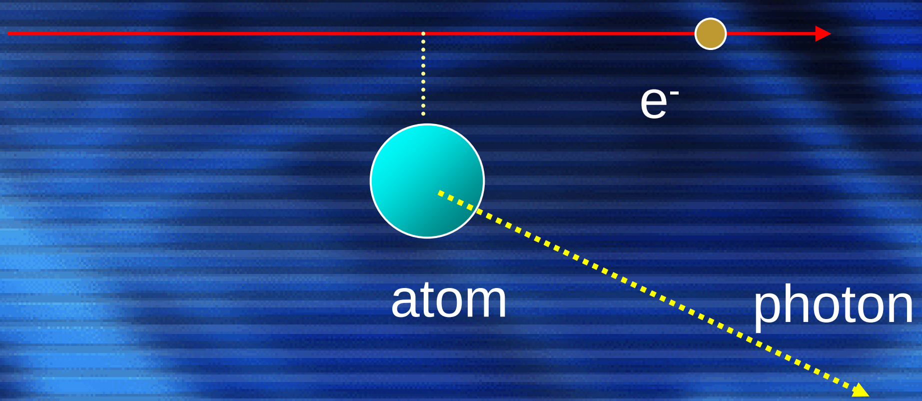
Bethe-Heitler

in crystals:

coherent bremsstrahlung (CB)

channeling radiation (CR)

2. Polarization radiation



in crystals:

“coherent polarization radiation” (CPR)

others:

transition rad., SP rad., Cherenkov rad.

Amusia Phys.Rep. **162**, 249 (1988)

Interference condition for CB and CPR

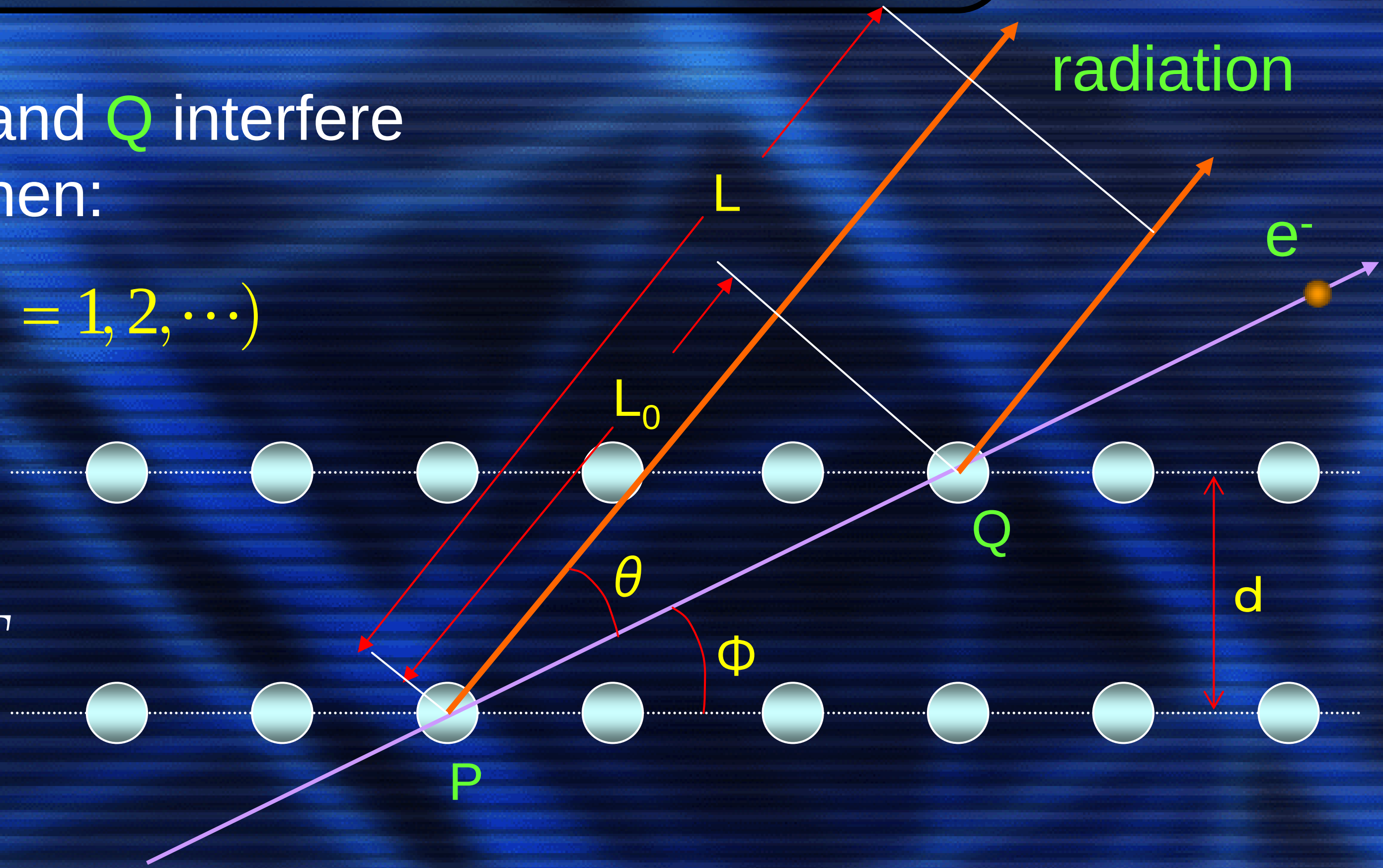
photons from P and Q interfere constructively when:

$$L - L_0 = n\lambda, \ (n = 1, 2, \cdots)$$

$$PQ = \frac{d}{\sin\phi}$$

$$PQ = vT, \ L = cT$$

$$L_0 = PQ\cos\theta$$



$$\frac{c}{v}(1 - \beta\cos\theta)\frac{d}{\sin\phi} = n\lambda \quad \text{or} \quad \omega = \frac{|\mathbf{g}\cdot\mathbf{v}|}{1 - \beta\cos\theta}$$

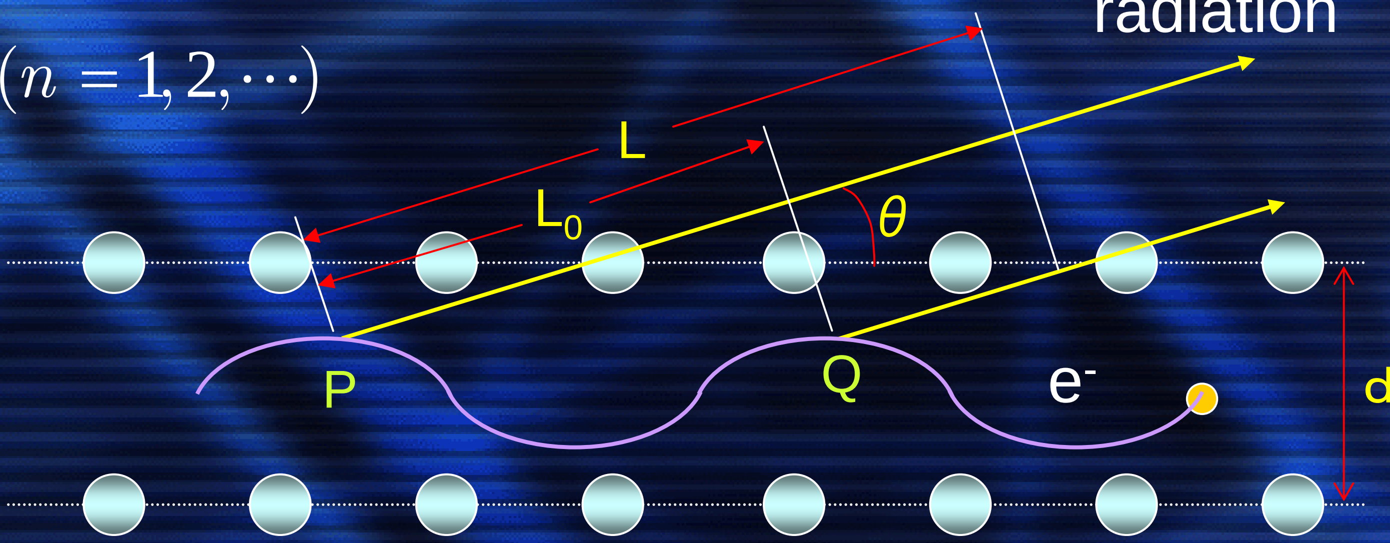
"dispersion relation" of both CB and PXR

Interference condition for CR

photons from P and Q interfere constructively when:

(actually $\theta < \gamma^{-1}$)

$$L - L_0 = n\lambda, \ (n = 1, 2, \cdots)$$



$$T = \frac{2\pi}{\Omega}$$

$$PQ = vT, \ L = cT$$

$$L_0 = PQ \cos\theta$$

$$\omega = \frac{n\Omega}{1 - \beta\cos\theta}$$

for θ =0:

$$\omega = \frac{n\Omega}{1-\beta} \approx 2\gamma^2 n\Omega$$

Diffracted Channeling Radiation (DCR)

Baryshevsky and Dubovskaya, J.Phys.C **16**, 3663 (1983).
General theory

Ikeda, Matsuda, Nitta, Ohtsuki, NIM **B115**, 380 (1996)
Kinematical theory

Yabuki, Nitta, Ikeda, Ohtsuki, Phys.Rev.B 63, 174112 (2001).
Dynamical theory

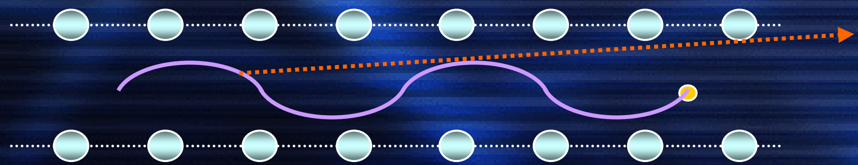
Korotchenko , Pivovarov, Tukhfatullin, NIM B (to appear).
Detailed dynamical calculations for LiF crystal.

(and presentations given by above authors and Fiks in this conference.)

DCR as intense monochromatic x-ray source

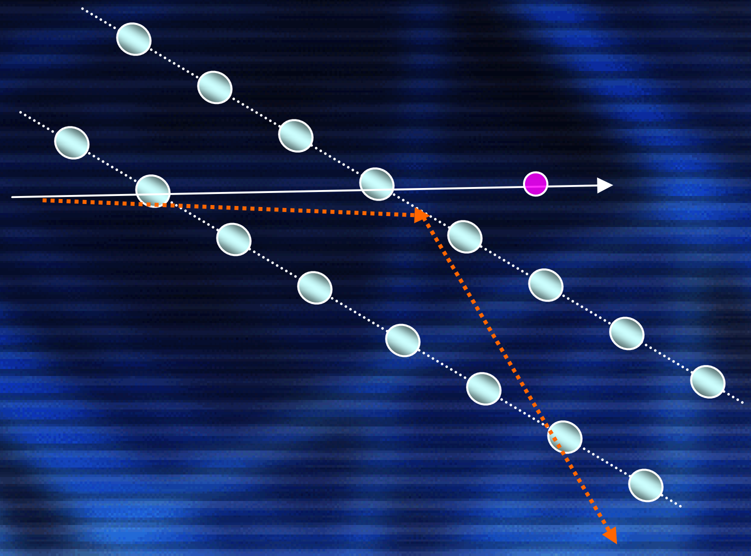
channeling radiation:

~0.1photons/e$^-$
large background
forward radiation



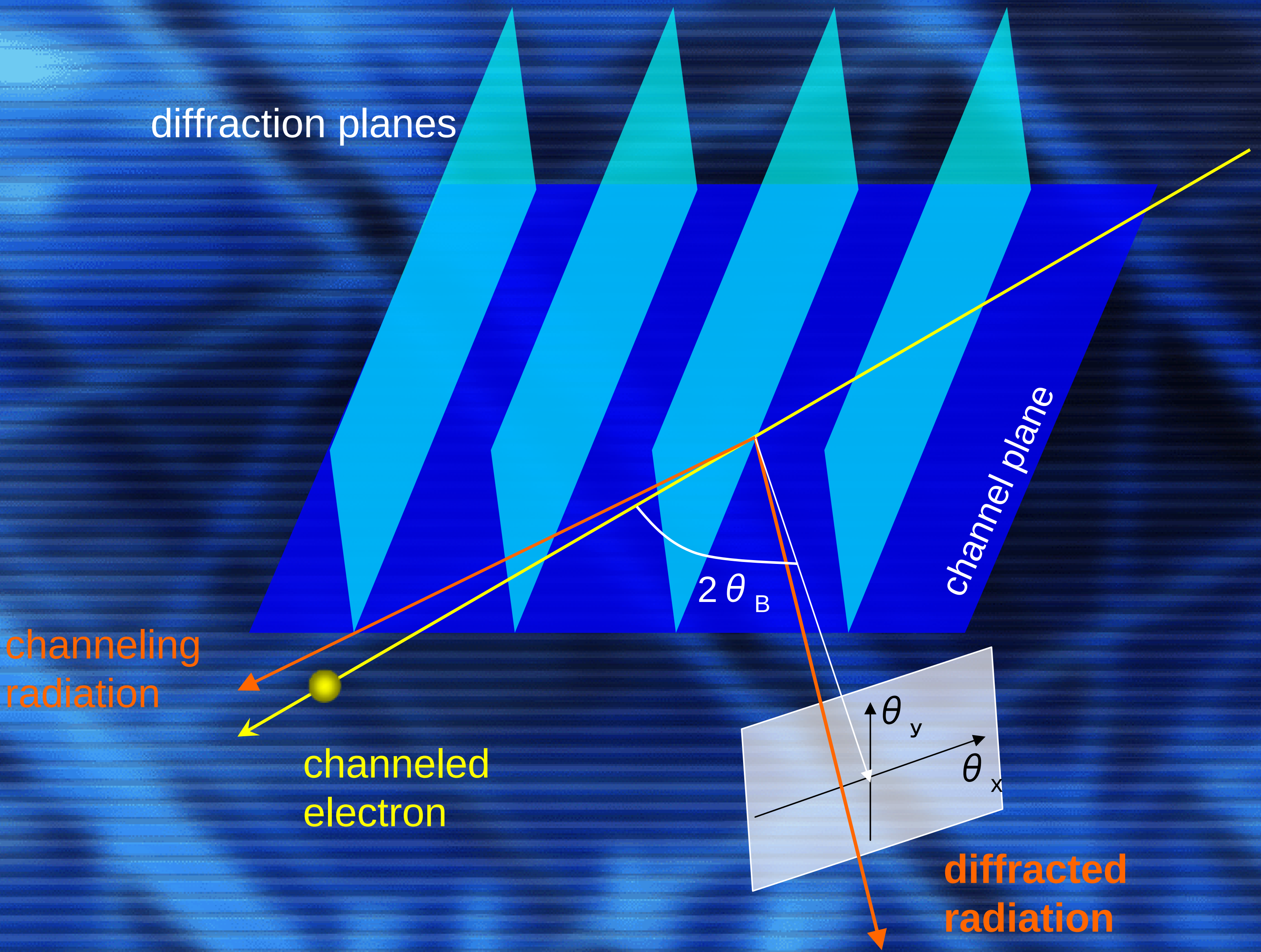
parametric x-ray radiation:

~10^{-5}photons/e$^-$
negligible background
emitted into a large angle



diffraction of channeling radiation:
(use of channeling crystal itself as a monochromater)

Diffracted Channeling Radiation (DCR)



Frequency of DCR

Energy-momentum conservation:

$$E_i - E_f = \hbar\omega$$

$$\mathbf{p}_{\|} - \mathbf{p}_{\|} = \hbar(\mathbf{k}_{\|} + \mathbf{g}_{\|})$$

initial and final energy of the particle

$$E_{i,f} \approx E(\mathbf{p}_{\|}) + E_{\perp i,,f} \qquad E(\mathbf{p}_{\|}) = \sqrt{(c\mathbf{p}_{\|})^2 + m^2c^4}$$

$$\omega = \frac{\mathbf{g}\cdot\mathbf{v}_{\|} + \Omega_{if}}{1 - \beta_{\|}^{*}\cos\Theta}$$

$$\beta_{\|}^{*} = v_{\|}/c^{*}$$

$$c^{*} = c/\sqrt{\varepsilon_0}$$

$$\Omega_{if} = (E_{\perp,i} - E_{\perp,f})/\hbar$$

Diffracted Channeling Radiation (DCR)

$$\omega = \frac{\mathbf{g}\cdot\mathbf{v}_{\|} + \Omega_{if}}{1-\beta_{\|}^{*}\cos\Theta}$$

$\Omega_{if} = 0$

$$\omega = \frac{\mathbf{g}\cdot\mathbf{v}_{\|}}{1-\beta_{\|}^{*}\cos\Theta}$$

PXR

$\mathbf{g} = 0$

$$\omega = \frac{\Omega_{if}}{1-\beta_{\|}^{*}\cos\Theta} \rightarrow \frac{2\Omega_{if}}{\gamma^{-2} + |\chi_0(\omega)|}$$

$$(\Theta \rightarrow 0)$$

CR

Radiation process and matrix elements

Fermi's golden rule:

$$w_{IF} = \frac{2\pi}{\hbar} |< F | H_{int} | I >|^2 \rho_F$$

The interaction Hamiltonian:

$$H_{int} = -\frac{e}{\gamma mc} \mathbf{A} \cdot \hat{\mathbf{p}}$$

Bloch wave "photon":

$$\mathbf{A}(\mathbf{r}) = \sum_{\mathbf{k}} \sum_{\mathbf{g}} \mathbf{A}_{\mathbf{g}} \exp[i(\mathbf{k}+\mathbf{g})\cdot\mathbf{r}] + c.c..$$

quantum states of channeled electron:

$$\psi^{(s)}(\mathbf{r}) = \frac{1}{\sqrt{L_x L_z}} \varphi_n(y) e^{i\mathbf{p}_{\parallel}\cdot\mathbf{r}_{\parallel}/\hbar}$$

$$\varphi_n(y) = \frac{1}{\sqrt{L_y}} \sum_G C_G^{(n)}(p_y) \exp[i(p_y/\hbar + G)y]$$

$$\left[\frac{\hat{\mathbf{p}}_{\perp}^2}{2\gamma m} + V(y)\right] \varphi_n(y) = E_{\perp,n} \varphi_n(y)$$

Diffracted Channeling Radiation (DCR)

DCR needs dynamical theory of diffraction

kinematical: $$\mathbf{A_g} \sim \frac{\chi_{\mathbf{g}}}{\mathbf{k}^2 - (\mathbf{k}+\mathbf{g})^2}\mathbf{A}_0$$

PXR: virtual photon diffraction (no divergence)

$\mathbf{k}, \omega$ $\quad$ $\mathbf{k+g}, \omega$

$$\omega = \mathbf{k}\cdot\mathbf{v} \qquad \omega = c^*\left|\mathbf{k}+\mathbf{g}\right|$$

DCR: (virtual) CR diffraction (divergence occurs)

$\mathbf{k}, \omega$ $\quad$ $\mathbf{k+g}, \omega$

$$\omega = c^*\left|\mathbf{k}\right| \qquad \omega = c^*\left|\mathbf{k}+\mathbf{g}\right|$$

Matrix elements

$$< F \,|\, H_{int} \,|\, I >=$$

$$-< \varphi_f , \mathbf{p}_{\|} \,|\, \frac{e}{\gamma mc} \sum_{\mathbf{g}(\neq 0)} \left[\mathbf{A}_{\mathbf{g}}^{*} \exp(-i(\mathbf{k}+\mathbf{g})\cdot \mathbf{r}) \right] \cdot \hat{\mathbf{p}} \,|\, \varphi_i , \mathbf{p}_{\|} >$$

$$-< \varphi_f , \mathbf{p}_{\|} \,|\, \frac{e}{\gamma mc} \left[\mathbf{A}_{0}^{*} \exp(-i\mathbf{k}\cdot \mathbf{r}) \right] \cdot \hat{\mathbf{p}} \,|\, \varphi_i , \mathbf{p}_{\|} >$$

$$\equiv M_0^{(if)} + \sum_{\mathbf{g}(\neq 0)} M_{\mathbf{g}}^{(if)} .$$

$$M_{-\mathbf{g}}^{(if)} = -\left(\frac{e}{c} \right)$$

$$\times \left[(\mathbf{A}_{-\mathbf{g}}^{*} \cdot \mathbf{v}_{\|}) < \varphi_f \,|\, e^{-i(\mathbf{k}-\mathbf{g})_y y} \,|\, \varphi_i > + \frac{1}{\gamma m} (\mathbf{A}_{-\mathbf{g}}^{*})_y < \varphi_f \,|\, e^{-i(\mathbf{k}-\mathbf{g})_y y} \hat{p}_y \,|\, \varphi_i > \right]$$

$$\times \delta(\mathbf{p}_{\|} - \mathbf{p}_{\|} \,|\, \hbar \mathbf{k}_{-\mathbf{g}\|})$$

intraband transition: i=f

$$M_{-\mathbf{g}}^{(ii)} = -\left(\frac{e}{c}\right)(\mathbf{A}_{-\mathbf{g}}^{*} \cdot \mathbf{v}_{\|})F_{ii}((\mathbf{k}-\mathbf{g})_y)\delta(\mathbf{p}_{\|} - \mathbf{p}_{\|} \mid \hbar\mathbf{k}_{-\mathbf{g}\|})$$

$$F_{ii}(q) = \langle \varphi_i \mid e^{-iqy} \mid \varphi_i \rangle$$ form factor

$$\left(\frac{dN}{d\theta_x d\theta_y dz}\right)_{PXR} = \frac{\alpha\omega_B}{4\pi c \sin^2\theta_B}\left(\frac{\theta_x^2}{4(1+W_{v\|}^2)} + \frac{\theta_y^2}{4(1+W_{v\perp}^2)}\right)$$

$$W_{v\sigma} \equiv \frac{1}{2|\chi_g|P_\sigma}\left[\theta_x^2+\theta_y^2+\theta_{kin}^2 - \frac{|\chi_g|^2 P_\sigma^2}{\theta_x^2+\theta_y^2+\theta_{kin}^2}\right], (\sigma = \|, \perp)$$

$$\theta_{kin}^2 = \gamma^{-2} + |\chi_0|$$

interbanded transition: i≠f

$$M_{-\mathbf{g}}^{(if)} = \left(\frac{e}{c}\right) < \varphi_f \,|\, y \,|\, \varphi_i > \left[i(\mathbf{k}-\mathbf{g})_y (\mathbf{A}^*_{-\mathbf{g}} \cdot \mathbf{v}_{\|}) + i(\mathbf{A}^*_{-\mathbf{g}})_y \Omega_{if} \right] \delta(\mathbf{p}_{\|} - \mathbf{p}'_{\|} \,|\, \hbar \mathbf{k}_{-\mathbf{g}\|}).$$

$$\left(\frac{dN}{d\theta_x d\theta_y dz}\right)_{DCR} = \frac{\alpha \omega_B^3 y_{if}^2}{4\pi c^3 \sin^2 \theta_B} \left(\frac{\theta_x^2 \theta_y^2}{4(1+W_{\|}^2)} + \frac{(\theta_y^2 - \frac{\Omega_{if}}{\omega_B})^2}{4(1+W_{\perp}^2)} \right)$$

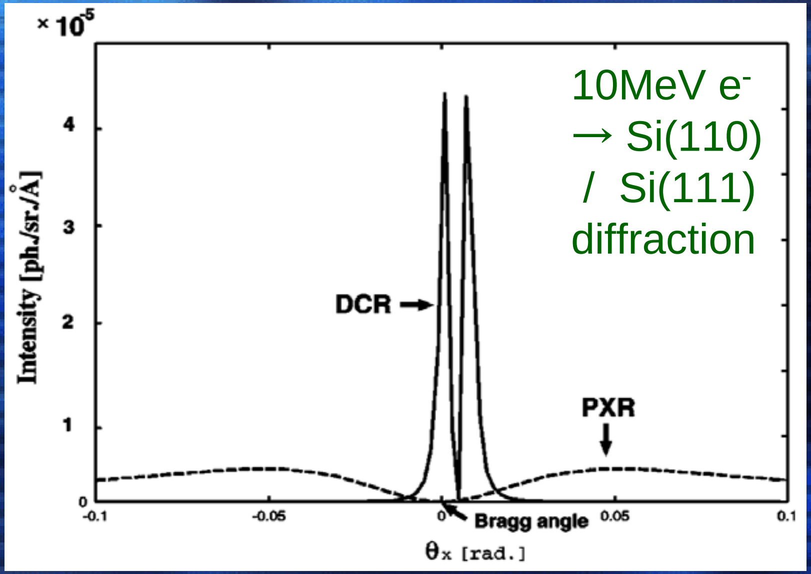
where

$$y_{if} = < \varphi_f \,|\, y \,|\, \varphi_i >$$

$$W_\sigma = \frac{1}{2|\chi_g| P_\sigma} \left[R - \frac{|\chi_g|^2 P_\sigma^2}{R} \right]$$

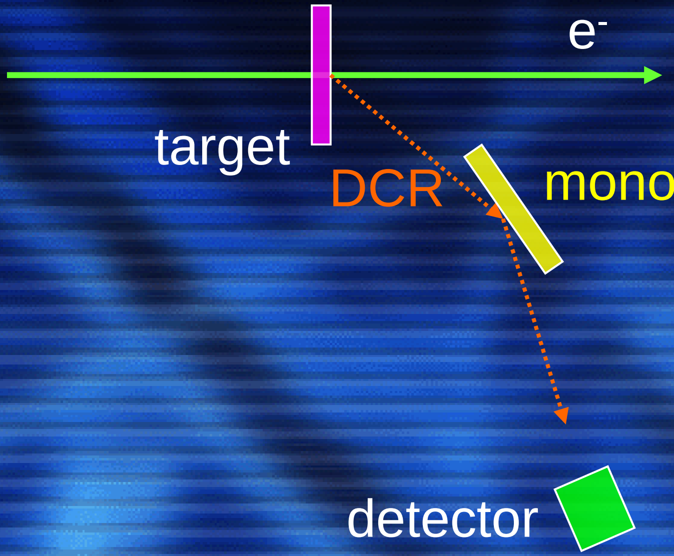
$$R = \left[\theta_x - \left(\frac{\Omega_{if}}{\omega_B}\right) \cot \theta_B \right]^2 + \theta_y^2 + \frac{1}{\gamma^2} + |\chi_0| - 2\left(\frac{\Omega_{if}}{\omega_B}\right)$$

Angular distribution of DCR and PXR

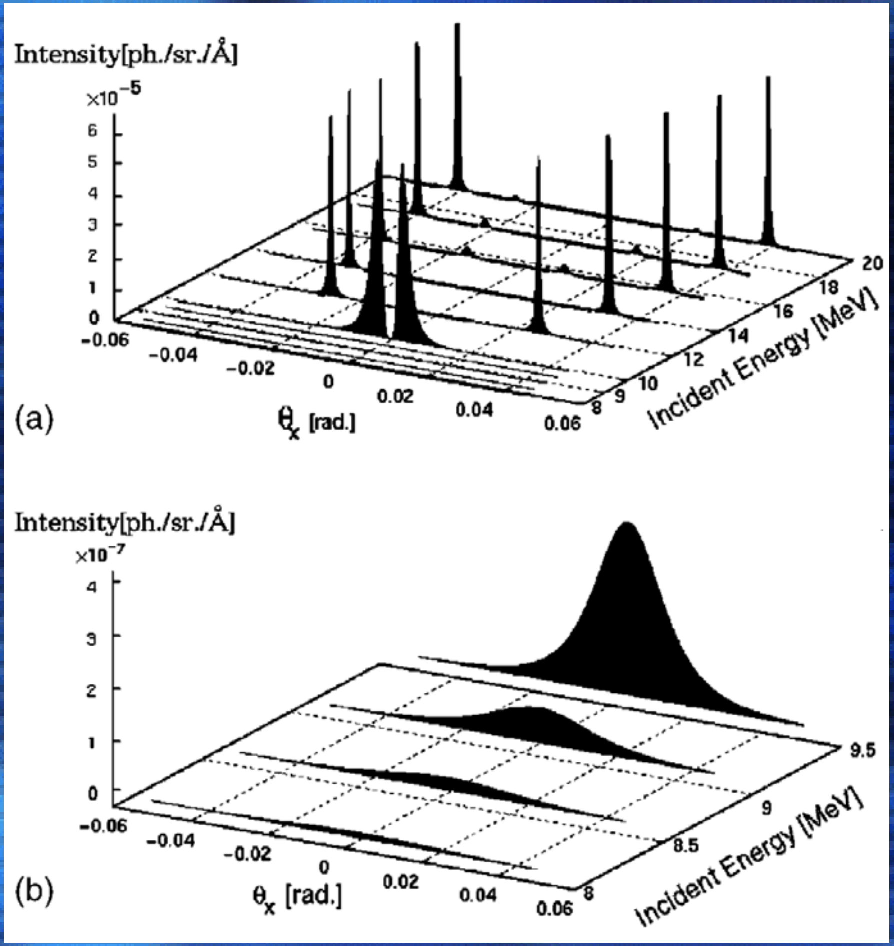


DCR peaks appear at $W_\sigma = 0$

$$\theta_x = \cot\theta_B \left(\frac{\Omega_{if}}{\omega_B}\right) \pm \sqrt{2\left(\frac{\Omega_{if}}{\omega_B}\right) - \frac{1}{\gamma^2} - |\chi_0| - \theta_y^2}$$



Electron energy and DCR intensity



if the electron energy is small, $W_\sigma = 0$ is not satisfied.

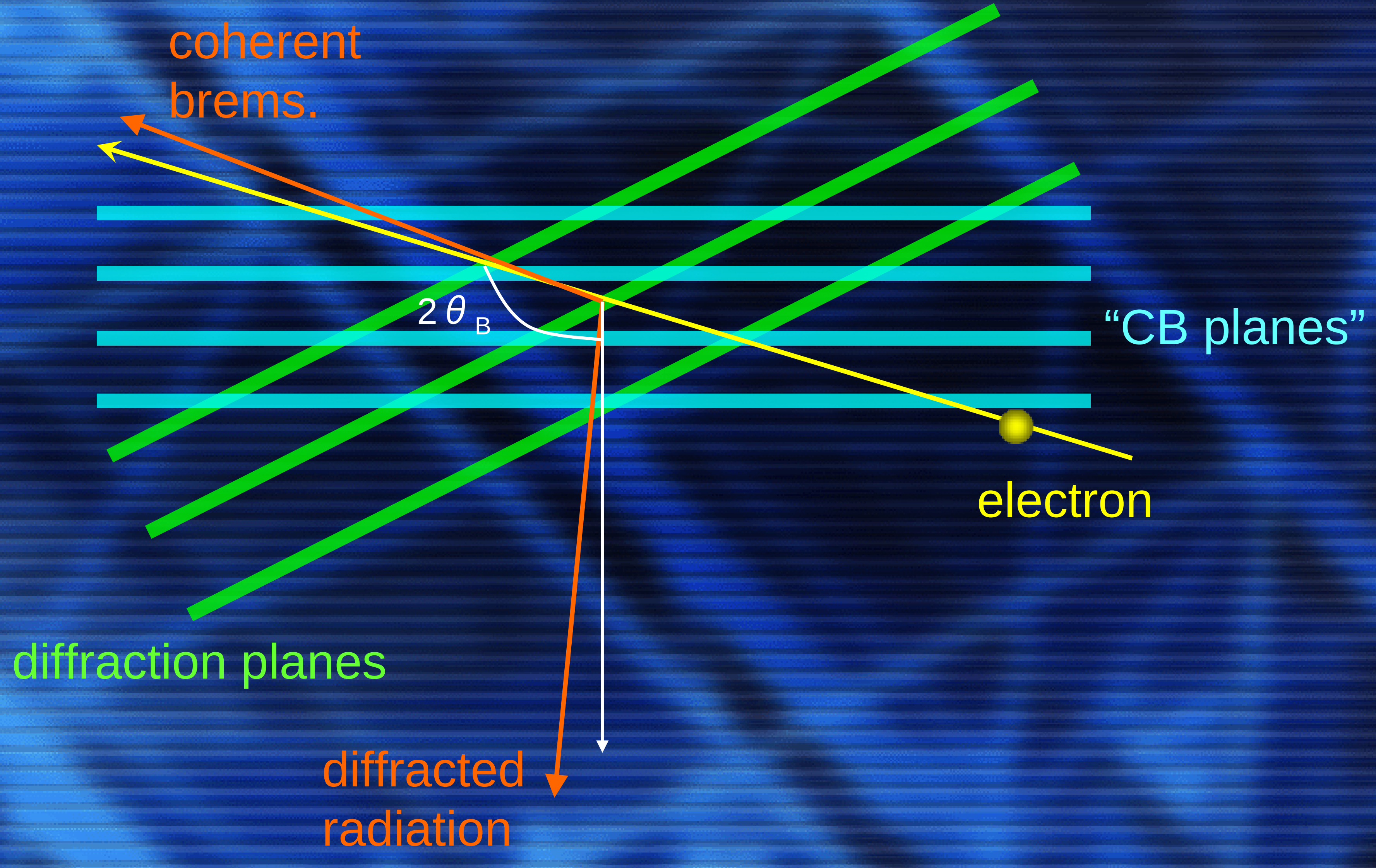
DCR threshold

$$\gamma > \gamma_{\text{th}}$$

where

$$\gamma_{th} = \left[2\left(\frac{\Omega_{if}}{\omega_B}\right) - |\chi_0| + |\chi_g| \right]^{-1/2}.$$

Diffracted Coherent Bremsstrahlung



Diffracted Coherent Bremsstrahlung (DCB)

$$\omega = \frac{(\mathbf{g}+\mathbf{h})\cdot\mathbf{v}}{1-\beta^{*}\cos\Theta}$$

$\mathbf{h}=0$

$$\omega = \frac{\mathbf{g}\cdot\mathbf{v}}{1-\beta^{*}\cos\Theta}$$

PXR

$\mathbf{g}=0$

$$\omega = \frac{\mathbf{h}\cdot\mathbf{v}}{1-\beta^{*}\cos\Theta}$$

$$\rightarrow \frac{2\mathbf{h}\cdot\mathbf{v}}{\gamma^{-2}+|\chi_0(\omega)|}$$

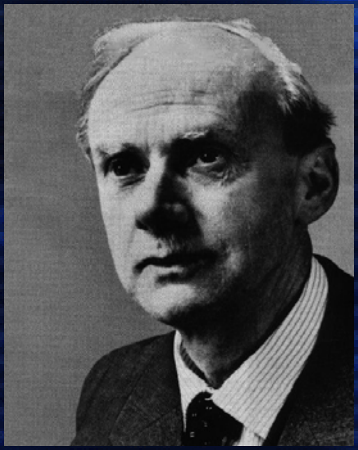
$$(\Theta \rightarrow 0)$$

CB

PXR emitted by Neutral Particles

Search for properties of neutrino

Majorana neutrino



Dirac neutrino

mass, magnetic moment, …

PR-type radiation by neutrino magnetic moment

Cherenkov radiation by neutrino

Grims and Neufeld, Phys.Lett.B **315** (1993) 129.

Transition radiation by neutrino

Classical theory:

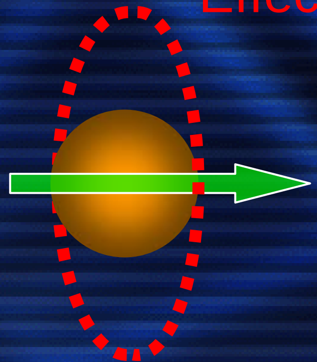
Sakuda, PRL **72** (1994) 804.

Quantum theory:

Sakuda and Kurihara, PRL **74** (1995) 1284.

Classical electrodynamics

effective current due to magnetic moment



$$\mathbf{j}'_{\mathrm{eff}}(\mathbf{r}') = c\nabla' \times \boldsymbol{\mu}\delta(\mathbf{r}')$$

Lorentz transformation

-**v**

$$\mathbf{j}_{\mathrm{eff}}(\mathbf{r}) = \frac{c}{\gamma}\nabla \times \boldsymbol{\mu}\delta(\mathbf{r} - \mathbf{v}t)$$

Classical PXR formula

$$\frac{dI}{d\Omega} = \frac{c}{64\pi^3 \varepsilon_0^{3/2}} \int_0^\infty d\omega \left| \sum_{\mathbf{g}\neq 0} \chi_{\mathbf{g}} \mathbf{k} \times \mathbf{k} \times \mathbf{E}_0(\mathbf{k}-\mathbf{g},\omega) \right|^2$$

(Ter-Mikaelian)

$$\mathbf{E}_0(\mathbf{k},\omega) = -i\left(\frac{4\pi\omega}{c^2}\right)\frac{\mathbf{j}_{\text{eff}}(\mathbf{k},\omega)}{(\omega/c^*)^2 - \mathbf{k}^2}$$

$$\mathbf{j}_{\text{eff}}(\mathbf{k},\omega) = \frac{(2\pi i)c}{\gamma}\mathbf{k} \times \boldsymbol{\mu}\,\delta(\mathbf{k}\bullet\mathbf{v}-\omega)$$

Intensity of PXR from magnetic moment

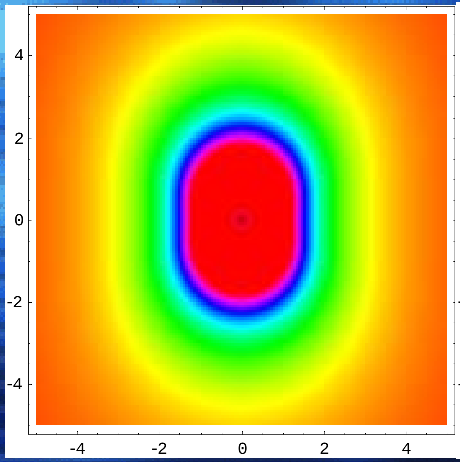
$$\frac{dN^{\mu}}{d\Omega dz} = \left(\frac{\mu^2}{4\pi\hbar v \varepsilon_0^{3/2}}\right)\frac{1}{\gamma^2}\frac{\left(\omega_B / c\right)^3}{\sin^2\theta_B}$$
$$\sum_{\mathbf{g}\neq 0}\left|\chi_{\mathbf{g}}\right|^2 \frac{\left(\cos 3\theta_B / \cos\theta_B\right)^2\theta_x^2 + \left(\cos 2\theta_B\right)^2\theta_y^2}{\left(\theta_x^2+\theta_y^2+1/\gamma^2+\chi_0^2\right)^2}$$

cf. PXR for an electron (F-I formula):

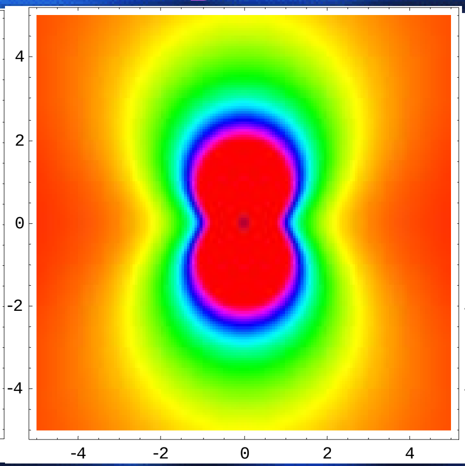
$$\frac{dN^{e}}{d\Omega dz} = \left(\frac{e^2}{4\pi\hbar c}\right)\frac{\left(\omega_B / c\right)}{\sin^2\theta_B}\sum_{\mathbf{g}\neq 0}\left|\chi_{\mathbf{g}}\right|^2 \frac{\left(\cos 2\theta_B\right)^2\theta_x^2 + \theta_y^2}{\left(\theta_x^2+\theta_y^2+1/\gamma^2+\chi_0^2\right)^2}$$

Angular distribution of PXR by magnetic moment

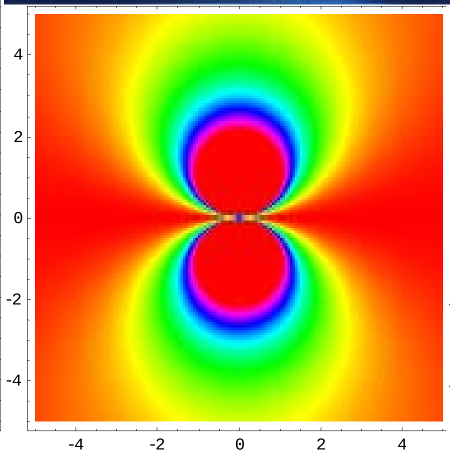
$\Theta_B = 20^\circ$



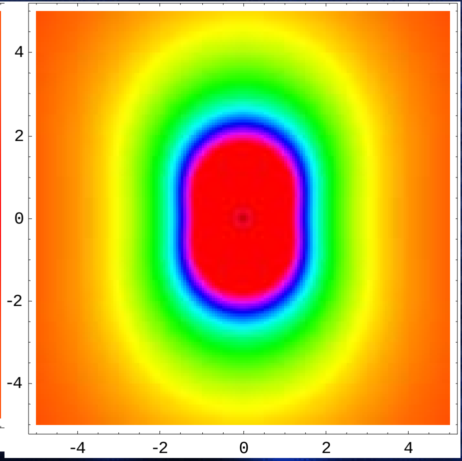
$\Theta_B = 25^\circ$



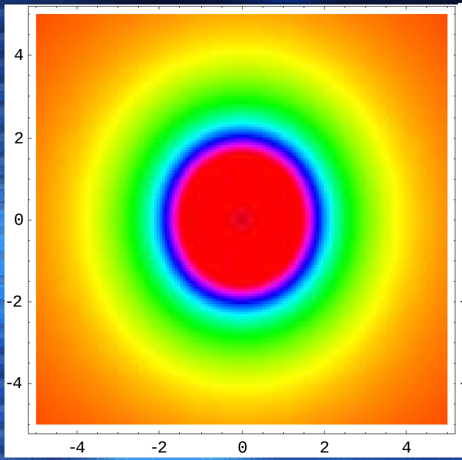
$\Theta_B = 30^\circ$



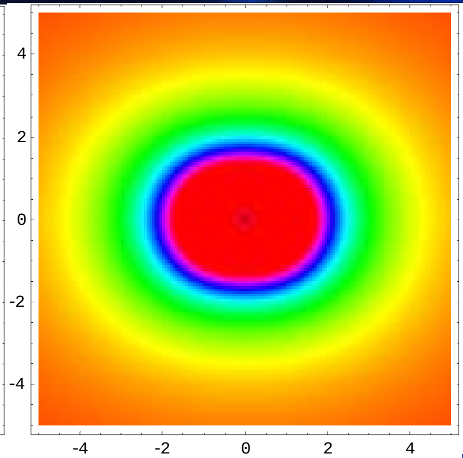
$\Theta_B = 34^\circ$



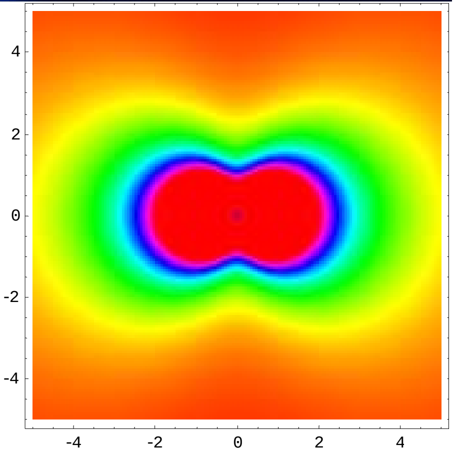
$\Theta_B = 35^\circ$



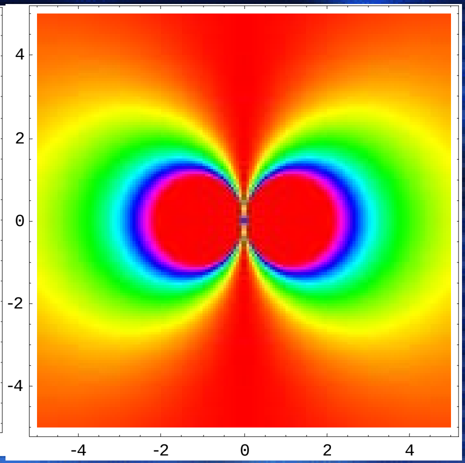
$\Theta_B = 36^\circ$



$\Theta_B = 38^\circ$



$\Theta_B = 45^\circ$



Comparison with electric-charge PXR

$$\frac{N^{\mathrm{e}}_{\mathrm{PXR}}}{N^{\mu}_{\mathrm{PXR}}} \sim \frac{\left(\dfrac{e^2}{4\pi\hbar c}\right)\dfrac{(\omega_B/c)}{\sin^2\theta_B}}{\left(\dfrac{\mu^2}{4\pi\hbar c}\right)\dfrac{1}{\gamma^2}\dfrac{(\omega_B/c)^3}{\sin^2\theta_B}} \sim \left(\frac{\mu}{\mu_B}\right)^2\left(\frac{\lambda_c}{(\omega_B/c)}\right)^2\frac{1}{\gamma^2}$$

μ_B : Bohr magneton $\quad$ λ_c : Compton wavelength

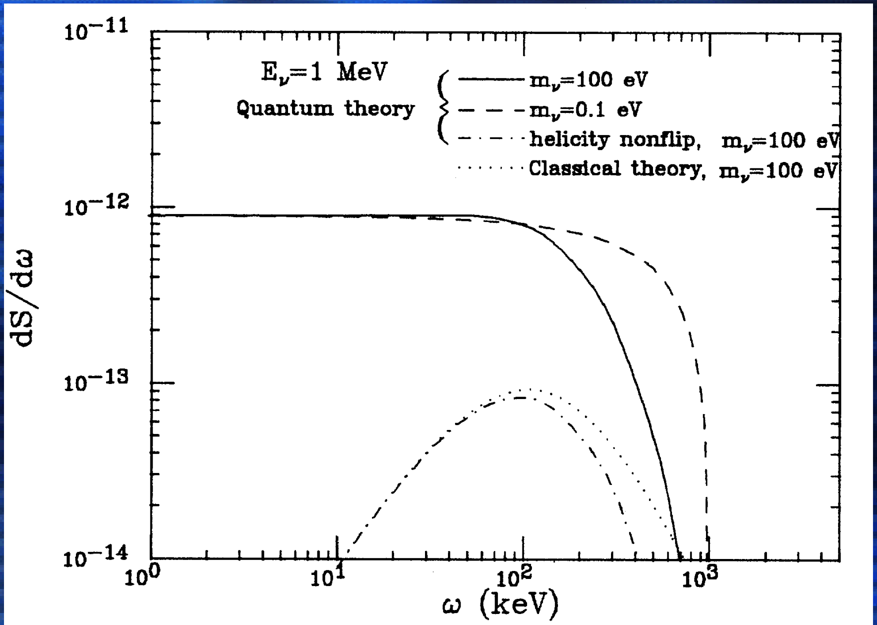
$$\gamma = E_\nu/(m_\nu c^2) \sim 10^3 \quad \Rightarrow \quad \frac{N^{\mathrm{el}}_{\mathrm{PXR}}}{N^{\mathrm{mg}}_{\mathrm{PXR}}} \sim \left(\frac{\mu}{\mu_B}\right)^2 \cdot 10^{-4} \cdot 10^{-6}$$

$$\mu/\mu_B \sim 10^{-10} \quad \Rightarrow \quad \boxed{\frac{N^{\mathrm{e}}_{\mathrm{PXR}}}{N^{\nu}_{\mathrm{PXR}}} \leq 10^{-30}}$$

!!!

Quantum effect: radiation due to spin flip

cf. TR (Sakuda and Kurihara 1995)



(numerical calculation)

$$H_{\mathrm{int}} = \frac{\mu_\nu}{2}\bar{\psi}\sigma_{\rho\lambda}\psi F^{\rho\lambda}$$

$$\sigma_{\mu\nu} = \frac{i}{2}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Analytical expression has been obtained for TR (Ueno and Nitta)

Summary

- Diffracted channeling radiation (DCR)
- Diffracted coherent bremsstrahlung (DCB)
- A neutral particle with magnetic moment may emits PXR as well as TR.