Challenging Safe Fast Reactor Based on a Nuclear Burning Wave

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Organic Fuel Consumption
Dynamics of Electricity Production
(forecast of IEA & Nuclear)
Nuclear Energy

- Chemical reactions
  \[ + \text{O}_2 = \text{C} \text{O}_2 \]
  \( \sim 1 \text{ eV} \) (C

- Nuclear reactions
  \( \sim 100 \text{ MeV} \) (10^8 times)
  \[ 1 \text{ g U} \sim 5 \text{ t Co} \]

\[ k_{\text{eff}} = 1 \]
Nuclear Power Problems

- Safety (after Chernobyl accident)
- Closed fuel cycle (fuel reproduction)
- Ecology problems (nuclear waste)
- Nuclear terrorism

A-Bomb House, Hiroshima
Explored reserves of Uranium
Breeding process in FR:

$$^{238}\text{U} + n \rightarrow ^{239}\text{U} \rightarrow ^{239}\text{Np} \rightarrow ^{239}\text{Pu}$$

$$T_{1/2} \approx 2.35\text{ days}$$
World needs for Uranium (forecast till 2100)

Thermal Reactors
- 240 kt/year
- 115 kt/year

Fast Reactors
- 50 kt/year


\[ ^{238}\text{U} \ (n,\gamma) \rightarrow ^{239}\text{U} \ (\beta) \rightarrow ^{239}\text{Pu} \ (n,fission) \ldots \]

\[ \frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial z^2} + vn \left( \sigma_{a8} N_8 - \left( \sigma_a + \sigma_f \right)_{\text{Pu}} N_{\text{Pu}} \right) \]

\[ \frac{\partial N_8}{\partial t} = -vn\sigma_{a8} N_8 \]

\[ \frac{\partial N_9}{\partial t} = vn\sigma_{a8} N_8 - \frac{1}{\tau_\beta} N_9 \]

\[ \frac{\partial N_{\text{Pu}}}{\partial t} = \frac{1}{\tau_\beta} N_9 - vn \left( \sigma_a + \sigma_f \right)_{\text{Pu}} N_{\text{Pu}} \]

\[ N_{\text{cr}}^{Pu} = \frac{\sum_i \sigma_{ai} N_i}{(v - 1)\sigma_f^{Pu}} \]

\[ N_{eq}^{Pu} > N_{cr}^{Pu} \]

\[ x = z + Vt \]
History

1988 - Russia:

1996 - USA:

2000 - Japan:


2001 - Ukraine:
A.I. Akhiezer, N.A. Khizhnyak, N.F. Shulga, V.V. Pilipenko, L.N. Davydov., PAST, 6 (2001) 272; 276; 279.
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1946 Kharkov - UPhTI (Lab #1)

A.I. Akhiezer  I.Ya. Pomeranchuk
ВВЕДЕНИЕ В ТЕОРИЮ НЕЙТРОННЫХ МУЛЬТИПЛИЦИРУЮЩИХ СИСТЕМ (РЕАКТОРОВ)
Our goal: To study the space-time evolution of neutron flux in FR in the framework of multigroup diffusion theory, and to clarify the initiation conditions and stability of the nuclear burning wave (NBW) regime in FR.

Model: The model of calculation includes the following set of nonlinear partial differential equations: the non-stationary diffusion equation of neutron transport, the burn-up equations for ten components of the nuclear transformation chain involved in U–Pu cycle and the equations of nuclear kinetics for describing precursor nuclei of delayed neutrons.

![Diagram]

Composition: U-Pu fuel - 40 %, Na - 25 %, Fe - 35 %

**Ignition zone**
- Fuel: Pu - 10 %
- $^{238}\text{U}$ - 90 %

**Breeding zone**
- Fuel: $^{238}\text{U}$ - 100 %

$^{239}\text{Pu} : \overset{240}{\text{Pu}} : \overset{241}{\text{Pu}} : \overset{242}{\text{Pu}} = 0.70 : 0.22 : 0.05 : 0.03$
Non-stationary case  (one-group approximation)

\[
\frac{1}{\nu} \frac{\partial \Phi}{\partial t} + \frac{\partial W}{\partial z} + (\Sigma_a + DB_r^2) \Phi - (1 - \overline{\beta})(\nu_f \Sigma_f) \Phi = \sum_i \sum_j \lambda^i \lambda^j C^i_j
\]

\[W(z) = -D \frac{\partial \Phi}{\partial z}; \quad B_r = \frac{2.405}{R + 0.71/\Sigma_{tr}}; \quad D(z) = 1/3\Sigma_{tr}(z)\]

\[\overline{\beta} = \sum_i \beta_i \frac{(\nu_f \Sigma_f)_i}{\nu_f \Sigma_f}; \quad \beta_l = \sum_i \beta^i_i\]

\[\Sigma_a(z) = \sum_j \sigma^j_\alpha N^j(z) \quad \nu_f \Sigma_f(z) = \sum_j \nu_f \sigma^j_f N^j(z)\]

\[\Phi(0) - 2W(0) = 2j_{ex} \quad \Phi(L) + 2W(L) = 0\]

\[\Phi(z, t = 0) = 0 \quad 0 \leq z \leq L \quad 0 \leq t \leq T\]

\[T_n \approx 10^{-7} \text{ s} \quad T_d \approx 1 \text{ min} \quad T_{1/2} \approx 2.35 \text{ days} \quad T \approx 10 \text{ days}\]
Equations of nuclear kinetics for the precursor nuclei of delayed neutrons

\[ \frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i (\nu_f \Sigma_f)_i \Phi \]

\[ C_i^0(z, t = 0) = C_{0i}^0(z) . \]

Dynamics of the FR nuclear composition

Table. The numeration of the nuclei in the $^{238}\text{U}$–$^{239}\text{Pu}$ transformation chain

<table>
<thead>
<tr>
<th>N</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleus</td>
<td>$^{238}\text{U}$</td>
<td>$^{239}\text{U}$</td>
<td>$^{239}\text{Np}$</td>
<td>$^{239}\text{Pu}$</td>
<td>$^{240}\text{Pu}$</td>
<td>$^{241}\text{Pu}$</td>
<td>$^{242}\text{Pu}$</td>
<td>$^{243}\text{Am}$</td>
<td>$^{241}\text{Am}$</td>
<td>FP</td>
</tr>
</tbody>
</table>

\[ \frac{\partial N_l}{\partial t} = -\left(\sigma_{cl} \Phi + \Lambda_l \right) N_l + \left(\sigma_{cl} \Phi + \Lambda_{(l-1)} \right) N_{(l-1)} , \]

\[ (l = 1 \div 8) , \]

\[ \frac{\partial N_9}{\partial t} = \Lambda_6 N_6 , \]

\[ \sigma_{cl} = \sigma_{cl} + \sigma_{f1} , \quad \Lambda_l = \ln 2 / T_{1/2} / \]

\[ \frac{\partial N_{10}}{\partial t} = \sum_{l=1,4,5,6,7} \sigma_{f1} N_f \Phi , \]

\[ N_l(z, t = 0) = N_{0l}(z) \]
Calculation technique

A rectangular mesh with steps $h$ and $\tau$ (uniform for $z$ and with variable step for $t$) was used. The solutions of the set of the algebraic equations were obtained by finite-difference technique using the implicit difference Crank-Nicolson scheme. This symmetric-in-time scheme has an unconditional stability at any relation between space and time steps. It is the unique implicit scheme that has approximation of the second order of accuracy in $h$ and $\tau$. Here $n$ numerates time layer, $K$ is the node of the space mesh.

$$-a^{(K)}\Phi^{n+1}(K-1) + \left(b^{(K)} + \frac{1}{\sigma}\right)\Phi^{n+1}(K) - c^{(K)}\Phi^{n+1}(K+1) =$$

$$= a^{(K)}\Phi^n(K-1) - \left(b^{(K)} - \frac{1}{\sigma}\right)\Phi^n(K) + c^{(K)}\Phi^n(K+1) + V_{n+1}(K) + V_n(K)$$

$$V_n(K) = \sum \lambda_i C^n_i(K), \quad \Phi^n(K) = \Phi(z_K, t_n), \quad \sigma = \nu\tau.$$  

Computing

On the basis of the developed approach a package of original numerical codes has been elaborated for determination of the critical parameters of assemblies under consideration in the one- and multi-group approximations and for simulation of the neutron flux space-time evolution inside FR in the one-group approximation. In calculations we used the BNAB 26-group constants.
Goldin’s solution in FR

$\phi$, $10^{17}$ cm$^2$s$^{-1}$  
$N_{pu}$, $10^{21}$ cm$^{-3}$
Nuclear burning wave in FR

$\Phi$, $10^{17} \text{ cm}^{-2}\text{s}^{-1}$

$N_{\text{Pu}}$, $10^{21} \text{ cm}^{-3}$

$t=40 \text{ days}$

Graph showing the change in $\Phi$ and $N_{\text{Pu}}$ over distance $x$ in cm.
Stability of the burning process

Integrated neutron flux as function of time \( t \) (days) at the initial stage of burning.
The solid curve has been calculated with \( j_{ex} \) turned off at \( t_{off} = 10 \) days and the dashed one – without turning \( j_{ex} \) off.

Reactivity:

\[
\rho = \frac{k_{eff} - 1}{k_{eff}}
\]

at \( t_{off} = 30 \) days, dashed blue line.
Breeding zone
Fuel: $^{238}$U -100 %

Ignition zone
Fuel: $^{238}$U - 90 %
+ Pu - 10 %

Composition: U-Pu fuel - 44 %, coolant Na – 36 %, constructional material Fe – 20 %

External neutron flux: $j_{ex} = 6 \cdot 10^{11} \text{ cm}^{-2}\text{s}^{-1}$
(a) scalar neutron flux \((\times 10^{16}\ \text{cm}^{-2}\ \text{s}^{-1})\);  
(b) power density \((\text{kW cm}^{-3})\); 
(c) concentration of \(^{239}\text{Pu} \times 10^{21}\ \text{cm}^{-3}\); 
(d) depth of fuel burn-up (%) 
for for \(t_1 = 5, t_2 = 100, t_3 = 2000, t_4 = 4000\) and \(t_5 = 5000\) days.
Nuclear burning wave in 500 cm length and 110 cm radius cylindrical FR

\[ \Phi, \, 10^{17} \text{ cm}^{-2}\text{s}^{-1} \]

\[ N_{\text{pu}}, \, 10^{21} \text{ cm}^{-3} \]
Nuclear burning wave in 5m length cylindrical FR for different reactor radius $R$

NBW velocity $V$, cm/day

Integral neutron flux $\Phi_I$, $\times 10^{17}$cm$^{-1}$s$^{-1}$

$R = 150$ cm (red line); $120$ cm (green line); $R = 110$ cm (blue line)
Dependence of the NBW velocity $V$ on the reactor radius $R$
Fuel burn-up

Fission products

$^{239}\text{Pu}$

$^{238}\text{U}$
Fuel burn-up

$t = 5500$ days

$N_{\text{fuel}}$

$x, \text{cm}$

Fission products

$^{239}\text{Pu}$

$^{238}\text{U}$
2D calculations of the neutron flux evolution in cylindrical fast reactor $R=120$ cm & $L=500$ cm.
$^{232}\text{Th} \rightarrow ^{233}\text{Th}$  
$^{233}\text{U} \rightarrow ^{234}\text{U} \rightarrow ^{235}\text{U} \rightarrow ^{236}\text{U} \rightarrow ^{237}\text{U}$  
$^{233}\text{Pa} \quad \text{T}_{1/2} = 22.2\text{ min}$  
$^{237}\text{Np} \quad \text{T}_{1/2} = 6.75\text{ days}$  
$^{238}\text{U} \rightarrow ^{239}\text{U}$  
$^{239}\text{Pu} \rightarrow ^{240}\text{Pu} \rightarrow ^{241}\text{Pu} \rightarrow ^{242}\text{Pu} \rightarrow ^{243}\text{Pu}$  
$^{241}\text{Am} \quad \text{T}_{1/2} = 14.3\text{ years}$  
$^{243}\text{Am} \quad \text{T}_{1/2} = 4.98\text{ hours}$  
$^{239}\text{Np} \quad \text{T}_{1/2} = 2.35\text{ days}$  
$^{237}\text{U} \quad \text{T}_{1/2} = 2.35\text{ days}$  
$^{239}\text{U} \quad \text{T}_{1/2} = 23.5\text{ min.}$
Nuclear Burning Wave in Th-U

medium

$\Phi$

t=0.00 years

$N_U$

Fr__Th-U_noHe__160_years
Conclusion

Main features of the reactor working in the NBW regime:

- Self-sustained critical state due to the negative reactivity feedback (internal safety)
- No necessity of permanent control & fuel reloading (underground location)
- Fuel is the natural or depleted uranium ($^{238}\text{U}$)
- Fuel burn-up depth is about 50 %
- Intensive accumulation of Pu ($\approx 10\%$ along the reactor) (fuel recycling)
- Nuclear waste incineration (ecology)
- NBW velocity is about 22 cm/year for reactor radius $R = 110$ cm
- Neutron flux at the stable NBW regime $\approx 2 \cdot 10^{16}$ cm$^{-2}$ s$^{-1}$
- Power density $\approx 2$ kW cm$^{-3}$
- Total power $\approx 2$ GW
- No NBW regime for Th-U fuel cycle
Thank you for attention!