COHERENT PROCESSES IN BENT SINGLE CRYSTALS

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Coherent process of scattering of ultrarelativistic particles by bent crystallographic planes (volume reflection) is considered. The main characteristics of the process (as mean and mean squared angles of scattering and different distributions of scattered particles) are obtained in the analytical form. Influence of incoherent stochastic process (volume capture) on the coherent particle scattering is studied.

Another coherent process known as radiation of particles moving in an area of volume reflection is investigated. Calculations show the high value of radiation energy losses of electron and positron beams in a relatively large angle range. The peculiarities of this process in comparing with characteristics of well known radiation processes in straight single crystals are discussed.

On the basis of published experimental results the comparison of calculated and measured data is presented.

Volume reflection

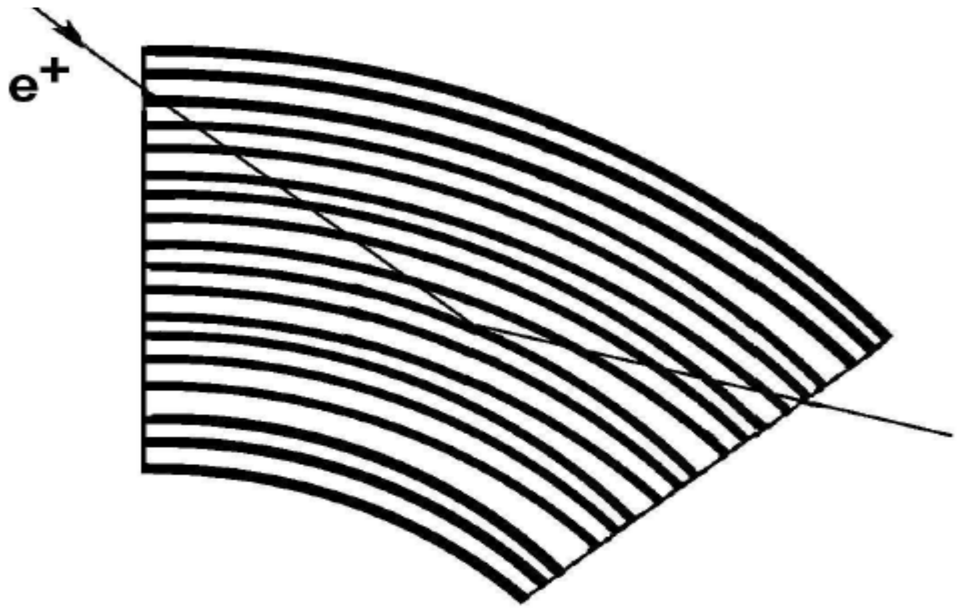
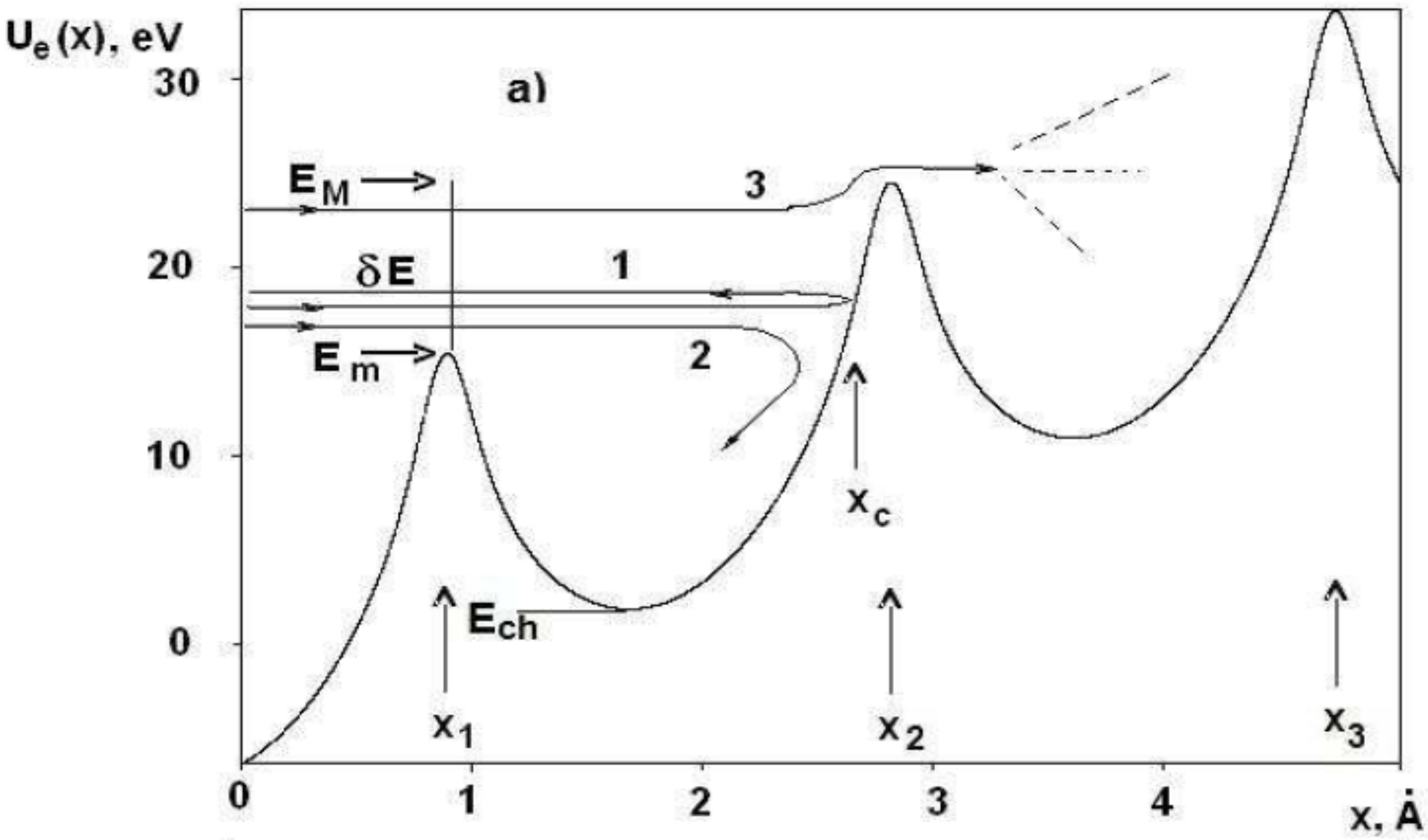


Fig. 1. Scheme of volume reflection process

Volume reflection presents the coherent scattering of charged particles by bent planes or axes of crystallographic structures.

$$E = E_0\beta^2\dot{x}^2/(2c^2) + U(x) + \beta^2 E_0 x/R,$$

U_e (x) Effect. potential



Volume reflection process was predicted (A. Taratin, S. Vorobiev) in MC calculations of charged particle passage through bent single crystals:

A.M. Taratin and S.A. Vorobiev Phys. Lett. A119, 425 (1987).

The analytical description of the process one can find in the papers:

V.A. Maisheev Phys. Rev. ST AB 10, 084701 (2008);
ArXiv: physics/0607009.
Yu.A. Chesnokov, V.A.Maisheev and I.A, Yazynin ArXiv:0808.1486.

Computer codes for calculations ov VR:

http://mail.ihep.ru/~maisheev

Recently, effect of volume reflection was observed in direct experiments in IHEP (Protvino), PNPI (Gatchina) and CERN:

Yu.M. Ivanov et. al. Phys. Rev. Lett. 97, 144801 (2006).
Yu.M. Ivanov et. al. JETP Lett. 84, 372 (2006).
W. Scandale et. al. Phys. Rev. Lett. 98, 154801 (2007).

On the 1st step we consider pure process (without multiple scattering on atoms). On the 2nd step we introduce multiple scattering and calculate real parameters of beam after interaction with a bent single crystal.

1st step

On the basis equations of motion in bent single crystals we get the following expression for angle of volume reflection:

$$\alpha(E) = \frac{2c}{R}\int_{x_0}^{x_c}\left[\frac{1}{\sqrt{\frac{2c^2}{E_0\beta^2}(E - U(x) - E_0\beta^2 x/R)}} - \frac{1}{\sqrt{\frac{2c^2}{E_0\beta^2}(E - U(x_c) - E_0\beta^2 x/R)}}\right]dx.$$

where E_0, E and β are the total and transversal energies of particle and its velocity divided by the velocity of light c,

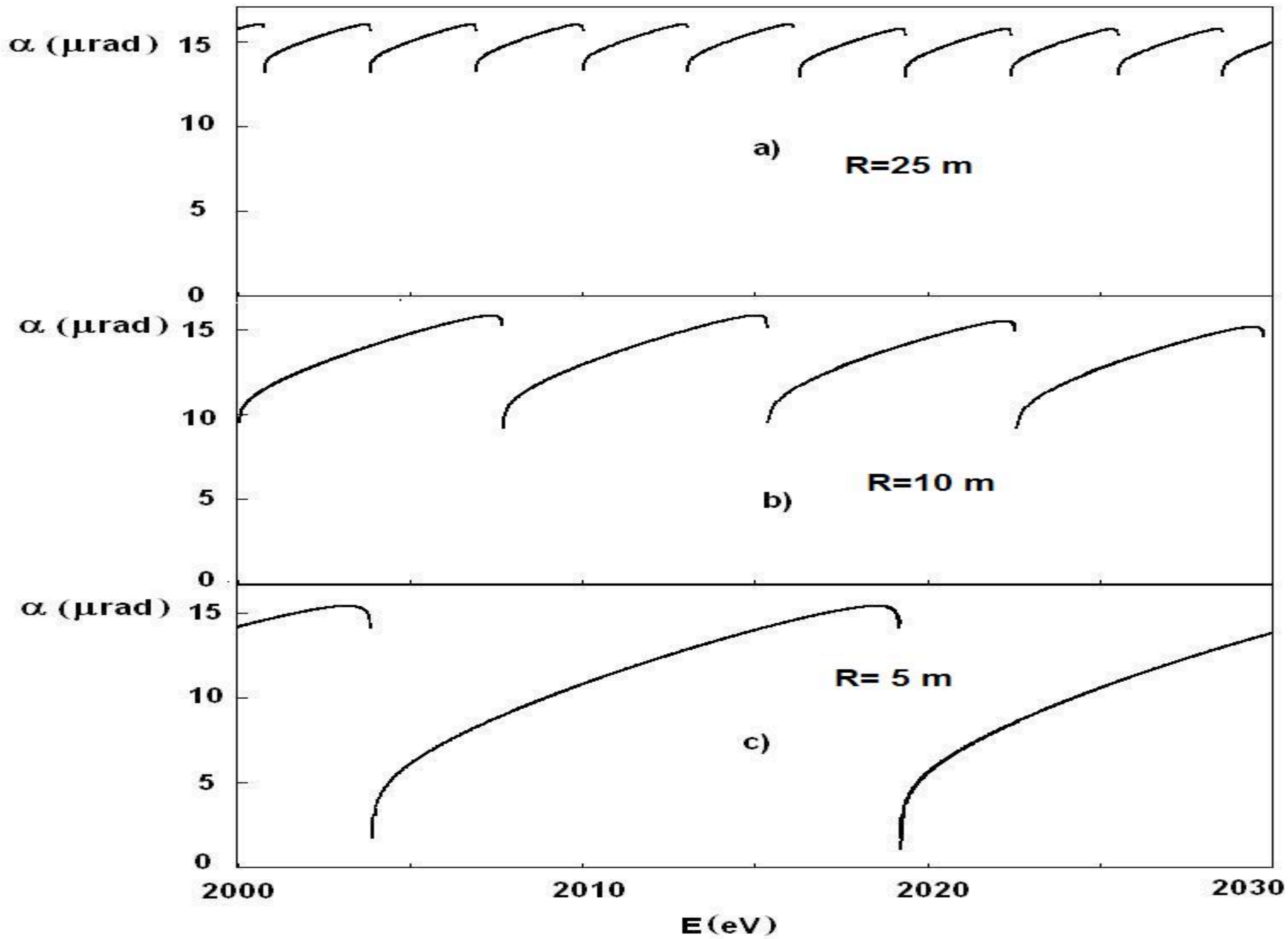
$U(x)$ is the periodic planar potential in the straight single crystal as a function of the coordinate,

R is the radius of bending. The critical point x_c is the solution of the equation:

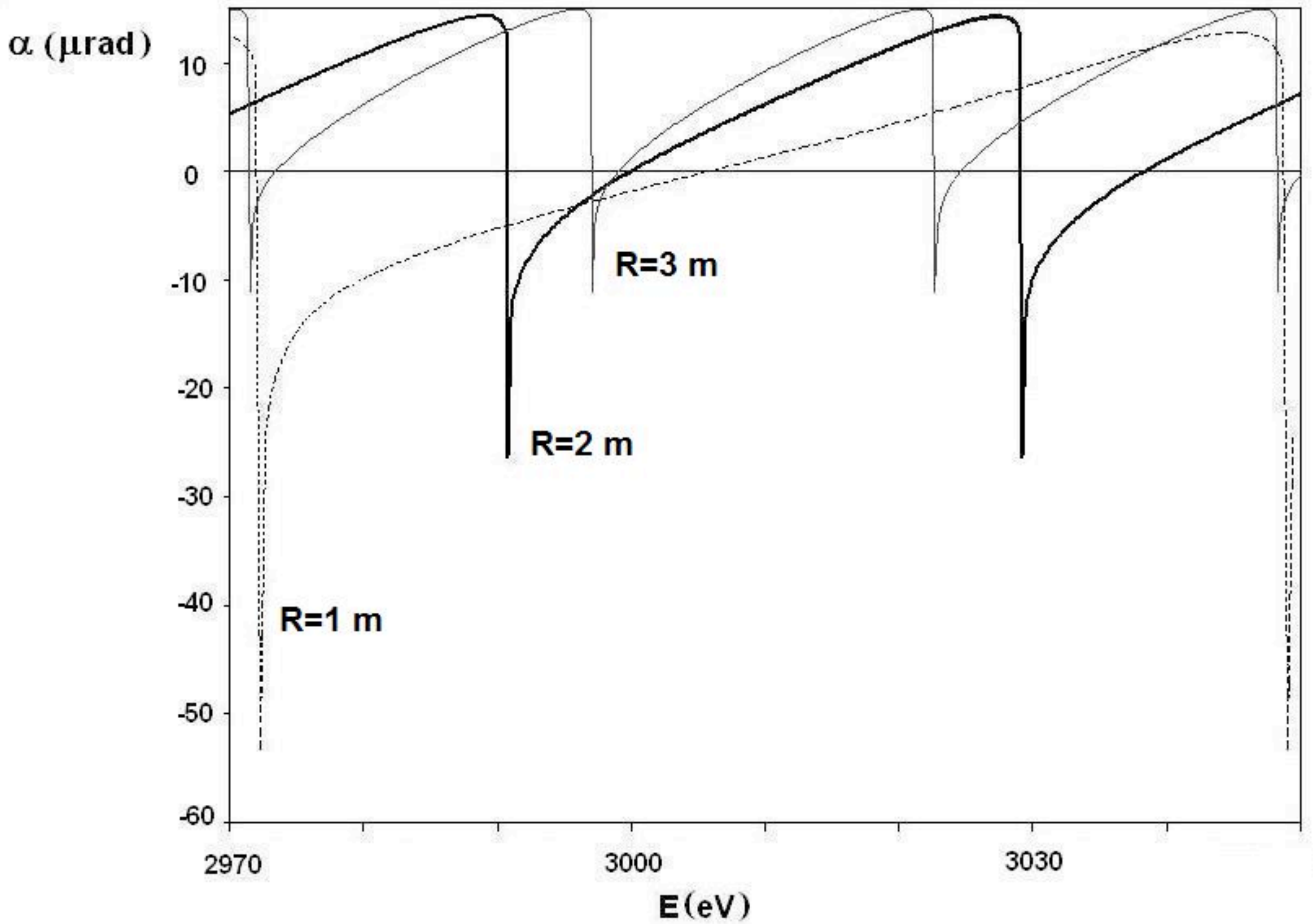
$$E - U(x_c) - E_0\beta^2 x_c/R = 0;$$

x_0 is the initial coordinate of a particle.

$\alpha(E)$ **is a periodic function of the transversal energy with the period** $\delta E = E_0\beta^2 d/R.$ **(d is the interplanar distance).**



α as a function E


α (μrad)
10
0
-10
-20
-30
-40
-50
-60
R=3 m
R=2 m
R=1 m
2970
3000
3030
E (eV)

Then we find the different distribution functions for particles scattered in the process. In particular the distribution function over transversal energy close to equiprobable one. It allows one to write for mean and mean square volume reflection angles:

$$\langle\alpha\rangle = \frac{1}{\delta E}\int_E^{E+\delta E} \alpha(E)dE,$$
$$\sigma_{vr}^2 = \frac{1}{\delta E}\int_E^{E+\delta E} (\alpha(E) - \langle\alpha\rangle)^2 dE$$

where δE **is the period.**

2nd step. Introduction of multiple scattering.

We got for beam with the narrow divergence the following equation:

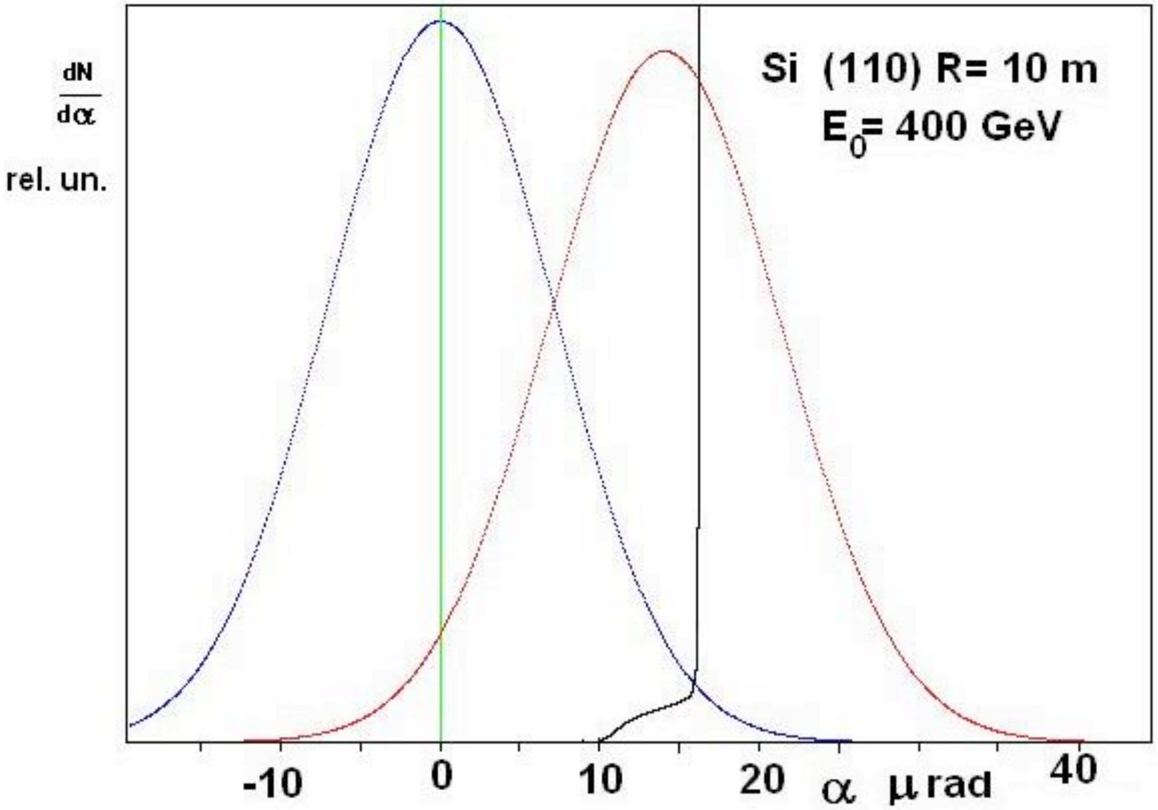
$$\rho_e(\alpha) = \int_{-\infty}^{\infty} \rho_m(\varphi)\rho_v(\alpha - \varphi)d\varphi$$

where $\rho_v(\alpha)$ **is the distribution over angle of VR (without multiple scattering)** $\rho_m(\alpha)$ **is the distribution due to multiple scattering, and** $\rho_e(\alpha)$ **is the result distribution.**

We got also important result, that at introducing multiple scattering (at suggestion of its gauss form) the equation for this angle is previous.
For mean squared value of volume reflection angle we got

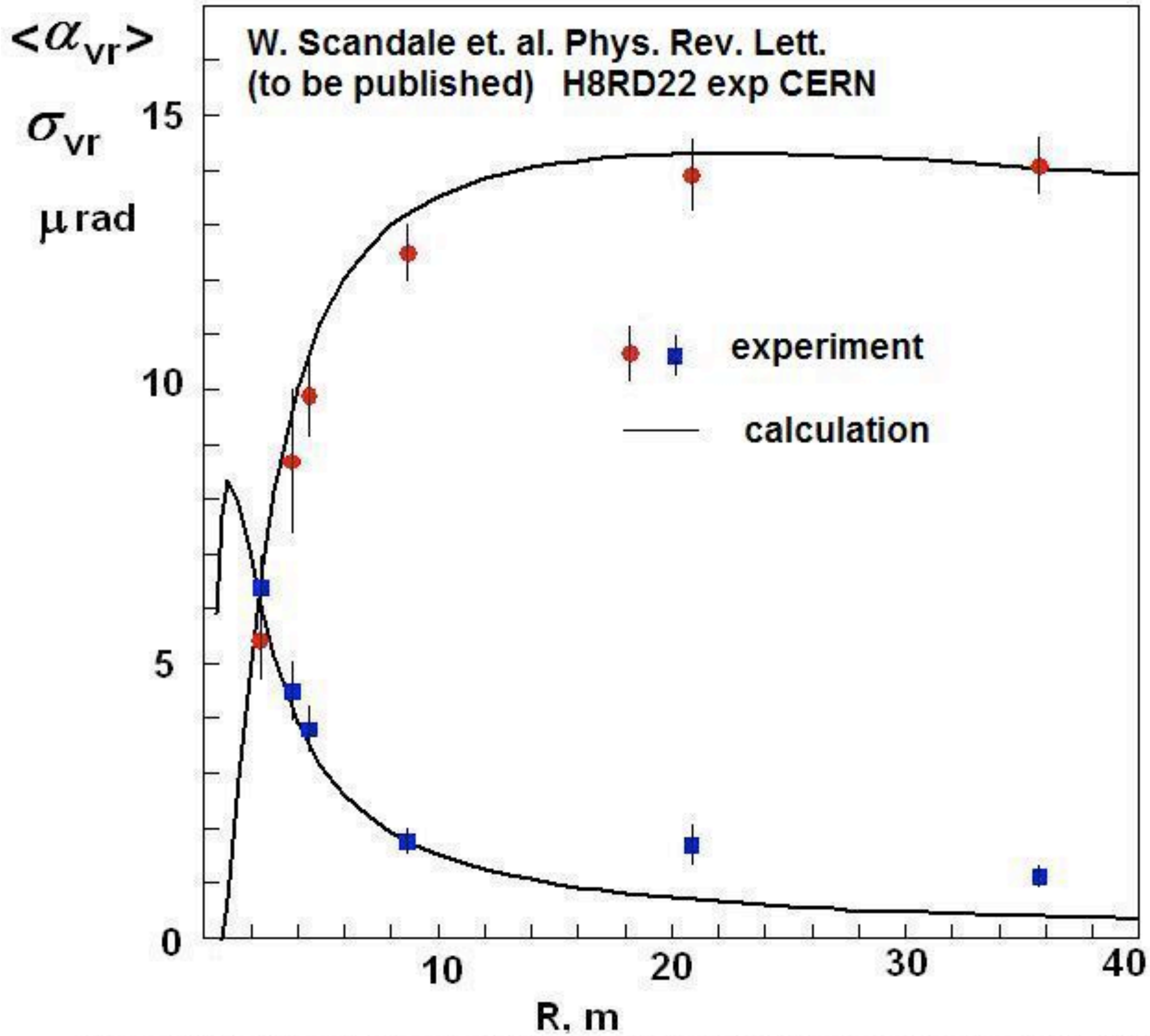
$$\sigma_T^2 = \sigma_{vr}^2 + \sigma_m^2 + \sigma_b^2,$$

where σ_{vr} is coherent (or potential) mean squared part,
σ_m is mean squared part due to multiple scattering on atoms,
σ_b is mean squared divergence of the beam.



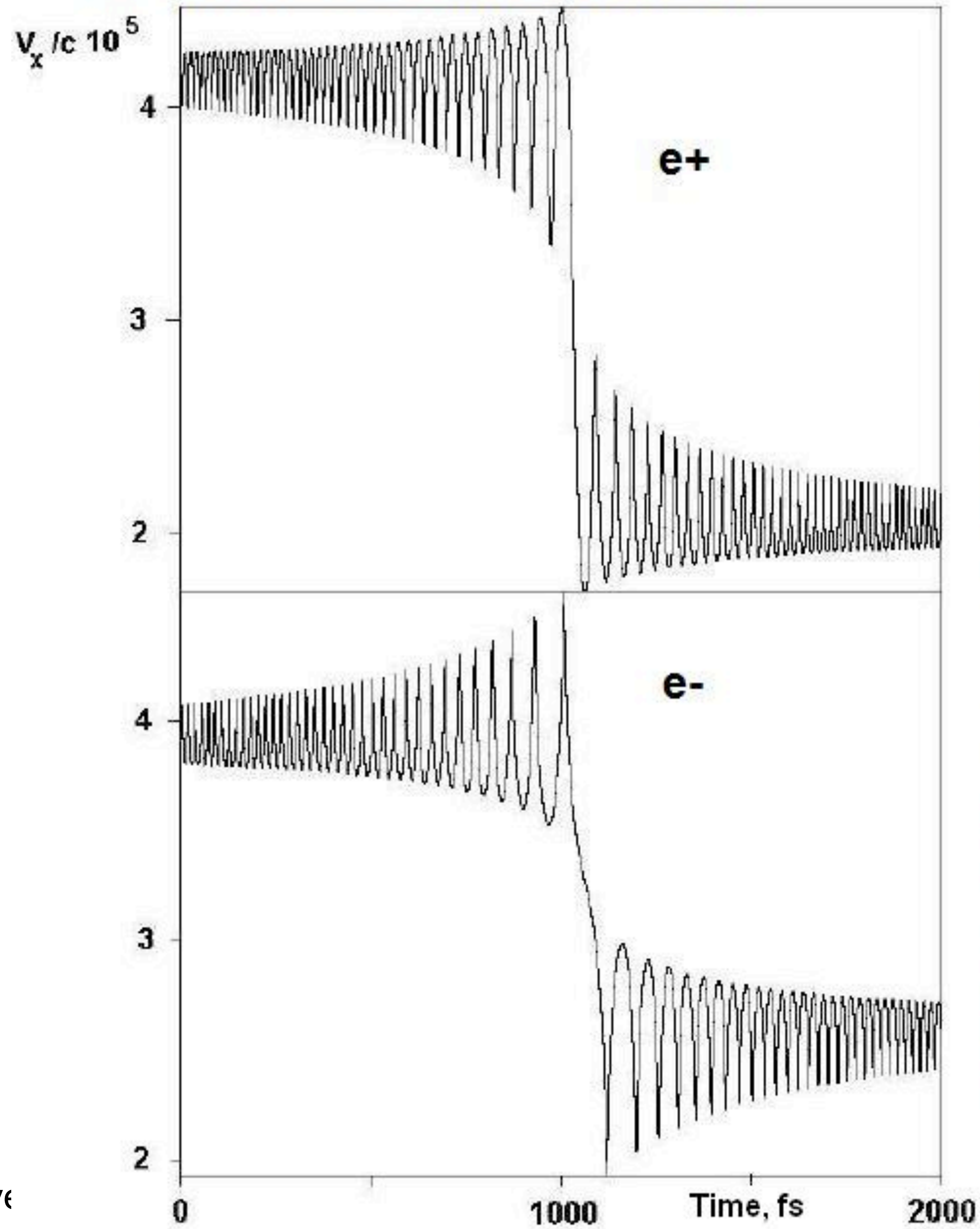
Recently in H8RD22 experiment (beam line N H8) in CERN
the parameters of voume reflection were measured at different radii.
In the experiment 2 mm silicon single crystal was used
with the (110) working plane. The energy of proton beam was 400 GeV.

The paper with the results of this experiment accepted for
publication in Phys. ReV. Lett.
In the first time these results were presented on recent FNAL
conference (W. Scandale)



In the first time these results were presented on the recent FNAL conference (W. Scandale):
http://tdserver1.fnal.gov/project/worksop/crystal_collimation/agenda.html

Radiation at volume reflection



Yu. A. Chesnokov,
V.I. Kotov,
V.A. Maisheev
I.A. Yazynin,
JINST 3, P02005 (2008)

Relative transversal velocities of 200 GeV positrons and electrons at Volume Reflection as functions of time. Si, (110),0.6 cm thickness

Main peculiarities

1) Aperiodicity
2) Variation of amplitude
3) Deflection on the angle more than charac-teristic radiation angle ($1/\gamma = 2.5\ \mu$rad).

The character of the radiation process is determined by ρ parameter.
When $\rho \ll 1$ process has the interference character,
When $\rho \gg 1$ process has the synchrotron- like character.
When $\rho \sim 1$ is the intermediate case.

For radiation at the quasiperiodic motion

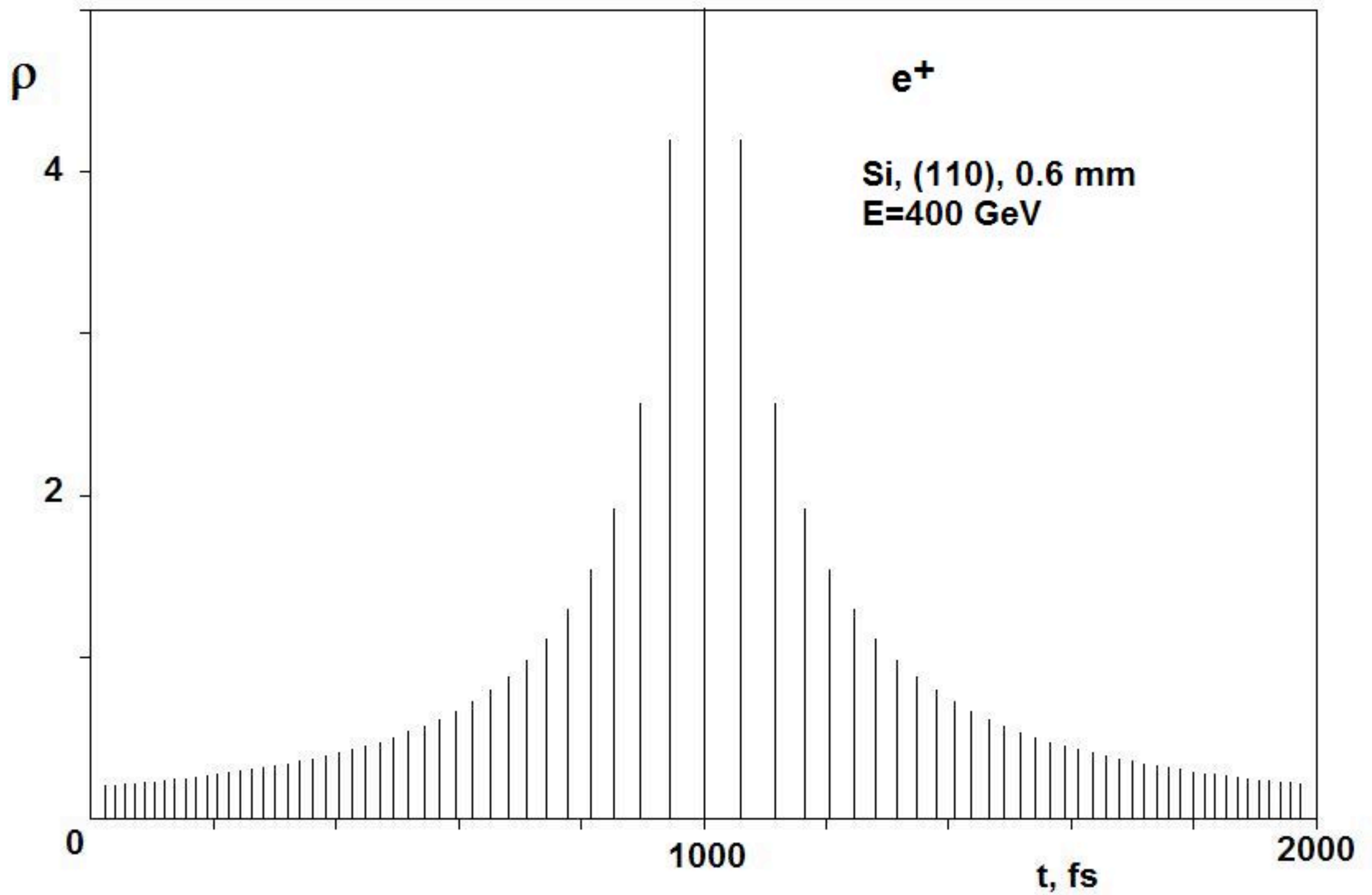
$$\rho = 2\gamma^2 < (v(t) - v_m)^2 > / c^2 \qquad v_m \text{ is the mean velosity}$$

In the planar case in thin straight crystals the character of the process is determined by initial angle of particle relative to plane direction θ and it is conserved at the passage through the crystal (approximately).

Here when $\theta \ll \theta_b$ $\rho \gg 1$ (synchrotron-like radiation)
when $\theta \gg \theta_b$ $\rho \ll 1$ (Coherent bremsstrahlung)

$$\theta_b = U/mc^2$$

In bent crystal crystal angle relative derection of plane for moving particle changed. In this case taking account the gradual variation of the period at particle aproaching to reflection point we can find the ρ -parameter for every separate oscillation. These calculations (for the case corresponding to the previous picture) are shown in the figure.



ρ
e+
Si, (110), 0.6 mm
E=400 GeV
4
2
0
1000
2000
t, fs

Thus, at motion of particle in bent crystal the character of radiation process is changed. This peculiarity depends on the thickness of the crystal and particle energy.
Our estimation of parameter in the area of reflection point is

$\rho = 0.5\,\gamma\,\theta_b$ for positrons

$\rho = \gamma\,\theta_b$ for electrons

$\theta_b = U/mc^2$

Besides , ρ =1 for 12 GeV positrons and 24 GeV electrons

Thus, at low energies electrons and positrons radiation process is similar to coherent bremsstrahlung.

One can expect that energy spectrum of emmited photons depends on current location of particles, hence we cannot use such conception as the intensity of radiation (as convinient in straight crystals).

The energy range of emmited photons one can estimate from the relations:

$$\omega = \frac{2\gamma^2\omega_0}{1+\rho/2}$$

$$E_{\gamma,max} = \frac{\hbar\omega E_0}{E_0+\hbar\omega}$$

where $\omega_0 = 2\pi/T$ and T is the period of every oscillation.

Our specific calculations were based on quasiclassical theory of QED processes developed by V.N. Baier with coworkers.

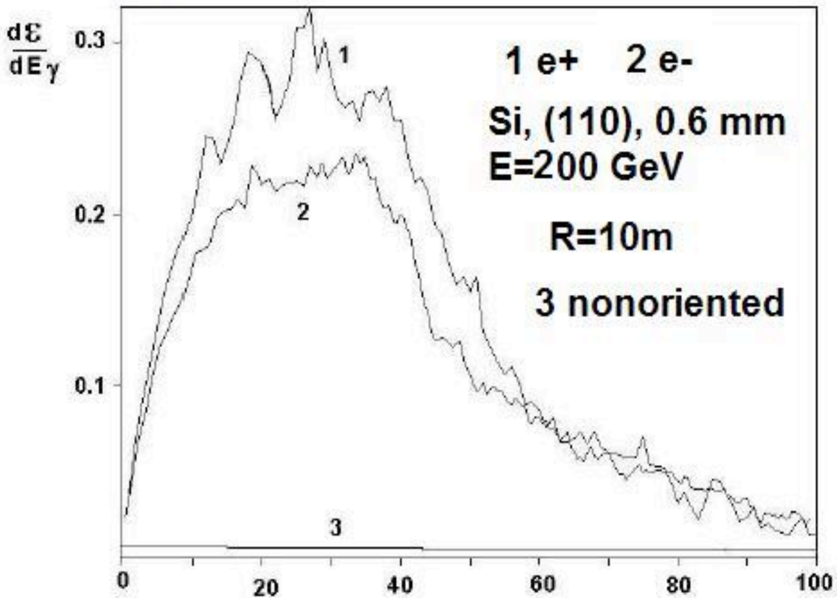
see:

V.N. Baier, V.M. Katkov and V.M. Strakhovenko Electromagnetic processes at high energies in oriented single crystals, Singapore, Singapore World Scientific, 1998.

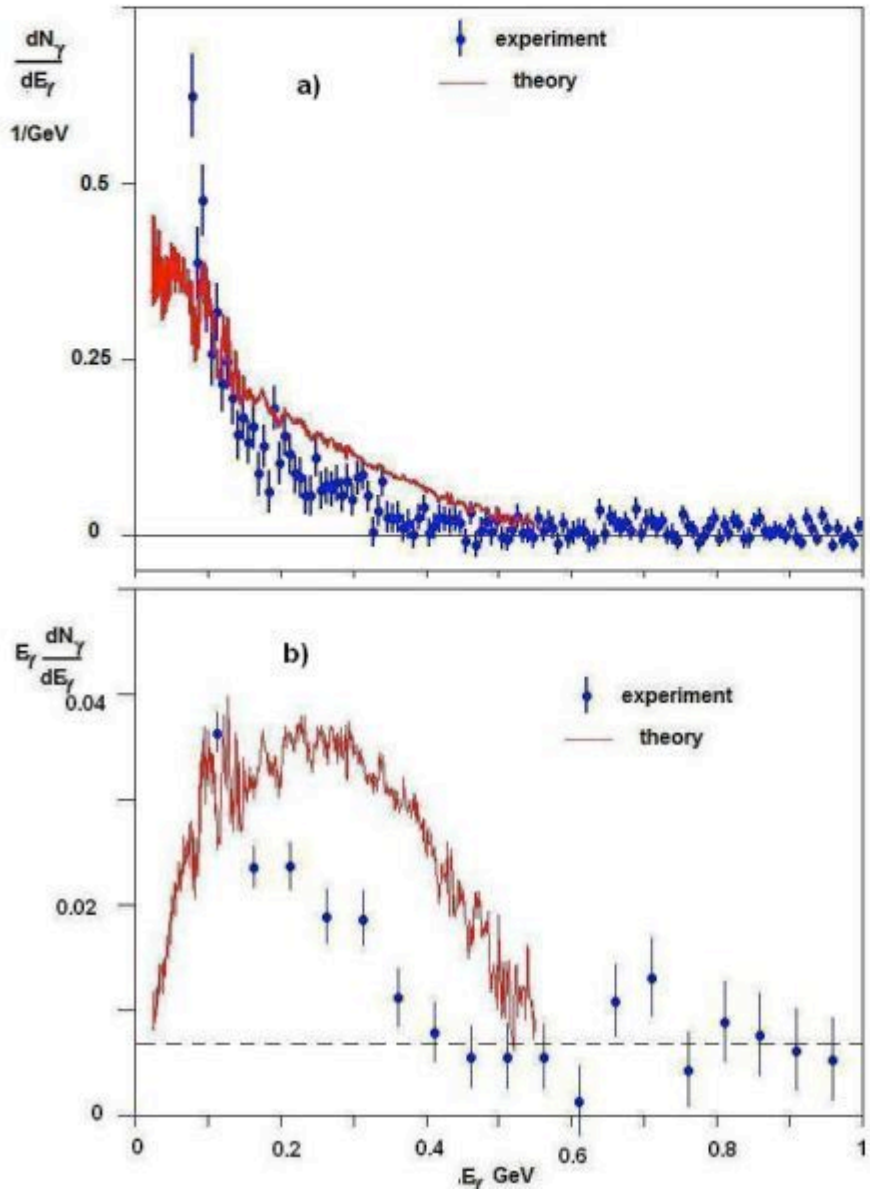
$$\frac{d\mathcal{E}}{dE_\gamma} = \frac{i\alpha m^2 c^4}{2\pi\varepsilon^2}\omega\int_{\mathbf{D}}\frac{dtd\tau}{\tau-0}\left\{1+\frac{\varepsilon^2+\varepsilon'^2}{4c^2\varepsilon\varepsilon'}\gamma^2[\Delta\mathbf{v}(t-\tau/2)-\Delta\mathbf{v}(t+\tau/2)]^2\right\}\exp -iA_1, \quad (3.1)$$

$$A_1 = \frac{\omega\varepsilon\tau}{2\varepsilon'}\left[\frac{1}{\gamma^2}+\frac{1}{\tau}\int_{-\tau/2}^{\tau/2}ds(\Delta\mathbf{v}(t+s)/c)^2-\left(\frac{1}{\tau}\int_{-\tau/2}^{\tau/2}ds\Delta\mathbf{v}(t+s)/c\right)^2\right], \quad (3.2)$$

where $\Delta\mathbf{v}(t,\mathbf{v}_0) = \mathbf{v}(t_1) - \mathbf{v}_0$ is the velocity variation as a function of time t_1, m and γ are the mass and Lorentz factor of particle, E_γ, ω are the energy and frequency of photon, ε is the particle energy, $\varepsilon = \varepsilon - E_\gamma$. The time variables t_1 and t_2 (t_2 is time variable as t_1) connected with variables t and τ by equations: $t_1 = t - \tau/2$ and $t_2 = t + \tau/2$. **D** is the domain of definition of integrand function.



Calculated radiation energy losses of positrons and electrons in bent single crystal.



Experiment on accelerator IHEP (Protvino)

IHEP - PNPI (Gatchina)

A.G. Afonin et. al.
JETP Letters 88, 488, 2008

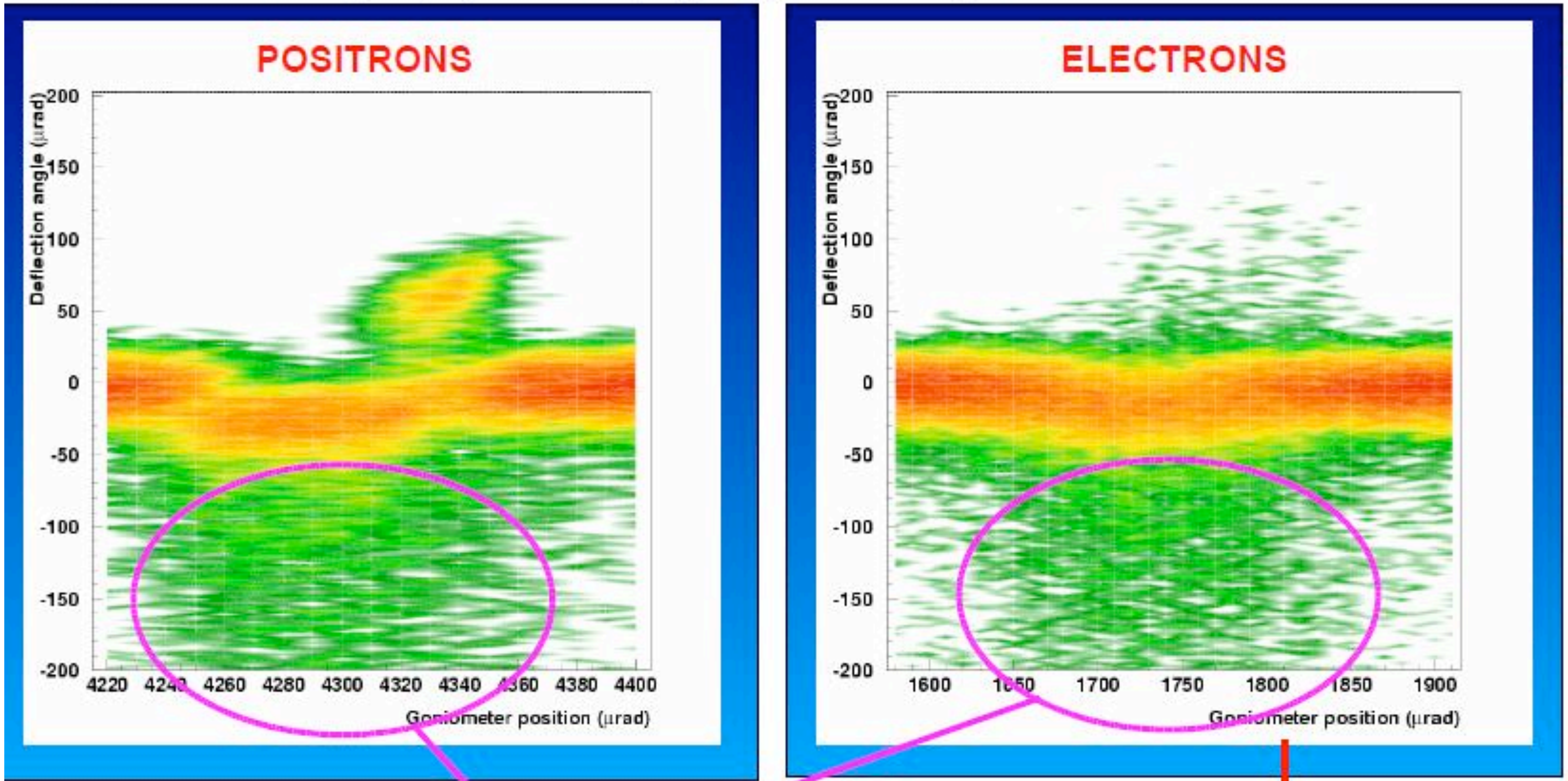
10 GeV positron beam
bent silicon crystal,
(111) plane, 0.65 mm thick.
R=1.3 m (bending radius)

Only coherent part of the spectra are presented

For comparison:
Dashed line in fig. b) represents the calculated incoherent contribution.

Experiment in CERN (H8RD22)
In the first time these results were presented on the recent FNAL conference (W. Scandale):
http://tdserver1.fnal.gov/project/worksop/crystal_collimation/agenda.html

W. Scandale et. al.
Submitted in Phys. Rev. A



P=180 GeV/c

Beam
23% e
65% μ
12% h

e+/ e- having lost energy via radiation emission

The crystal is not ideal but it's there!!!!!!

W. Scandale, FNAL-06/12/07

Experiment in CERN (H8RD22)

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W. Scandale et. al.
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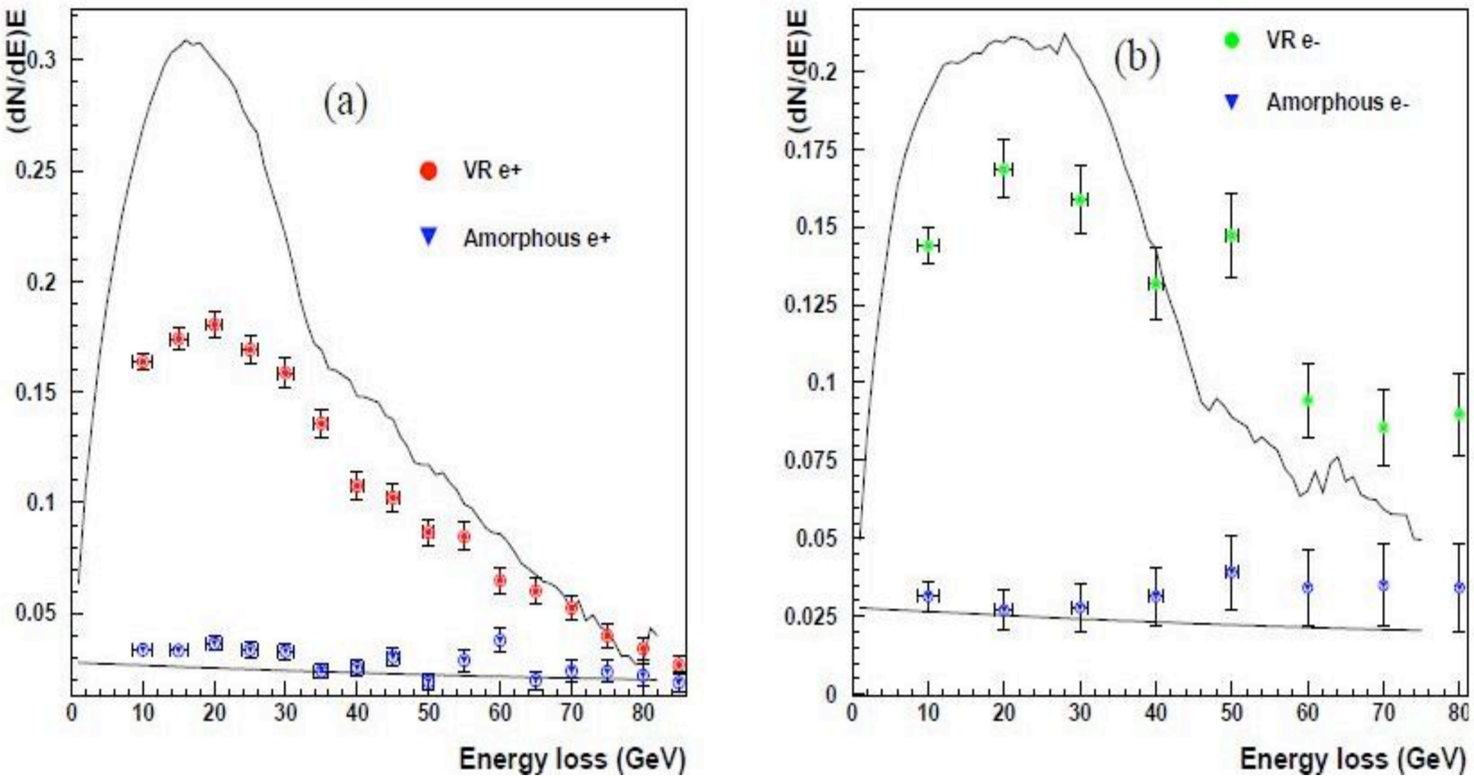
P = 180 GeV/c

Si (111) Thickness 0.9 mm (electrons)
0.84 mm (positrons)

R=8m electrons 12 m positrons



the observed disagreement can be explained as follows:

1. the multiple scattering of particles on the crystal atoms has not been considered in the analytical calculations;

2. the presence of a non homogeneity of the quasi-mosaic crystal structure and in particular the dependence of the bending radius from the particle hit position and the torsional effect due to the holder;

3. a consistent number of particles ($\sim 10 - 15\%$) may be captured in the channeling regime; even if they can exit channeling quickly, the motion changes;

4. the radiation of two or more γ-quanta by one particle has not been considered in the calculation.

Conclusions:

1) the presented analytical description of Volume reflection allow one to calculate different characteristics of the process and are in a good agreement with measurements.
2) Using generalizing parameters in description one can extend the experimental (or computing) data on on others energies.
3) the calculations of radiation at volume reflection show high level of energy losses of light leptons. This fact was confirmed experimentally.
4) The further development of methods of calculation is needed for the radiation process in the area of volume reflection for its better describing.