Methods of charged particle beam cooling

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- **Introduction.**
- **Methods of the beam cooling.**
- **Possible applications.**
- **Conclusion.**

Introduction

Beam cooling is the increase of the six dimensional (6D) phase space density of the beam $\rho(\vec{r},\vec{p}) = dN/dV_{6D}$ due to the reduction of its 6D volume (emittance).

Brightness of the beam (usually measured as the current per unit volume, per unit solid angle and per unit energy spread) is proportional to the phase space density. It determines the luminosity of the colliding beams and brilliance of the light sources based on particle beams.

Thus, cooling of beams is of high importance for many applications.

The normalized emittance of the beam

$$\in_{n,\,6D} = \int\int\int\int dV_{6D} = \int\int\int\int dx\cdot dy\cdot dz\cdot dp_x\cdot dp_y\cdot dp_z$$

There are two ways for the particle beam cooling: cooling based on friction & on finite interparticle spacing.

Cooling based on friction:

1) Radiation cooling,
2) Ionization cooling by energy losses in media (bremsstrahlung, ionization, excitation),
3) Electron cooling,
4) Cooling of relativistic ion beam by counter propagated monochromatic laser beam,
5) Radiative (Stimulated Radiation) ion cooling by broadband laser beam.

Cooling based on finite interparticle spacing:

6) Stochastic cooling (SC),
7) Optical stochastic cooling (OSC),
8) Transit-time OSC,
9) Enhanced optical cooling (EOC).

Cooling based on friction:

According to the Robinson damping criterion the rate of the beam density change in circular machines is determined by the 6D damping increment

$$\alpha_{6D} = \overline{div\ \overrightarrow{F}_{Fr}} = \frac{2}{\beta^2}\frac{\overline{P}_{Fr}(p)}{\varepsilon}\Big|_{\varepsilon=\varepsilon_s} + \frac{\partial \overline{P}_{Fr}(p)}{\partial \varepsilon}\Big|_{\varepsilon=\varepsilon_s},$$

where $\overrightarrow{F}_{Fr} = -\alpha(\vec{r},p,t)\cdot\vec{n}_p$ **is the friction force,** $\alpha(\vec{r},p,t) = P_{Fr}(p,t)/c\beta$ **is the frictional coefficient,** $\vec{n}_p = \overrightarrow{p}/|\overrightarrow{p}|$ **is the unite vector directed along the particle velocity,** $\overrightarrow{p}$ **is the particle momentum,** $p = |\overrightarrow{p}|$ **,** $\overline{P}_{Fr}(p)$ **is the averaged rate of the particle energy loss owing to friction,** ε_s **is the synchronous energy of the particle.**

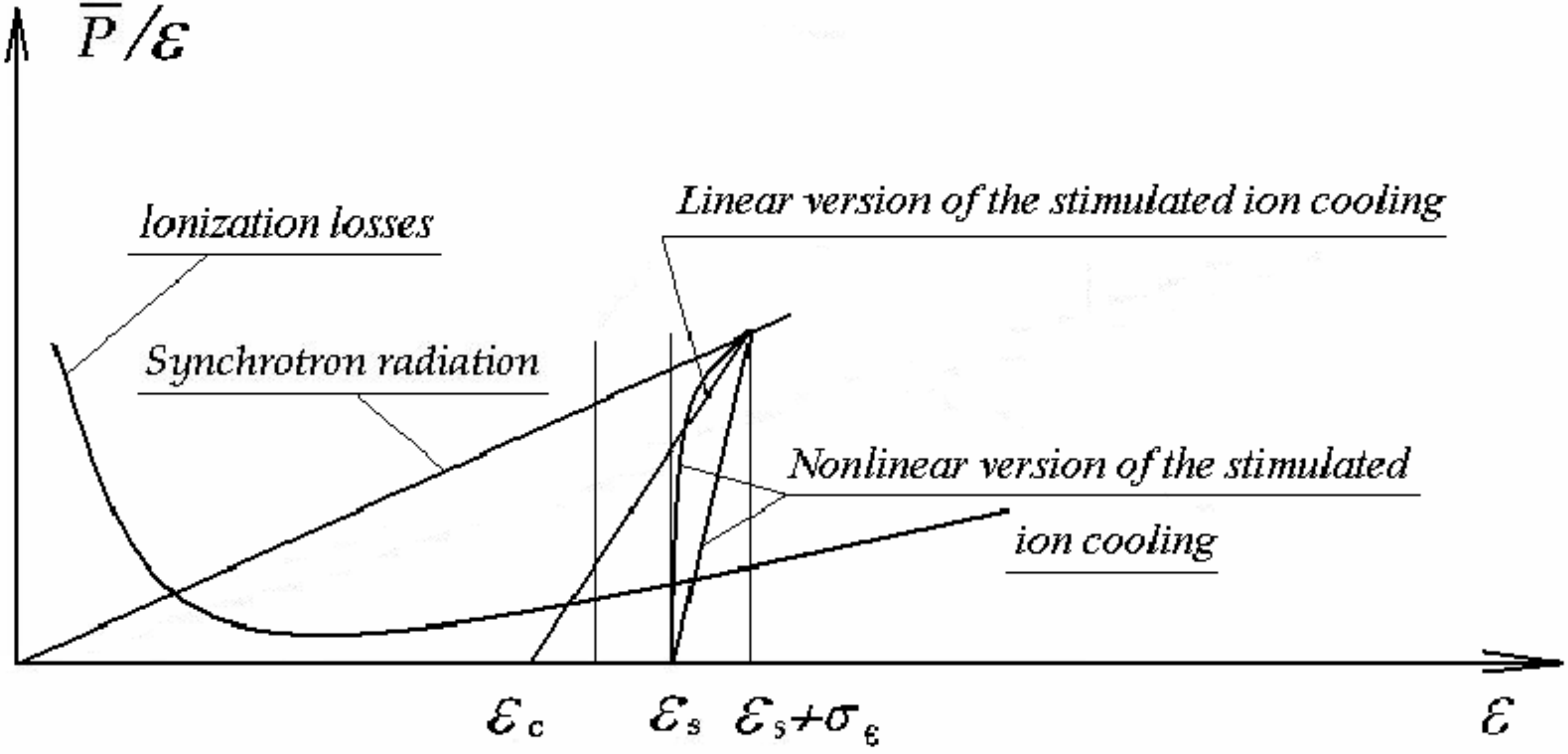
Robinson’s damping criterion follow from the continuity equation for the particle beam density ρ **in the 6D phase space:**

$$\frac{d\rho}{dt} + \rho\ div\ \vec{v} = 0$$

Here components of the 6D velocity $\vec{v} = (\vec{v}_r, \vec{v}_p)$ **are** $\dot{x}, \dot{y}, \dot{z}, \dot{p}_x, \dot{p}_y, \dot{p}_z$ **, where** $\dot{r}_i = dr_i / dt,\ \ \dot{p}_i = dp_i / dt$**. The divergence of the 6D-velocity in (1) is** $div\ \vec{v} = div_r \overrightarrow{v_r} + div_p \overrightarrow{v_p} = div_p \overrightarrow{v_p}$ **. The values** $\dot{p}_i$ **are the components of the force** $\vec{F}$ **acting upon the particle (**$\vec{F} = \vec{F}_H + \vec{F}_{Fr} = \dot{\vec{p}} = \dot{\vec{p}}_H + \dot{\vec{p}}_{Fr}$**). The friction power is** $P_{Fr} = \overrightarrow{F}_{Fr} \cdot \vec{v} = \alpha(\vec{r}, p, t) \cdot \vec{n}_p \cdot \vec{v} = \alpha(\vec{r}, p, t)\ c\ \beta$ **, where** $\beta = v/c$**. It follows that** $\alpha(\vec{r}, p, t) = P_{Fr}(\vec{r}, p, t)/c\beta$ **,** $-div\ \overrightarrow{F}_{Fr} = 2P_{Fr}(p)/\beta^2\varepsilon + \partial P_{Fr}(p)/\partial\varepsilon$**. The value** $div_p \overrightarrow{F}_H = 0$**. Finally the above equation can be written in the form** $d\ln\rho/dt + div_p \overrightarrow{F}_{Fr} = 0$**. The solution of this equation is**

$$\rho = \rho_0 e^{-\int div_p \overrightarrow{F}_{Fr}\ dt} \ => \ \rho_0 e^{-\alpha_{6D}\cdot t}.$$

$$\alpha_{6D} = \frac{2}{\beta^2}\frac{\overline{P}_{Fr}(p)}{\varepsilon}\Big|_s + \frac{\partial \overline{P}_{Fr}(p)}{\partial \varepsilon}\Big|_s, \quad \tau = \frac{1}{\alpha_{6D}}, \quad \tau^{ord} \approx \frac{\varepsilon}{\overline{P}_{Fr}(p)}\Big|_s, \quad \tau^{enh} \approx \frac{2\sigma_\varepsilon}{\overline{P}_{Fr}(p)}\Big|_s .$$



Linear version of the stimulated ion cooling

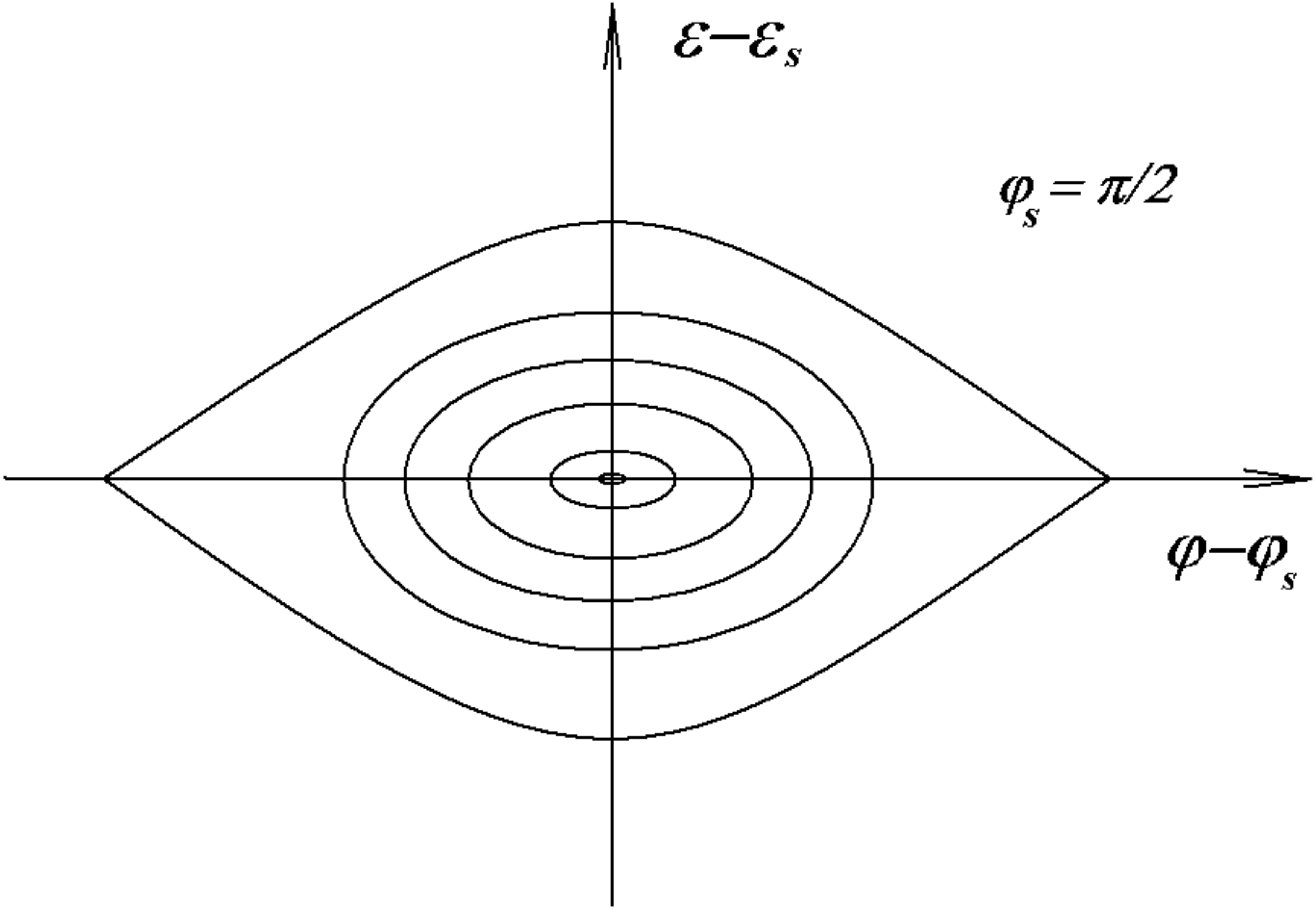
$$\overline{P} = \overline{P}_m \frac{\varepsilon - \varepsilon_c}{\varepsilon_s - \varepsilon_c + \sigma_\varepsilon} \quad \text{at} \quad \varepsilon > \varepsilon_c , \qquad \varepsilon_c \leq \varepsilon_s - \sigma_\varepsilon ;$$

$$\overline{P} = 0 \quad \text{at} \quad \varepsilon < \varepsilon_c , \qquad \varepsilon > \varepsilon_s + \sigma_\varepsilon \; .$$

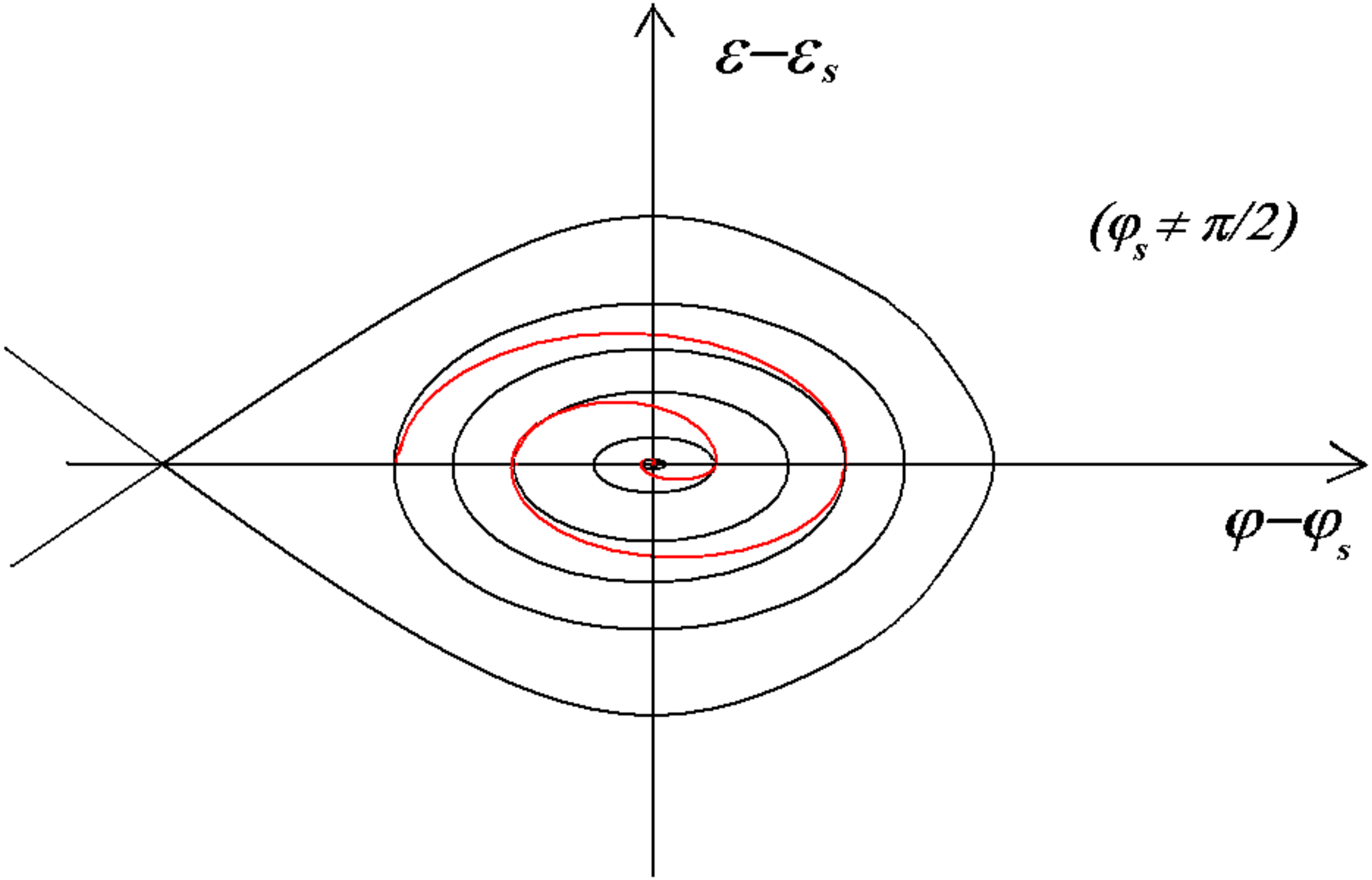
Nonlinear version of the stimulated ion cooling

$$\overline{P} = \overline{P}_m \frac{\varepsilon - \varepsilon_c}{\varepsilon_s - \varepsilon_c + \sigma_\varepsilon} \quad \text{at} \quad \varepsilon_c = \varepsilon_s , \qquad \varepsilon > \varepsilon_c ,$$

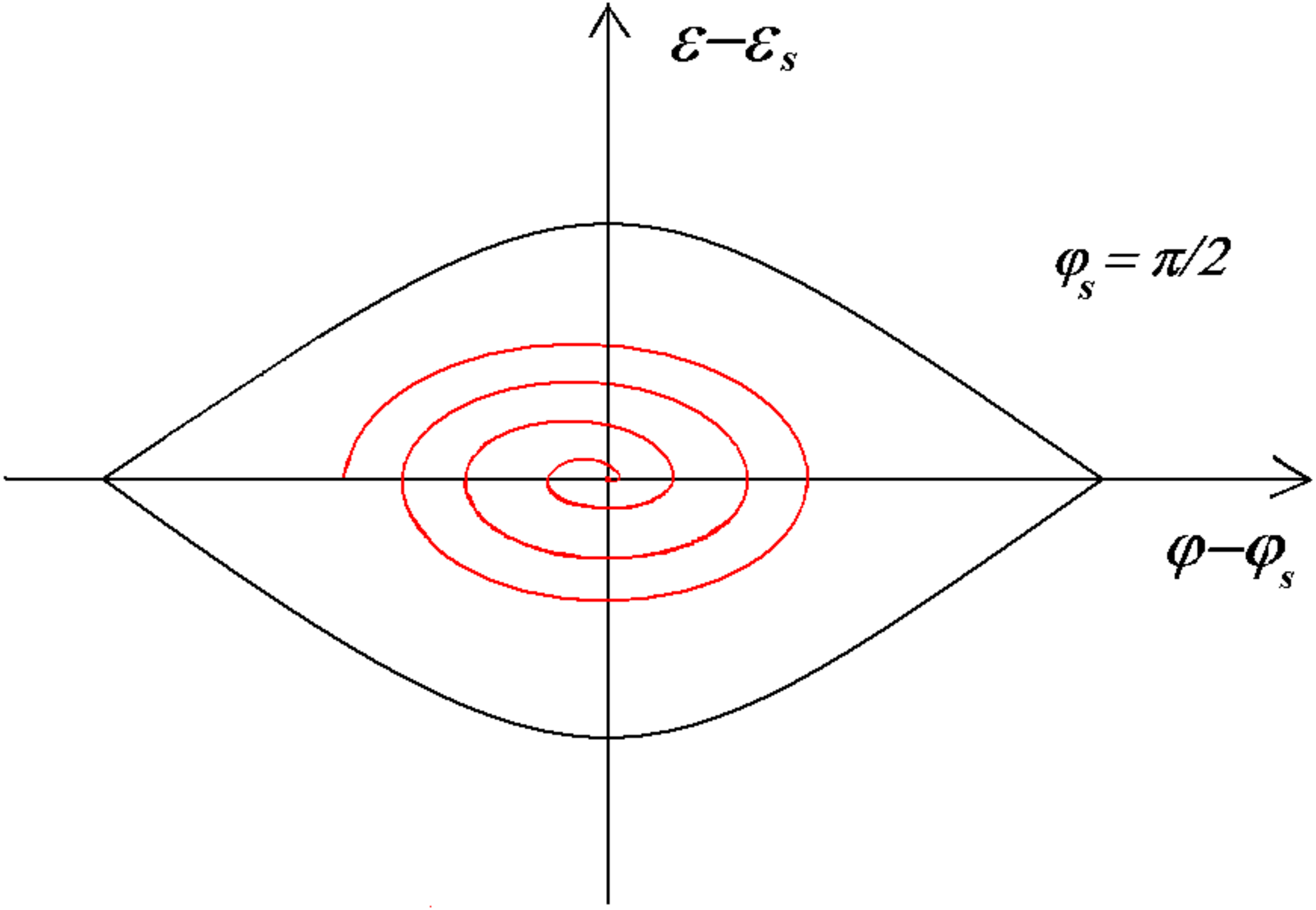
$$\overline{P} = 0 \quad \text{at} \quad \varepsilon < \varepsilon_c , \qquad \varepsilon > \varepsilon_s + \sigma_\varepsilon \; .$$


$\varepsilon-\varepsilon_s$
$\varphi_s = \pi/2$
$\varphi-\varphi_s$

Linear version of the stimulated ion cooling



Nonlinear version of the stimulated ion cooling



Consequences from the quantum nature of the light scattering

The equilibrium energy spread of the ion beam in the linear case is

$$\sigma_{\varepsilon,eq} = \hbar\overline{\omega}\sqrt{\overline{P}_s\tau_\varepsilon/\hbar\overline{\omega}} = \hbar\overline{\omega}\sqrt{2\sigma_\varepsilon/\hbar\overline{\omega}} >> \hbar\overline{\omega},$$

where $\hbar\overline{\omega} = \hbar\omega_l\gamma^2$ **is the average energy of scattered photons,** $\hbar\omega_l$ **is the energy of laser photons,** $\gamma = \varepsilon/m_ic^2$ **and** m_ic^2 **are the relative energy and the rest energy of the ion.**

The equilibrium energy spread of the ion beam in the linear case is

$$\sigma_{\varepsilon,eq} = \hbar\overline{\omega},$$

The nonlinear version of the laser cooling considered above is not optimal one. Faster increase of the power losses like

$$\overline{P} = \overline{P}_m \frac{\varepsilon - \varepsilon_s}{\varepsilon - \varepsilon_s + \sigma_c} \qquad \text{at} \qquad \varepsilon_s < \varepsilon < \varepsilon_s + \sigma_\varepsilon ,$$

$$\overline{P} = 0 \qquad \text{at} \quad \varepsilon < \varepsilon_s , \qquad \varepsilon > \varepsilon_s + \sigma_\varepsilon ,$$

and at $\sigma_c << \sigma_\varepsilon$ **is preferable. Optimization of the nonlinear version is the topic for the future search.**

METHODS OF FORMATION OF THE BROADBAND LASER BEAMS WITH GIVEN SPECTRAL DISTRIBUTION

1) The necessary power can be generated by broadband lasers filtered and then amplified by optical parametric amplifiers.

2) An undulator with a deflecting parameter $K \sim 1$ together with narrowband laser light can be used for the ion excitation in the interaction region (equivalent to the broadband laser light). Tapered undulator with the magnetic field varying by definite law and monochromatic laser beam will be equivalent to laser with a broad band and a very sharp frequency cutoff.

3) Successive frequency shift of a single mode laser by an acusto-optic modulator coupled to a passive ring cavity and other methods can also be used.

Friction along particles trajectories in the storage ring leads to violation of the Liouville's theorem and, in case of linear system to appearance of damping decrements. There is a correlation of damping decrements determined by Robinson's damping criterion. This criterion limits the rate of particle cooling in storage rings.

$$\alpha_x = -\frac{1}{2}\left[\frac{\overline{P}}{\varepsilon_s} + \frac{\partial \overline{P}}{\partial \varepsilon}\Big|_s - \frac{d\overline{P}}{d\varepsilon}\Big|_s\right], \qquad \alpha_y = -\frac{1}{2}\frac{\overline{P}}{\varepsilon_s}, \qquad \alpha_s = -\frac{1}{2}\frac{d\overline{P}}{d\varepsilon}\Big|_s,$$

$$\sum_i^3 \alpha_i = -\frac{\overline{P}}{\varepsilon_s} - \frac{1}{2}\frac{\partial \overline{P}}{\partial \varepsilon}\Big|_s, \qquad \overline{P} = \frac{1}{T_s}\oint P(x,t)dt.$$

Coefficients α_i and their sum can be both negative and positive. Cooling in 6D phase space can be followed by damping in one direction and anti-damping in another one. Fast 6D cooling occur if

$$\partial \overline{P} / \partial \varepsilon \,\big|_s \gg \overline{P} / \varepsilon_s > 0.$$

The energy loss function must be increasing with energy for longitudinal cooling. The theorem works if $\alpha_i < \omega_i$, where ω_i is the mode frequency.

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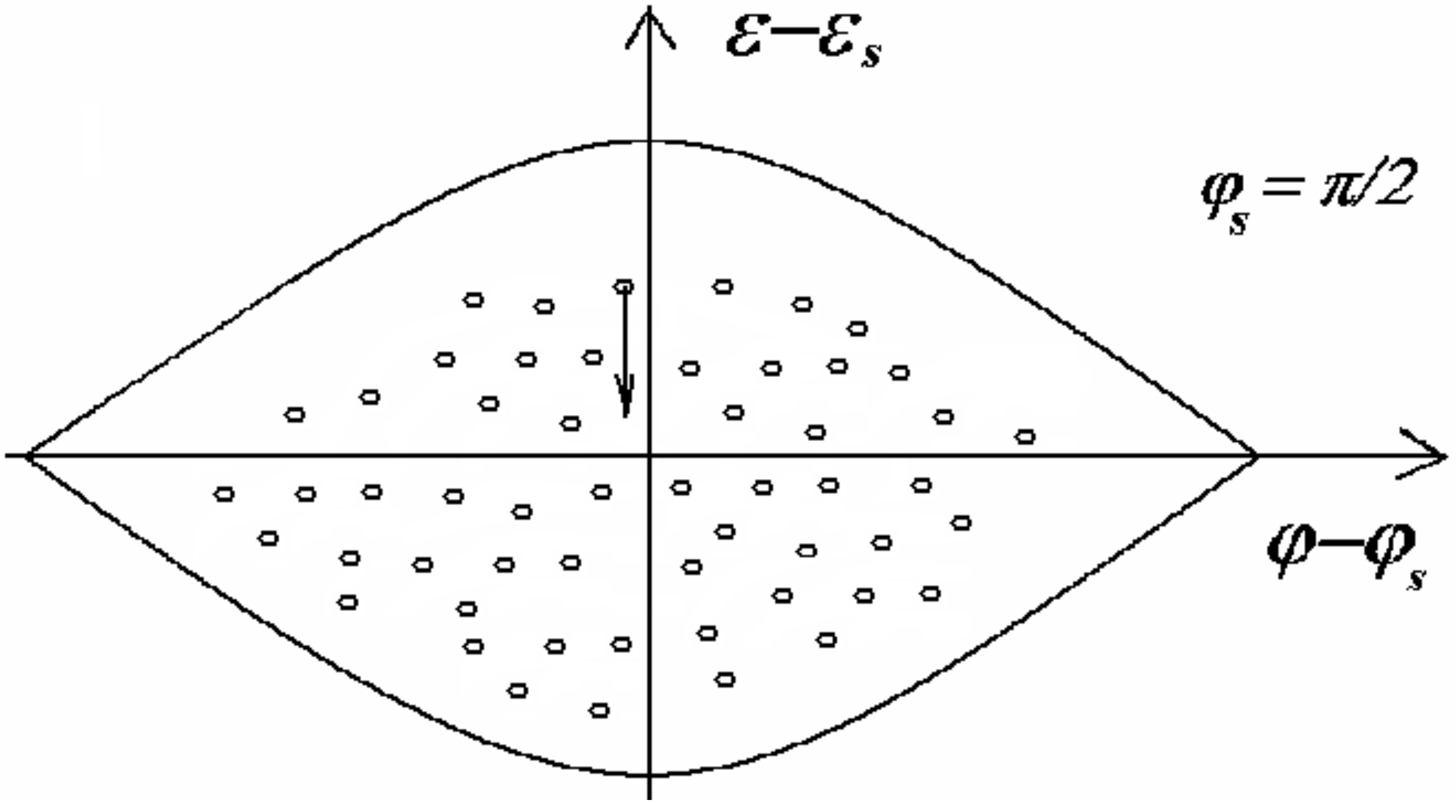
Cooling based on the finite interparticle spacing

In methods based on the interparticle spacing the spacing between the particles must be sufficient for the delicate work to insert peripheral particles of the beam between central particles by some unique instruments and do not disturb the central ones. E.g., in the EOC method the length of the URW is ~ 10 λ ~ 10mkm.



The idea is due to Simon van der Meer (1972). Pick-up takes signals from particles of the beam, which are amplified and go across the span of a storage ring so as to put an appropriate signal upon a kicker just as the particle arrives on its circular route to decrease the amplitude of the particle oscillation. Typical frequencies employed in stochastic cooling are ~5 GHz (~5cm). The distance between pick-up and kicker is $l=(2n+1)\lambda_b/4$, where λ_b is the wavelength of betatron oscillations. Isochronous bend between pick-up and kicker must be used.

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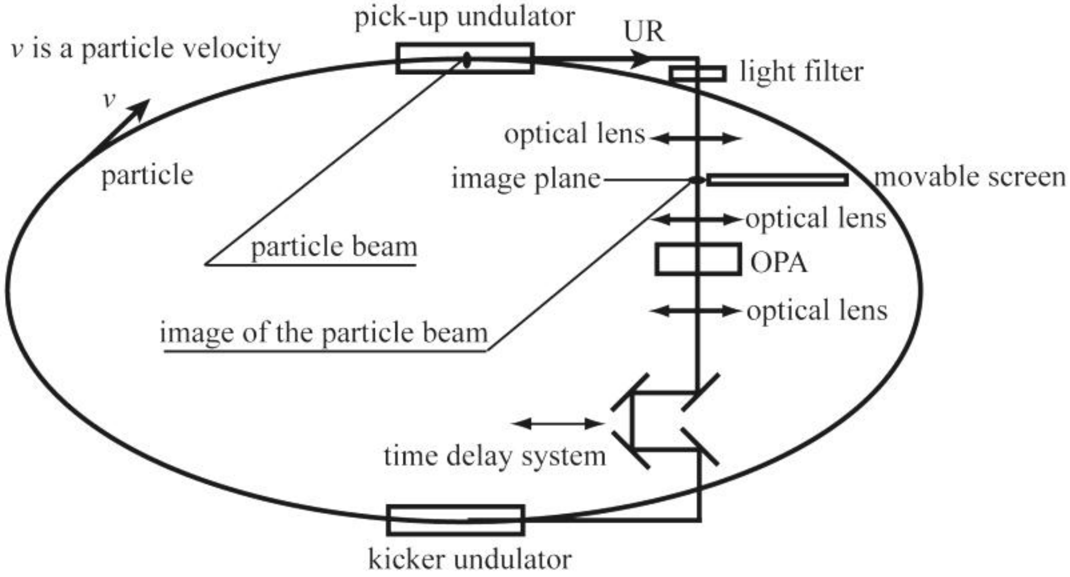
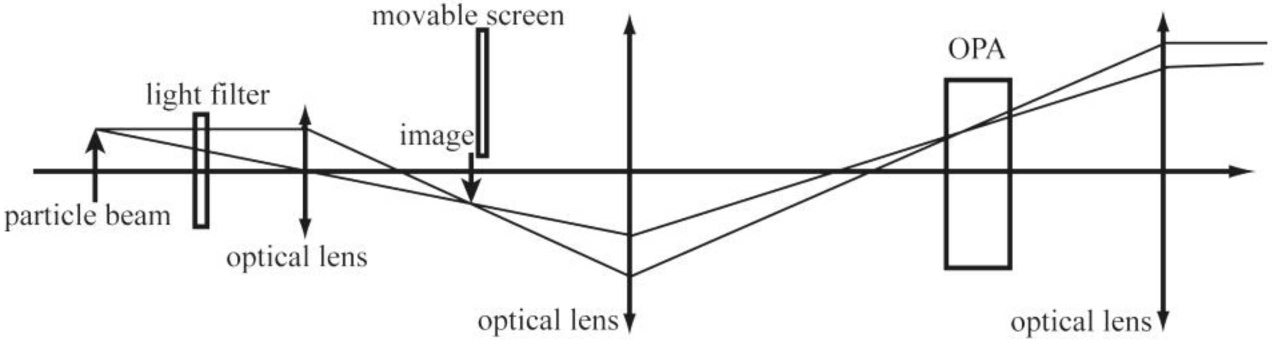
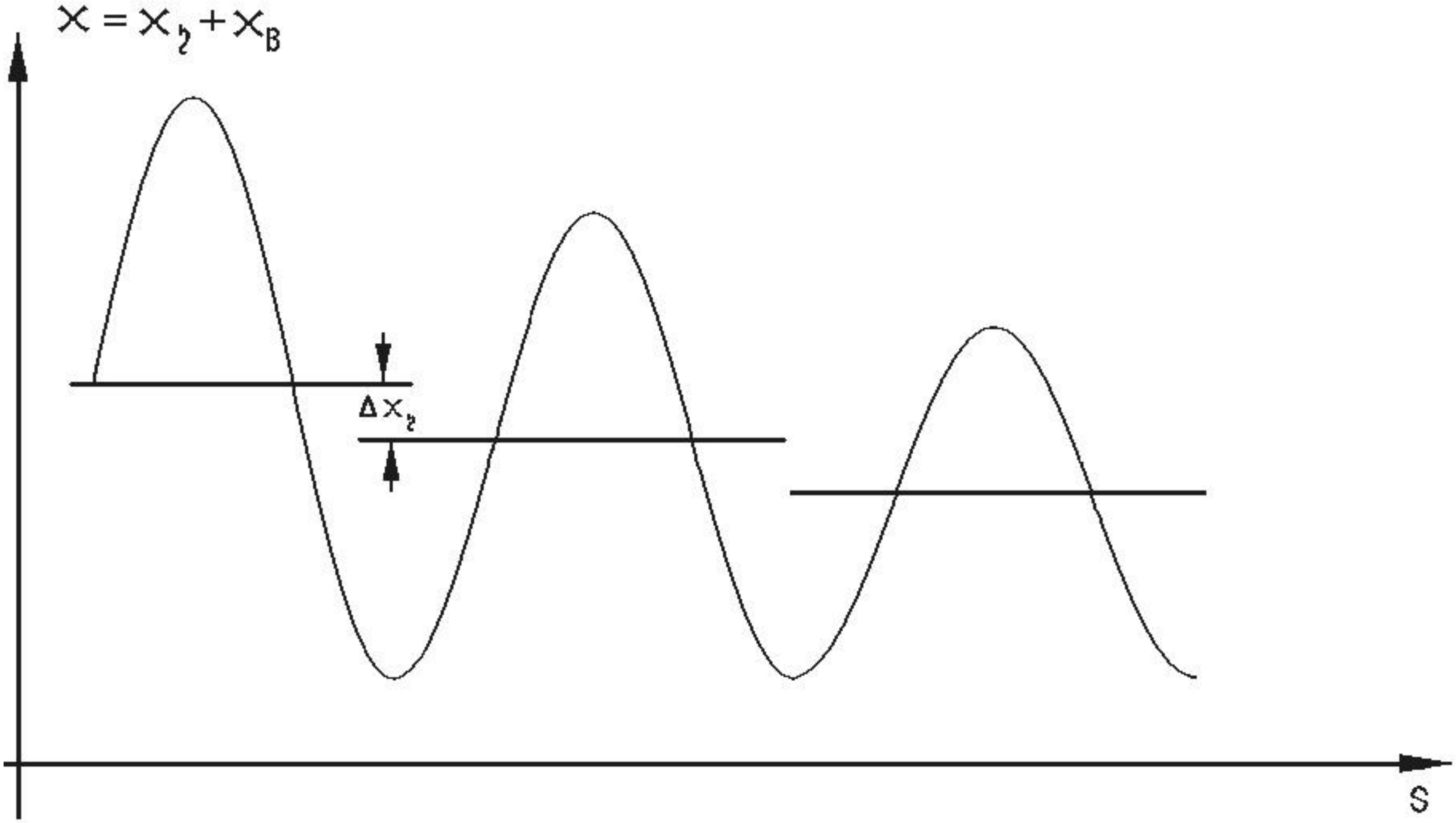
van der Meer S., CERN Internal Report CERN/ISR-PO/72-31 (1972). D.Mohl, CERN 95-06, v.II, p.587.


pick-up undulator
UR
v is a particle velocity
light filter
v
optical lens
particle
image plane
movable screen
optical lens
particle beam
OPA
optical lens
image of the particle beam
time delay system
kicker undulator


movable screen
OPA
light filter
image
particle beam
optical lens
optical lens
optical lens


$X = X_{\zeta} + X_B$
ΔX_{ζ}
S

3. Possible applications of being cooled beams

1. Colliding beams ($\overline{p}$, $\mu^{\pm}$, e^{+} , ion beams).
2. Light sources based on relativistic particle beams (backward Rayleigh scattering sources and quantum generators based on being cooled ion beams; SR, UR and backward Compton scattering sources based on e-beams).
3. Inertial fusion.
4. Crystalline ion beams.

The cooling rings supply very dense beam either for direct use in precision experiments or for injection into big, high luminosity machines like the Tevatron or the projected next Linear Colliders and muon Colliders or the proposed Neutrino Factories.

D.Mohl, A.M.Sessler, NIM A532 (2004), p.1-10

4. Conclusion

1. **All discoveries made with e+e- machines since 1960s (including charm, tau, synchrotron light physics) with radiation cooling.**
2. **W, Z, top, anti-H with stochastic cooling.**
3. **Beams of unprecedented brightness by electron cooling.**
4. **Special ion beams of still higher brightness by laser cooling.**
5. **A wide field of fascinating accelerator physics.**
6. **New generations of light sources can be based on ion storage rings if stimulated radiation cooling is used.**

In summary, we can say, beam cooling has led, leads, and will lead to spectacular results. In addition beam cooling was , is, and will be fun! What more do you want?

D.Mohl, A.M.Sessler, NIM A532 (2004), p.1-10.

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