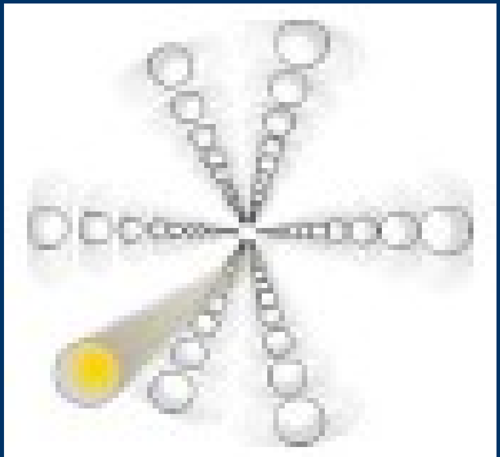


On the connection between diffraction radiation and transition radiation

D.V. Karlovets and **A.P. Potylitsyn**

Tomsk polytechnic university, Tomsk, Russia

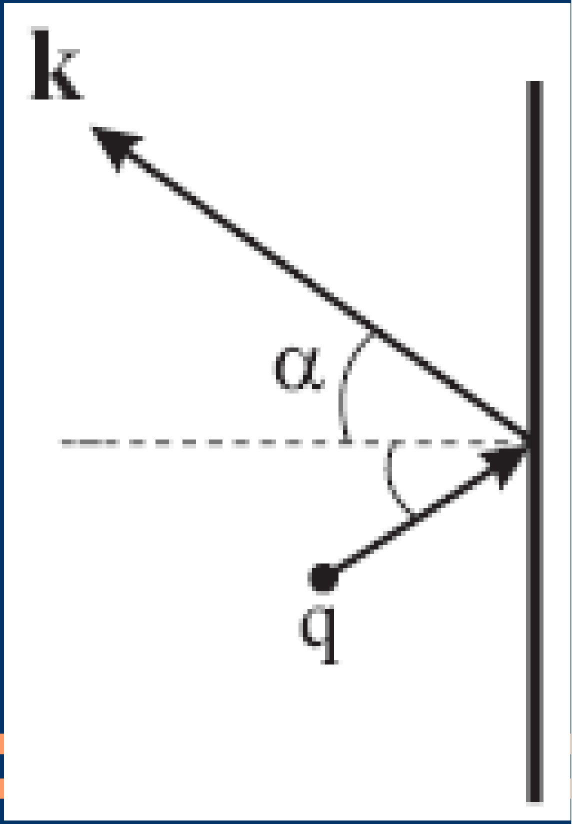
TR and DR
on ideally conducting surfaces

1. Backward TR at oblique incidence:

for the first time by
N.A. Korkhmazyan, Proceedings of Armenian Academy of Sci. ***11****, No. 6, p. 87, 1958 (in Russian)*

then was considered by Garibyan, Pafomov *et al*.

e.g.:
V.E. Pafomov, Proc. Lebedev Inst. Phys. ***44****, p. 25, 1971*

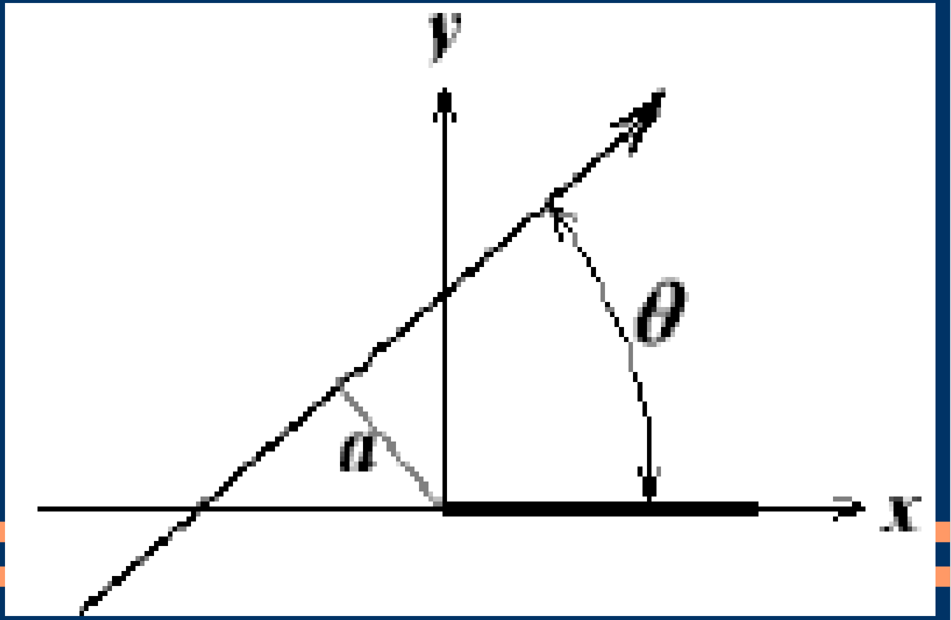


TR and DR on ideally conducting surfaces

2. DR at oblique incidence:

for a semi-plane

by A.P. Kazantsev and G.I. Surdutovich,
Sov. Phys. Dokl. 7, p. 990, 1963



Shall we get TR when the slit width *b* approaches zero?

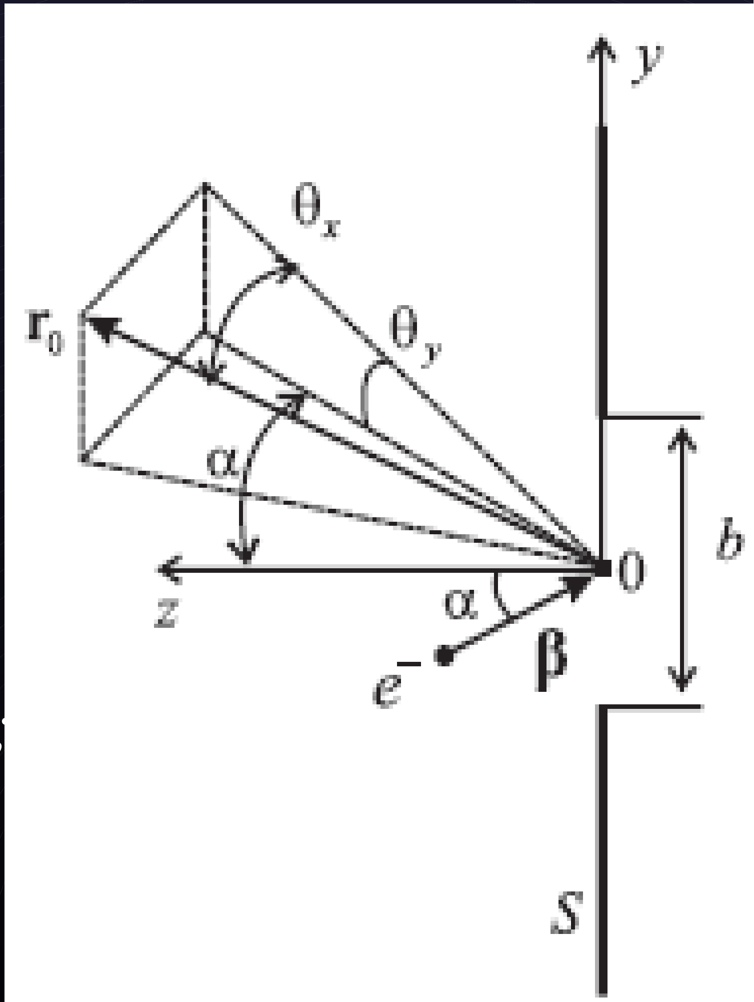
There were several works on DR from a slit:

1. A.P. Potylitsyn, N.A. Potylitsyna, Russ. Phys. J. ***43****, No.4, p.303, 2000;*

2. R.B. Fiorito, D.W. Rule, Nucl. Instrum. Meth. B ***173****, p.67, 2001;*

3. N. Potylitsyna-Kube, X. Artru, Nucl. Instrum. Meth. B ***201****, p.172, 2003;*

et al.



Based on Kazantsev-Surdutovich's resultes for a zero-width slit:

$$E_x = \frac{ie}{2\pi^2}\frac{\theta_x}{\gamma^{-2}+\theta^2}, \quad E_y = \frac{ie}{2\pi^2}\frac{\theta_y}{\gamma^{-2}+\theta^2}$$

But only for ultrarelativistic case *or* for normal incidence.

The little-known fact:

In general case this is not actually true !

Backward TR at oblique incidence

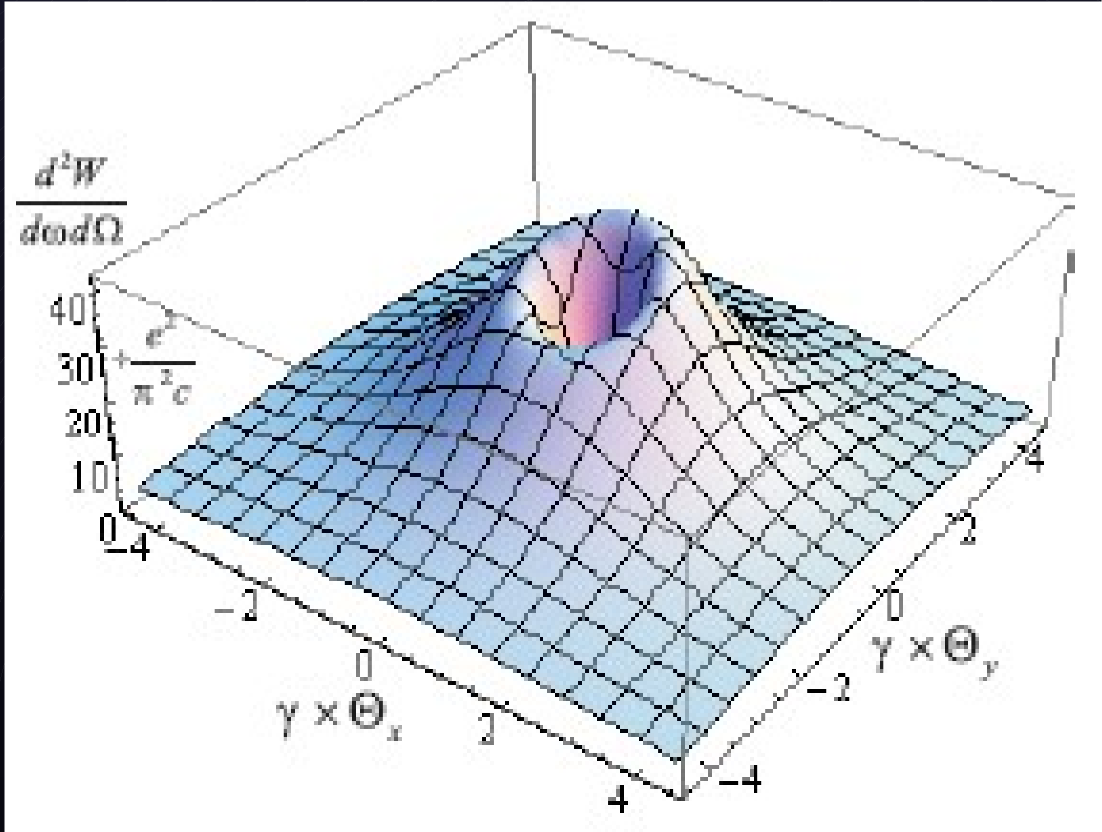
$$\acute{\mathbf{E}}^0(\acute{\mathbf{r}},\omega) = \frac{e\omega}{\pi v^2\gamma}\left(\frac{\acute{\rho}}{\acute{\rho}}K_1\left[\frac{\omega\acute{\rho}}{v\gamma}\right] - \frac{i}{\gamma}\frac{\mathbf{V}}{v}K_0\left[\frac{\omega\acute{\rho}}{v\gamma}\right]\right)e^{i\frac{\omega}{v}\acute{z}}, \quad \acute{\rho} = \{\acute{x},\acute{y}\}$$

Plays role at moderate energies and at oblique incidence

See e.g.:

D.V. Karlovets, A.P. Potylitsyn, *JETP* ***106****, No.6, p. 1045, 2008*

$$\gamma = 12, \alpha = \pi/4$$



Is the solution by Kazantsev and Surdutovich exact ?

Simple check of the surface current method:

$$\mathbf{E}^R(\mathbf{r}_0,\omega) = -\frac{i}{\omega}\frac{e^{ikr_0}}{r_0}\mathbf{k}\times\mathbf{k}\times\int \mathbf{j}_s^e(\mathbf{r},\omega)e^{-i(\mathbf{kr})}dS.$$

Surface current is found from the ordinary boundary conditions:

$$\mathbf{j}_s^e(\mathbf{r},\omega) = \frac{c}{2\pi}\mathbf{n}\times\mathbf{H}^0 - \\ -\frac{1}{2\pi}\mathbf{n}\times\int \mathbf{j}_s^e(\acute{\mathbf{r}},\omega)\times(\mathbf{r}-\acute{\mathbf{r}})\left(ik-\frac{1}{|\mathbf{r}-\acute{\mathbf{r}}|}\right)\frac{e^{ik|\mathbf{r}-\acute{\mathbf{r}}|}}{|\mathbf{r}-\acute{\mathbf{r}}|^2}d\acute{S},$$

See e.g. ***V.A. Fock****, Electromagnetic diffraction and propagation problems, Pergamon press, London, 1965*

Surface current method for backward TR at normal incidence

$$\frac{d^2W}{d\omega d\Omega} = cr_0^2|\mathbf{E}^R(\mathbf{r}_0,\omega)|^2 = \frac{e^2}{\pi^2 c}\frac{\beta^4 \sin^2\theta \cos^2\theta}{(1-\beta^2\cos^2\theta)^2},$$

$\beta^2 \cos^2\theta$, **- extra term !**

For TR at the oblique incidence the error of this method is **much greater** !

Particularly, there is no «right» asymmetry in TR angular distributions

The supposed reason:

the ordinary (Fock's) surface current method is suitable
only for plane electromagnetic waves but
not suitable for the solution
of ***in***homogeneous Maxwell's equations

The proof:

The Fock's integral equation for surface current
is found using the so-called
Double current sheet method

see e.g.:

J.A. Cullen, Phys. Rev. ***109****, No.6, p.1863, 1958,*
and also: W.R. Smythe, Phys. Rev. ***72****, No. 11, p.1066, 1947*

Double current sheet method

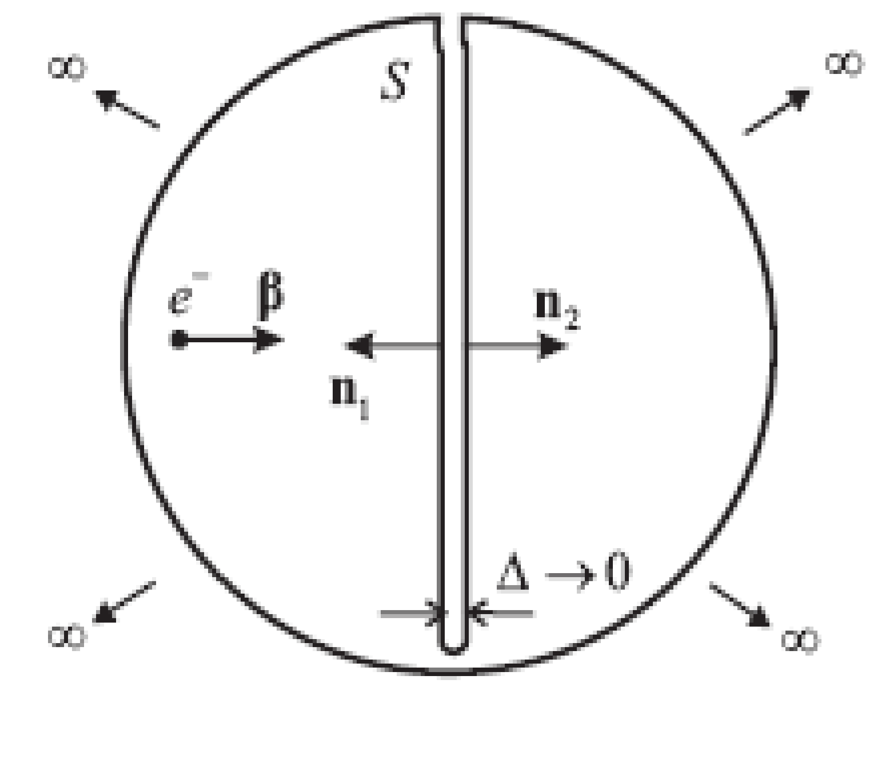
The physical sense:

1. Double *electric* sheet – corresponds to the surface current of ***electric dipoles***,
2. Double *magnetic* sheet – corresponds to the surface current of ***magnetic dipoles***

See e.g.:

I.E. Tamm, Fundamentals of the theory of electricity, Moscow, MIR, 1979

The Fock's surface current method corresponds to double *electric* sheet



Double current sheet method

$$\mathbf{E}^R(\mathbf{r}_0,\omega) = \frac{i}{k}\left(\vec{\nabla}_0(\vec{\nabla}_0) + k^2\right)\frac{1}{2\pi}\int_S [\mathbf{n},\mathbf{H}^R] g\, dS,$$

$$\mathbf{H}^R(\mathbf{r}_0,\omega) = \vec{\nabla}_0 \times \frac{1}{2\pi}\int_S [\mathbf{n},\mathbf{H}^R] g\, dS,$$

Integral equations for double **electric** sheet *(ordinary Fock's method)*

$$\mathbf{E}^R(\mathbf{r}_0,\omega) = \vec{\nabla}_0 \times \frac{1}{2\pi}\int_S [\mathbf{n},\mathbf{E}^R] g\, dS,$$

$$\mathbf{H}^R(\mathbf{r}_0,\omega) = -\frac{i}{k}\left(\vec{\nabla}_0(\vec{\nabla}_0) + k^2\right)\frac{1}{2\pi}\int_S [\mathbf{n},\mathbf{E}^R] g\, dS,$$

Integral equations for double **magnetic** sheet *(dual method)*

See similar method in:

J.D. Jackson, Classical electrodynamics, 3d ed., Willey, 1999

and more detailed:

*D.V. Karlovets, A.P. Potylitsyn, JETP **134**, No. 5, p. 887, 2008*

Double current sheet method

In the case of TR or DR the total fields consist of radiation field and particle's own field:

$$[\mathbf{n}, \mathbf{E}^R] = [\mathbf{n}, \mathbf{E}] - [\mathbf{n}, \mathbf{E}^0], [\mathbf{n}, \mathbf{H}^R] = [\mathbf{n}, \mathbf{H}] - [\mathbf{n}, \mathbf{H}^0]$$

The integral equations for current density **no more** correspond to the ordinary Fock's equation !

$$\mathbf{j}_s^e(\mathbf{r},\omega) = \frac{c}{2\pi}[\mathbf{n}, \mathbf{H}^0] - \frac{1}{2\pi}\mathbf{n} \times \int [\mathbf{j}_s^e(\acute{\mathbf{r}},\omega), \vec{\nabla} g] d\acute{S}_{sc} + \frac{c}{4\pi^2}\mathbf{n} \times \int [[\mathbf{n}, \mathbf{H}^0], \vec{\nabla} g] d\acute{S}_{sc}$$

<u>Additional terms:</u>

$$\mathbf{j}_s^m(\mathbf{r},\omega) = \frac{c}{2\pi}[\mathbf{n}, \mathbf{E}^0] - \frac{1}{2\pi}\mathbf{n} \times \int [\mathbf{j}_s^m(\acute{\mathbf{r}},\omega), \vec{\nabla} g] d\acute{S}_{ap} + \frac{c}{4\pi^2}\mathbf{n} \times \int [[\mathbf{n}, \mathbf{E}^0], \vec{\nabla} g] d\acute{S}_{ap}$$

The use of magnetic currents here
is physically possible due to the following fact:

In vacuum a magnetic dipole is completely equivalent to a Dirac dipole i.e. a pair of magnetic charges

See details in:

I.M. Frank, Sov. Phys. Usp. **27**, p. 772-785, 1984,

and our poster:
«On a dual symmetry in some-boundary-value problems of classical electrodynamics» by D.V. Karlovets

This fact was indicated **neither** in the recent work by Shkvarunets and Fiorito (Phys. Rev. ST – AB 11, p. 012801, 2008)
nor in the classical works on diffraction
(e.g. S.A. Schelkunoff, Phys. Rev. **56**, p. 308, 1939)

Double current sheet method

Solution of *new* integral equations allows to write down the final expressions for radiation field

$$\mathbf{E}^R_{ap}(\mathbf{r}_0,\omega) = -\frac{i}{k}\left(\vec{\nabla}_0(\vec{\nabla}_0) + k^2\right)\frac{1}{2\pi}\int\limits_{S_{ap}}[\mathbf{n},\mathbf{H}^0]g\,dS_{ap},$$

$$\mathbf{H}^R_{ap}(\mathbf{r}_0,\omega) = -\vec{\nabla}_0 \times \frac{1}{2\pi}\int\limits_{S_{ap}}[\mathbf{n},\mathbf{H}^0]g\,dS_{ap},$$

For double **electric** sheet

$$\mathbf{E}^R_{sc}(\mathbf{r}_0,\omega) = -\vec{\nabla}_0 \times \frac{1}{2\pi}\int\limits_{S_{sc}}[\mathbf{n},\mathbf{E}^0]g\,dS_{sc},$$

$$\mathbf{H}^R_{sc}(\mathbf{r}_0,\omega) = \frac{i}{k}\left(\vec{\nabla}_0(\vec{\nabla}_0) + k^2\right)\frac{1}{2\pi}\int\limits_{S_{sc}}[\mathbf{n},\mathbf{E}^0]g\,dS_{sc},$$

For double **magnetic** sheet

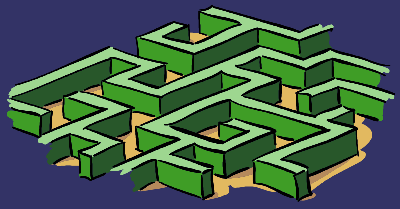
Double current sheet method

In the presence of external sources (particle)
the dual symmetry for *secondary* sources
is broken !

And one of the methods
(ordinary – electric *or* dual – magnetic)
doesn't work

See details in:

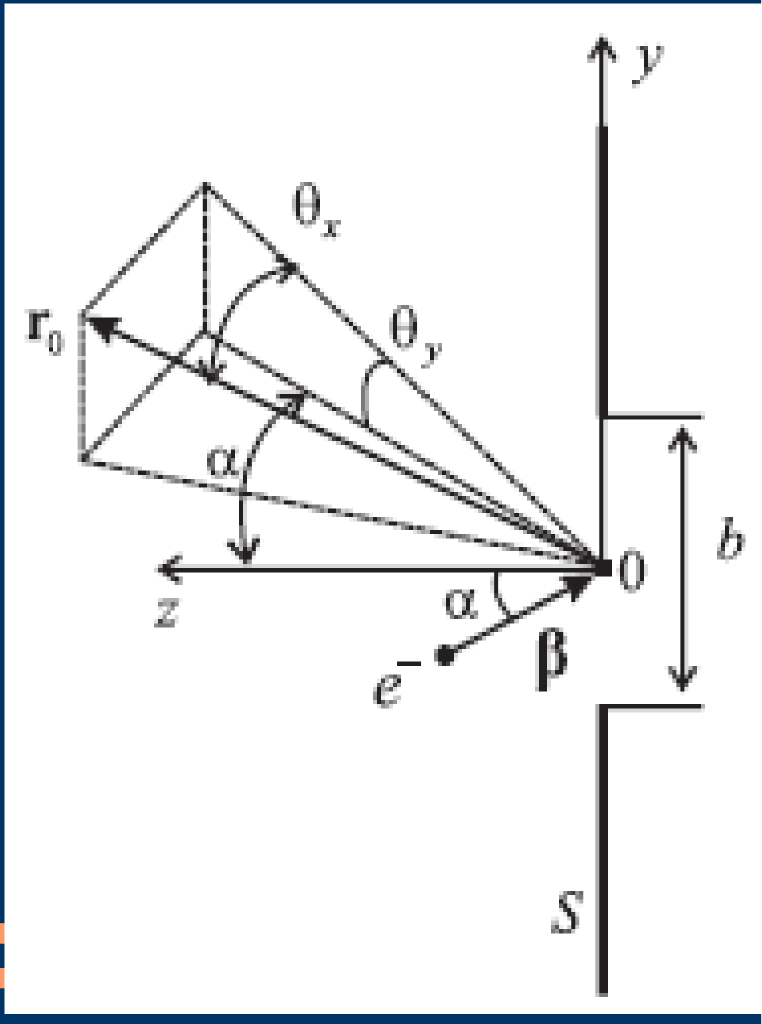
D.V. Karlovets, A.P. Potylitsyn, JETP ***134****, No. 5, p. 887, 2008*



Example 1:
DR for a slit

$$\left.\frac{d^2W}{d\omega d\Omega}\right|_{b=0} = \frac{e^2}{\pi^2 c}\frac{\beta^2\cos^2\alpha(e_x^2+e_y^2-2\beta e_y\sin\alpha+\beta^2\sin^2\alpha(e_y^2+e_z^2))}{[(\sin\alpha-\beta e_y)^2+\cos^2\alpha(1-\beta^2(e_y^2+e_z^2))]^2}$$

Solution for a zero-width slit
is found to be

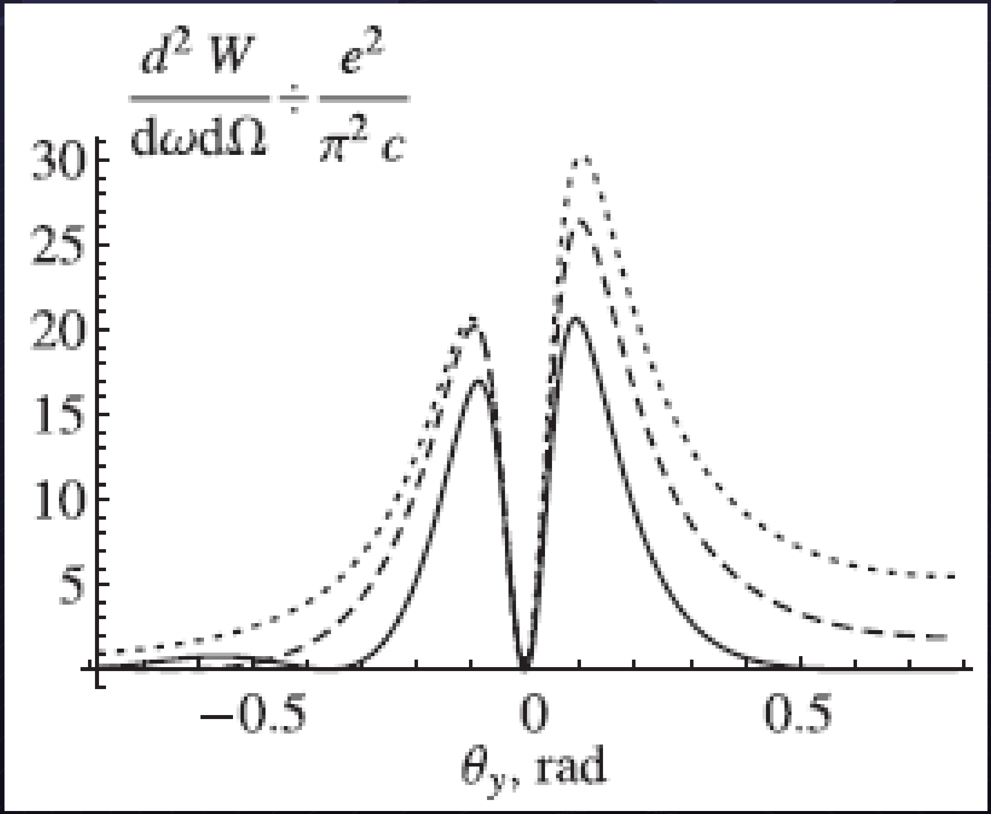


This is exactly the same formula
derived by Korkhmazyan
and Pafomov !

Example 1:

DR for a slit

$\gamma = 10$ $\alpha = \pi/4$ $\theta_x = 0$



Solid curve - $b = 0.2\gamma\lambda$

Dashed curve - $b = 0.1\gamma\lambda$

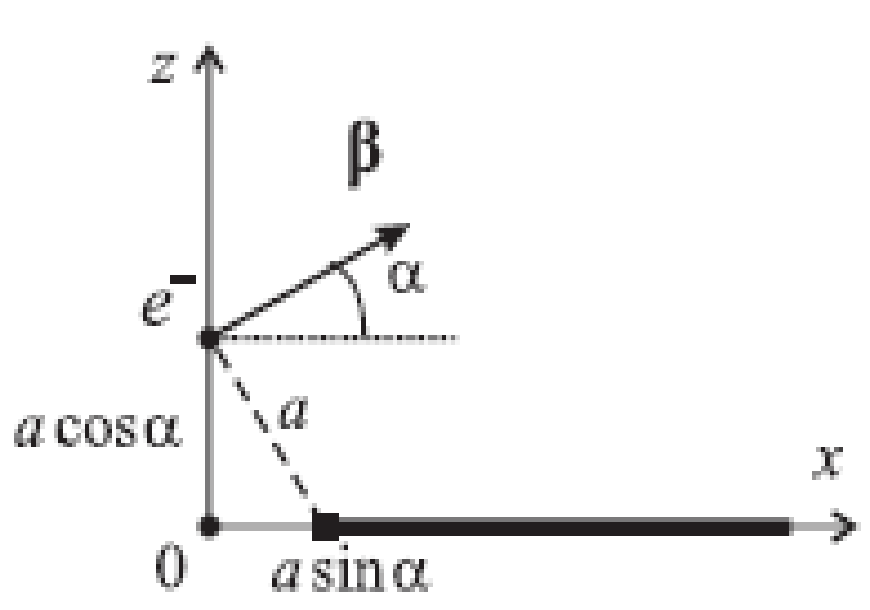
Dotted curve - $b = 0.01\gamma\lambda$

Unlike DR and TR by surface current method, **this** method provides complete agreement with TR theory and predicts **«right» asymmetry**

Example 2:

<u>DR for a semi-plane</u>

Let's compare results
for a semi-plane
by the *double current sheet method*
with those ones
by Kazantsev and Surdutovich
(*surface current method*)



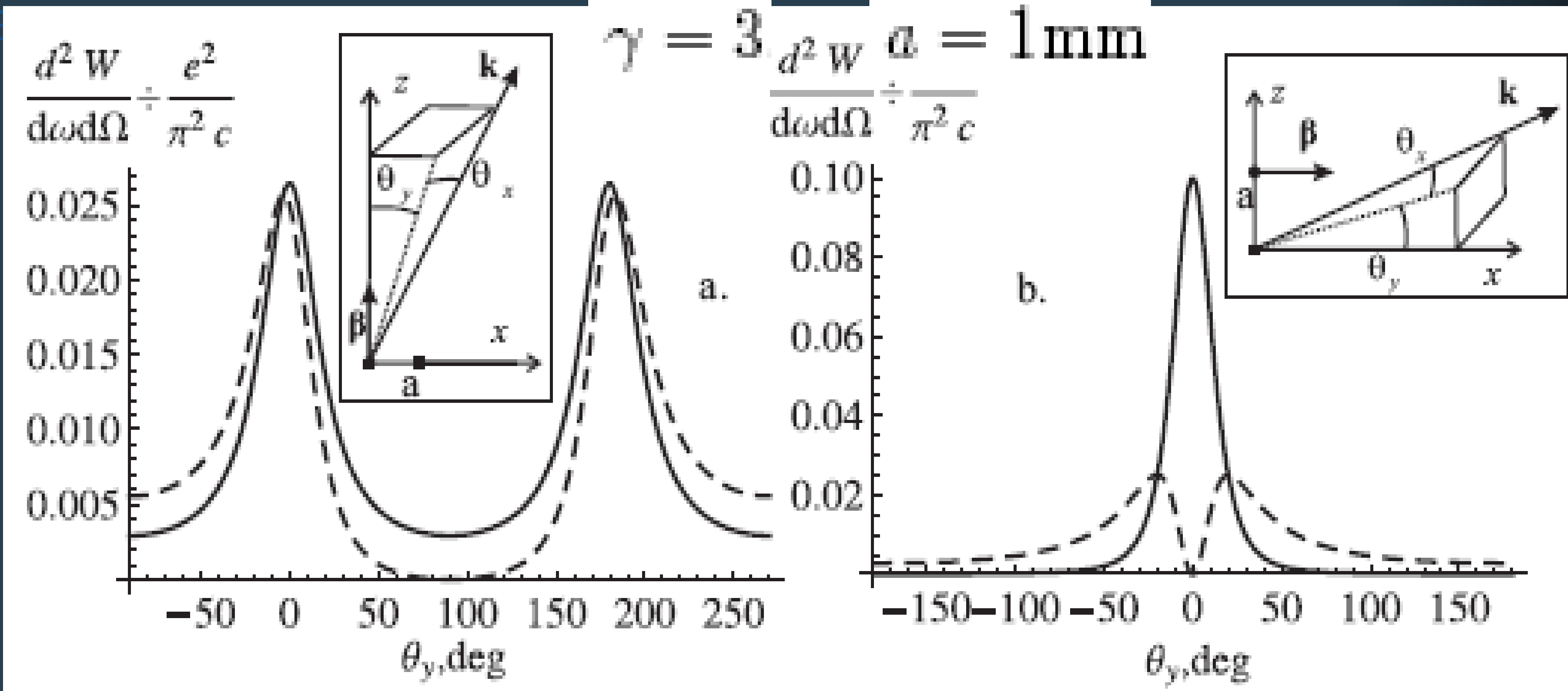
By the d. c. s. method in the wave zone:

$$\frac{d^2W}{d\omega d\Omega} = c r_0^2 |\mathbf{E}^R|^2 = \frac{e^2}{\pi^2 c} \frac{1}{4(1+(\beta\gamma e_y)^2)[(\cos\alpha/\beta - e_x)^2 + \sin^2\alpha((\beta\gamma)^{-2} + e_y^2)]}$$
$$\times\Big[(e_x^2+e_z^2)(\cos^2\alpha(\beta\gamma)^{-2} + \beta^{-2}\sin^2\alpha(1+(\beta\gamma e_y)^2)) +$$
$$+\gamma^2 e_y^2(e_y^2+e_z^2) + 2\beta^{-1} e_x e_y^2 \cos\alpha\Big] e^{-a\frac{2\omega}{v\gamma}\sqrt{1+(\beta\gamma e_y)^2}}, \quad (2.41)$$

Comparison

(solid line — d.c.s. method, dashed line — s.c. method)

$\gamma = 3 \quad a = 1\,\mathrm{mm}$



$$\frac{d^2W_2}{d\omega d\Omega} \Big/ \frac{d^2W_1}{d\omega d\Omega} = \beta(1 - \sin\theta)$$

$$\frac{d^2W_2}{d\omega d\Omega} \Big/ \frac{d^2W_1}{d\omega d\Omega} = \frac{\beta(1 - \cos\theta)}{1-\beta}\Big|_{\gamma \gg 1} \approx$$

$$\approx 2\gamma^2(1 - \cos\theta)$$

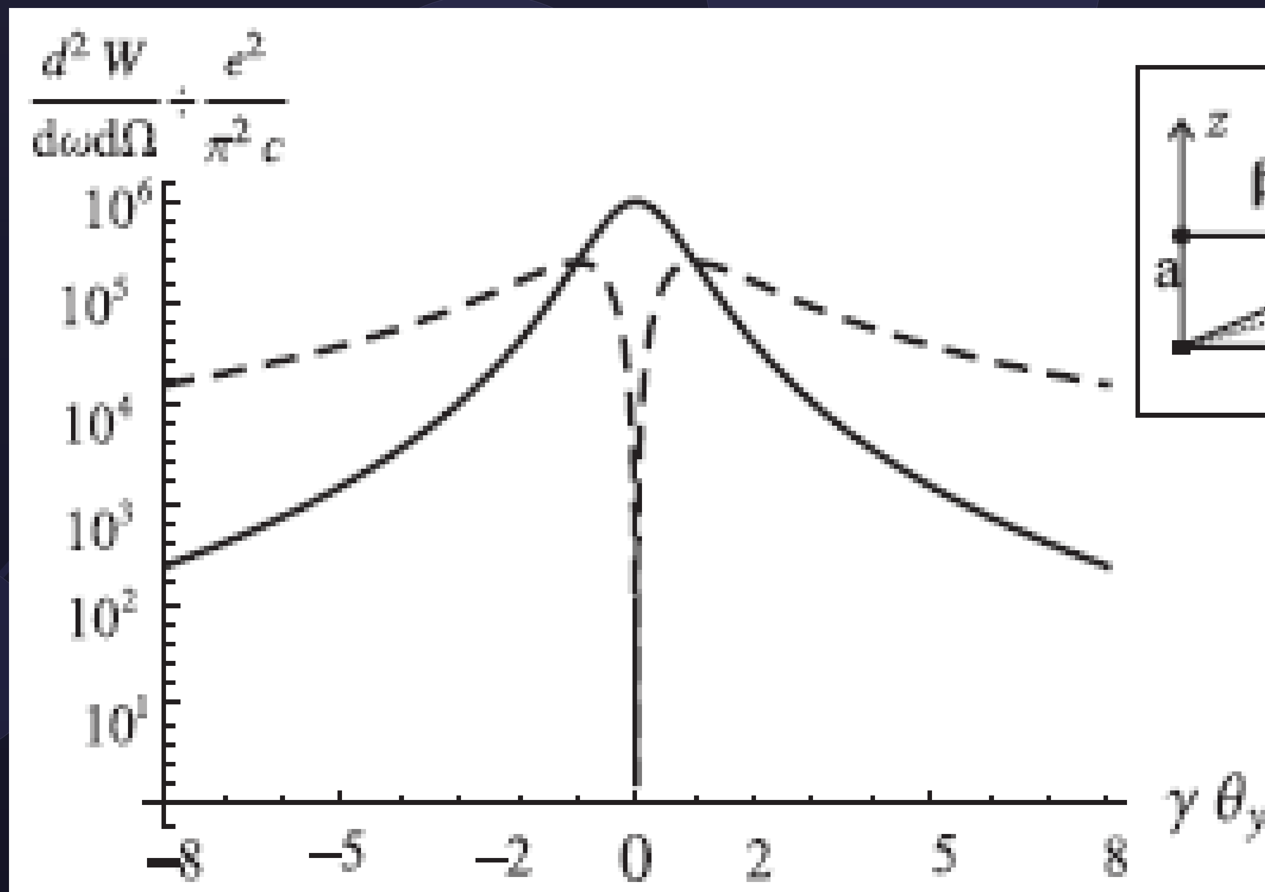
S.C. method / D.C.S. method

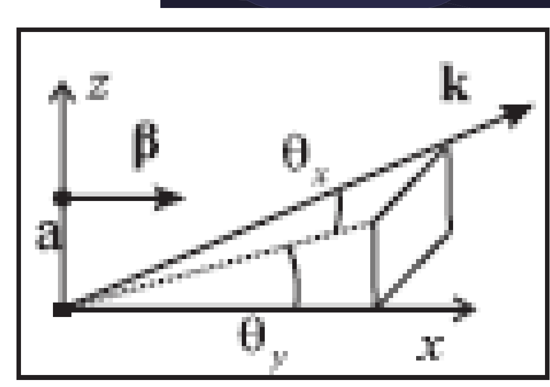
Comparison

(solid line — d.c.s. method, dashed line — s.c. method)

$\gamma = 10^3$

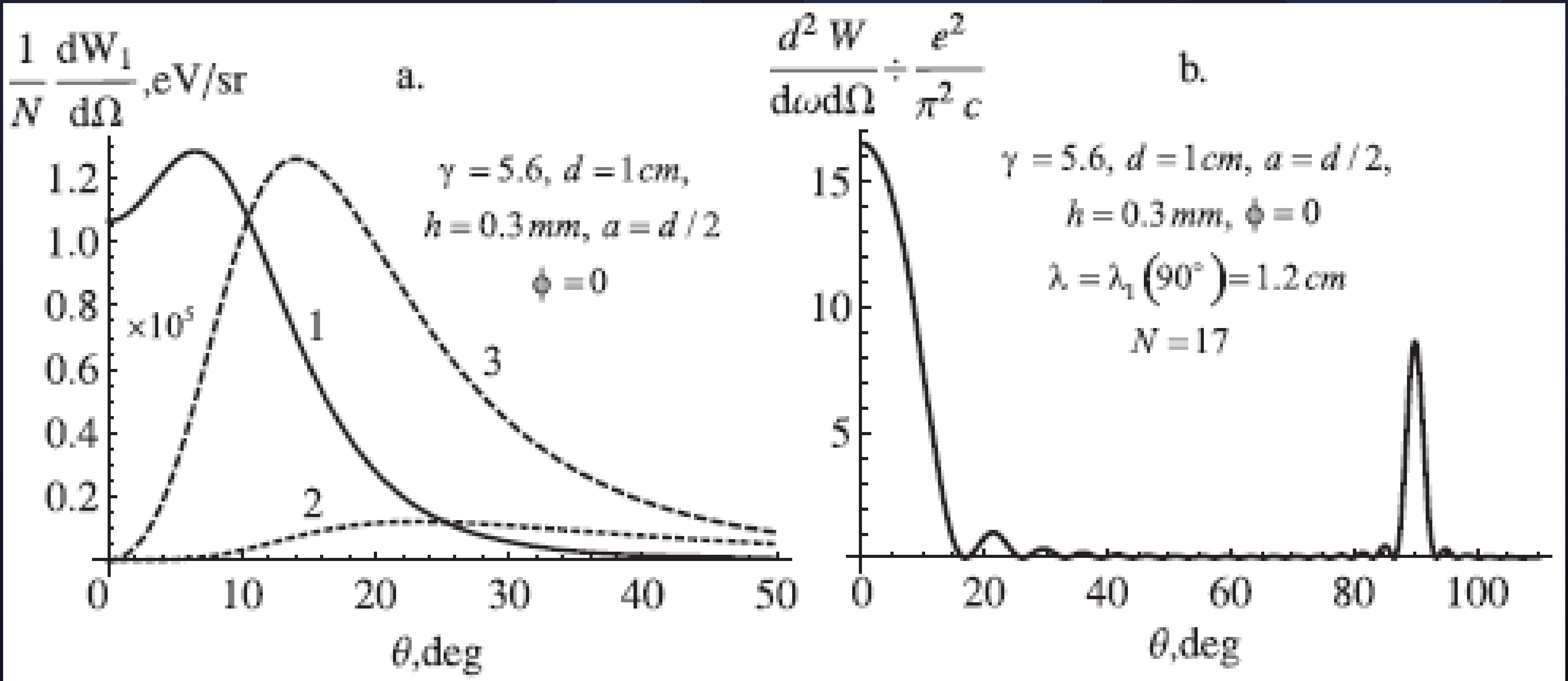
$a = 1\text{mm}$





Orders of magnitude,
even for the ultrarelativistic case !

Example 3:
Smith-Purcell radiation



Solid line — double current sheet method,

Dashed — surface current model by Brownell, Walsh and Doucas, Phys. Rev. E **57**, No.1, p.1075, 1998;

Dotted — surface current method by Fock (used by Kazantsev and Surdutovich).

Example 3:

Smith-Purcell radiation

High intensity forward directed radiation !

Similar behavior was observed by Woods et al.
Phys. Rev. Lett. ***74****, No. 19, p. 3808, 1995*
within angles ***Theta > 20 deg***.

$\gamma = 5.6,$

More recently by Kalinin et al.
Nucl. Instrum. Meth. B ***252****, p. 62, 2006*
within angles ***Theta < 20 deg***.

$\gamma = 12$

Conclusion

1. The well-known surface current method is suitable ***only*** for plane-wave diffraction,

2. Solutions for TR and DR obtained by this method are ***approximate*** and the region of their validity is defined by the equalities:

$$(\beta, \mathrm{n}) \approx 1, \quad \cos\Theta \approx 1,$$

3. The exact solutions are found for DR from:
a. Semi-plane, b. Slit, c. Hole in a screen, d. Grating (SPR).

These solutions are found using proposed **double current sheet** method, where the role of the radiation source is played by the surface current of electric or magnetic dipoles.

4. The ***exact transition*** between DR and TR formulas is found for the case of a zero-width slit in the infinite screen.