On dynamic effects in coherent X-radiation of relativistic electron in Bragg scattering geometry

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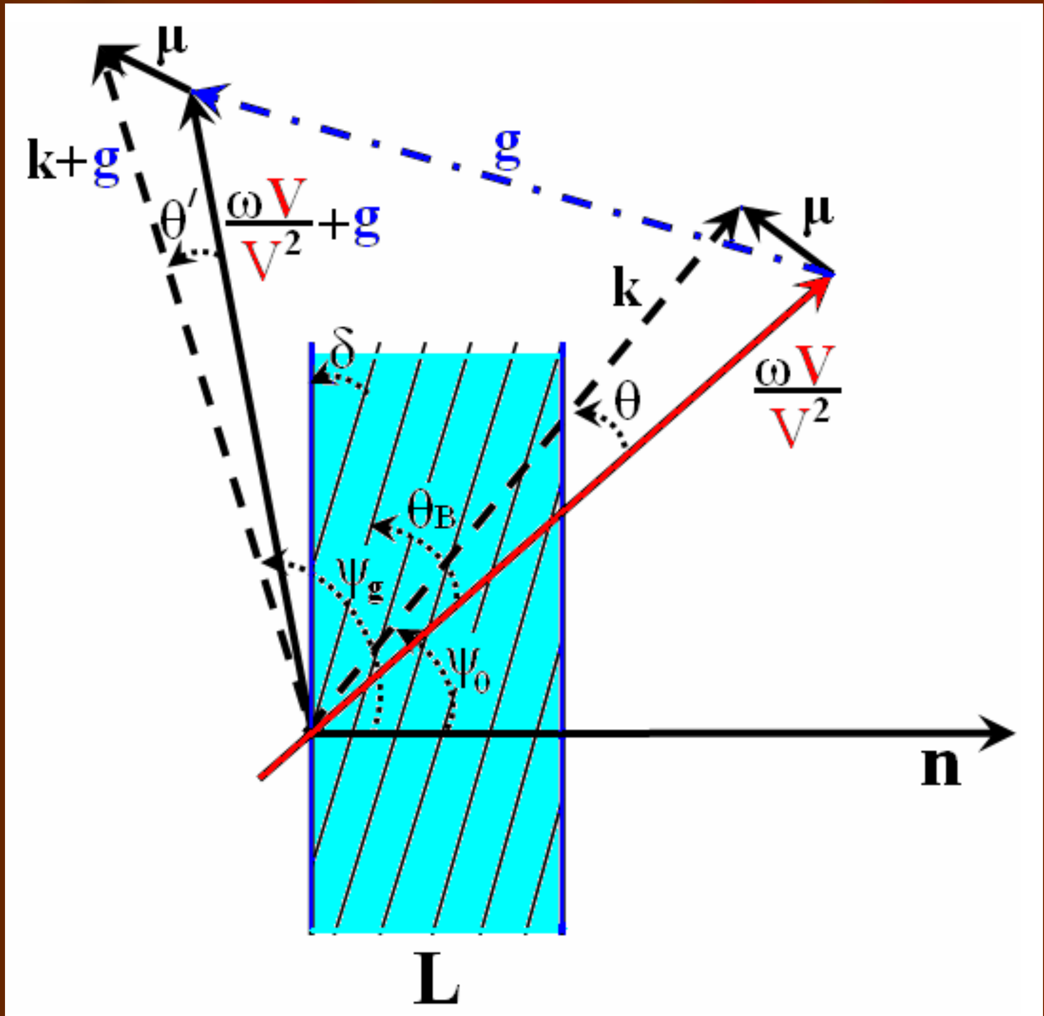

Fig.1. Radiation process geometry

θ' – radiation angle, θ_B– Bragg angle, **k** end **k_g=k+g** - Wave vectors of incedent and diffracted photons accordingly, δ - the angle between plate surface and atomic planes, V – velocity of incident electron,

$\varepsilon=\sin(\theta_B-\delta)/\sin(\theta_B+\delta)$

belongs to the formulas for X- radiation

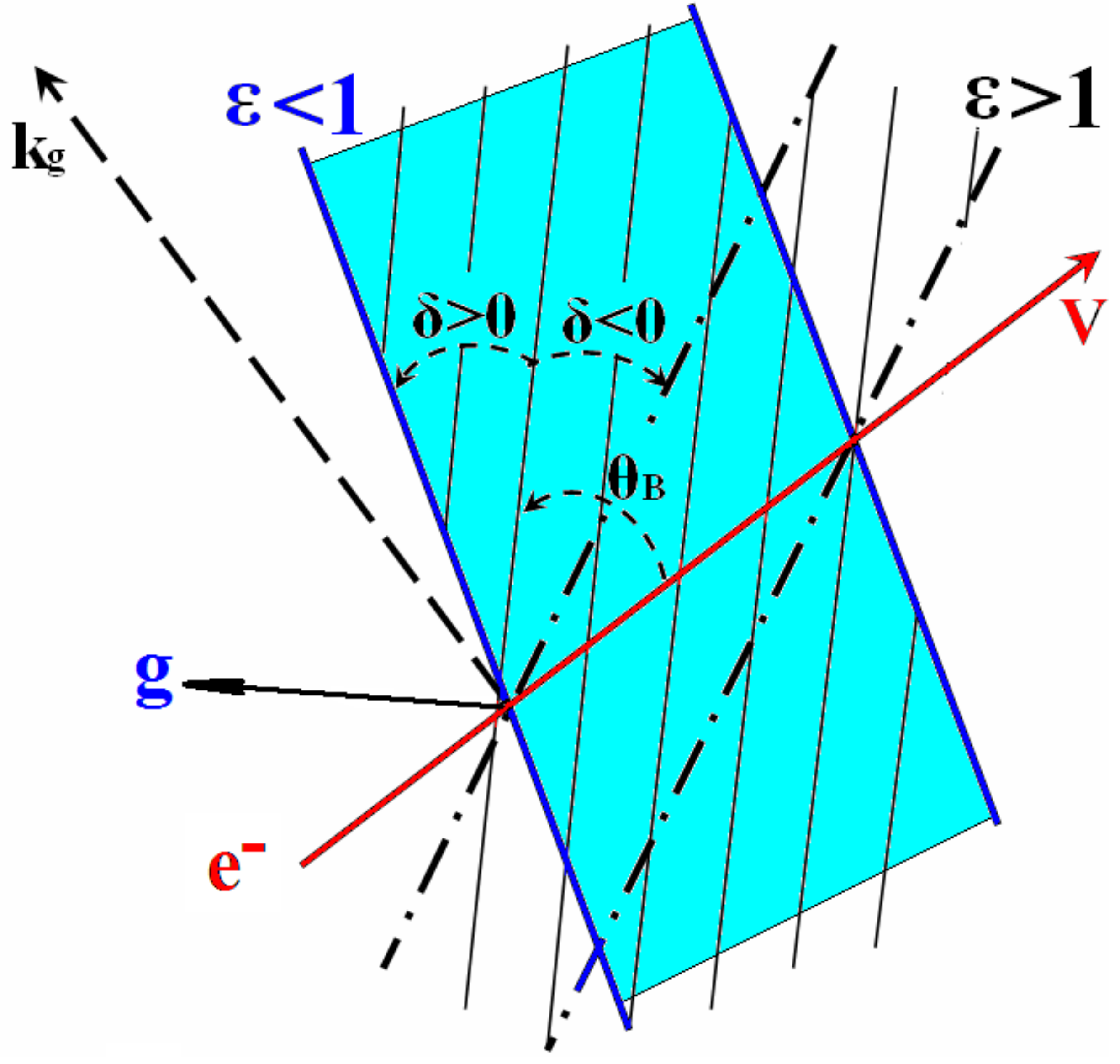

Fig.2. Asymmetric ($\varepsilon>1$, $\varepsilon<1$) radiation reflection from crystal plate. In the case of symmetric reflection $\varepsilon=1$, $\delta=0$.

Spectral-angular distributon of PXR

$$\omega\frac{d^2N_{PXR}^{(s)}}{d\omega d\Omega}=\frac{e^2}{\pi^2}\frac{P^{(s)^2}\theta^2}{(\theta^2+\gamma^{-2}-\chi_0')^2}R_{PXR}^{(s)},$$

$$R_{PXR}^{(s)}=\left|\frac{\Omega_+^{(s)}}{\Delta^{(s)}}\frac{1-\exp\left(-ib^{(s)}\Delta_+^{(s)}\right)}{\Delta_+^{(s)}}-\frac{\Omega_-^{(s)}}{\Delta^{(s)}}\frac{1-\exp\left(-ib^{(s)}\Delta_-^{(s)}\right)}{\Delta_-^{(s)}}\right|^2,$$

where

$$\Delta^{(s)}=\left(\xi^{(s)}-K^{(s)}-i\rho^{(s)}\frac{1+\varepsilon}{2}\right)\exp\left(-ib^{(s)}\Delta_+^{(s)}\right)-\left(\xi^{(s)}+K^{(s)}-i\rho^{(s)}\frac{1+\varepsilon}{2}\right)\exp\left(-ib^{(s)}\Delta_-^{(s)}\right)$$

$$\Omega_\pm^{(s)}=\varepsilon\left((\sigma^{(s)}-i\rho^{(s)})\cdot\exp\left(-ib^{(s)}\Delta_\mp^{(s)}\right)+\Delta_\pm^{(s)}\right)\qquad \Delta_\pm^{(s)}=\frac{\xi^{(s)}\pm K^{(s)}}{\varepsilon}-\sigma^{(s)}+i\frac{\rho^{(s)}(\varepsilon-1)}{2\varepsilon}$$

$$K^{(s)}=\sqrt{\xi^{(s)2}-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4}-\kappa^{(s)2}\varepsilon\right)}$$

$$b^{(s)}=\frac{\omega|\chi_g'|C^{(s)}}{2}\cdot\frac{L}{\gamma_0},\qquad C^{(1)}=1,\qquad C^{(2)}=|\cos 2\theta_B|\qquad \varepsilon=\sin(\theta_B-\delta)/\sin(\theta_B+\delta)$$

The conditions of significance of both PXR branches contribution are

$$\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2-\varepsilon}}{\varepsilon}-\sigma^{(s)}=0$$

$R^{(1)}_{PXR}$

$$\frac{\xi^{(s)}(\omega)+\sqrt{\xi^{(s)}(\omega)^2-\varepsilon}}{\varepsilon}-\sigma^{(s)}=0$$

$R^{(2)}_{PXR}$

If $\varepsilon \geq \nu^{(s)}$ then works the branch $R^{(2)}_{PXR}$ only, otherwise work both the branches.

Let note, that $\nu^{(s)}=\left(\frac{\chi'_g C^{(s)}}{\chi'_0}\right)^2 \leq 1$ always.

$\rho=0,\ \varepsilon \geq \nu^{(s)2},$

$$\omega \frac{d^2 N_{PXR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{P^{(s)^2} \theta^2}{(\theta^2 + \gamma^{-2} - \chi_0')^2} R_{PXR}^{(s)},$$

$$R_{PXR}^{(2)(s)} = \frac{\left(\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}\right)^2}{\xi^{(s)2} - \varepsilon + \varepsilon \sin^2\left(\frac{b^{(s)}\sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon}\right)} \cdot$$

$$\cdot \frac{\sin^2\left(\frac{b^{(s)}}{2}\left(\frac{\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon} - \sigma^{(s)}\right)\right)}{\left(\frac{\xi^{(s)} + \sqrt{\xi^{(s)2} - \varepsilon}}{\varepsilon} - \sigma^{(s)}\right)^2},$$

where

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{(1+\varepsilon)}{2\nu^{(s)}}$$

Widening effect of the PXR spectrum

Let us consider

- L is thickness of crystal plate,
- $L/\cos(\Psi_0)$ is pass of electron in the plate
- $1/\chi_g' C^{(s)}$ is an extinction length of photon in the crystal

Then the condition of dynamic effects manifestation will be

$$b^{(s)} = \frac{L}{2\cdot\cos(\psi_0)} \Big/ \frac{1}{\omega\left|\chi_g' C^{(s)}\right|} \gg 1$$

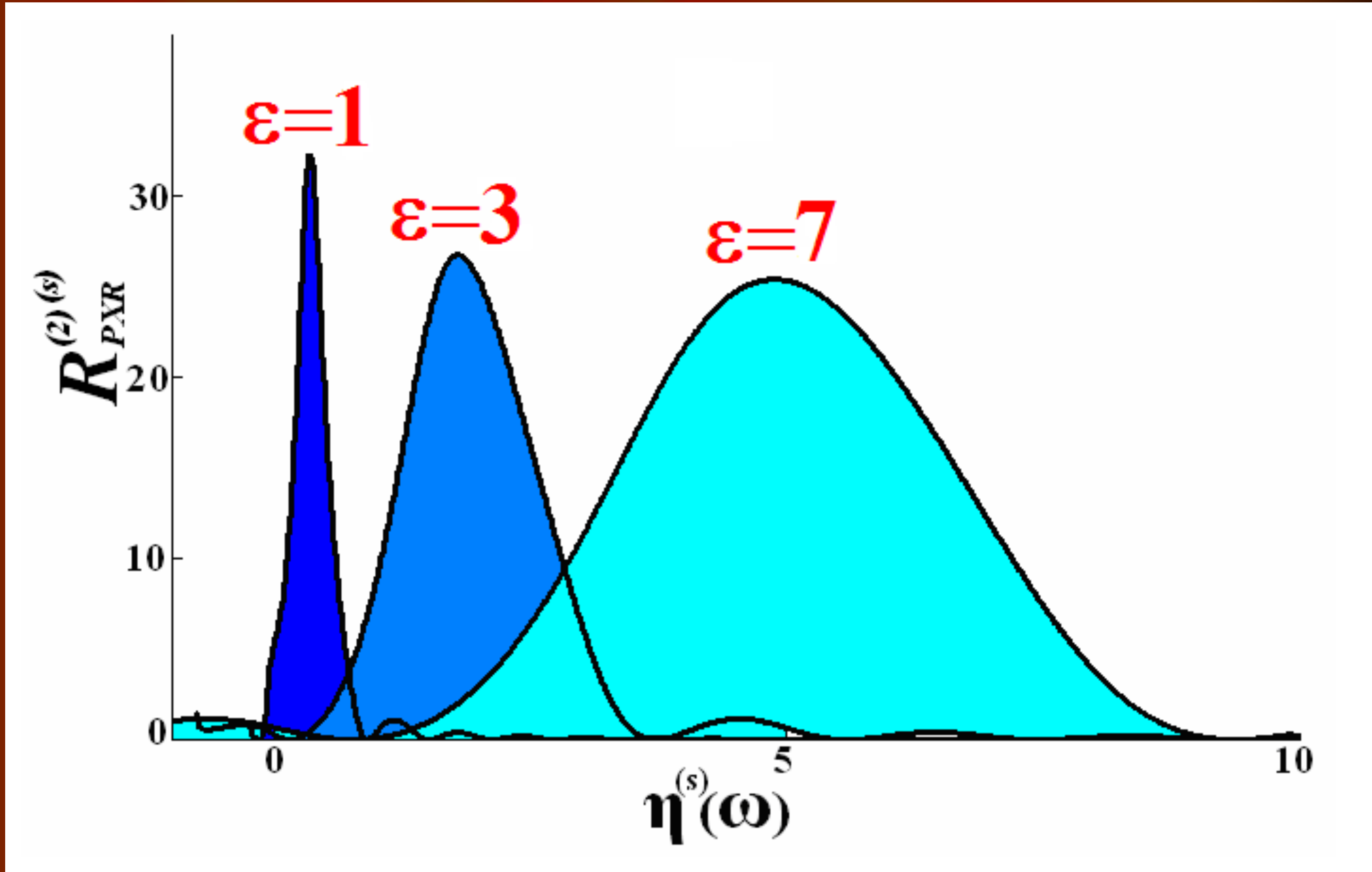

Fig.3. Widening of spectrum under asymmetry parameter increase. $\theta/\sqrt{|\chi'_0|}=1$, $1/(\gamma\sqrt{|\chi'_0|})=0.5$, $b^{(s)}=5$, $\nu^{(s)}=0.8$,

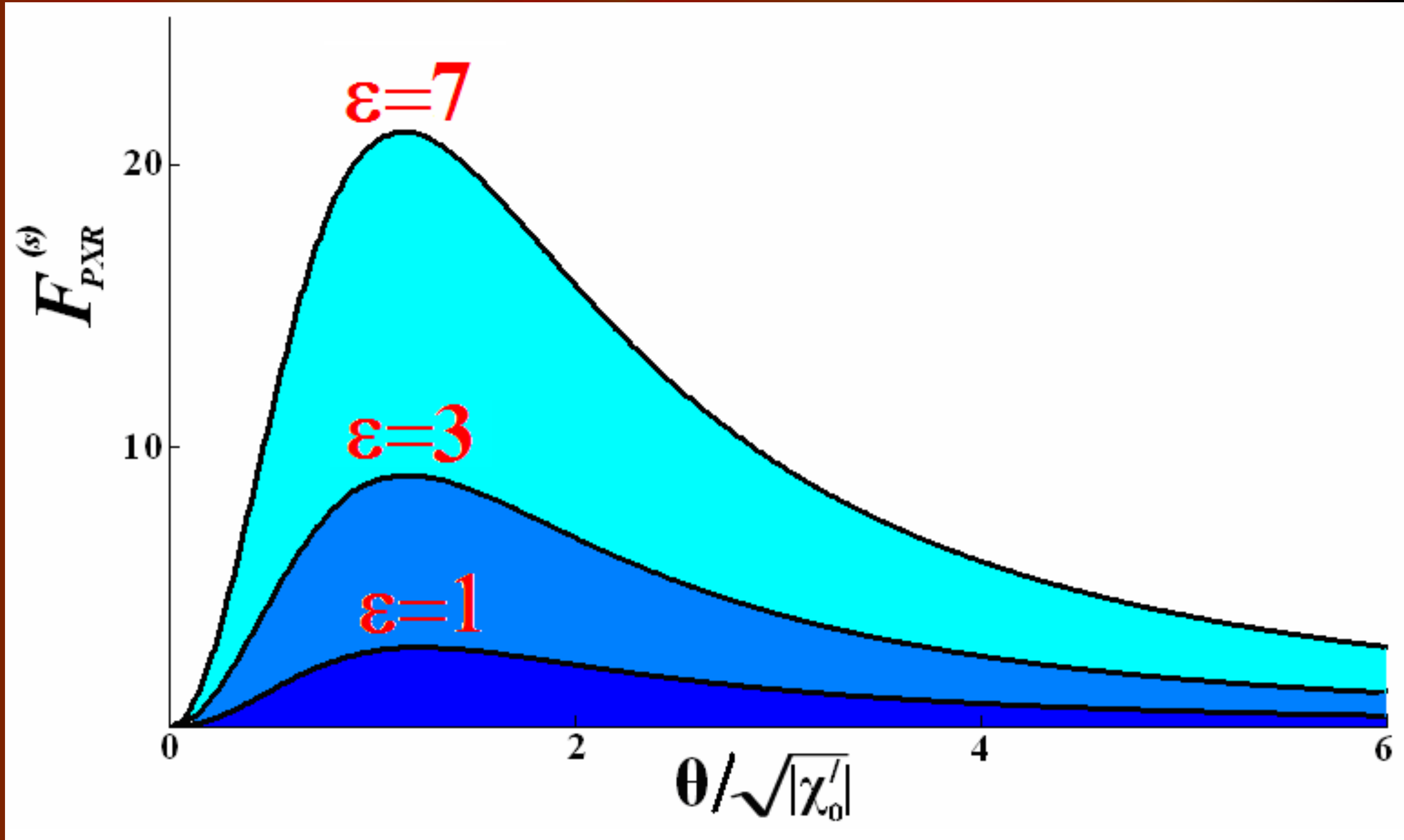

Fig.4. PXR angular density. Value of parameters $b^{(s)}$, $\nu^{(s)}$, $\theta/\sqrt{|\chi'_0|}$ and $1/(\gamma\sqrt{|\chi'_0|})$ are the same as in Fig.3.

- In the case $\varepsilon < \nu^{(s)}$ both equation

$$\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2-\varepsilon}}{\varepsilon}-\sigma^{(s)}=0$$

$$\frac{\xi^{(s)}(\omega)+\sqrt{\xi^{(s)}(\omega)^2-\varepsilon}}{\varepsilon}-\sigma^{(s)}=0$$

are valid and branches $R^{(1)}_{PXR}$, d $R^{(2)}_{PXR}$ and interference term give the contribution in the radiation

$$R^{(s)}_{PXR}=R^{(2)^{(s)}}_{PXR}+R^{(1)^{(s)}}_{PXR}+R^{(ИНТ)^{(s)}}_{PXR}$$

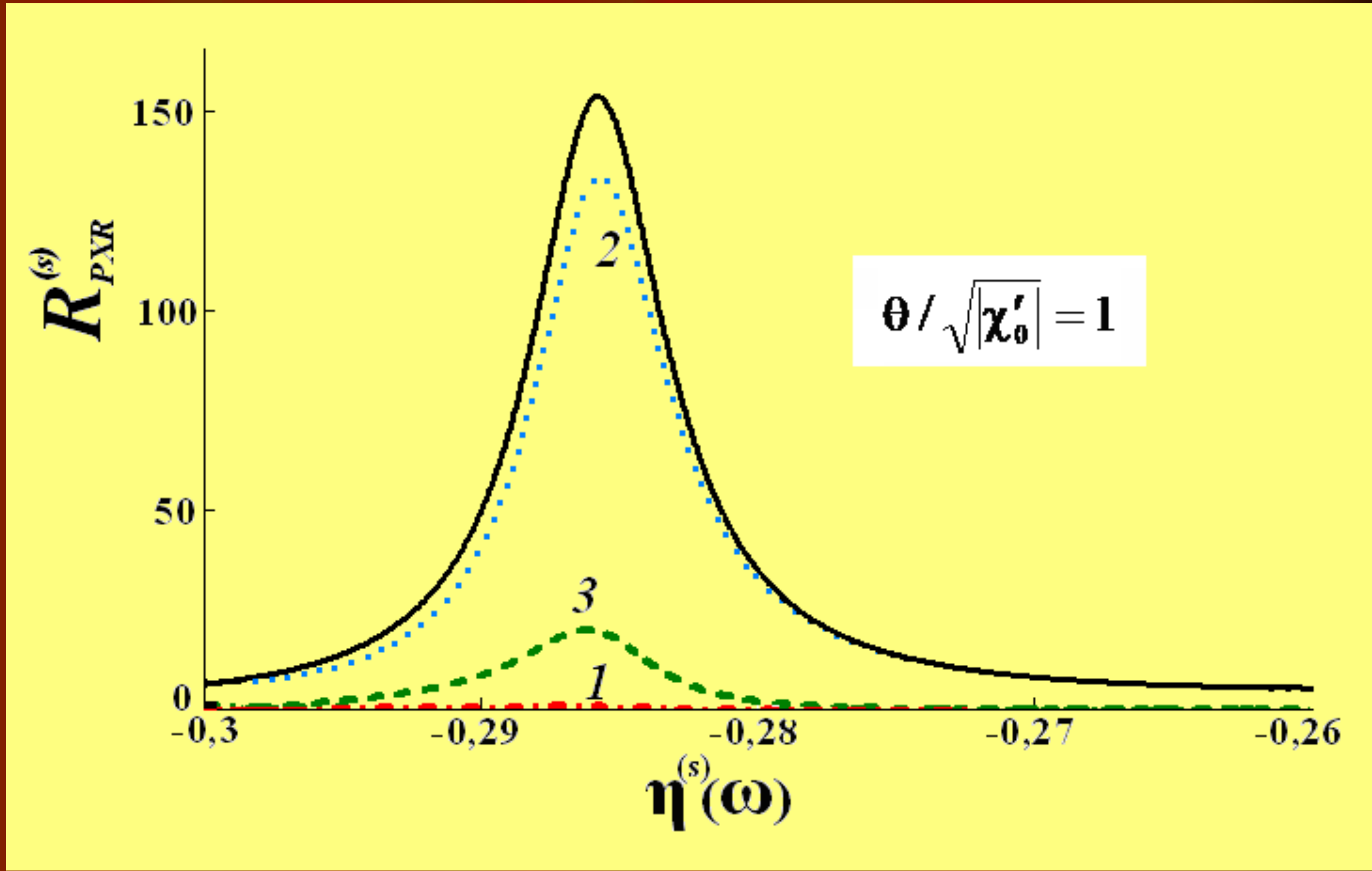

Fig.5. The contributions of the PXR branches and their interference item: 1 – $R^{(1)}_{PXR}$, 2 - $R^{(2)}_{PXR}$, 3 – $R^{(int)}_{PXR}$ $\varepsilon = 0.2$

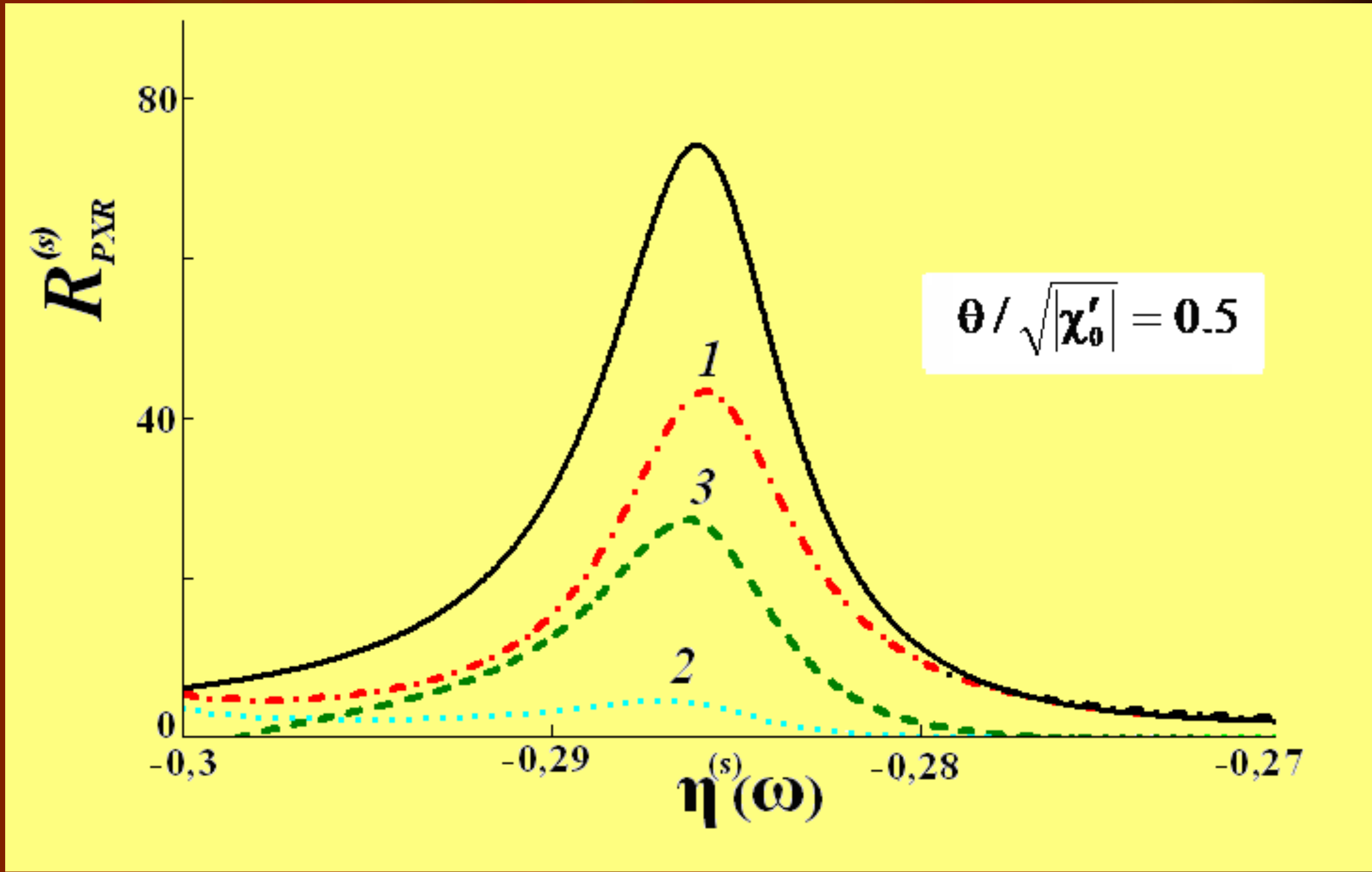

Fig.6. The same as in the Fig.5.

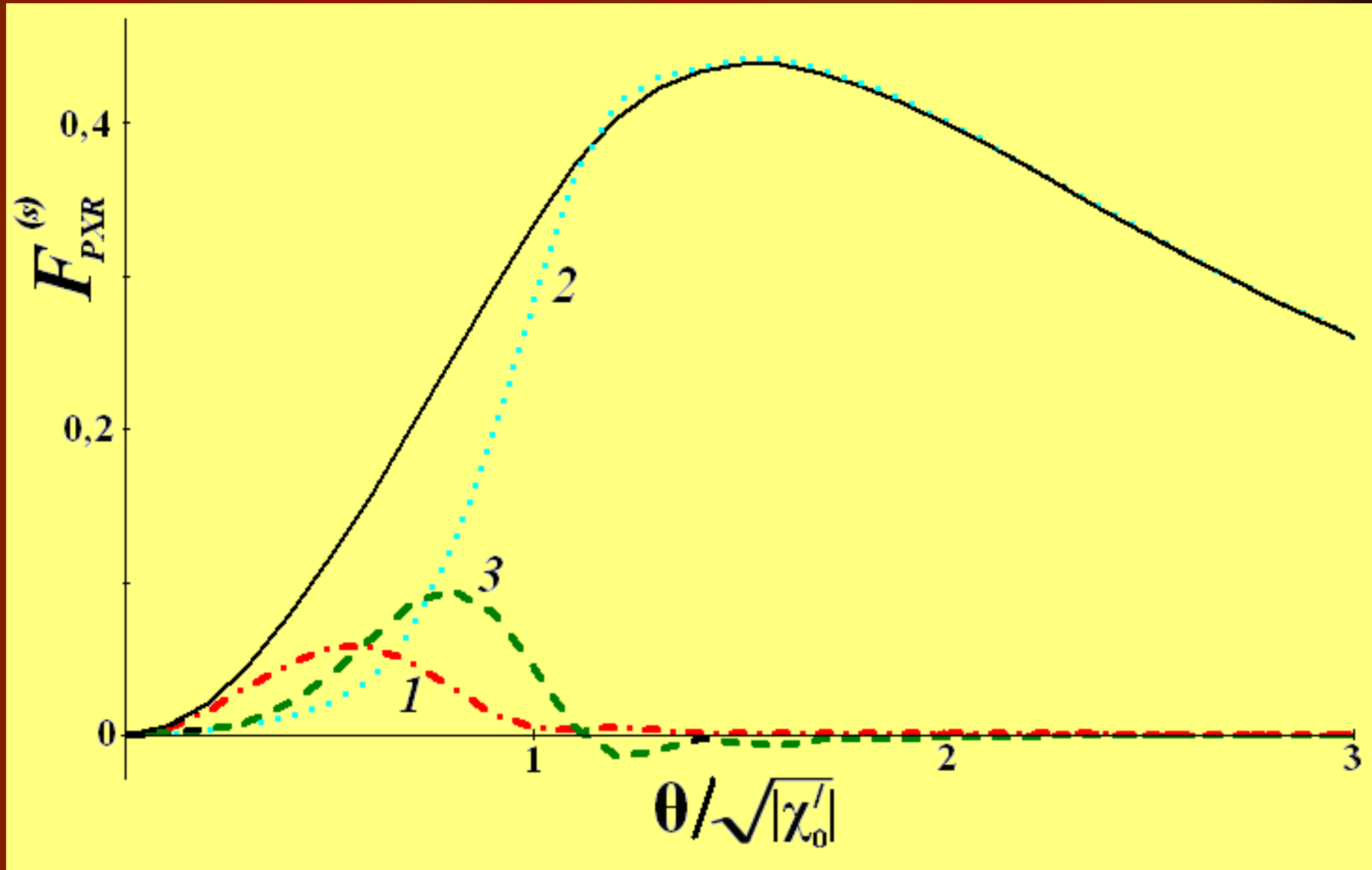

Fig.7. Contribution of the PXR branches and their interfirence item to angle density of the radiation. All the symbols are the same as in Fig.5.

Widening of DTR spectrum

$$E_{\mathrm{Rad}}^{(s)} = E_{PXR}^{(s)} + E_{DTR}^{(s)}$$

$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{\mathrm{Rad}}^{(s)} \right|^2$$

$$\omega \frac{d^2 N_{DTR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} P^{(s)^2} \theta^2 \left(\frac{1}{\theta^2 + \gamma^{-2} - \chi_0'} - \frac{1}{\theta^2 + \gamma^{-2}} \right)^2 R_{DTR}^{(s)},$$

For thin crystal plate

$$R_{DTR}^{(s)} = \frac{\varepsilon^2}{\xi^{(s)^2} - \left(\xi^{(s)^2} - \varepsilon \right) \coth^2 \left(\frac{b^{(s)} \sqrt{\varepsilon - \xi^{(s)^2}}}{\varepsilon} \right)},$$

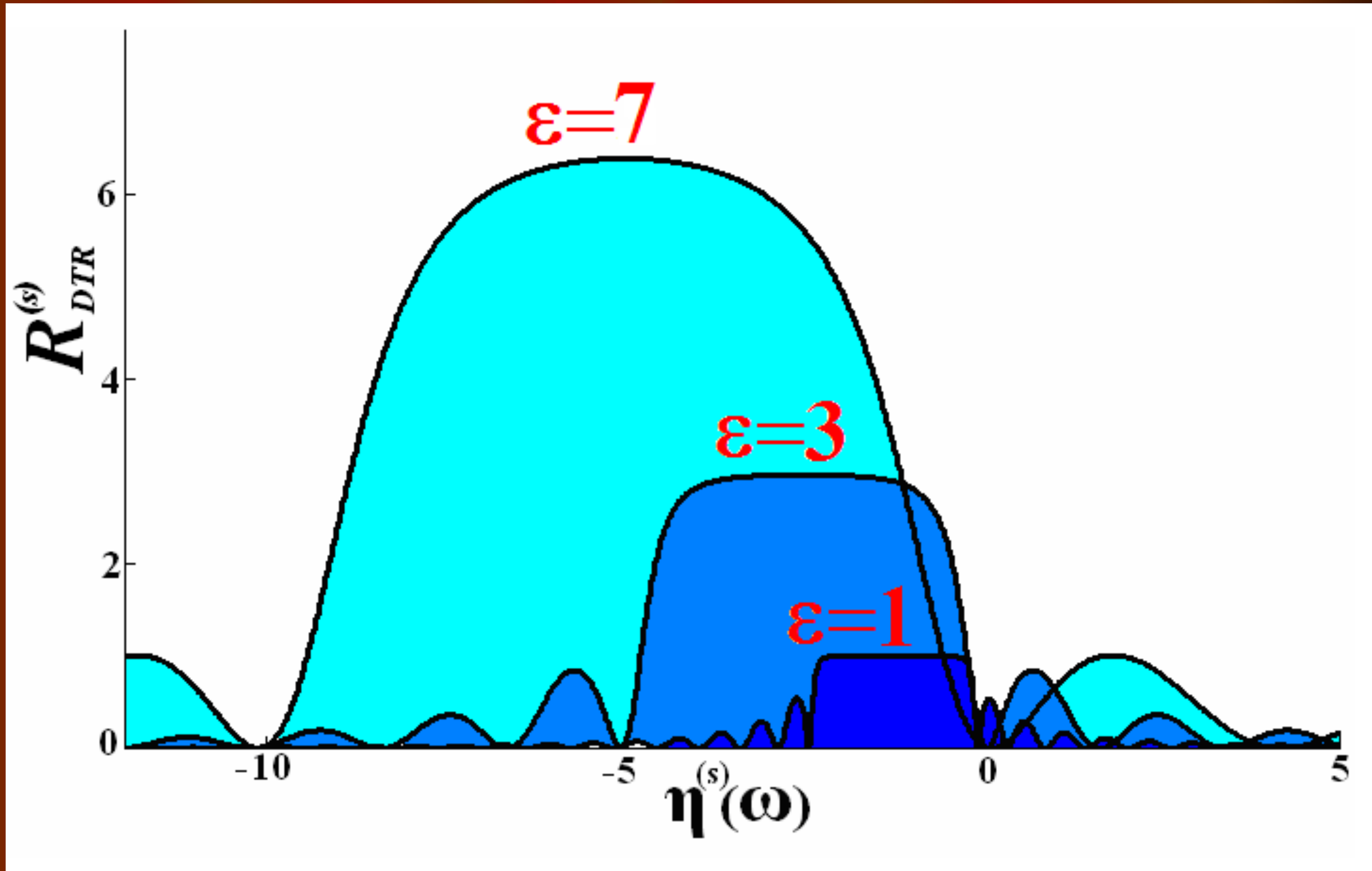

Fig.8. Widening of DTR spectrum.

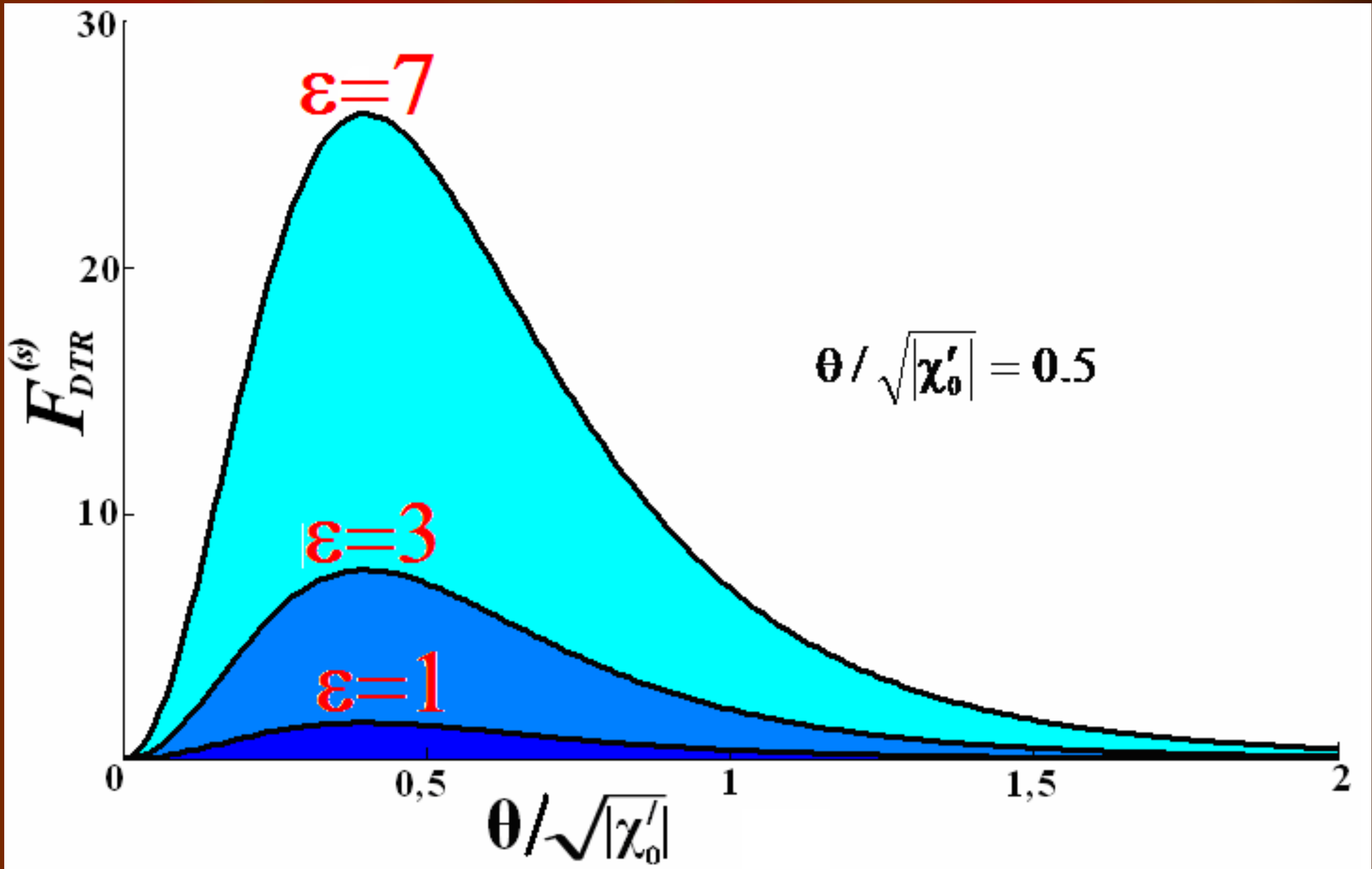

Fig.9. Angle density of DTR. All parameters are the same as in Fig.5.

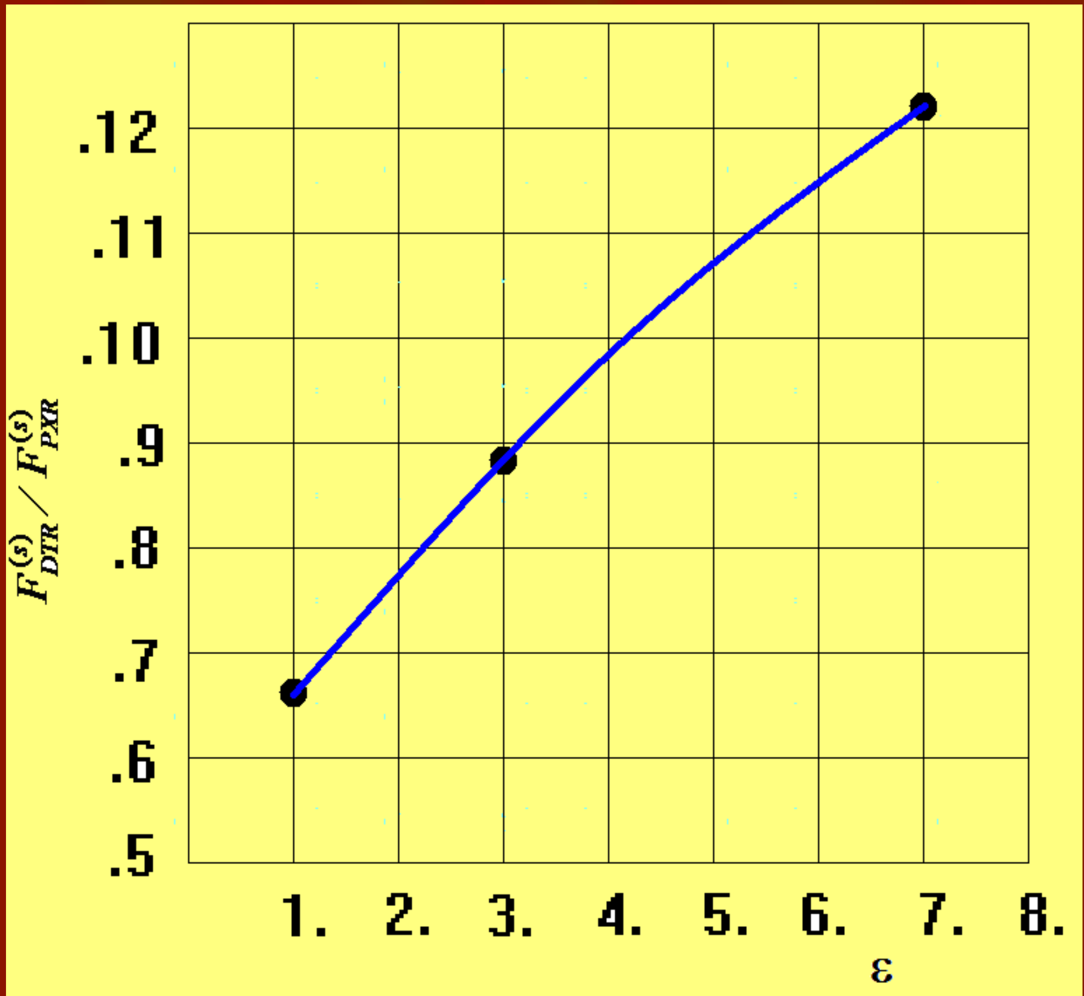

Fig.9a.

The relateve increase of DTR and PXR angular density

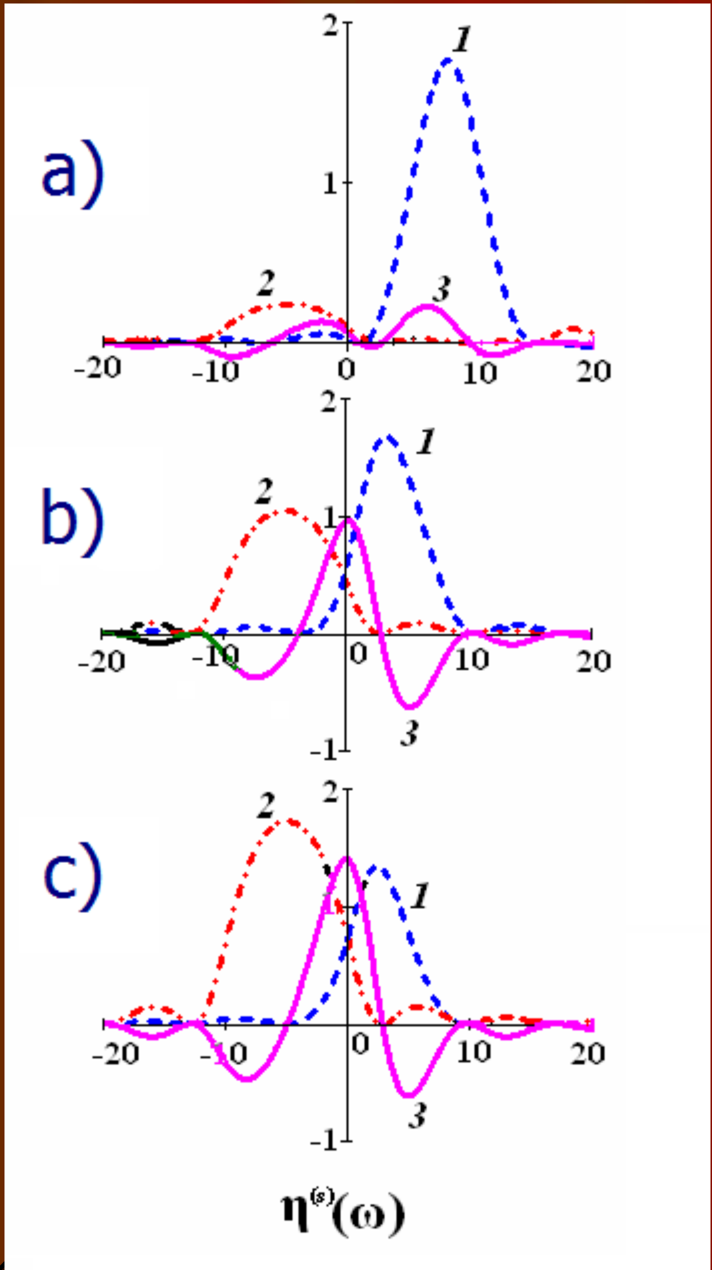

Fig.10. The relateve contributions PXR and DTR to total coherent radiation

1 – $T^{(s)}_{PXR}$, 2 - $T^{(s)}_{DTR}$

3 – $T^{(s)}_{INT}$

$\varepsilon=7$, $b^{(s)}=3$, $\nu^{(s)}=\underline{0.8}$,

$1/(\gamma\sqrt{|\chi'_0|})=0.5.$

a) $\theta/\sqrt{|\chi'_0|}=1.3,$

b) $\theta/\sqrt{|\chi'_0|}=0.8,$

c) $\theta/\sqrt{|\chi'_0|}=0.3,$

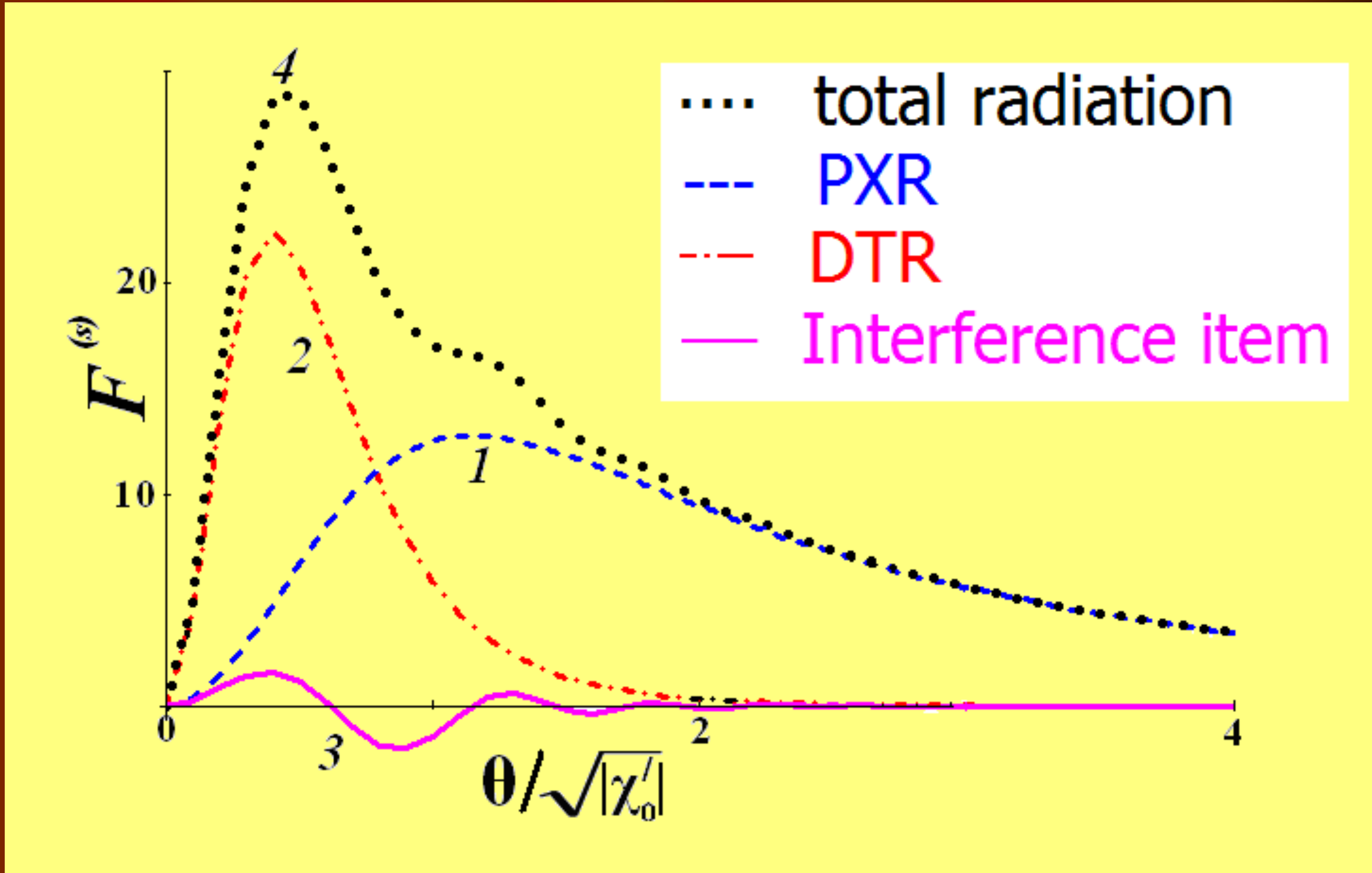

Fig.11. Relative contributions of PXR, DTR and their interference item to total radiation angle density

Thank you for attention

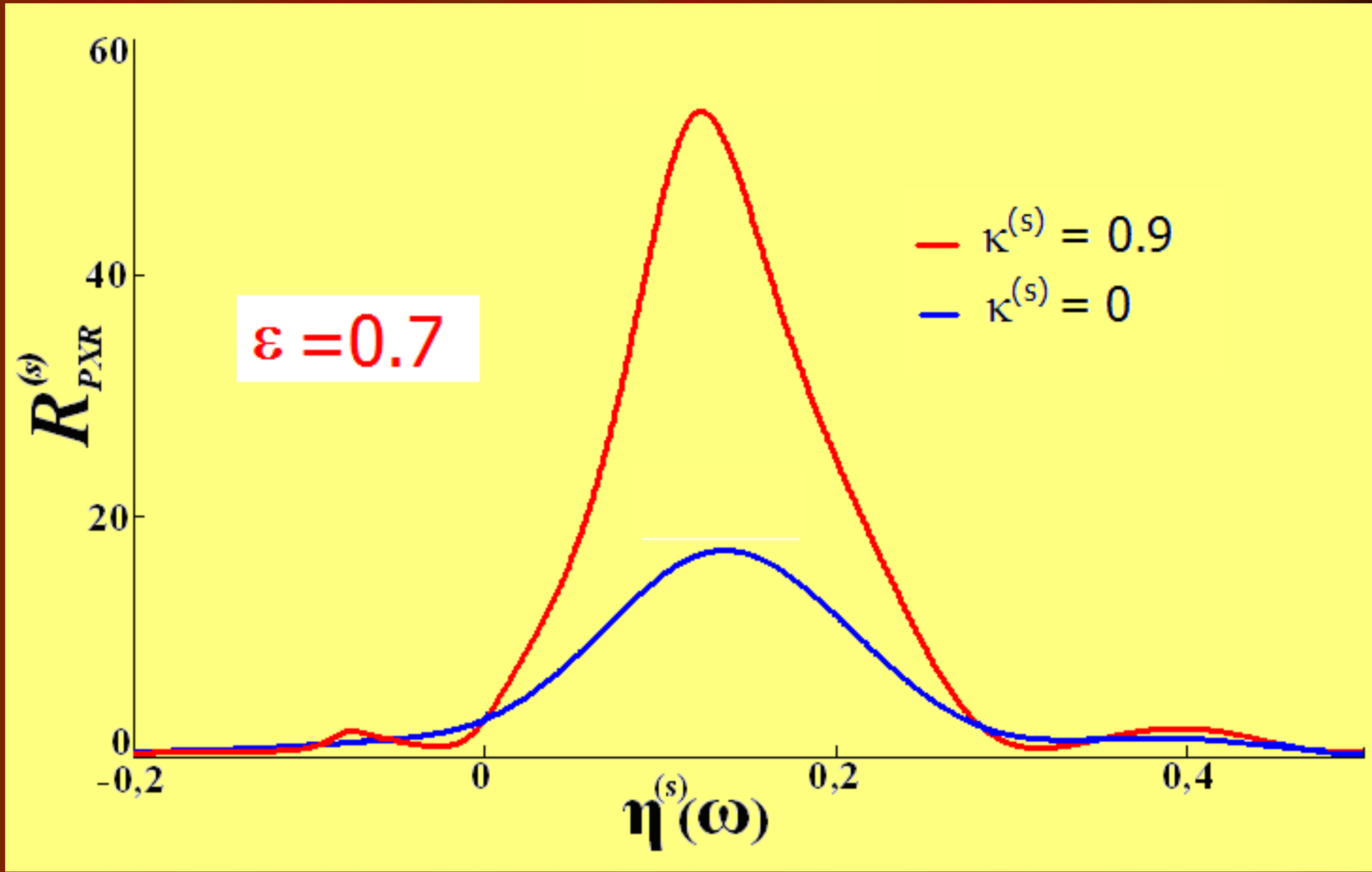

Fig. 12. The Borrmann effect manifestation degree. ε=0.7, b(s)=10, ν(s)=0.9, ρ(s)=0.1, others are the same as in Fig.3.

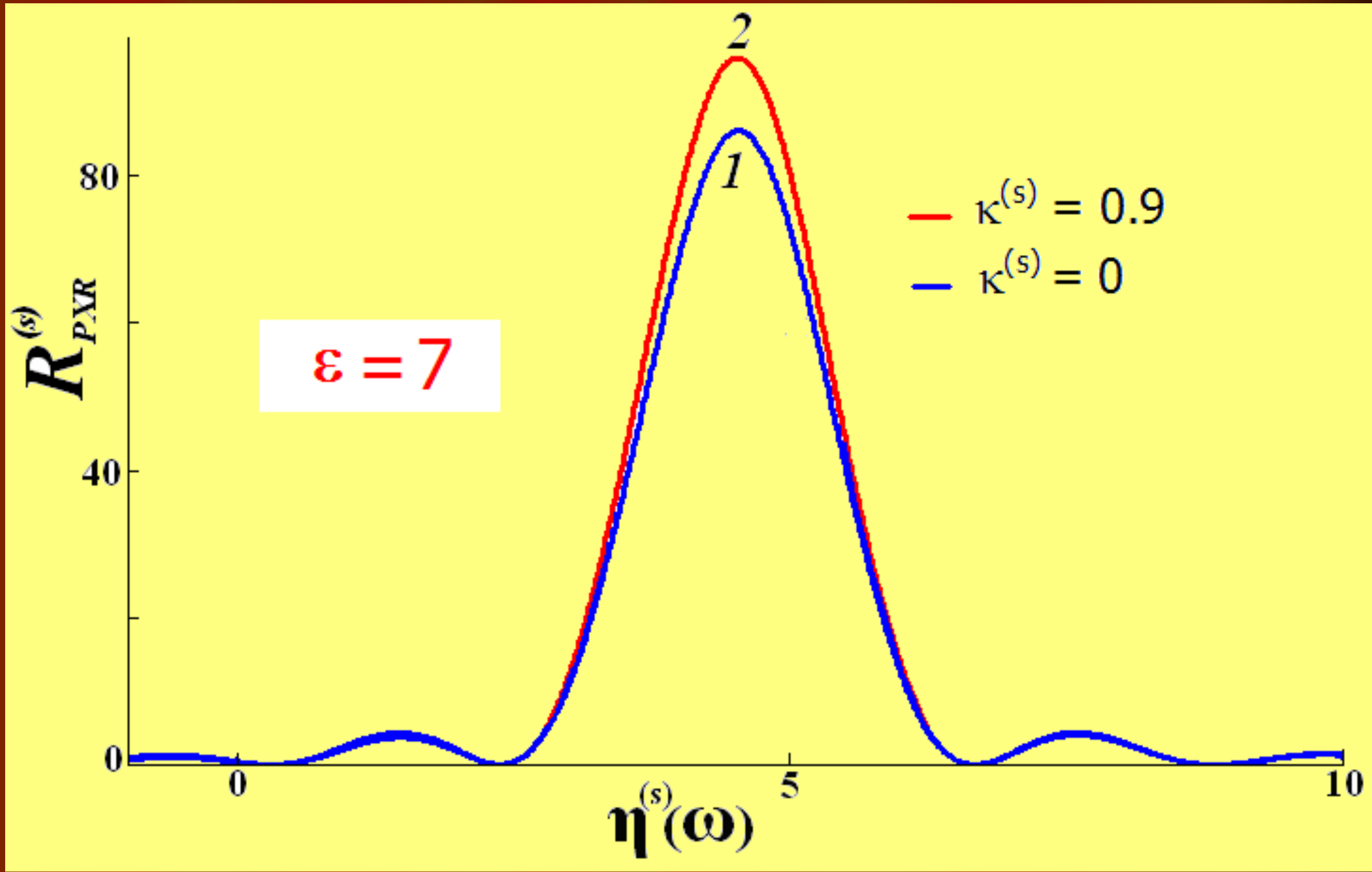

Fig.14. The Borrmann effect manifestation ointo PXR spectrum. The conditions are the same as in Fig. 12.

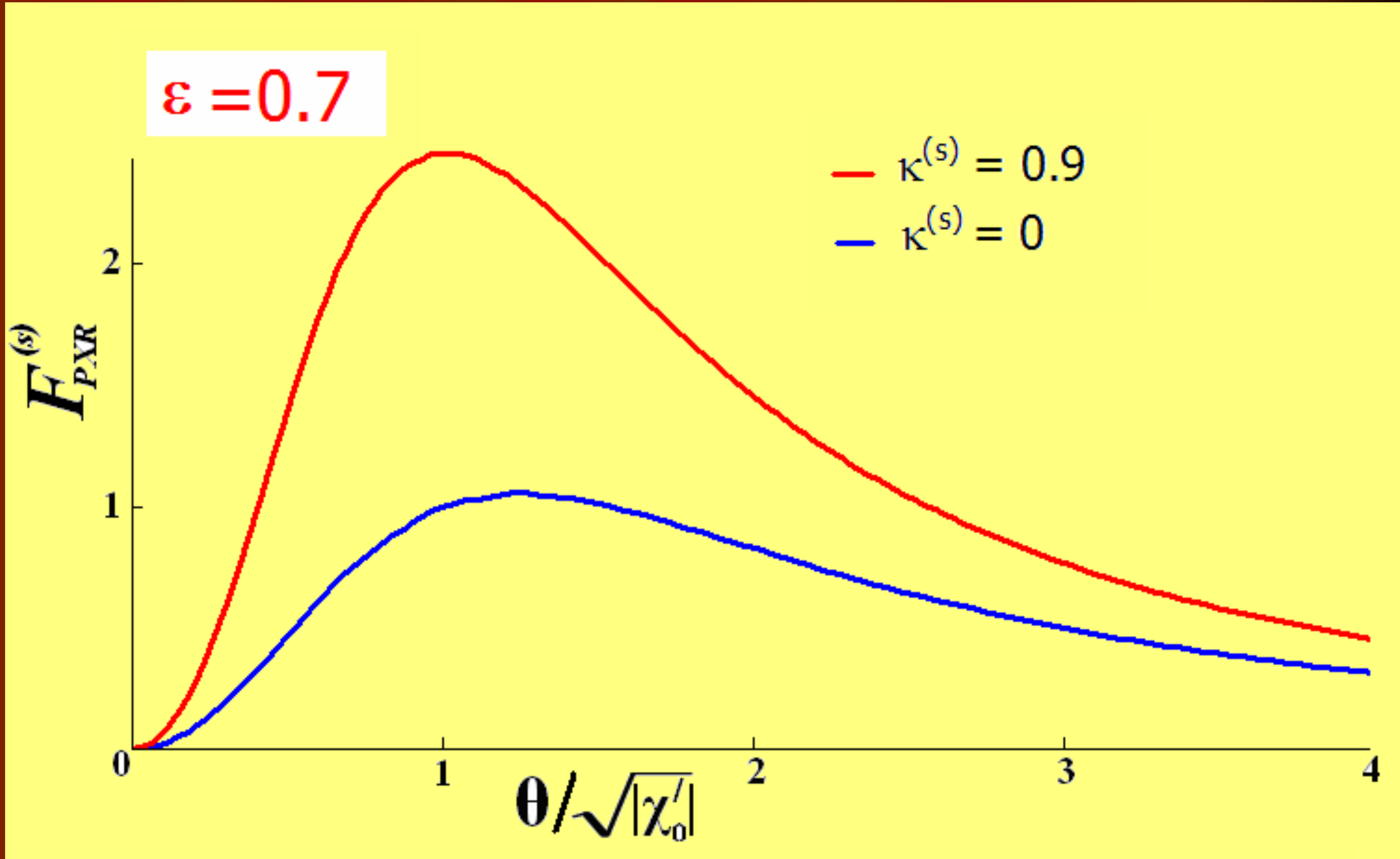

Fig.14. The Borrmann effect manifestation into PXR angular density. The conditions are the same as in Fig. 12.

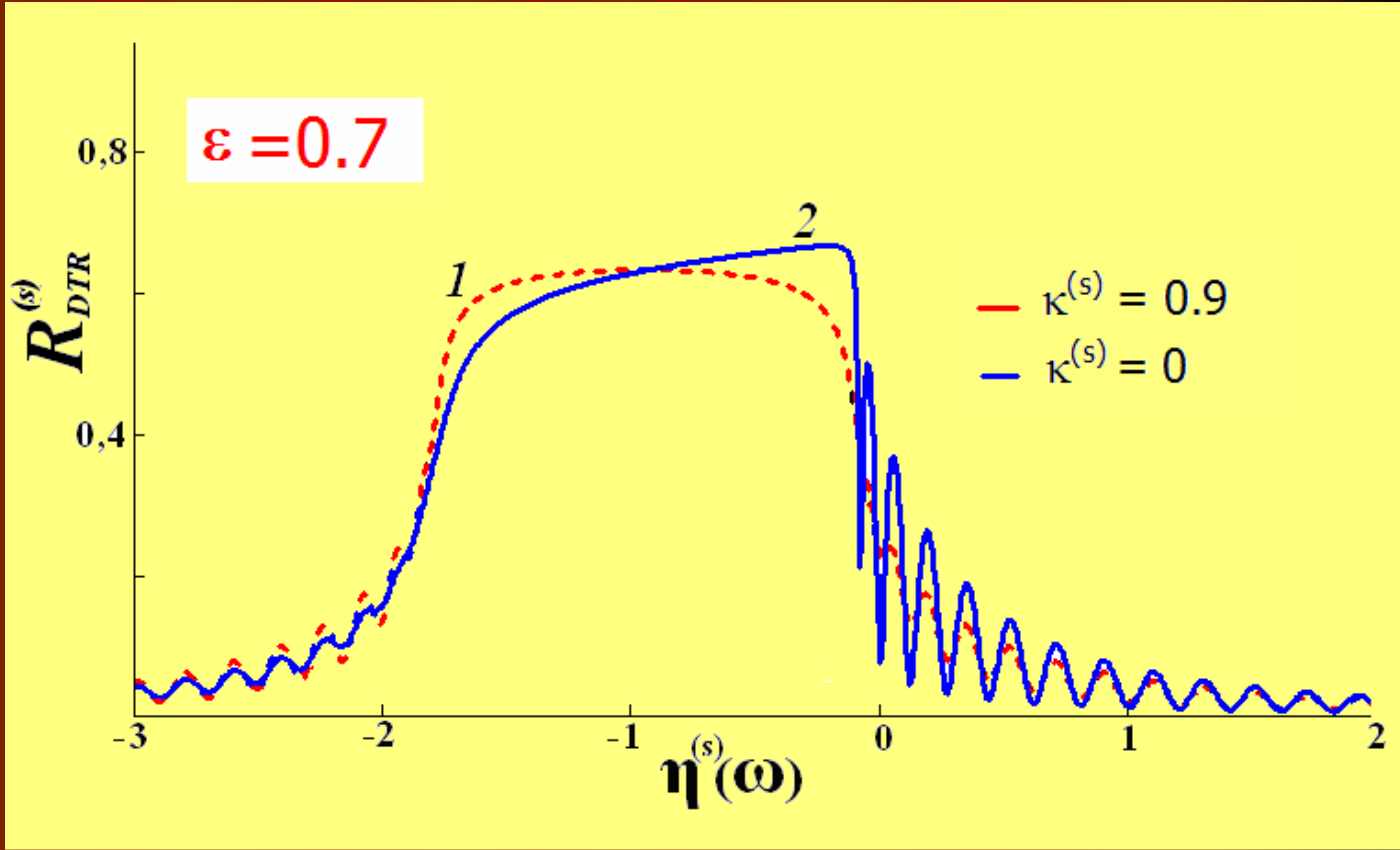

Fig.15. The Borrmann effect in DTR

DTR angular density. Condition is the same as in Fig. 12.

Conclusion

- Analytical expressions for spectral-angular distribution of parametric X-radiation (PXR) and diffracted transition radiation (DTR) of relativistic electron crossing a single crystal plate in Bragg geometry are derived on the basis of two-wave approximation of dynamic diffraction theory [1]. The expressions for the general case of asymmetrical reflection of relativistic electron Coulomb field on the plate are obtained. It was shown that in this geometry the essential increase of PXR angular density is possible due to a dynamic effect of PXR spectrum broadening just as in Laue geometry [2]. The considered Bragg geometry is interesting because of an interference effect of extinction, as the wave vector becomes complex even when an absorption is absent, and then the incident wave energy is transferred into reflected wave. A frequency region where the extinction effect manifests itself is named "region of total absorption". It is shown that decrease of angle of electron incidence to the plate under fixed Bragg angle (asymmetric change) gives rise to a considerable increase of DPR angle density. The possibility of dynamical Borrmann effect manifestation in PXR and DPR for a thick single crystal is also considered in this work. It is shown that by the change of reflection asymmetry one can create such conditions in which the length of the electron path in the plate will become small enough for the electron multiply scattering neglect and the length of photon path will become more then the photo absorption length resulting in bright manifestation of Borrmann effect in PXR.