

# **EFFECT OF HEAVY ION CHARGE FLUCTUATIONS ON CHERENKOV RADIATION**

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# ENERGY LOSS

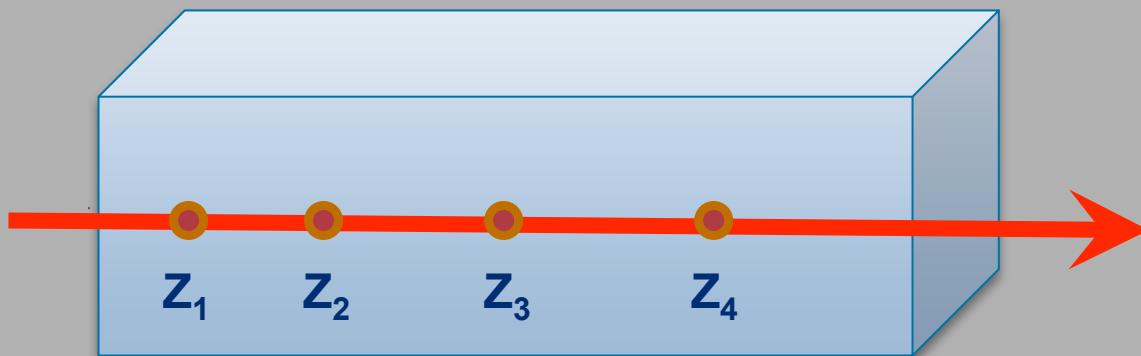
$$W = \int \int Q(\mathbf{k}, \omega) d\mathbf{k} d\omega$$

$$Q(\mathbf{k}, \omega) = -\frac{1}{8\pi^4} \text{Im} \left[ \frac{4\pi |\mathbf{j}_L|^2}{\omega \epsilon} + \frac{4\pi \omega}{c^2} \frac{|\mathbf{j}_T|}{\omega^2 \epsilon - k^2} \right]$$

$\epsilon(\mathbf{k}, \omega) = 0$  - Longitudinal waves

$\frac{\omega^2}{\tilde{n}^2} \epsilon(\mathbf{k}, \omega) \mu(\mathbf{k}, \omega) - k^2 = 0$  - Transverse waves

**When heavy ions penetrate into the target, the ion-atom collisions cause fluctuations of the projectile charge due to electrons loss or capture.**



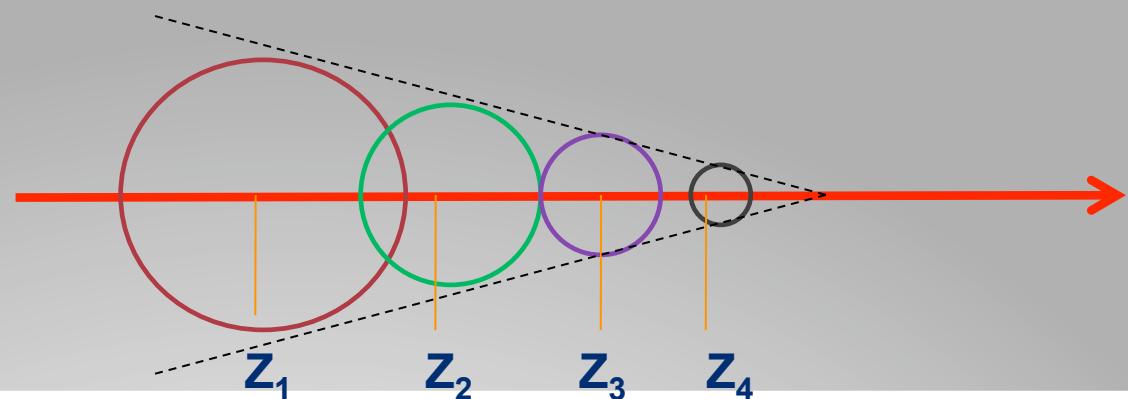
$$\mathcal{Q}(\mathbf{k}, \omega) = \left\langle -\frac{1}{8\pi^4} \text{Im} \left[ \frac{4\pi}{m\epsilon} |\mathbf{j}_\perp|^2 + \frac{4\pi\omega}{c^2} \frac{|\mathbf{j}_\perp|^2}{\omega^2 - k^2} \right] \right\rangle$$

**The correlation between various charge state values can arise in the stopping power, bremsstrahlung, transition and Cherenkov radiation**

**LONGITUDINAL WAVES :**

Z.I. Miskovic, S.G Davison., F.O Goodman., W.K. Liu Phys. Rev.1999, B60, 14478-1483.

**TRANSVERSE WAVES (HUYGENS PRINCIPLE):**



# Correlation effects

TRANSVERSE WAVES:

$$\frac{d^2W}{d\omega d\Omega} = \left\langle \frac{\omega k}{(2\pi)^2} |\mathbf{n} \times \mathbf{j}(\mathbf{k}, \omega)|^2 \right\rangle, \quad \mathbf{j}(\mathbf{k}, \omega) = e \int dt Z(t) \mathbf{v}(t) e^{i[(\omega t - \mathbf{k}\mathbf{r}(t))]},$$

$$\mathbf{n} = \mathbf{k} / k, \quad k = \omega \sqrt{\epsilon'(\omega)},$$

$$\frac{d^2W}{d\omega d\Omega} = \frac{\omega^2 v^2 e^2 \sqrt{\epsilon'(\omega)} \sin^2 \theta}{(2\pi)^2} \Delta(\omega - \mathbf{k}\mathbf{v}),$$
$$\Delta(\omega - \mathbf{k}\mathbf{v}) = \int dt dt' \langle Z(t) Z(t') \rangle \exp [i(\omega - \mathbf{k}\mathbf{v})(t - t')].$$

Autocorrelation  
function

# Correlation effects

AUTOCORRELATION FUNCTION:

$$\langle Z(t)Z(t') \rangle = Z_{eq}^2 + \langle \xi(t)\xi(t') \rangle$$

$$Z_{eq} = \langle Z(t) \rangle \quad \xi(t) = Z(t) - Z_{eq}$$

$$\langle \xi(t)\xi(t') \rangle = \Lambda^2 \exp(-\Gamma|t-t'|)$$

$\Lambda^2$  - DISPERSION OF A RANDOM VARIABLE OF A CHARGE

$$\Gamma \approx n(\sigma_c + \sigma_l)v$$

# Correlation effects

The spectral-angular density of the radiation

$$\frac{d^3W}{d\omega d\Omega dl} = \frac{d^3W^{(TF)}}{d\omega d\Omega dl} + \frac{\Lambda^2 e^2 \omega \sin^2 \theta}{2\pi^2} \frac{xy}{x^2 y^2 + (y - \cos \theta)^2}$$

$$x = \Gamma / \omega \quad y = c_p / v$$

The spectral density of the radiation

$$\begin{aligned} \frac{d^2W^{(COR)}}{d\omega dl} = & \frac{\Lambda^2 e^2 \omega}{\pi} \left[ \left( x^2 y^2 - y^2 + 1 \right) \left( \operatorname{arctg} \frac{1-y}{xy} + \operatorname{arctg} \frac{1+y}{xy} \right) \right] - \\ & \frac{\Lambda^2 e^2 \omega}{\pi} xy \left[ 2 + y \ln \left| \frac{x^2 y^2 + (1-y)^2}{x^2 y^2 + (1+y)^2} \right| \right], \end{aligned}$$

# Correlation effects

If the threshold condition is satisfied but characteristic time of charge exchange is much more than period of an electromagnetic wave :

$$x = \Gamma / \omega \ll 1 \quad y = c_p / v < 1$$

$$\frac{d^2W}{d\omega dl} \approx \frac{d^2W^{(TF)}}{d\omega dl} \left( 1 + \Lambda^2 / Z_{eq}^2 \right)$$

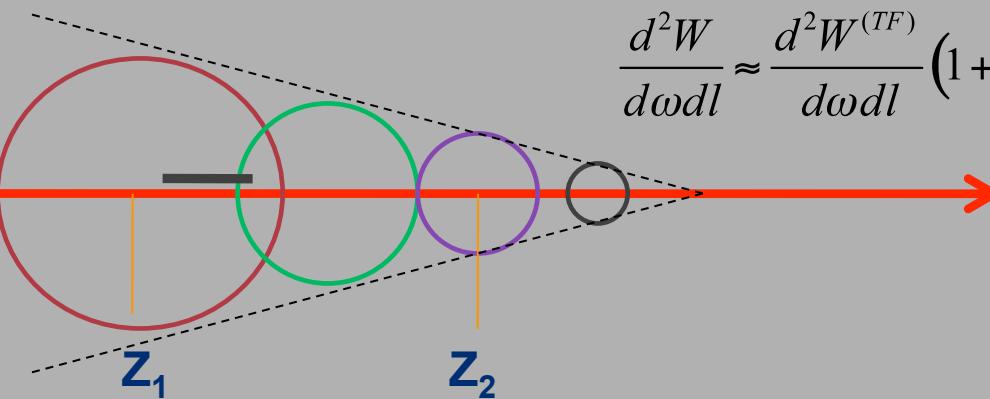
$$Z_{eff} = Z_{eq} \left( 1 + \Lambda^2 / Z_{eq}^2 \right)^{1/2}$$

# Correlation effects

The threshold condition is satisfied

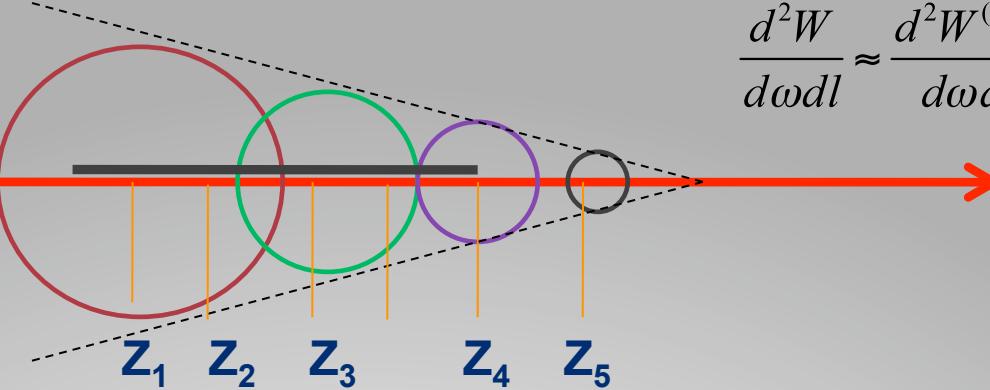
$$y = c_p / v < 1$$
$$x = \Gamma / \omega \ll 1$$

$$\frac{d^2W}{d\omega dl} \approx \frac{d^2W^{(TF)}}{d\omega dl} \left( 1 + \Lambda^2 / Z_{eq}^2 \right)$$

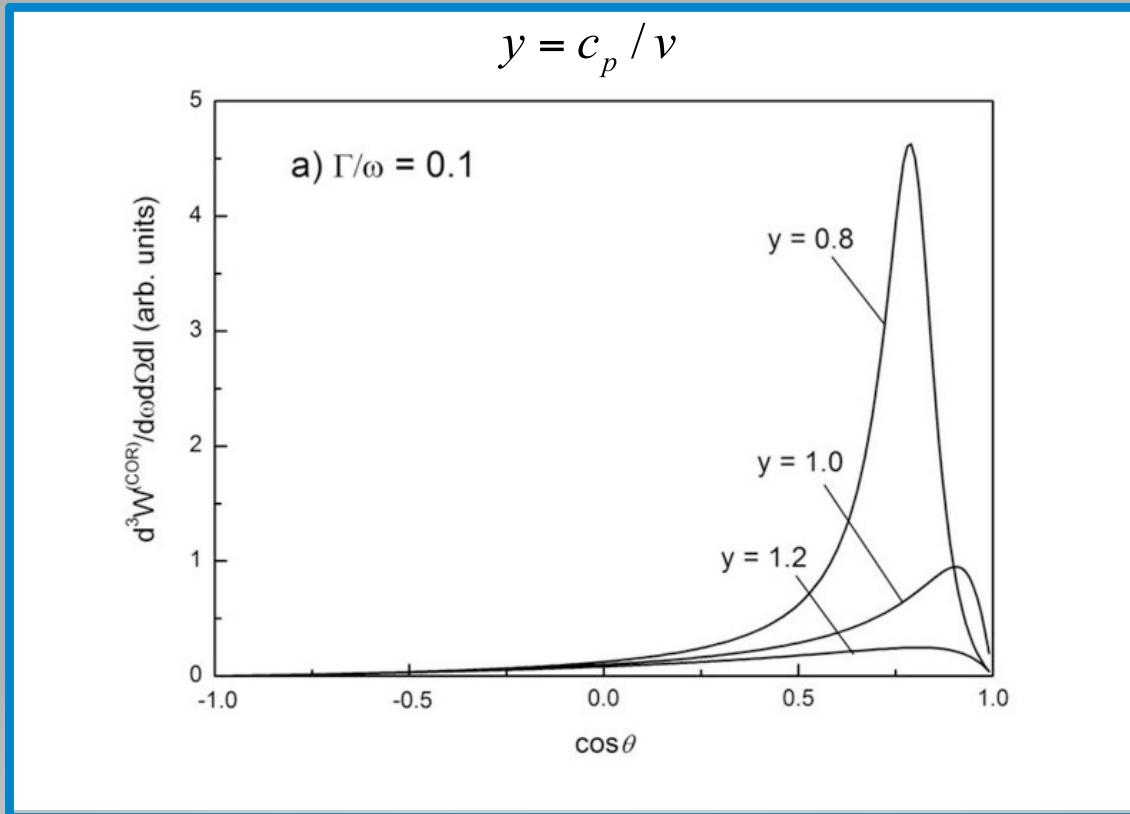


$$y = c_p / v < 1$$
$$x = \Gamma / \omega \gg 1$$

$$\frac{d^2W}{d\omega dl} \approx \frac{d^2W^{(TF)}}{d\omega dl}$$

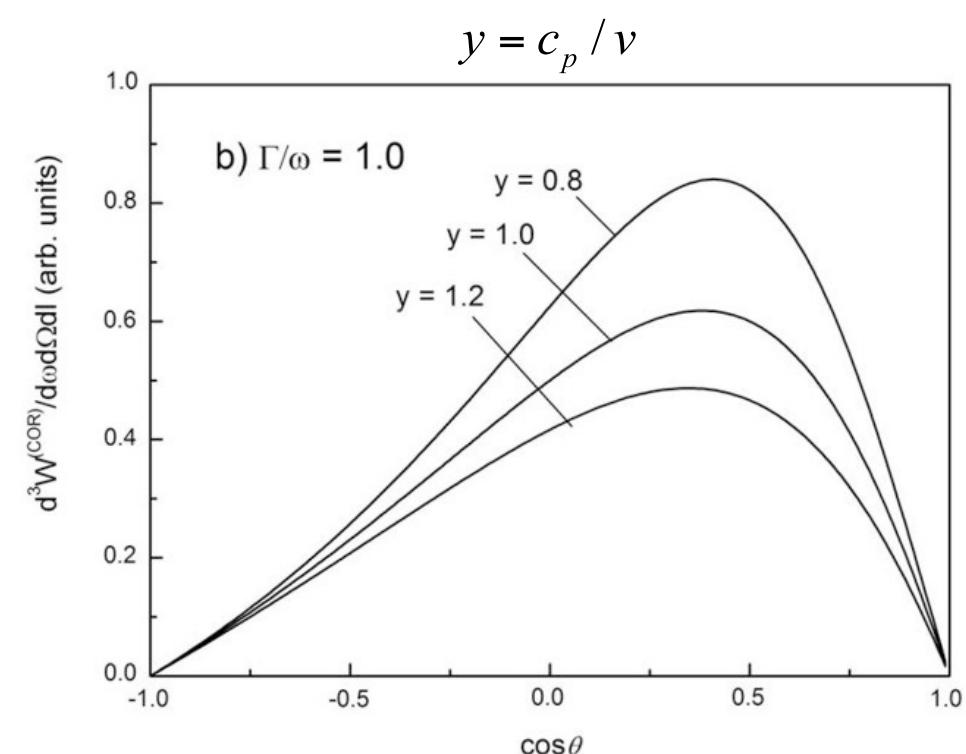


# Correlation effects

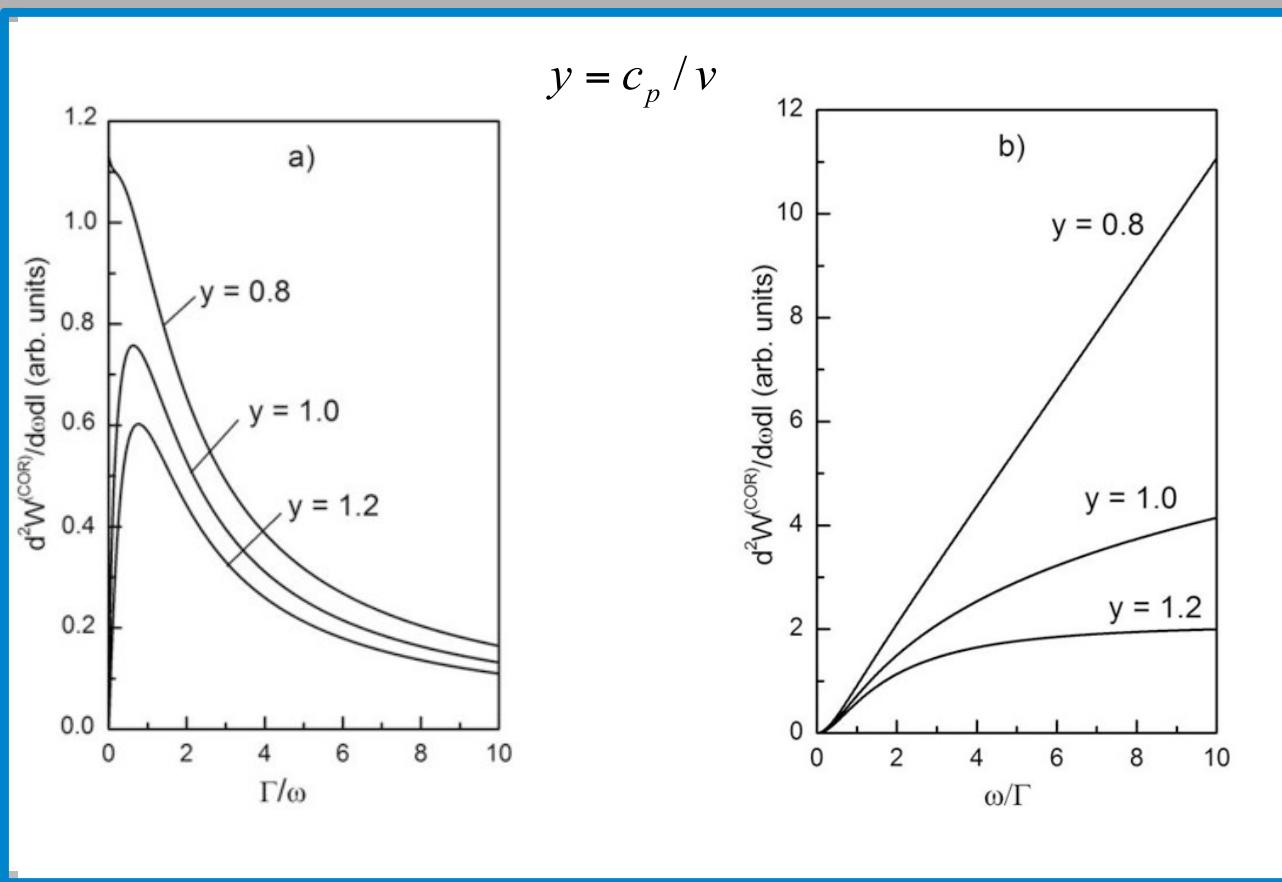


The charge exchange correlation gives the nonzero contribution to the radiation yield at the violation of the Cherenkov threshold condition

# Correlation effects



# Correlation effects



The spectral density of the radiation

# Autocorrelation function:

$$\langle Z(t_0)Z(t_0 + \tau) \rangle = \sum_i \sum_j Z_i Z_j \pi_{ji}(\tau) P_i(t_0)$$

$$\frac{\partial}{\partial \tau} \|\pi(\tau)\| = \|A\| \cdot \|\pi(\tau)\|$$

$$A = N \nu \begin{pmatrix} -\sigma_L & \sigma_C & 0 & 0 & \cdots & 0 \\ \sigma_L & -(\sigma_L + \sigma_C) & \sigma_C & 0 & \cdots & 0 \\ 0 & \sigma_L & -(\sigma_L + \sigma_C) & \sigma_C & \cdots & 0 \\ 0 & 0 & \sigma_L & -(\sigma_L + \sigma_C) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \sigma_C \end{pmatrix}$$

# Autocorrelation function:

$$P(t_0 + \tau) = \pi(\tau) P(t_0)$$

Equilibrium charge

$$Z_{eq} = \sum_i Z_i P_i$$

Dispersion

$$\Lambda^2 = \sum_i (Z_i - Z_{eq})^2 P_i$$

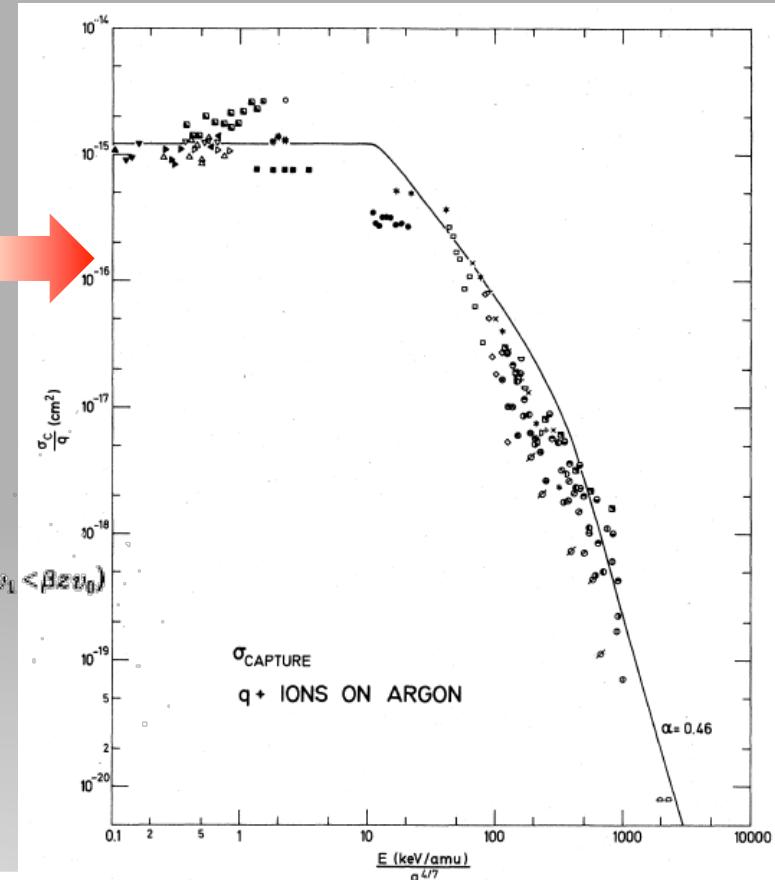
$$\langle Z(t_0) Z(t_0 + \tau) \rangle = Z_{eq}^2 + \Lambda^2 e^{-\Gamma \tau}$$

# Cross sections $\sigma_L$ and $\sigma_C$

N. Bohr, J. Lindhard, Mat. Fys. Medd.  
Dan. Vid. Selsk. 28, no 7 (1954).

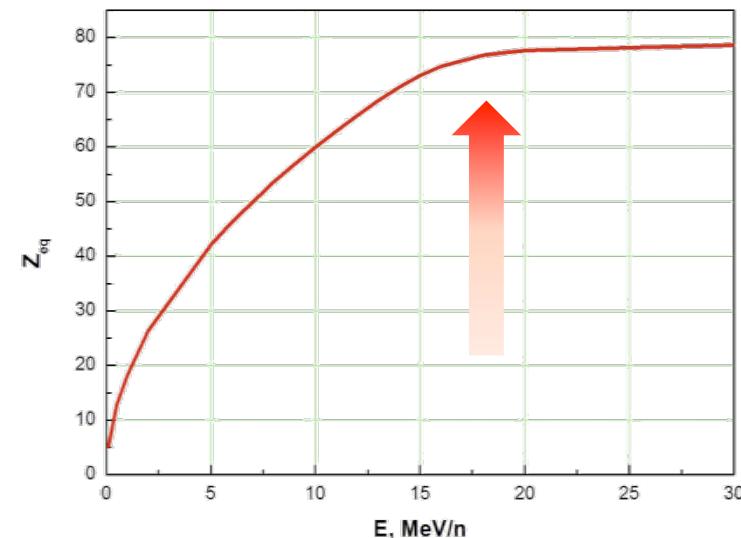
H.Knudsen, H.K.Haugen, P.Hvelplund,  
Phys.Rev., A23 (1981) 597-610.

$$\frac{\sigma_C}{\pi \alpha_0^2 q} = \begin{cases} \frac{1}{2} z^{2/3} \left[ \left( \frac{\alpha v_a}{v_0} \right)^{-2} - (\beta z)^{-2} \right] & (v_1 < \alpha v_a) \\ \frac{8}{3} \xi^{-5} \left[ \left( \frac{z^{2/3}}{8} \xi^5 \right)^{3/5} - \left( \frac{\alpha v_a}{v_0} \right)^3 \right] + \frac{1}{2} z^{2/3} \left[ \left( \frac{z^{2/3}}{8} \xi^5 \right)^{-2/3} - (\beta z)^{-2} \right] & (\alpha v_a < v_1 < \beta z v_0) \\ \frac{8}{3} \xi^{-5} \left[ (\beta z)^3 - \left( \frac{\alpha v_a}{v_0} \right)^3 \right] & (\beta z v_0 < v_1) \end{cases}$$

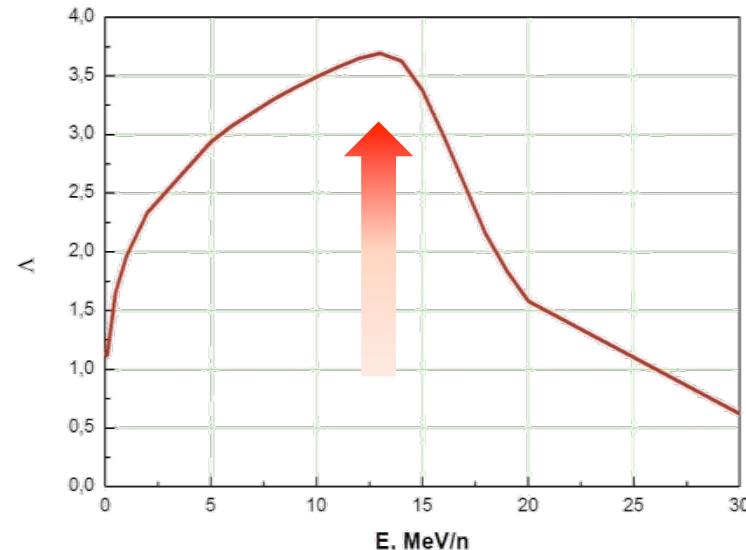


# The results of the calculations for Au ions in Carbon

Equilibrium charge

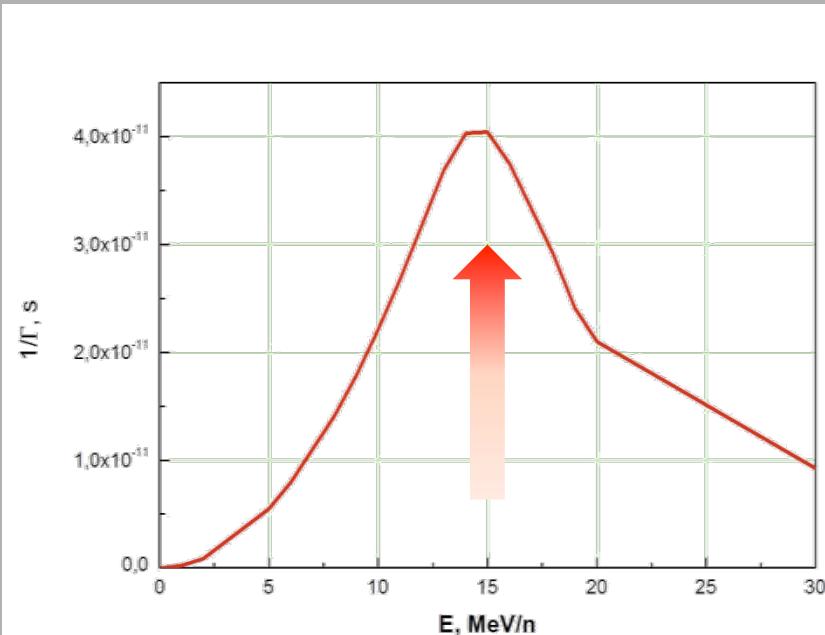


Dispersion

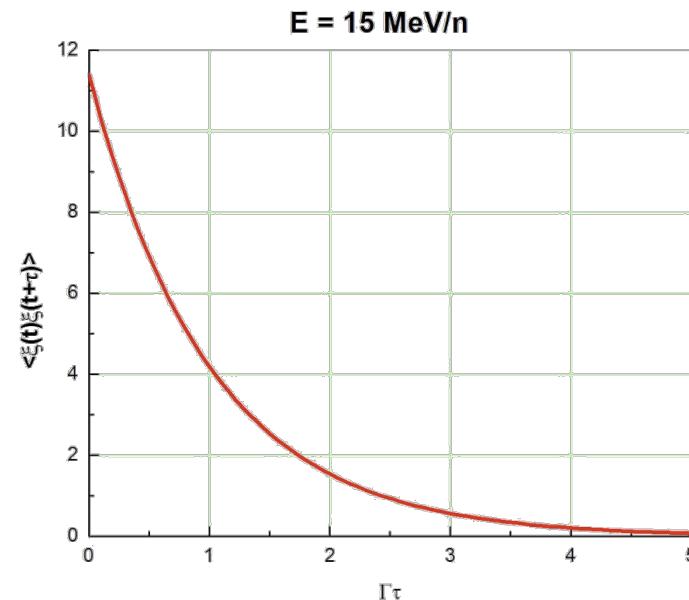


# The results of the calculations for Au ions in Carbon

$1/\Gamma$ , s

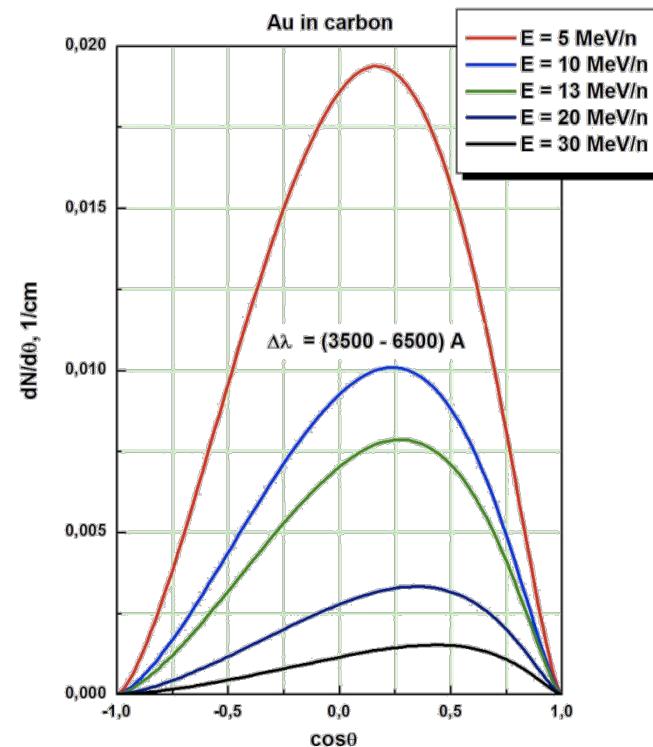
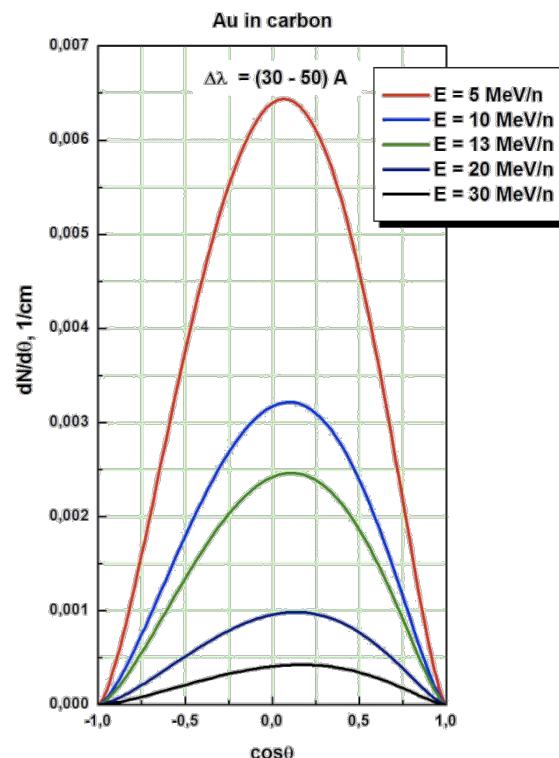


Autocorrelation function



# The results of the calculations for Au ions in Carbon

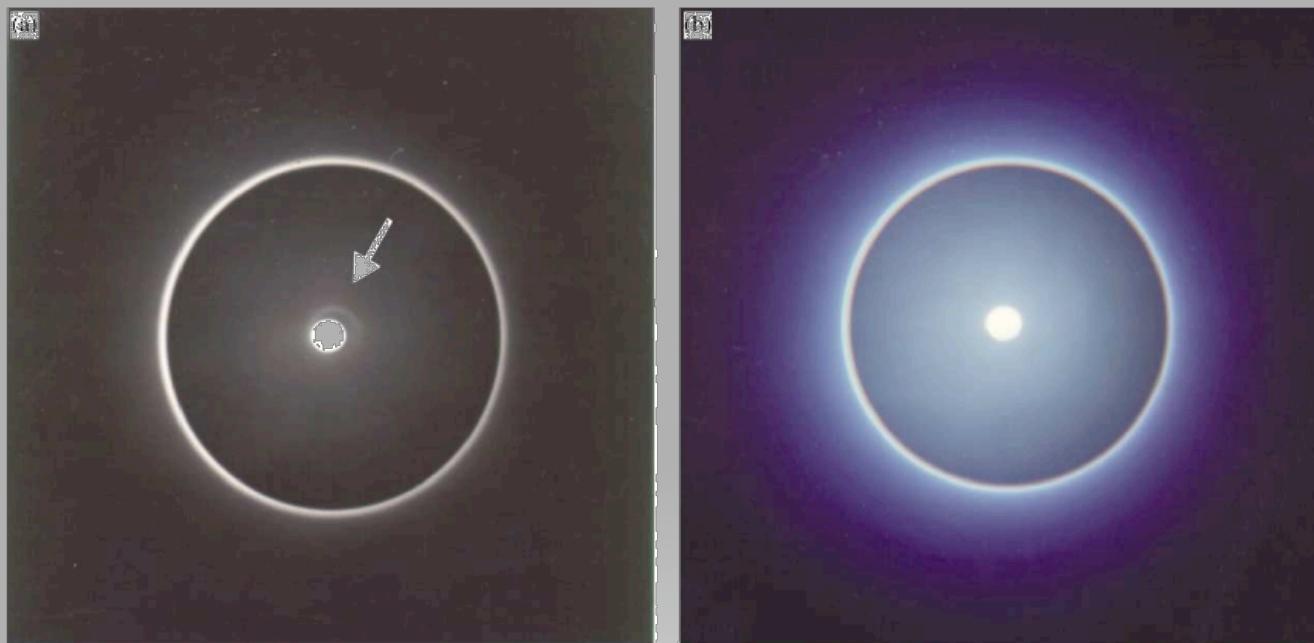
The threshold condition is not satisfied



$$y = c_p / v > 1$$

# Vavilov-Cherenkov radiation emitted by heavy ions near the threshold

J. Ruzicka et al. Vacuum 63 (2001) 591-595



Photographs of the radiation emitted by heavy ions Au in SiO-aerogel radiator: (a) maximal energy 991.52 MeV/u,  $1 \times 10^7$  ions; (b) minimal energy 641.1 MeV/u,  $6 \times 10^7$  ions.

# **Abstract**

Correlation effects in the Cherenkov radiation due to ions charge fluctuations in the matter are considered. Stochastic process of charge exchange leads to the washout of a radiation wave front and to the transformation of spectral-angular density. The effect is determined by a root-mean-square deviation of an ion charge. The additional radiation yield gives the nonzero contribution at the violation of the Cherenkov threshold condition. The interference of an electromagnetic field generated in the matter by the different ion's charge states along the trajectory is the cause of the additional radiation.

**Thanks' !**