Ray Tracing Calculation of PXR Produced in Curved and Flat Crystals by Electron Beams with Large Emittance

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1. Introduction

A short excursion into publications

a) First prediction

1. **M.L. Ter-Mikaelian, The Influence of the Medium on High Energy Processes at High Energies, Publishing House of Academy of Science of Armenia, Yerevan, 1969; High Energy Electromagnetic Processes in Condensed Media, Wiley Interscience, New York, 1972.**

b) Monographs

2. **G.M. Garibian and Yan Shi, Rentgenovskoe Perekhodnoe Izluchenie, Publ. of Academy of Sc. Of Armenia, Yerevan, 1983.**
3. **V.A. Bazilev, N.K. Zhevago, Izluchenie Bistrikh chastits v Veshchestve i vo Vneshnikh Polyakh, FizMat. Literatura, Moscow,1987.**
4. **P. Rullhusen, X. Artru and P. Dhez, "Novel Radiation Sources Using Relativistic Electrons", world Scientific, Singapore ,1998.**
5. **V.G. Barishevski, I.D. Feranchuk, A.P. Ulyanov, Parametric X-Ray Radiation in Crystals: Theory, Experiments and Application, Springer Tracts in Modern Physics, Heidelberg, 2005.**

c) First experimental observation

6. **S.A. Vorobev et al, Pisma Zh. Eksp. Teor. Fiz. 41,3 , 1985.**

d) Existing permanent PXR station (To our knowledg)

7. Y. Hayakawa et al, Nucl. Instr. and Meth. B252, 102, 2006.

8. B. Sones et al, Nucl. Instr. and Meth. B227, 22, 2005; 261, 98, 2007.

e) Applications of curved crystal as monochromators and for particle deflection

9. R. Caciuffo, S. Melone, F. Rustichelli, Phys. Rep. 152, N1, 1, 1987.

10. V.M. Biryukov, Yu. A. Chesnokov, V.I. Kotov, Crystal Channeling and Its Appl. at High-Energy Accelerators, Springer-Verlag, Berlin, Heidelberg, 1997.

f) Curved crystals for focusing PXR of channeled particles(disadvantages low Acceptance and low emitance electron beams to be channeled)

11. A.V. Shchagin, Pisma Zh. Eksper. Teor. Fiz. 80, 535, 2004.

12. A.S. Gogolev, A.P. Potilitsin, Proc. RuPAC06, Novosibirsk, Russia, 2006.

g) Bragg refraction (XRD) in bent crystals using or without using Tagaki – Taupin like equations and ray tracing methods for XRD.

13. 16. T. Missalla, I. Uschmann, E. Forster, Rev. Sc. Instr. 70, 1288, 1999.

14. S.G. Podorov, A. Nazarkin, Opt. Commun.278, 340, 2007.

15. http://www.esrf.eu/computing/scientific/raytracing/PDF/primer.pdf.

The purpose of this work is: for the first time using ray tracing MC methods (postponing the development of accurate theory)

1) To calculate the properties of PXR produced in curved crystals.

2) To show:due to focusing in Rowland circle beams with significant emittance for microtrons etc can be used for prod. of intense PXR beams.

2. Theoretical background

We shall use the results of Kinematical model

As first results

a) It will be neglected multiple scattering, which can be done following work

16. V.G. Baryshevsky, A.O. Grubich, Le Tien Hai, Zh. Eksp. Teor. Fiz. 94, 51, 1988.

And models

17. S. Asano et al, Phys. Rev. Lett. 70, 3247, 1993; H. Backe et al, Proc. Of NATO ARW, Electron - Photons Interaction in Dense Media, Nor Amberd, Armenia, June, 2001.NATO Sc. Ser. Math. Phys. Chem. Vol. 49, p. 183, 2002.

b) Only crystal with 1D (cylindrical) curvature R providing 1D focusing will be considered postponing the 3D focusing by crystals with toroidal or eliptic curvature.

c) Johann geometry Rowland circle will be used ($R_{cr}=2R_{Rowl}$).

d) As in XRD the only assumption is: at the reflection point the curved crystal is flat.

e) The formulae (can be missed)

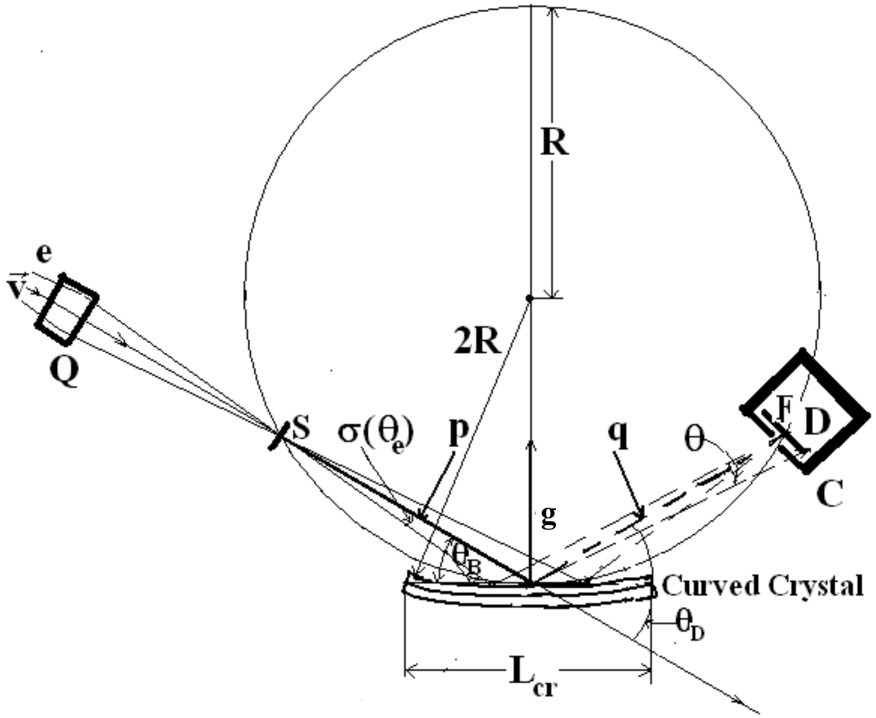
$$\frac{d^2N}{dL\,d\Omega} = \frac{z^2e^2\omega^3}{2\pi\varepsilon^{3/2}\hbar c^4\beta(1-\sqrt{\varepsilon}\mathbf{n}^*\mathbf{v}/c)}\sum_{\mathbf{g}}|\chi_{\mathbf{g}}|^2\left|\frac{\left[\mathbf{n}^*,\left[\mathbf{n}^*,\left(\varepsilon\omega\mathbf{v}/c^2-\mathbf{g}\right)\right]\right]}{\left((\mathbf{k}^*+\mathbf{g})^2-k^{*2}\right)}\right|^2 \qquad (1)$$

$$\hbar\omega = \frac{\hbar\mathbf{g}\cdot\mathbf{v}}{1-\sqrt{\varepsilon}\,\mathbf{n}^*\cdot\mathbf{v}/c} = \frac{2\pi\hbar c}{d}\frac{\beta\sin\theta_{\mathrm{B}}}{1-\sqrt{\varepsilon}\beta\cos\theta} \qquad (2)$$

$$\chi_{\mathbf{g}}(\omega) = -\frac{\omega_{\mathrm{p}}^2}{z_{\mathrm{c}}\omega^2}S(\mathbf{g})\exp[-W]F(\mathbf{g}) \qquad (3)$$

$$L_{\mathrm{ef}} = L_{\mathrm{abs}}(\omega)\left|\frac{\mathbf{n_t}\cdot\mathbf{n}^*}{\mathbf{n_t}\cdot\mathbf{n}}\right|(1-\exp[-L/(L_{\mathrm{abs}}(\omega)|\mathbf{n_t}\cdot\mathbf{n}^*|)]) \qquad (4)$$

3. Arrangement



$$p = q = 2R_{Rowl} \sin\theta_B \quad \textbf{(5)}$$

At the plane S one has an electron Gaussian beam with «source» cross section S_e and radial and angular dispersions

$\sigma(r)$ **and** $\sigma(\theta_e)$

At the plane F one has PXR «image» and low or large Acceptance (surface) detector D.

3. Ray Tracing Procedure (Simulation method)

1) For E_e = 20 MeV; Si (111) planes of a Si crystal (convenient for bending). hw = 8 keV; From (4) $\theta_B = 14.41^\circ$, the angle for the central electrons with $\theta_e = r = 0$. Choosing R_{Rowl} = 50 with the help of (5) one determines p = q = 25 cm.

2) Having $\sigma(r)$ simulates r_i and φ_{ri} (isotropic azimuthal distribution) and determines the coordinates x_{ie}, y_{ie}, z_{ie} of the out coming point on the plane S, choosing a coordinate frame XYZ (It is reasonable to take the central electron direction as OZ, and the axis OX in the scattering plane).

3) Having $\sigma(\theta_e)$ one simulates θ_{ei} and φ_{ei} and finds 3a) the equation of the straight line along which the electron moves with velocity vector $\vec{v}_i$; 3b) the coordinates $\Delta_x, \Delta_y, \Delta_z$ i.e…

3c) the incidence Bragg angle θ_{iB} and g_i

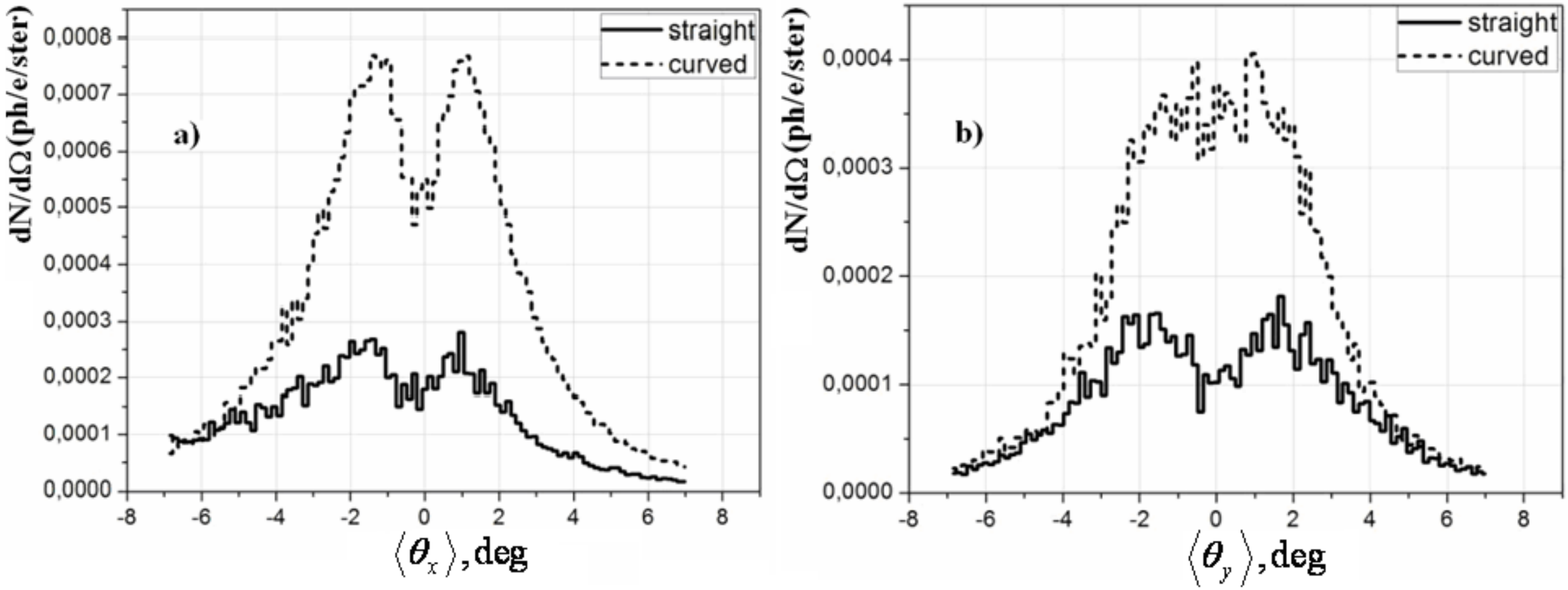
4) Calculated the angular density of PXR in the interaction act.
5) By repeating 2) - 4) for N_e incident electrons up to when the total Number of the produced PXR becomes 10^4 - 10^5, one obtains the Following 4 massives i) $x_F^{'}$ or $\langle\theta_x\rangle$, ii) $y_F^{'}$ or $\langle\theta_y\rangle$, iii) $\hbar\omega_i$, and iv) $dN/d\Omega$

Having these massives one can obtain:

a) The PXR images (scatter diagrams or the 3D distributions) at F and define, the magnification for flat and curved radiators. These images can be detected with the help of a large surface position sensitive X-ray detector.

b) The total spectral distribution of PXR photons (independently from the place of hitting) which can be detected with the help of The above mentioned large acceptance position sensitive.

c)The angular $dN/<d\theta_x>$ and $dN/d<\theta_y>$ - distributions of PXR photons which can be detected as usually by moving a small surface X-ray detector accepting a small solid angle, $\Delta x', \Delta y'$, along a line keeping constant value of one of the angles.



d) The spectral distribution $dN/d(\hbar\omega)$ of PXR photons for the given $\langle\theta_x\rangle$ and $\langle\theta_y\rangle$ with the help of small acceptance detector.

4. Results

For transparency and reasoability calculations are carried for 4 types of el. beams.

Table1.

NN	Electron Beam	Crystal	(mm) $\sigma(r)$	$\sigma(\theta_e)$
1	Ideal Beam	Flat bent	0 0	0.0008 0.0008
2	0 cross section diverging	Flat bent	0 0	0.008 0.008
3	Thick Pencil	Flat bent	2 2	0 0
4	General case	Flat bent	2 2	0.008 0.008

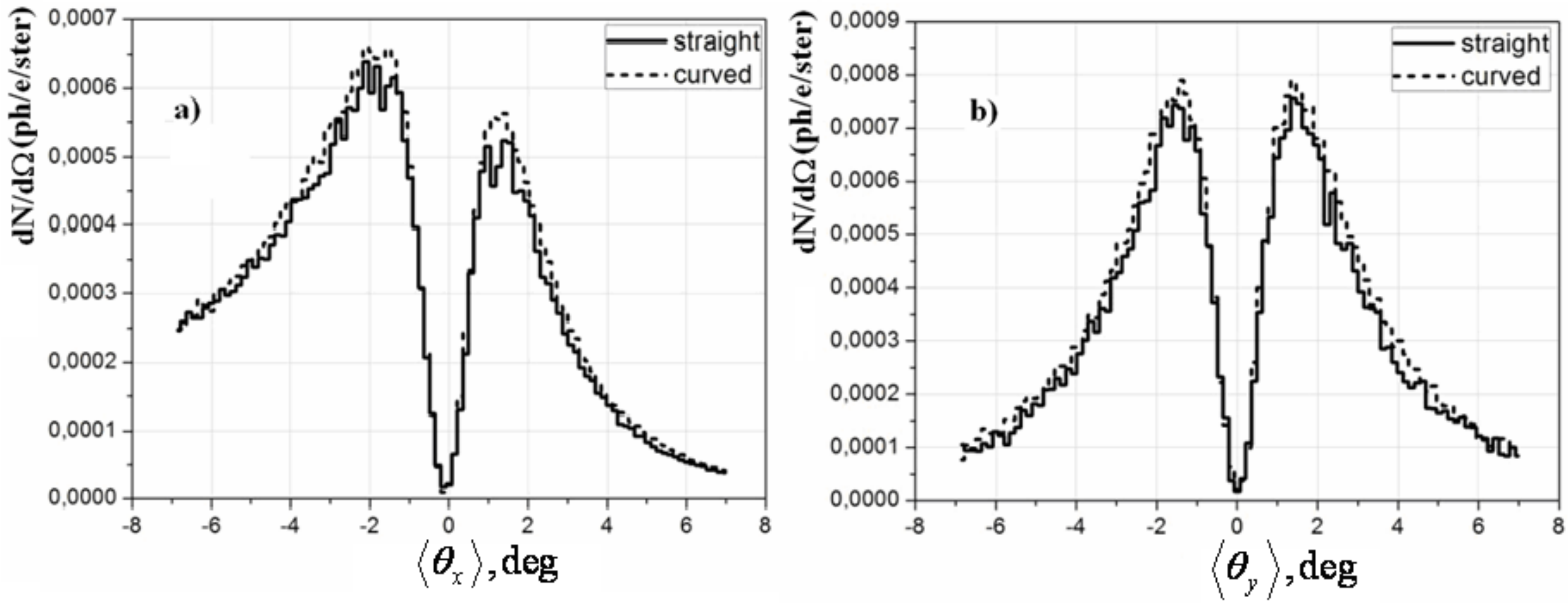


Fig.1. The PXR angular distribution of produced in flat and curved Si crystals by an ideal electron beam (1-st case).

No difference between flat and curved crystal.

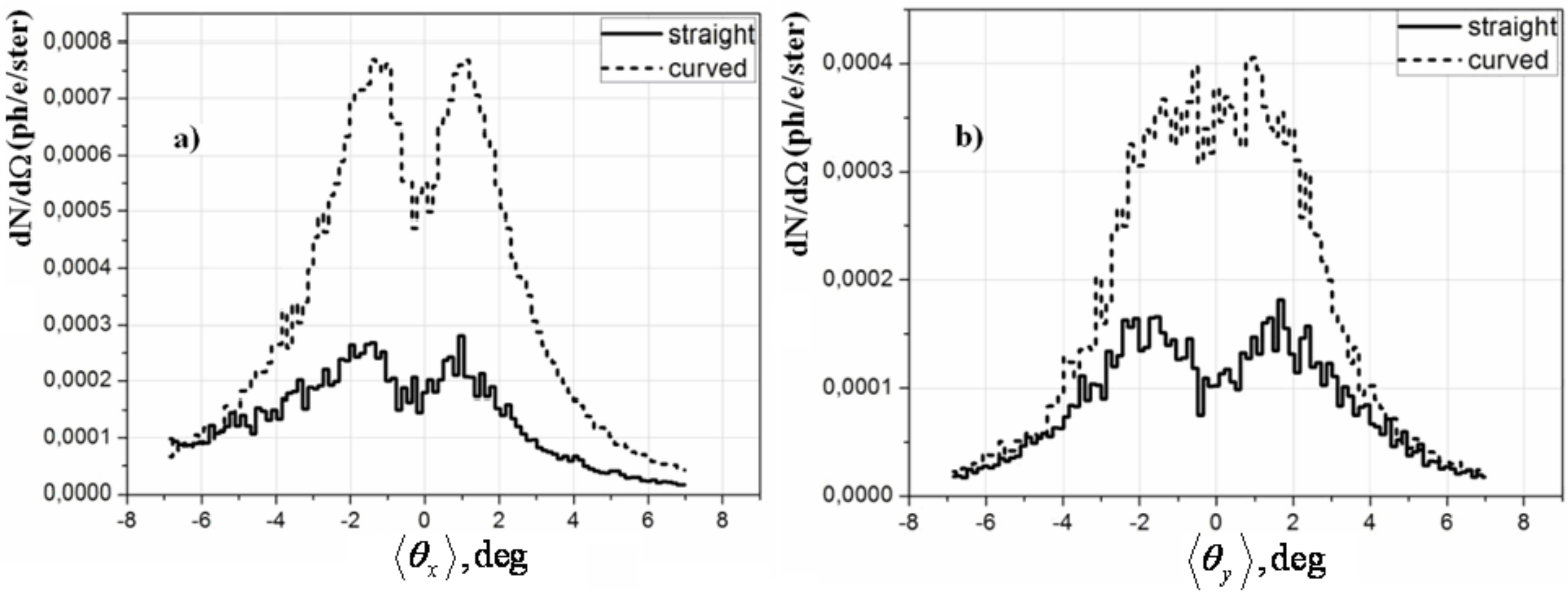


Fig.2. Are the same as in Fig. 1 for electron beam with 0 cross section and finite angular spread (2-d case).

Due to angular divergence there is focusing and density Increase in the case of curved crystal

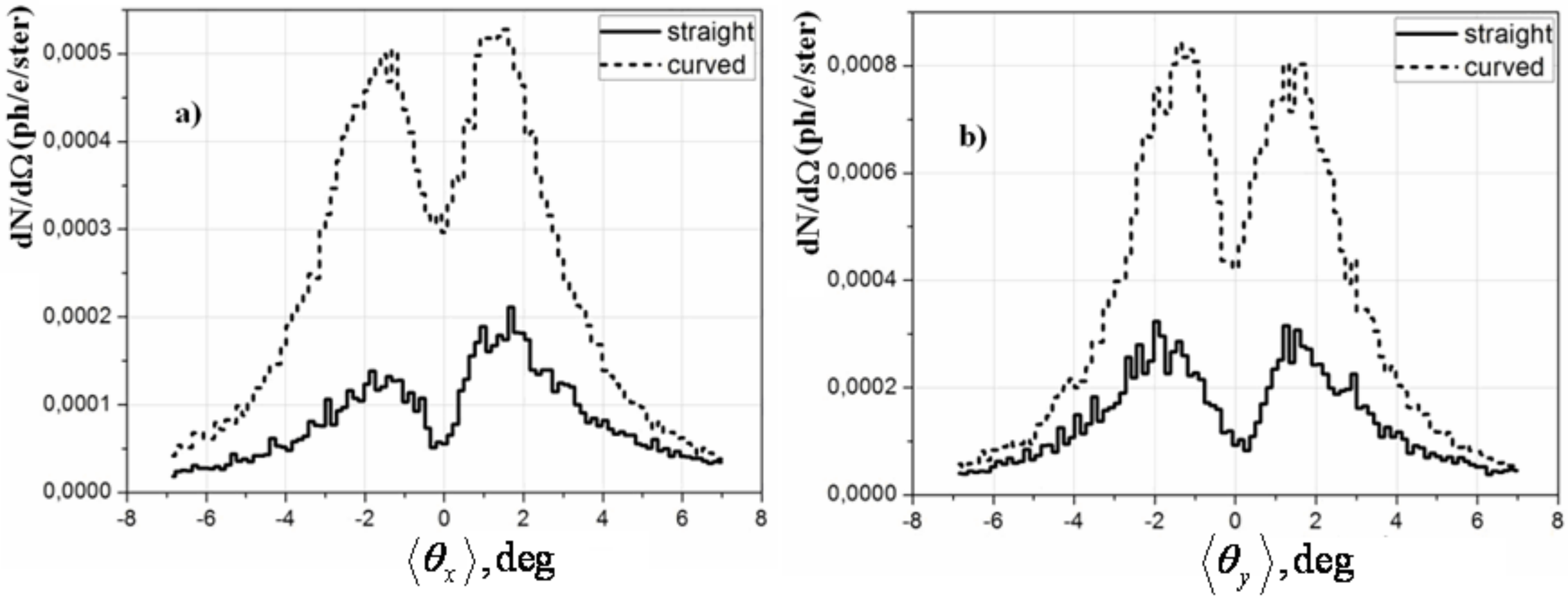


Fig.3. Are the same as in Fig. 1 for «thick pencil» type electron beam with 2 mm cross section and 0 rad angular spread (3-d case).

Due to cross section there is focusing and density increase in the case of curved crystal

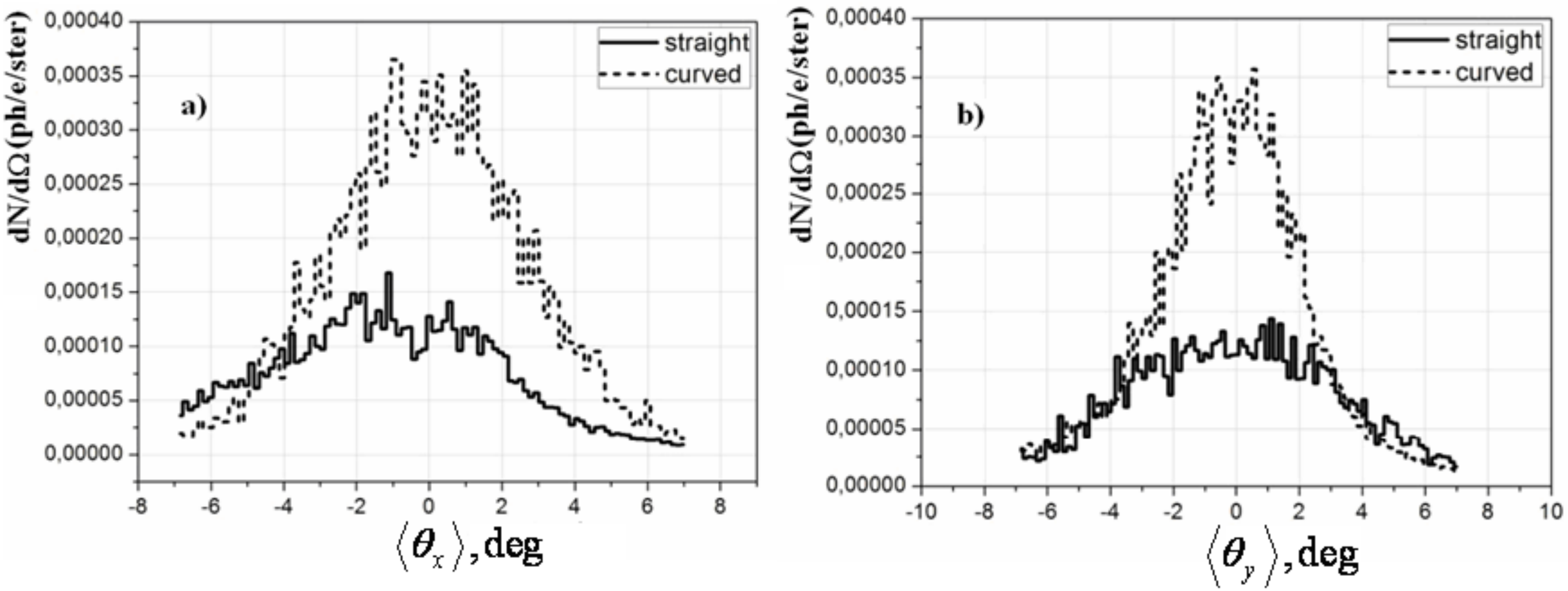


Fig.4. Are the same as in Fig. 1 for general type electron beam with finite cross section and angular spread (4-th case).

Due to cross section and angulat divergence there is focusing and density increase in the case of curved crystal

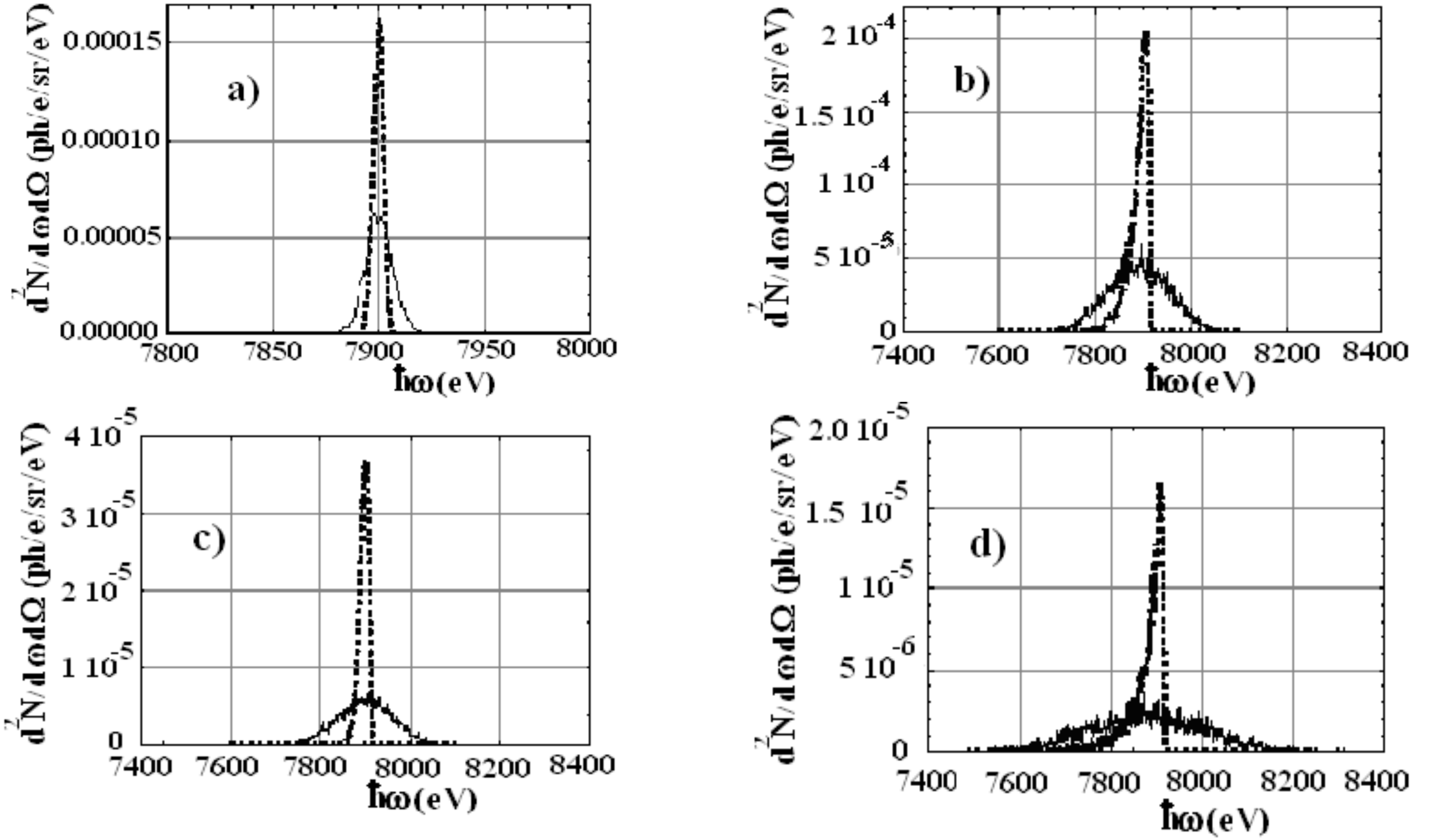


Fig. 5. The total PXR spectral lines for the cases a) “ideal beam”; b) “0 cross section diverging”; c) “thick pencil”; d) “general case” measured by small surface detector. The PXR intensity is equal to $8\cdot10^{-7}$ ph/e^- in “general case”, detector size = 1 cm^2 $\longrightarrow$ $d\Omega = 1.6\cdot10^{-3}$ sr at the focal point.

Line narrowing is evident

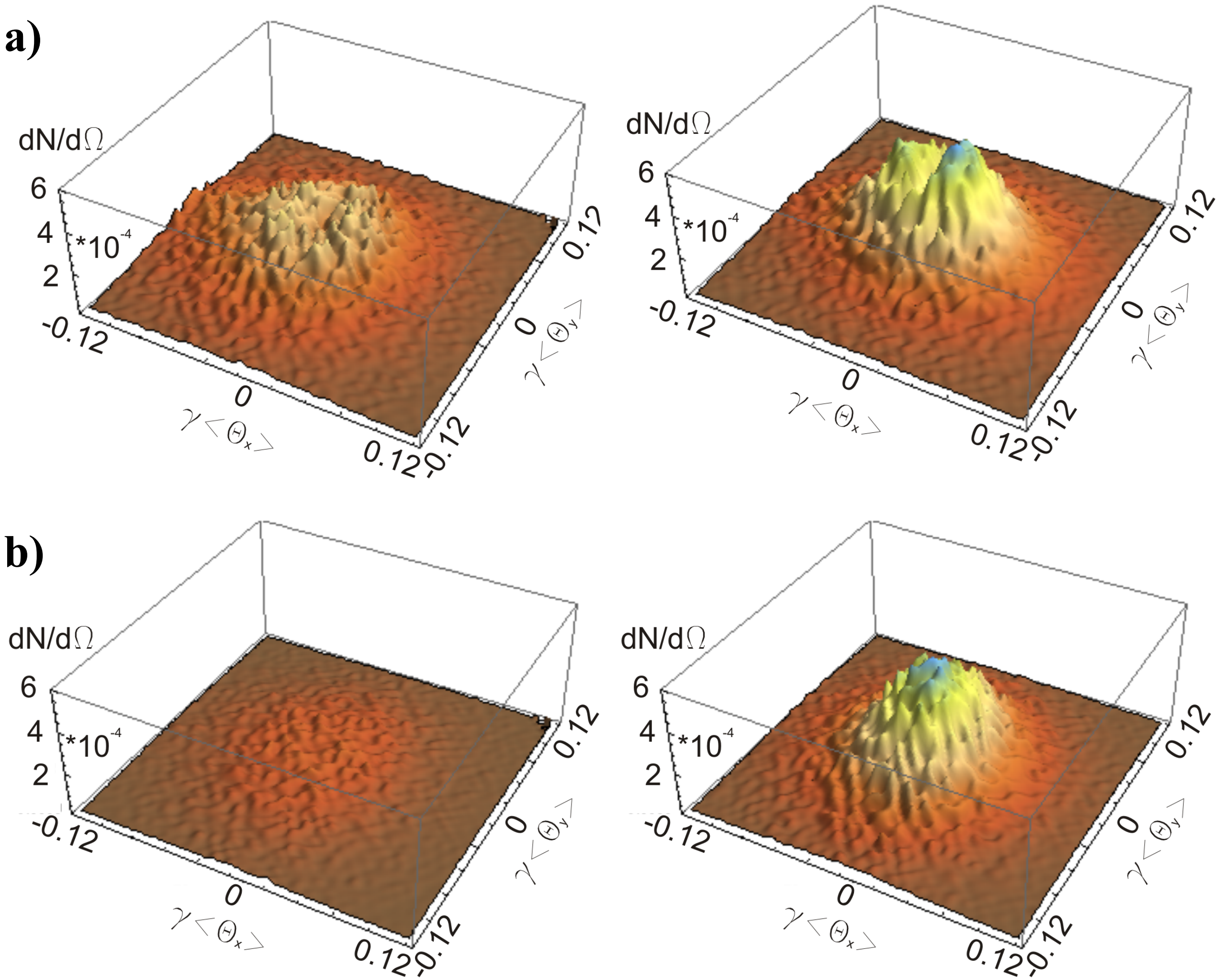


Fig. 6. The angular density of PXR produced by electron beam of "0 cross section diverging" a) and "general case" b) in flat (left) and curved (right) crystals measured by large surface detector.

Narrowing and density increase of regions is seen.

5. Experimental Consideration

a) Electron beam

Table 2. Parameters of the electron beam at the YerPhI microtron M-25.

Beam parameter	Value (unit)
Electron energy	(10-25) MeV
Macropulse length	$2\ 10^{-6}$ s
Repetition rate	400Hz
Micropulse length	$\sim 2\ 10^{-11}$ s
Number of electrons per micropulse	$3\ 10^{7}$
Number of micropulses per macropulse	6000
Electron beam cross section	~4x2 mm
Emittance (horizontal and vertical)	$3\ 10^{-2}$ and $1.5\ 10^{-2}$ mm rad

b) The arrangement

After ~90^o deflection by a magnet and 4 quads was shown in scheme.

c) The X-ray detectors

i) The small surface detector is an 2x3 mm^2 Amptek XR-100CR Detector with ~150 eV FWHM

ii) The large surface detector will be either 10x10 cm^2 MPC either Hamamatsu X-ray camera with CCD having energy resolutions ~15 % (at ~8 keV) and efficiencies (%, hw(keV))

$$\varepsilon_{MPC} = 0.0009(\hbar\omega)^3 - 0.0381(\hbar\omega)^2 + 0.4262(\hbar\omega) - 0.5087$$

$$\varepsilon_{CCD} = 0.0042(\hbar\omega)^3 - 0.0972(\hbar\omega)^2 + 0.6121(\hbar\omega) - 0.2281$$

5. Conclusions

It has been not considered multiple scattering and toraidal or elliptic curvature of the crystal. Nevertheless, in this work it has been shown that due to focusing many existing microtrons with average currents I_{av} ~ 10 μA can be used as PXR beam centers as the mentioned Japan and USA (100 and 56 MeV linacs with I_{av}~1 μA) ones.

Acknowledgements

One of the authors (K.A.I) thanks M.A. Aginian for multiple discussions and M. Petrossian for information on the YerPhI microtron.