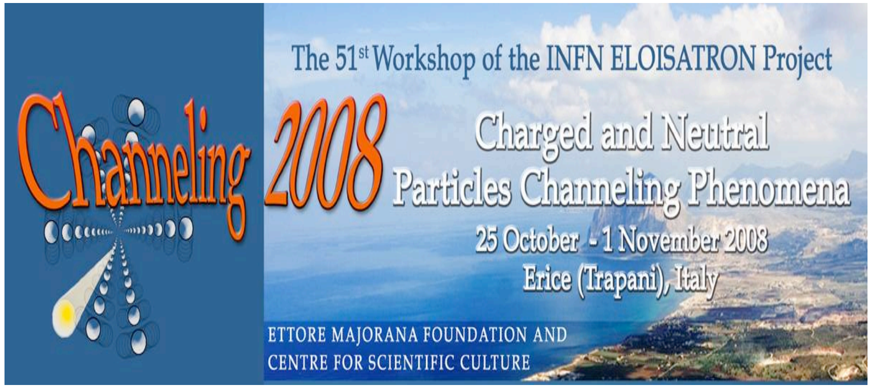
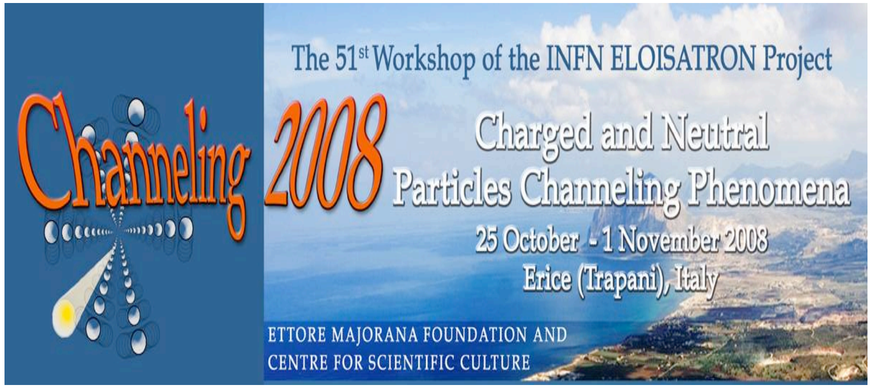


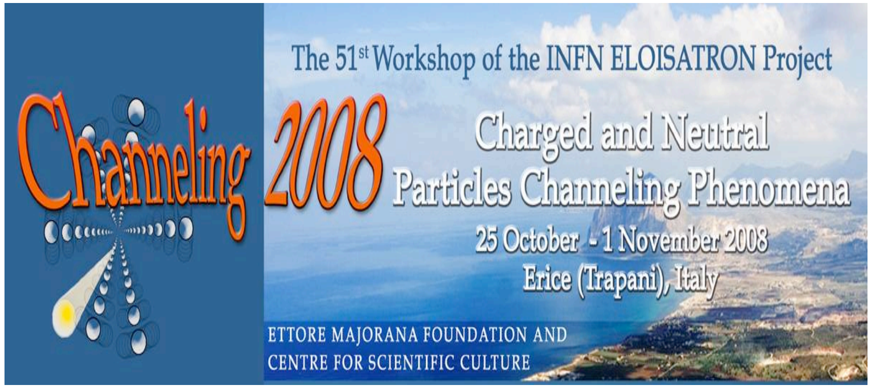
Creation of Relativistic Positronium Atom by Relativistic Axially Channeled Electron

Yu.P. Kunashenko[1,2], Yu.L. Pivovarov[1]
[1]Tomsk Polytechnic University
[2]Tomsk State Pedagogical University





- Historical review
- Theoretical consideration
- Results of calculation
- Conclusions

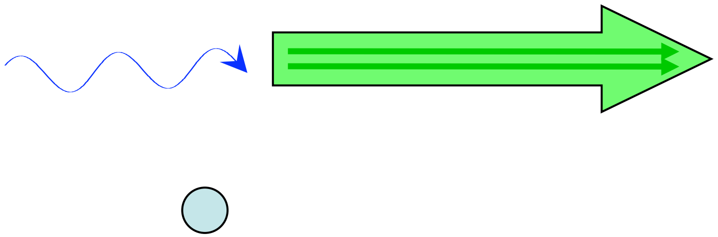


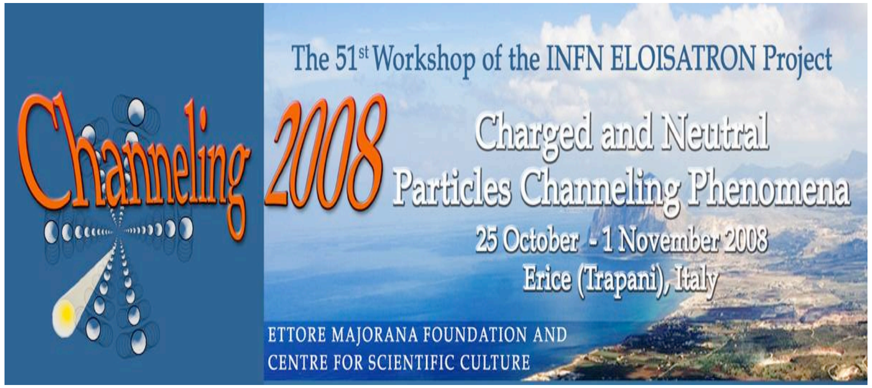
Positronium atom (Ps) production by photon and electron in Coulomb Field

1. G.V. Meledin, V.G. Serbo and A.K. Slivkov, Pis'ma Zh. Eksp. Teor. Fiz 13 (1971) 98 [JETP Lett. 13 (1971) 68].

2. H.A. Olsen, Phys.Rev.D. 33, (1986) 2033.
3. V.L. Lyuboshits, Yad.Fiz. 45 (1987) 682.

4. E. Holvik, H.A. Olsen, Phys.Rev.D. 35 (1987) 2124.

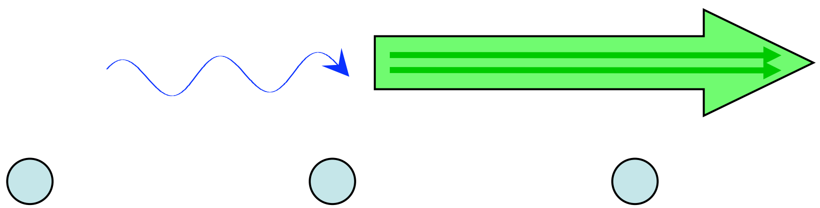




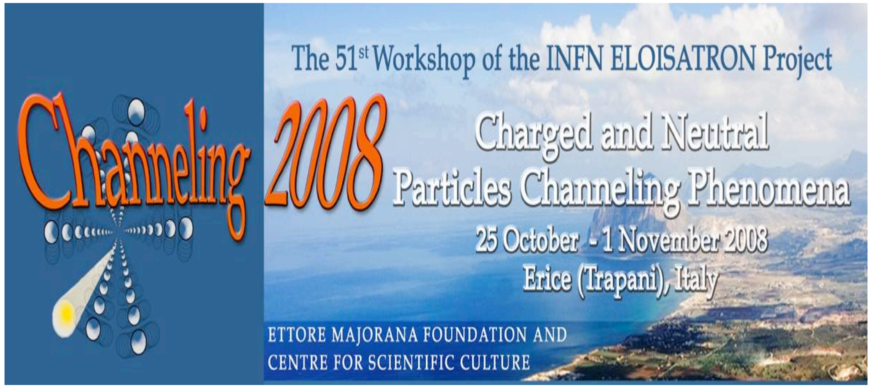
Coherent Ps production by photon and electron in a crystal

When high energy photon pass through a crystal parallel crystal axis coherent effect in Ps photo-production take place.

1. Yu.P. Kunashenko, Yu.L. Pivovarov, Yad.Fiz. 51 (1990) 627.
2. G.I. Sandnes, H.A. Olsen, Phys.Rev.A 48 (1993) 3725.
3. Yu.L.Pivovarov, Yu.P.Kunashenko, I.Endo, T.Isshiki, NIM B 145 (1998) 80.
4. Yu.P. Kunashenko, NIM B 229 (2005) 219.

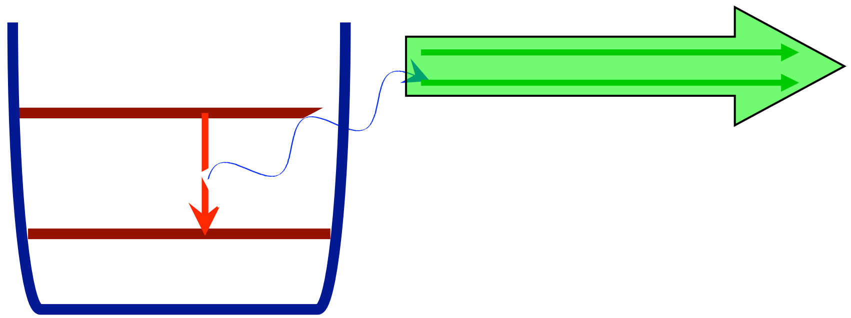


As a result sharp peaks appear in dependence of Ps cross-section production on created Ps energy and emission angles.

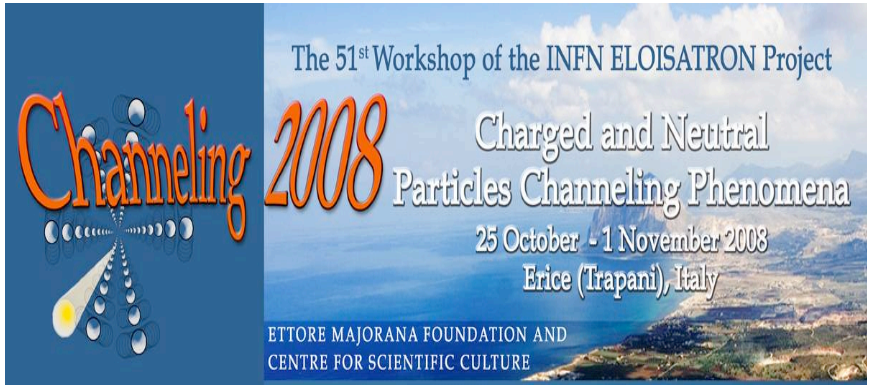


The Ps atom can appear also under conversion transitions in nuclei.

V.M. Kolomiets, S.N. Fedotkin, Yad.Fiz. 56 (1993) 92.



Ps atom production

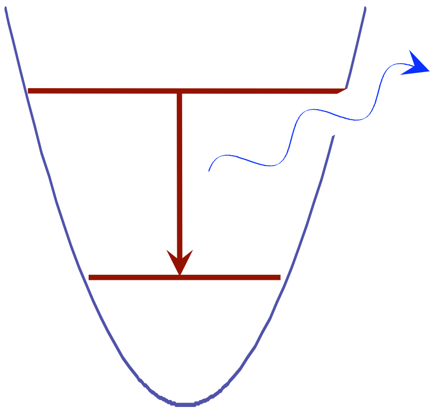


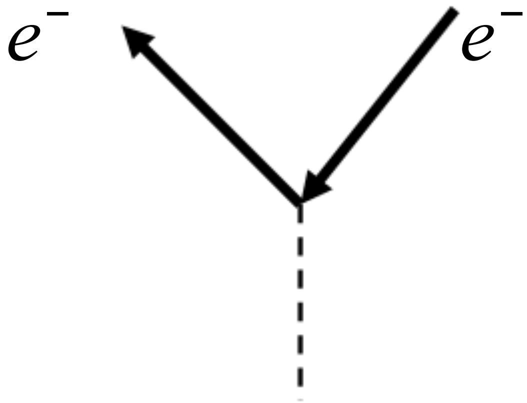
Channeling Radiation

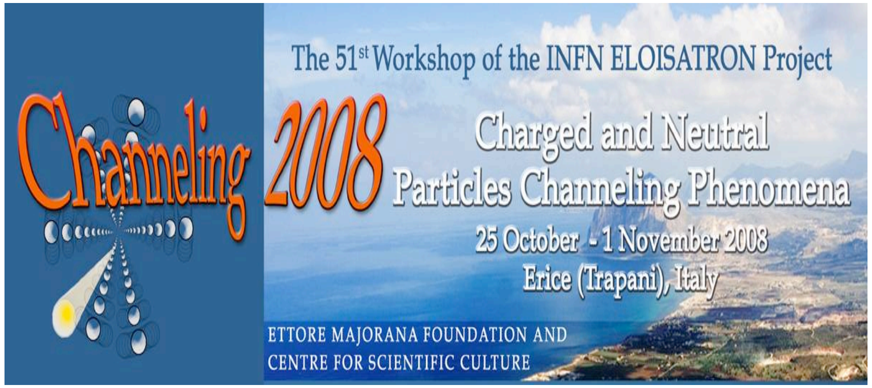
When relativistic electron pass through an alignment crystal with small angle with respect to crystal axis or plane it can be captured in a channeled states. Channeled electron has discreet transverse energy levels. During transition from one energy level to another it can emit photon.

V. N. Baier, V. M. Katkov, V. M. Strakhovenko. Electromagnetic Processes at High Energies in Oriented Single Crystals, World Scientific Publishing Co, Singapore, 1998.

Bazulev V.A. Zhevago N.K. Radiation of Fast Particles in Matter and External Fields Moscow., Nauka.,1987. (in Russian)

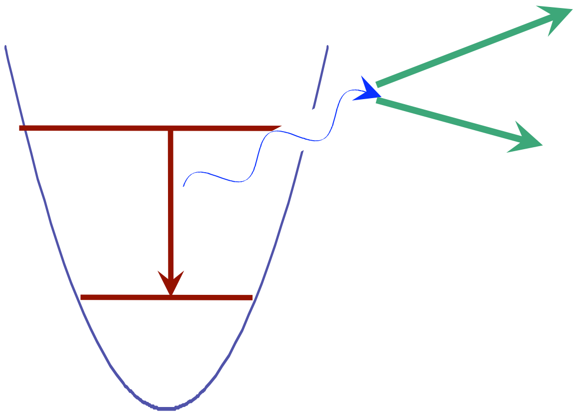


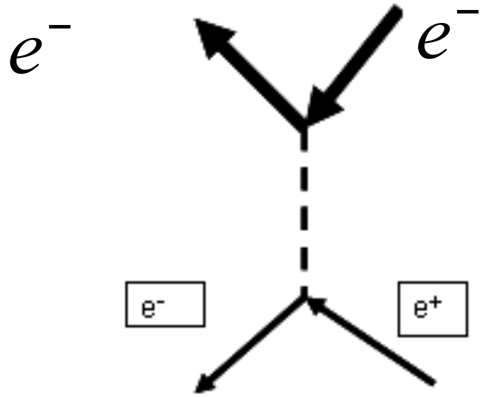


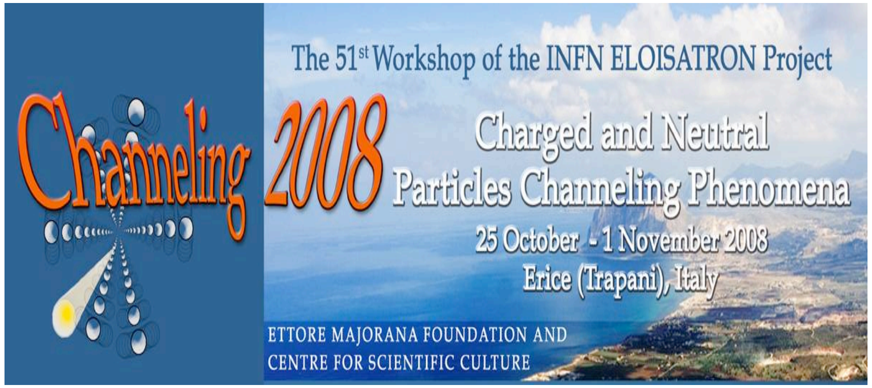


Pair Production

Cue N., Kimball J .C.// Phys.Lett. 1987 -124-191.
Kimball J .C., Cue N., Roth L.M. and March.B.B.// Phys.Rev.Lett. 1983 -50-950.
Belkacem A.,Bologna G. et.al. Observation of Enhanced//Phys. Rev. Lett.-1984-v.53, n 25 -p.2371-2373.
Belkacem A.,Bologna G.et.al. //Nucl.Instr.and Meth.-1986-v.B13-p.9-14. 1196-1199.





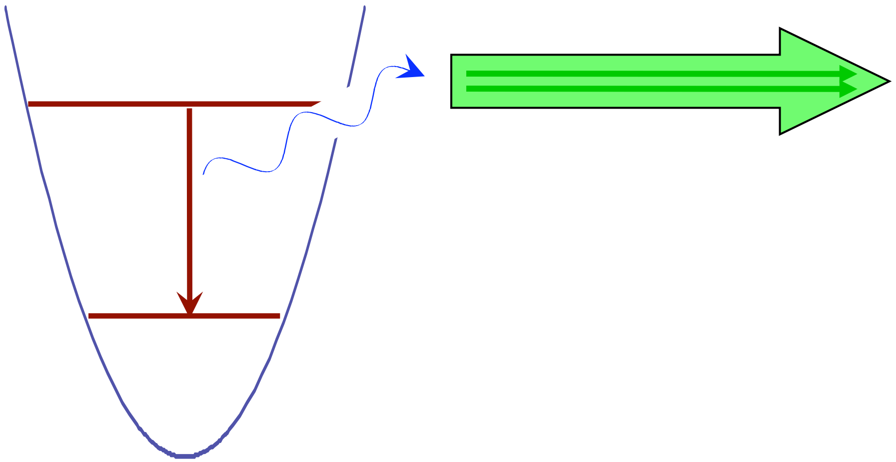


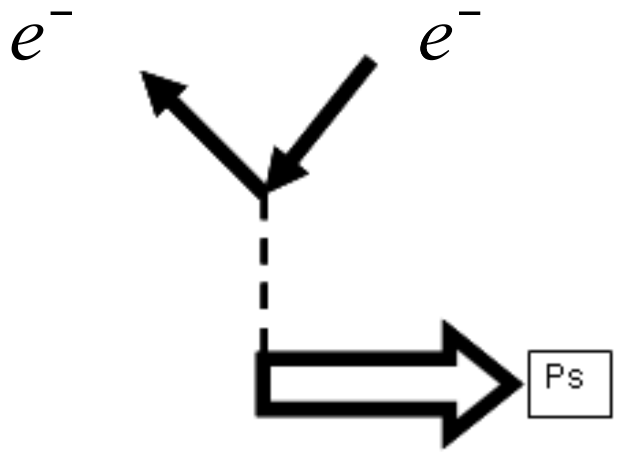
Ps atom production

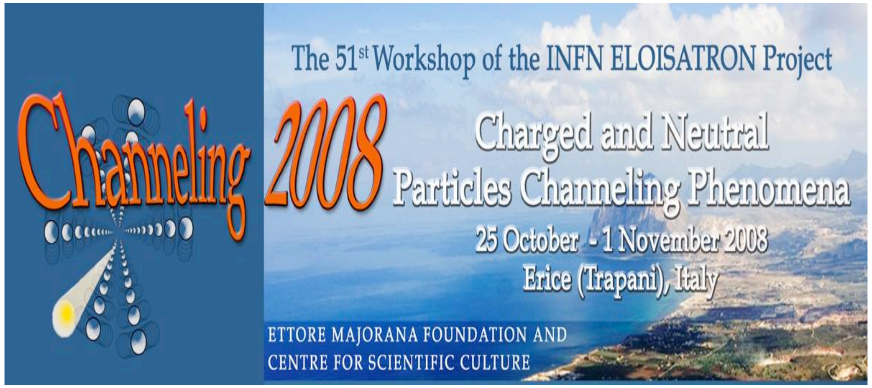
Present report

Due to conservation of charge parity only singlet Ps atom can be produced in a first Born approximation

Ps cross – section production is connected with e+e- pair cross – section production.







Electron – positron pair production probability by axially channeled electron

$$dW = 2\pi |M|^2 \delta\left(E_i - E_f - E_+ - E_-\right) d\nu$$

Matrix element $M = \int d\vec{r}_1 d\vec{r}_2 \overline{\Psi}_f(\vec{r}_1)\gamma\Psi_i(\vec{r}_1) D \overline{\Psi}_-(\vec{r}_2)\gamma\Psi_+(\vec{r}_2)$

D – is a photon propagator

$$D_{\mu\nu}(\vec{r}_1, \vec{r}_2) = g_{\mu\nu} D(\vec{r}_1 - \vec{r}_2), \quad D(\vec{r}) = 4\pi \int \frac{e^{i\vec{k}\vec{r}}}{\omega^2 - k^2 + i0} \frac{d\vec{k}}{(2\pi)^3}$$

$$d\nu = \frac{d\vec{p}_{||}}{(2\pi)^2} \frac{d\vec{p}_- d\vec{p}_+}{(2\pi)^3 (2\pi)^3}$$ - Phase volume

$$V(\rho) = -\frac{V_0}{\rho}$$

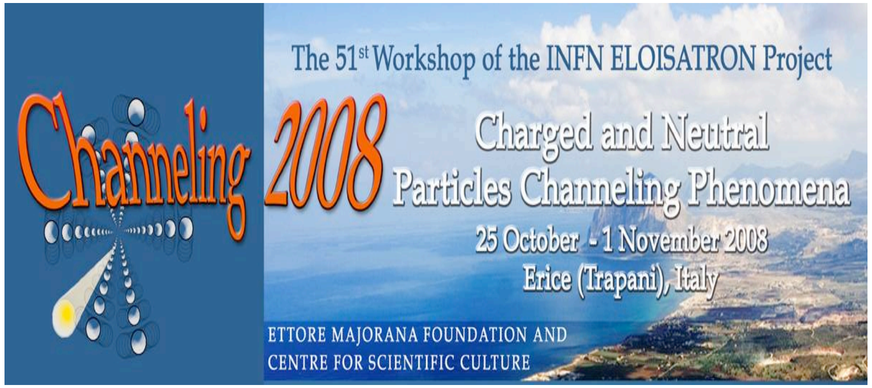
Axially channeled e- wave function

$$\Psi_{i(f)}(\vec{r}) = \frac{1}{\sqrt{2E_{i(f)}}} \varphi_{i(f)}(\vec{\rho}) \exp(i\vec{p}_z \vec{r}_z) u(\vec{p})$$

$\varphi_{i(f)}(\rho)$ – is a wave function of bound state of e- transverse motion.

$u(\vec{p})$ - is free particle spinor.

$\Psi_{\pm}(\vec{r})$ - plane wave, which describes created e+ (e-).



$$dW = \alpha^2 \frac{1}{2\pi^3} \frac{1}{Q^4} \left| J_\perp \right|^2 T_{if}^2 \, \delta\left(E_i - E_f - E_- - E_+\right)$$

$$\delta\left(\vec{p}^{\,\|}{}_i - \vec{p}^{\,\|}{}_f - \vec{p}_+{}^{\|} - \vec{p}_-{}^{\|}\right) d\vec{p}_+ d\vec{p}_- d\vec{p}_f$$

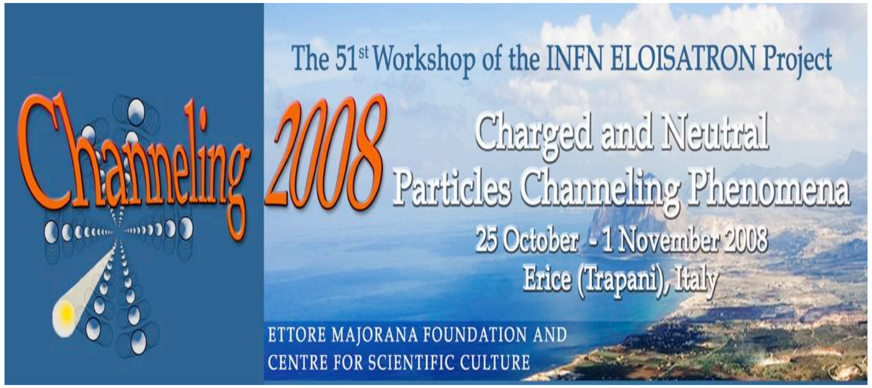
,

$$T_{if}^2 = \frac{1}{E_i E_f E_- E_+} \left[g^{\mu\nu} \left(m^2 - P_i P_f \right) + P_i^\mu P_f^\nu + P_i^\nu P_f^\mu \right]$$

$$\left[P_+^\mu P_-^\nu + P_+^\nu P_-^\mu - g^{\mu\nu} \left(m^2 + P_+ P_- \right) \right]$$

$$Q^2 = \left(E_i - E_f\right)^2 + \left(\vec{p}_+ + \vec{p}_-\right)^2$$

$$\left(J_{if}\right)_\perp = \int \varphi_i^*(\vec{\rho}) \exp\left(i\vec{p}_\perp \vec{\rho}\right) \varphi_f(\vec{\rho}) \, d\vec{\rho} \qquad \vec{p}_\perp = \left(\vec{p}_+ + \vec{p}_-\right)_\perp$$



Ps production probability

$$\vec{p}_+ = \vec{p}_- = \vec{p}\big/2 \,,$$
$$\mathring{A}_+ = \mathring{A}_- = \mathring{A}_{Ps}\big/2 \,.$$

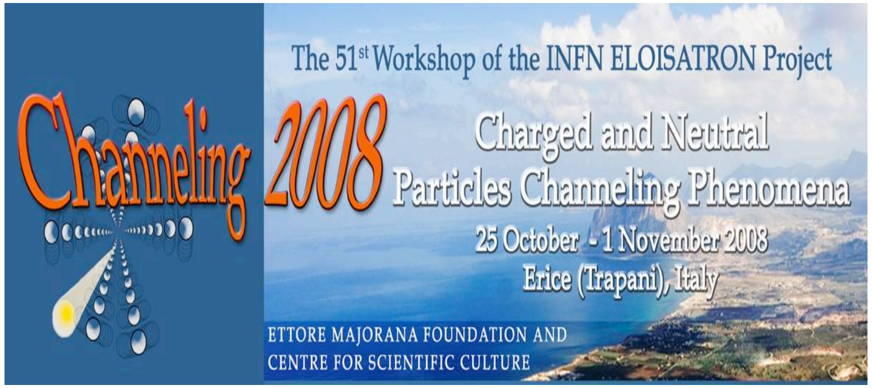
The probability of Ps creation can be obtain from one for e+e- production for equal energy and moments e+ and e-.

dividing by e+e- phase factor $$\frac{d^3\vec{p}_+ d^3\vec{p}_-}{(2\pi)^6 dE_-}\frac{E_+}{m}\frac{E_-}{m}$$

and multiplying by Ps phase factor $$\frac{d^3\vec{p}}{(2\pi)^3 dE}\frac{E}{m_{Ps}}\left|\Psi_{Ps}(0)\right|$$

Ψ_{Ps} - Ps wave function

H.A. Olsen, Phys.Rev.D. 33, (1986) 2033.
V.L. Lyuboshits, Yad.Fiz. 45 (1987) 682.



Ps production probability by axial channeled electron

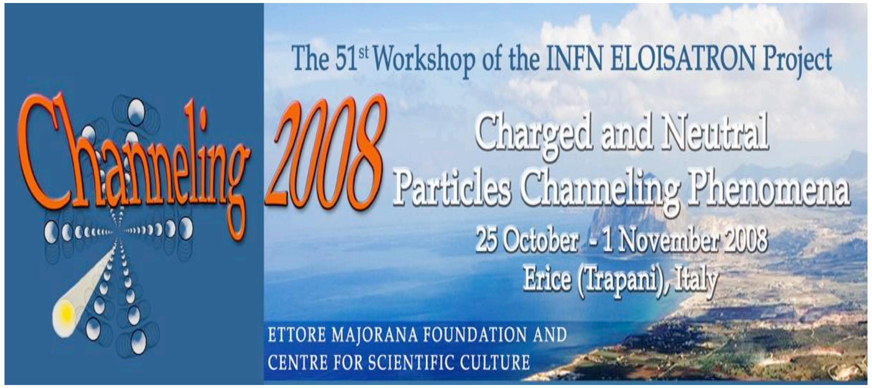
$$dW_{if} = \alpha^2 \frac{1}{m^2{}_{Ps} m^2} \left|\Psi_{Ps}(0)\right| \cdot \left|J_{if\perp}\right|^2 T_{if}{}^2$$
$$\delta\left[E_i - E_f - E\right]\delta\left[\vec{p}^{\|}{}_i - \vec{p}^{\|}{}_f - \vec{p}^{\|}\right] d^3\vec{p}\, d^2\vec{p}^{\|}{}_f$$

Here $J_{if\perp}$
- is a transverse matrix element:

$$J_{if\perp} = \int \varphi_i^*(\vec{\rho}) \exp(i\vec{p}_\perp \vec{\rho})\, \varphi_f(\vec{\rho})\, d\vec{\rho}$$

$p_\perp = p \sin\theta$ is Ps momentum in direction perpendicular to the crystal axis.

$$T_{if}{}^2 = \frac{1}{E_i E_f}\Big(E^2 E_i E_f + 2E_i^2 m^2 - 6m^4 - 2p_f p_i m^2$$
$$- EE_f(\vec{p}\vec{p}_f) - EE_f(\vec{p}\vec{p}_i) + (\vec{p}\vec{p}_i)(\vec{p}\vec{p}_i)\Big)$$



Conservation laws:

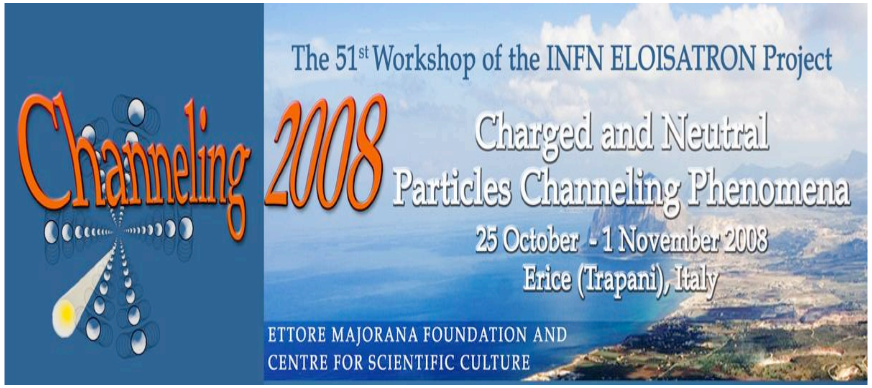
Energy $$\delta\left[E_i - E_f - E\right],$$

Longitudinal momentums: $$\delta\left[\vec{p}^{\parallel}{}_i - \vec{p}^{\parallel}{}_f - \vec{p}^{\parallel}\right],$$

$$E_i - \underbrace{\sqrt{m^2 - \left(P_i^2 - P_{Ps}\cos\theta\right)}}_{=E_f} + \underbrace{\Delta\varepsilon_{\perp}}_{=\varepsilon_i - \varepsilon_f} = E_{Ps},$$

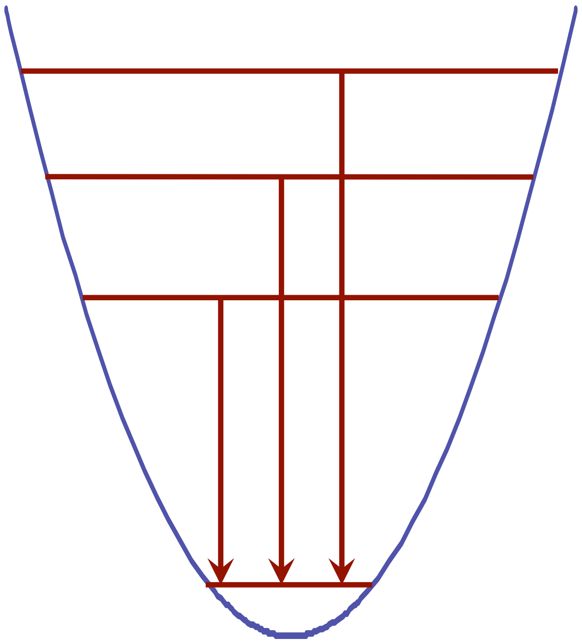
$$E_i > E_f$$

Different wave function for initial and final electron states.



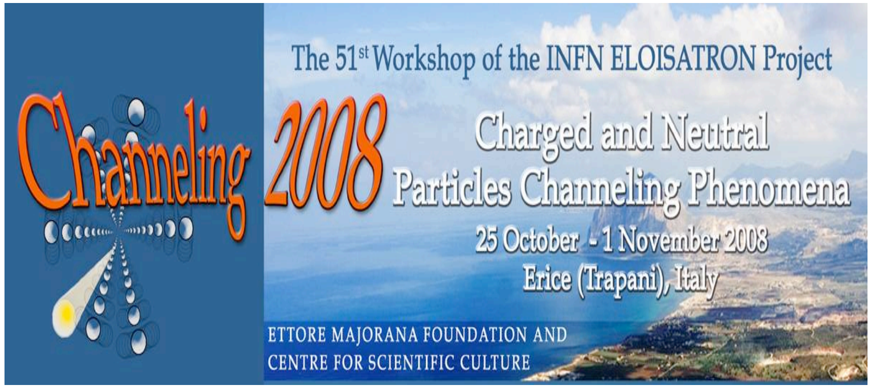
Ps production probability by axially channeled positron

$$\frac{dW}{dE_{Ps}d\Omega dt} = \sum_i \frac{dW_{if}}{dE_{Ps}d\Omega dt} P_i$$

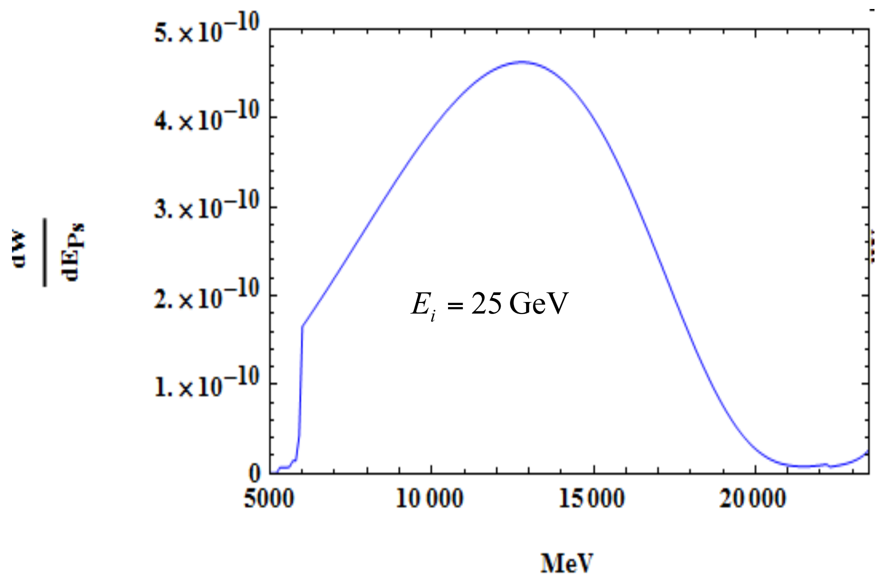


P_i - Population of energy level i

$$P_i = \left| \frac{2}{d} \int e^{i\vec{p}_{\perp}\vec{\rho}} \varphi_i(\vec{\rho}) d\vec{\rho} \right|^2$$

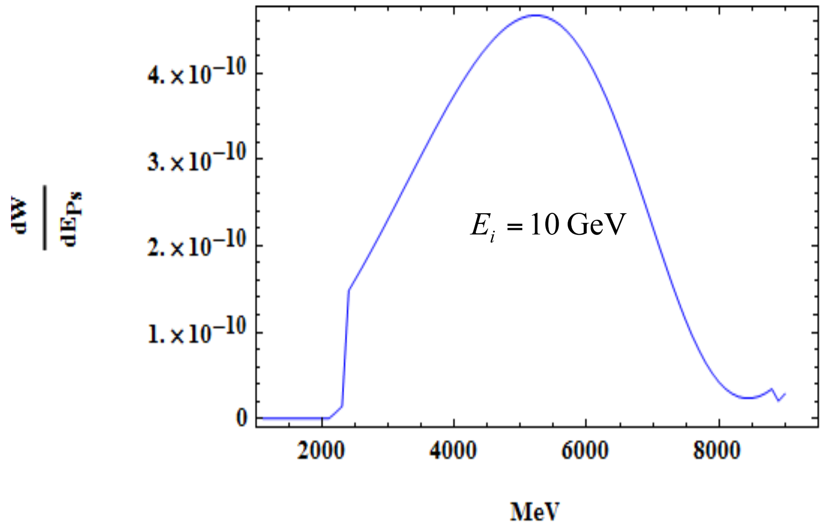


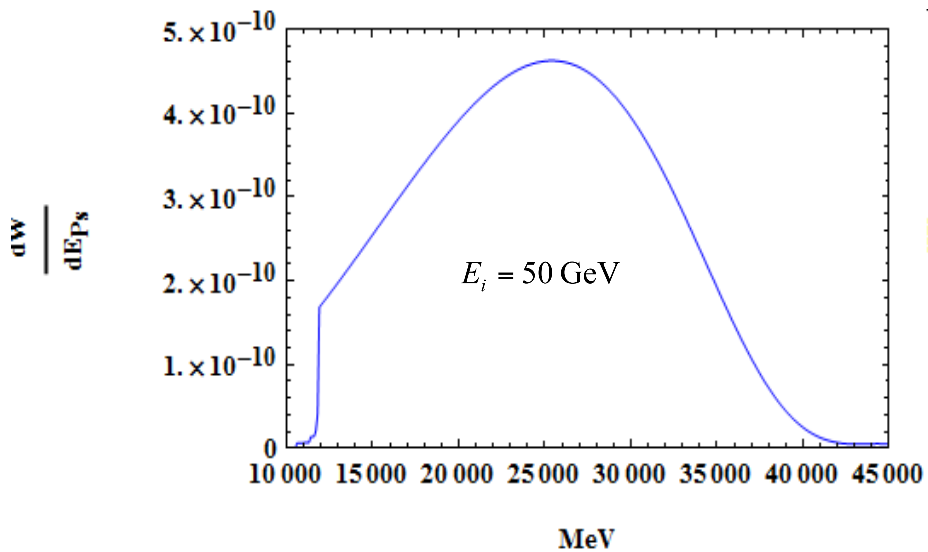
Dependence on initial electron energy.



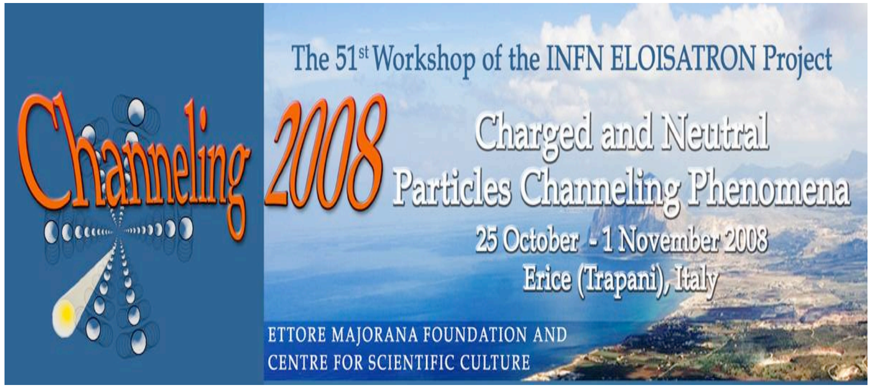
$$l_{cr} = 1\,\text{mkm},\ \Delta t = l_{cr}/v \approx l_{cr}/c$$

$$N = 25,\quad \vartheta_{\max} = 0.1\,\text{rad},\ \Theta_{in} = 0\,\text{rad}.$$





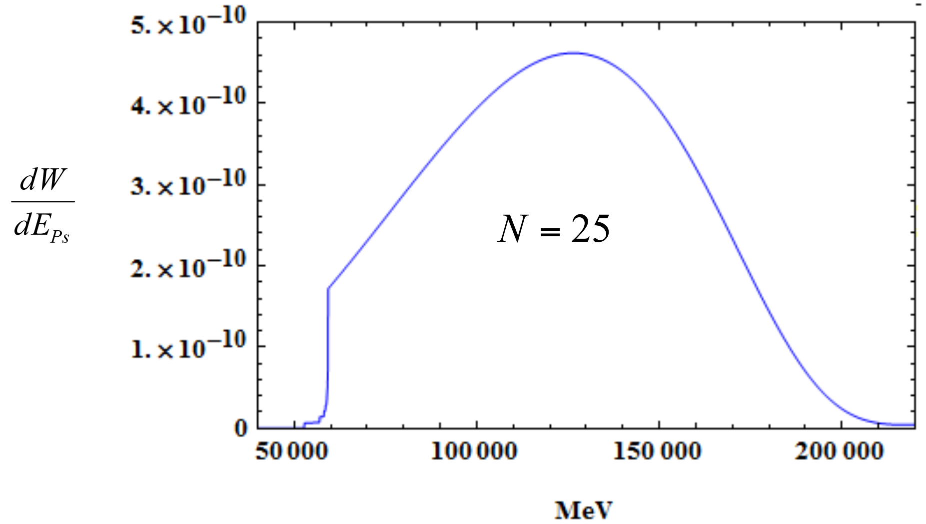
$$\frac{dW}{dE_{Ps}} = \int_{\vartheta_{\max}} \frac{dW}{dE_{Ps}\,d\Omega\,dt}\,d\Omega\Delta t = \int_{\vartheta_{\max}} \sum_i \frac{dW_{if}}{dE_{Ps}\,d\Omega\,dt} P_i\,d\Omega\Delta t$$

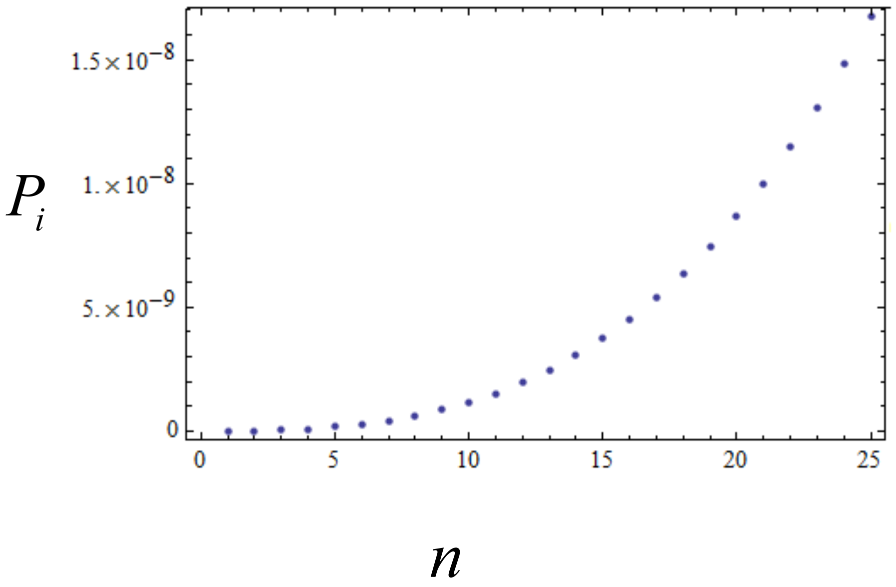


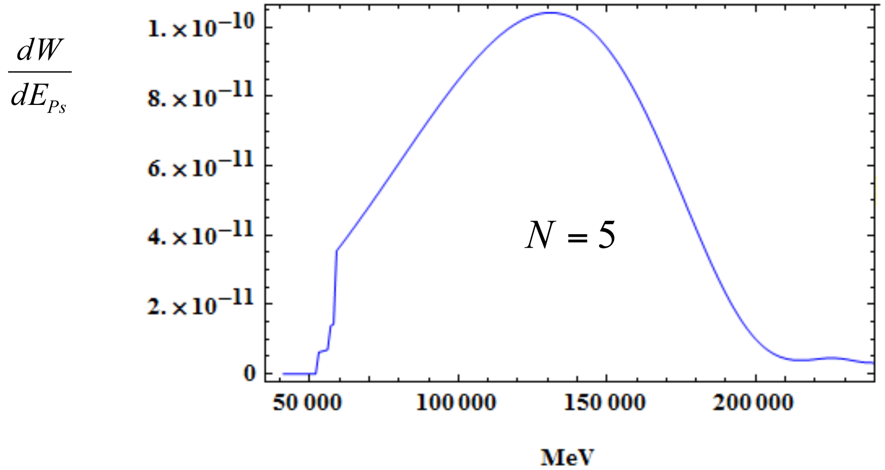
Dependence on number of transitions ***N***.

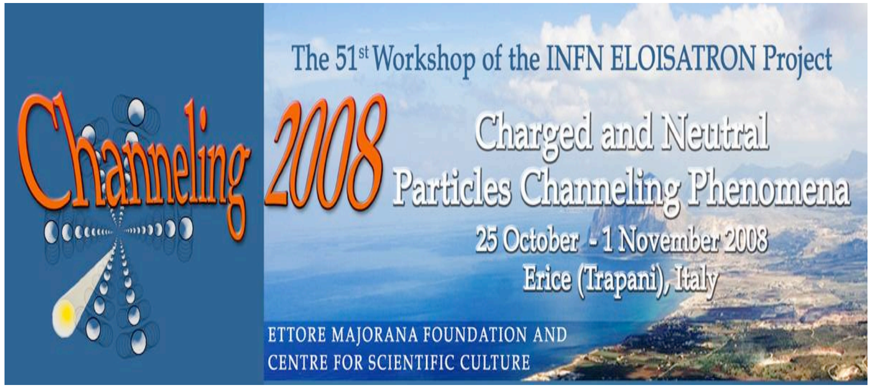
$$l_{cr} = 1\ \text{mkm}, \Delta t = l_{cr} / v \approx l_{cr} / c$$

$$E_i = 250\ \text{GeV},\ \vartheta_{\max} = 0.1\ \text{rad}, \Theta_{in} = 0\ \text{rad}.$$







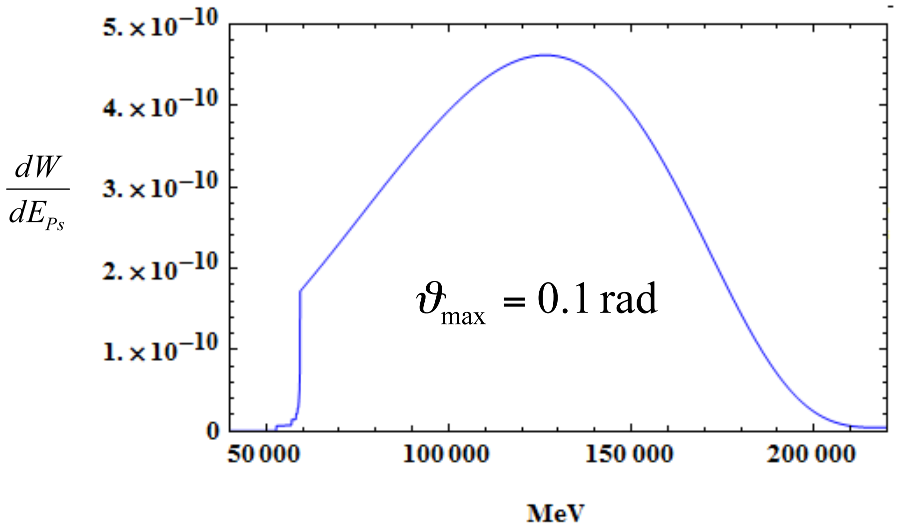


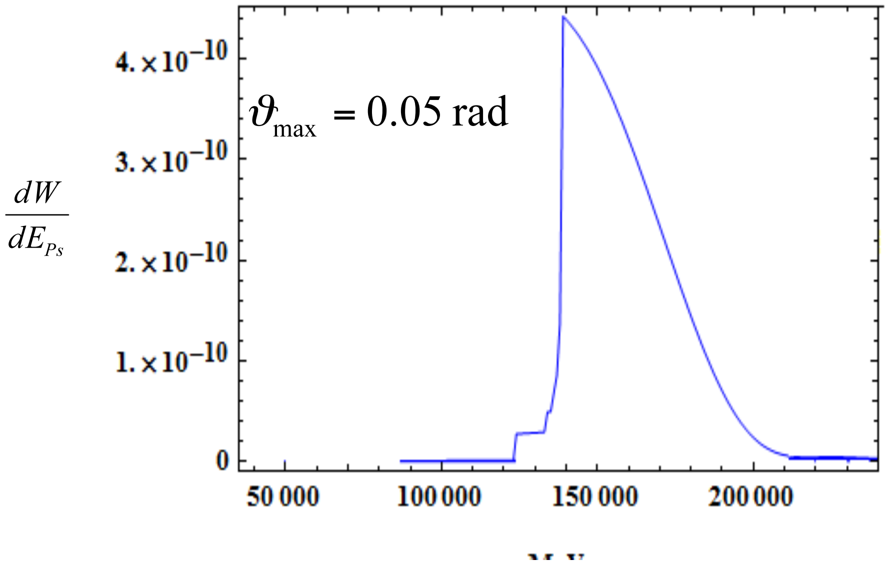
Dependence on maximal emission angle ϑ_{max}.

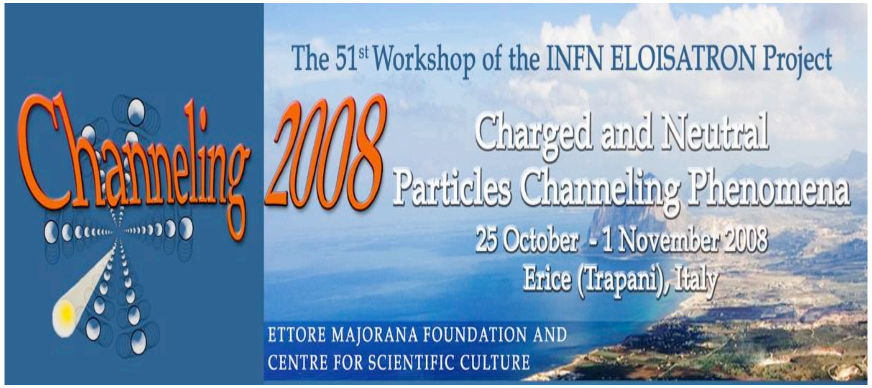
$$N = 25$$

$$l_{cr} = 1\,\text{mkm}, \Delta t = l_{cr}/v \approx l_{cr}/c$$

$$E_i = 250\,\text{GeV}, \Theta_{in} = 0\,\text{rad}.$$



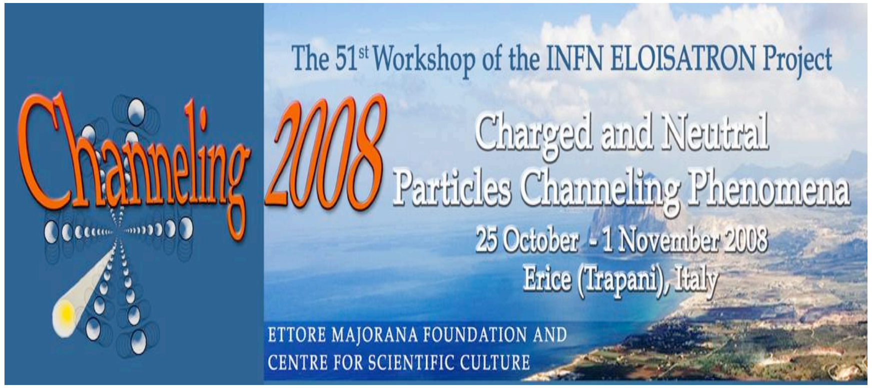




Conclusions

Relativistic Ps can be produced due to the transition of axially channeled electron from one transverse energy level to another.

Ps creation probability by axially channeled electron shows very pronounced threshold character in dependence of created Ps energy.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Charged and Neutral Particles Channeling Phenomena
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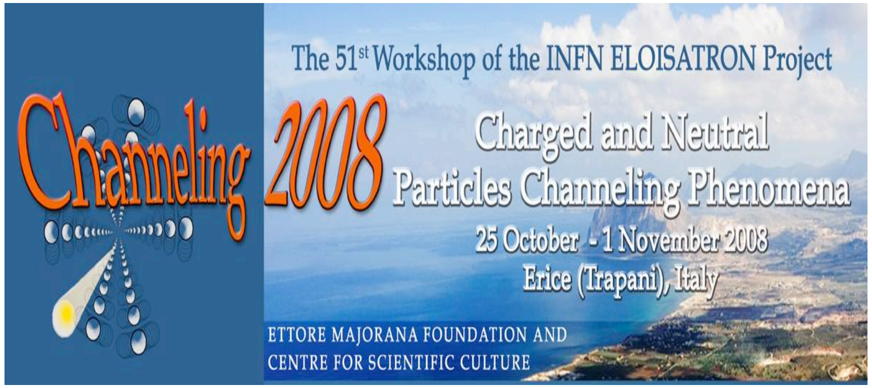
Thank You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