Spectra of radiation and created particles at intermediate energy in oriented single crystal

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Phys.Lett.A 372 (2008) 2904

Channeling 2008
Erice, Sicily, Italy,
October 25-November 1, 2008

The spectral distribution of electron-positron pair created by photon and the spectral distribution of photons radiated from electron in an oriented single crystal at intermediate energy (a few GeV for heavy elements) are calculated. The used method permits inseparable consideration both of the coherent and incoherent mechanisms of two relevant processes. The interplay of the coherent and the incoherent contributions is essential for formation of the spectra. Just in this situation the effects of multiple scattering of charged particles appear (the Landau-Pomeranchuk-Migdal (LPM) effect). The method includes the action of field of axis (or plane) as well as the multiple scattering of radiating electron or particles of the created pair. The influence of scattering on the coherent mechanism and the influence of field on the incoherent mechanism are analyzed.

The multiple scattering distorts the spectrum of radiating electron or particles of the created pair. In tungsten, axis ¡111¿ for the pair creation process at temperature T= 100 K the distortion of electron (positron) spectrum (the LPM effect) attains 8 % at photon energy 5 GeV and for the radiation process at T= 293 K the LPM effect reaches 7 % at electron energy 10 GeV. In amorphous medium only the distortion of the radiation spectrum was observed.

Introduction

When the formation length of pair creation by a photon

$$l_f = \frac{\omega}{2m^2} \tag{1}$$

becomes comparable to the distance over which the multiple scattering becomes important, the probability of the process will be suppressed. This is is the Landau- Pomeranchuk -Migdal (LPM) effect. The characteristic energy ω_e when the LPM effect affects the whole spectrum is

$$\frac{q_f^2}{m^2} = \frac{\omega}{\omega_e}, \quad \omega_e = \frac{m}{4\pi Z^2\alpha^2\lambda_c^3 n_a L_0}, \quad L_0 = \ln(ma) + \frac{1}{2} - f(Z\alpha),$$
$$a = \frac{111Z^{-1/3}}{m}, \quad f(\xi) = \sum_{n=1}^{\infty} \frac{\xi^2}{n(n^2+\xi^2)}, \tag{2}$$

where Z is the charge of nucleus, n_a is the mean atom density, $f(\xi)$ is the Coulomb correction.

The energy ω_e is very high even for heavy elements: $\omega_e = 10.9$ TeV for tungsten and $\omega_e = 17.5$ TeV for lead.

For radiation process the formation length

$$l_r = \frac{2\varepsilon(\varepsilon - \omega)}{m^2\omega}, \tag{3}$$

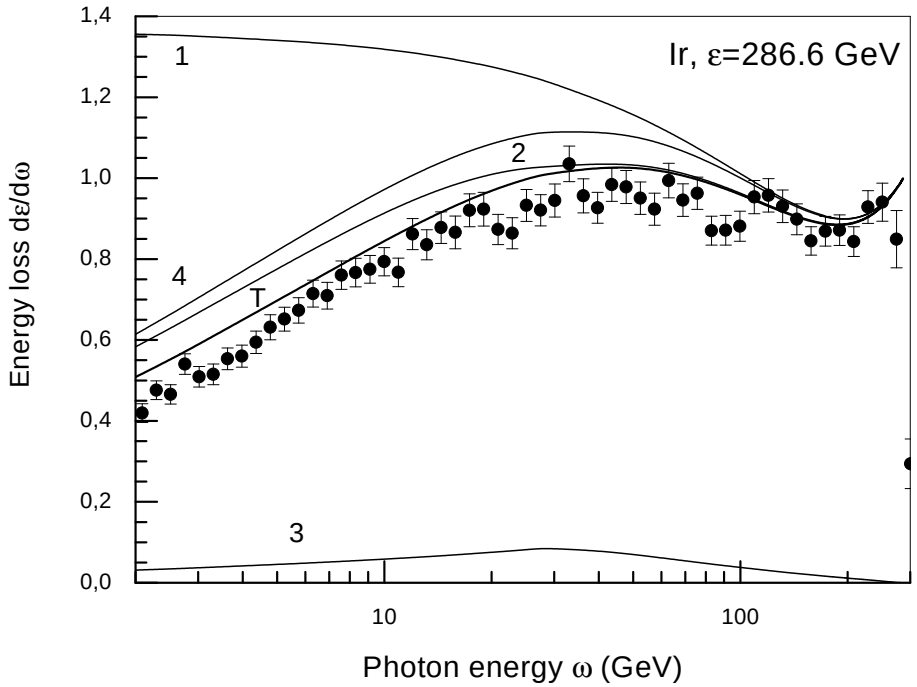

Figure 1: The energy losses $\frac{d\varepsilon}{d\omega}$ in iridium target with thickness $l = 0.128$ mm in units of inverse radiation length is for the initial electrons energy $\varepsilon = 287$ GeV CERN (2003). The Coulomb corrections are included. Curve 1 is the Bethe-Maximon intensity spectrum, curve 2 is the contribution of the main (Migdal) term, curve T is the final theory prediction with regard for multiphoton emission

Spectrum of particles created by photon in a crystal

For axial orientation of a crystal the ratio of the atom density $n(\varrho)$ in the vicinity of the axis to the mean atom density n_a is

$$\frac{n(\varrho)}{n_a} = \frac{e^{-\varrho^2/2u_1^2}}{2\pi d n_a u_1^2}, \quad \frac{n(0)}{n_a} \sim \left(\frac{d}{u_1}\right)^2, \quad \omega_0 = \omega_e \frac{n_a}{n(0)}, \tag{4}$$

where ϱ is the distance from the axis, u_1 is the amplitude of the thermal vibrations, d is the mead distance between atoms forming the axis. The strength of the electric field in the vicinity of the axis is

$$F \sim \frac{2V_0\varrho}{\varrho^2 + 2u_1^2}, \quad F_{max} \sim \frac{V_0}{u_1}, \quad V_0 \simeq \frac{Ze^2}{d}. \tag{5}$$

The formation length of pair creation in the field is $l_m \sim m/F_{max}$. The ratio of l_f and l_m is

$$\frac{l_f}{l_m} \sim \frac{V_0}{m u_1}\frac{\omega}{m^2} \sim \frac{\omega}{\omega_m} \equiv \kappa_m \sim 1, \quad l_f \sim l_m. \tag{6}$$

It is useful to compare the characteristic energy ω_0 with energy ω_m for which the probability of pair creation in the field becomes comparable with the Bethe-Maximon probability.

Here we consider the case when angle of incidence $\vartheta_0 \ll V_0/m$. This is the condition that the distance $\varrho \sim u_1$ as well as the atom density and the transverse field of axis can be considered as constant over the formation length

$$\frac{\Delta\varrho}{u_1} = \frac{\vartheta_0 l_f}{u_1} \sim \frac{\vartheta_0 l_m}{u_1} \ll 1. \tag{7}$$

The general expression for the spectral distribution of particles of pair created by a photon

$$dW(\omega,y)=\frac{\alpha m^2}{2\pi\omega}\frac{dy}{y(1-y)}\int_0^{x_0}\frac{dx}{x_0}G(x,y),\quad G(x,y)=\int_0^{\infty}F(x,y,t)dt+s_3\frac{\pi}{4},$$

$$F(x,y,t)=\mathrm{Im}\left\{e^{f_1(t)}\left[s_2\nu_0^2(1+ib)f_2(t)-s_3f_3(t)\right]\right\},\quad b=\frac{4\kappa_1^2}{\nu_0^2},\quad y=\frac{\varepsilon}{\omega},$$

$$f_1(t)=(i-1)t+b(1+i)(f_2(t)-t),\quad f_2(t)=\frac{\sqrt{2}}{\nu_0}\tanh\frac{\nu_0 t}{\sqrt{2}},$$

$$f_3(t)=\frac{\sqrt{2}\nu_0}{\sinh(\sqrt{2}\nu_0 t)}, \tag{8}$$

where

$$s_2=y^2+(1-y)^2,\ s_3=2y(1-y),\ \nu_0^2=4y(1-y)\frac{\omega}{\omega_c(x)},\ \kappa_1=y(1-y)\kappa(x), \tag{9}$$

ε is the energy of one of the created particles.

The situation is considered when the photon angle of incidence ϑ_0 (the angle between photon momentum **k** and the axis (or plane)) is small $\vartheta_0 \ll V_0/m$. The axis potential is taken in the form

$$U(x) = V_0 \quad \ln \quad 1 + \frac{1}{x+\eta} \quad - \ln \quad 1 + \frac{1}{x_0+\eta} \quad , \tag{10}$$

where

$$x_0 = \frac{1}{\pi d n_a a_s^2}, \quad \eta_1 = \frac{2u_1^2}{a_s^2}, \quad x = \frac{\varrho^2}{a_s^2}, \tag{11}$$

Here ϱ is the distance from axis, u_1 is the amplitude of thermal vibration, d is the mean distance between atoms forming the axis, a_s is the effective screening radius of the potential. The parameters in Eq.(10) were determined by means of fitting procedure, see Table.

Table

Parameters of the pair photoproduction and radiation processes in the tungsten crystal, axis $< 111 >$ and the germanium crystal, axis $< 110 >$ for two temperatures T
$(\varepsilon_0 = \omega_0/4, \varepsilon_m = \omega_m, \varepsilon_s = \omega_s)$

Crystal	T(K)	V_0(eV)	x_0	η_1	η	ω_0(GeV)	ε_m(GeV)	ε_s(GeV)	h
W	293	417	39.7	0.108	0.115	29.7	14.35	34.8	0.348
W	100	355	35.7	0.0401	0.0313	12.25	8.10	43.1	0.612
Ge	293	110	15.5	0.125	0.119	592	88.4	210	0.235
Ge	100	114.5	19.8	0.064	0.0633	236	50.5	179	0.459

The local value of parameter $\kappa(x)$ which determines the probability of pair creation in the field is

$$\kappa(x) = -\frac{dU(\varrho)}{d\varrho}\frac{\omega}{m^3} = 2\kappa_s f(x), \quad f(x) = \frac{\sqrt{x}}{(x+\eta)(x+\eta+1)}, \quad \kappa_s = \frac{V_0\omega}{m^3 a_s} \equiv \frac{\omega}{\omega_s}. \tag{12}$$

For an axial orientation of crystal the ratio of the atom density $n(\varrho)$ in the vicinity of an axis to the mean atom density n_a is

$$\frac{n(x)}{n_a} = \xi(x) = \frac{x_0}{\eta_1}e^{-x/\eta_1}, \quad \omega_0 = \frac{\omega_e}{\xi(0)}, \quad \omega_e = 4\varepsilon_e = \frac{m}{4\pi Z^2\alpha^2\lambda_c^3 n_a L_0}. \tag{13}$$

The functions and values in Eqs. above are

$$\omega_c(x) = \frac{\omega_e(n_a)}{\xi(x) g_p(x)} = \frac{\omega_0}{g_p(x)} e^{x/\eta_1}, \quad L = L_0 g_p(x),$$

$$g_p(x) = g_{p0} + \frac{1}{6L_0}\left[\ln \quad 1+\kappa_1^2 \quad + \frac{6D_p\kappa_1^2}{12+\kappa_1^2}\right], \quad g_{p0} = 1 - \frac{1}{L_0}\left[\frac{1}{42} + h \quad \frac{u_1^2}{a^2}\right)\right],$$

$$h(z) = -\frac{1}{2}\left[1 + (1+z)e^z \mathrm{Ei}(-z)\right], \tag{14}$$

where L_0 is defined in Eq.(2), the function $g_p(x)$ determines the effective logarithm using the interpolation procedure, $D_p = D_{sc} - 10/21 = 1.8246$, $D_{sc} = 2.3008$ is the constant entering in the radiation spectrum at $\chi/u \gg 1$ (or in electron spectrum in pair creation process at $\kappa_1 \gg 1$), Ei(z) is the integral exponential function.

The expression for $dW(\omega, y)$ includes both the coherent and incoherent contributions as well as the influence of the multiple scattering (the LPM effect) on the pair creation process. The probability of the coherent pair creation is the first term ($\nu_0^2 = 0$) of the decomposition of Eq.(8)

over ν_0^2

$$dW^{coh}(\omega,y)=\frac{\alpha m^2}{2\sqrt{3}\pi\omega}\frac{dy}{y(1-y)}\int_0^{x_0}\frac{dx}{x_0}\quad 2s_2K_{2/3}(\lambda)+s_3\int_\lambda^\infty K_{1/3}(z)dz\quad ,$$

$$\lambda=\lambda(x)=\frac{2}{3\kappa_1}, \tag{15}$$

where $K_\nu(\lambda)$ is MacDonald's function. The probability of the incoherent pair creation is the second term ($\propto \nu_0^2$) of the mentioned decomposition

$$dW^{inc}(\omega,y)=\frac{4Z^2\alpha^3 n_a L_0}{15m^2}dy\int_0^\infty\frac{dx}{\eta_1}e^{-x/\eta_1}f(x,y)g_p(x), \tag{16}$$

where $g_p(x)$ is defined above

$$f(x,y)=f_1(z)+s_2f_2(z),\quad f_1(z)=z^4\Upsilon(z)-3z^2\Upsilon'(z)-z^3,$$
$$f_2(z)=(z^4+3z)\Upsilon(z)-5z^2\Upsilon'(z)-z^3,\quad z=z(x,y)=\kappa_1^{-2/3}. \tag{17}$$

Here

$$\Upsilon(z) = \int_0^\infty \sin \quad zt + \frac{t^3}{3} \Bigg) \, dt \tag{18}$$

is the Hardy function.

The next terms of decomposition of the pair creation probability $dW = dW(\omega, y)$ over ν_0^2 describe the influence of multiple scattering on the pair creation process, the LPM effect. The third term ($\propto \nu_0^4$) of the mentioned decomposition has the form

$$\begin{aligned}
&\frac{dW^{(3)}(\omega, y)}{dy} = -\frac{\alpha m^2 \omega \sqrt{3}}{5600 \pi \omega_0^2 x_0} \int_0^{x_0} \frac{g_p^2(x)}{\kappa(x)} \Phi(\lambda) e^{-2x/\eta_1} dx \\
&\Phi(\lambda) = \lambda^2 \left(s_2 F_2(\lambda) - s_3 F_3(\lambda) \right), \\
&F_2(\lambda) = (7820 + 126\lambda^2)\lambda K_{2/3}(\lambda) - (280 + 2430\lambda^2) K_{1/3}(\lambda), \\
&F_3(\lambda) = (264 - 63\lambda^2)\lambda K_{2/3}(\lambda) - (24 + 3\lambda^2) K_{1/3}(\lambda),
\end{aligned} \tag{19}$$

where λ is defined above.

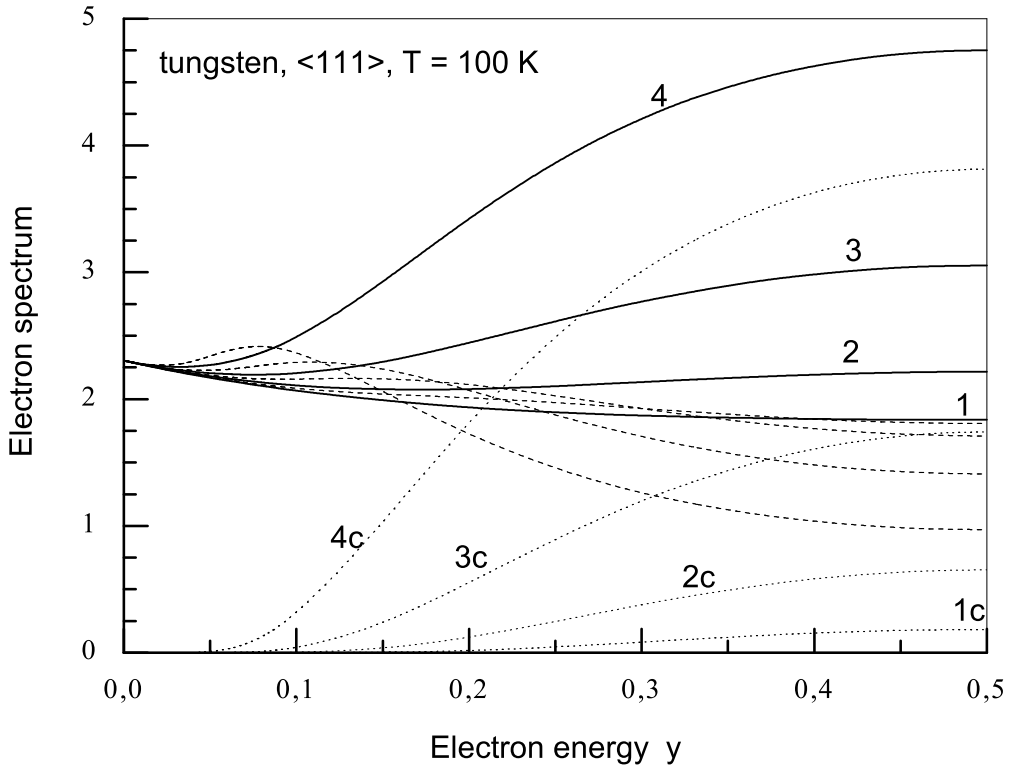

Figure 2: The spectral distribution of created by a photon pair (in units cm^{-1}) vs the electron energy $y = \varepsilon/\omega$ in tungsten, axis $< 111 >$, temperature T=100 K. (a) The curves 1, 2, 3, 4 are the theory prediction $dW(\omega, y)/dy$ for photon energies $\omega = 5,\ 7,\ 10,\ 15$ GeV respectively, The doted curves are the corresponding coherent contributions, the dashed curves present the incoherent contributions.

For $\omega = 7$ GeV the interplay of the coherent and incoherent contributions is leading to the nearly flat final spectrum (the variation is less than 10 %, this is quite unusual). It should be noted that for $\omega = 7$ GeV the right end ($y = 0.5$) is slightly lower than the left end of spectrum ($y \to 0$): $dW/dy(y \to 0) = 2.303\ \mathrm{cm}^{-1}$ and $dW(y = 0.5) = 2.215\ \mathrm{cm}^{-1}$, while the sum of the incoherent and coherent contributions is slightly higher: $dW^{inc}/dy(y = 0.5) + dW^{coh}/dy(y = 0.5) = 2.365\ \mathrm{cm}^{-1}$. The arising difference is the consequence of the LPM effect. This property may be very useful in experimental study.

We define the contribution of the LPM effect into the spectral distribution of created pair as

$$\Delta_p(\omega, y) = -\frac{dW(\omega, y) - dW^{coh}(\omega, y) - dW^{inc}(\omega, y)}{dW(\omega, y)}. \qquad (20)$$

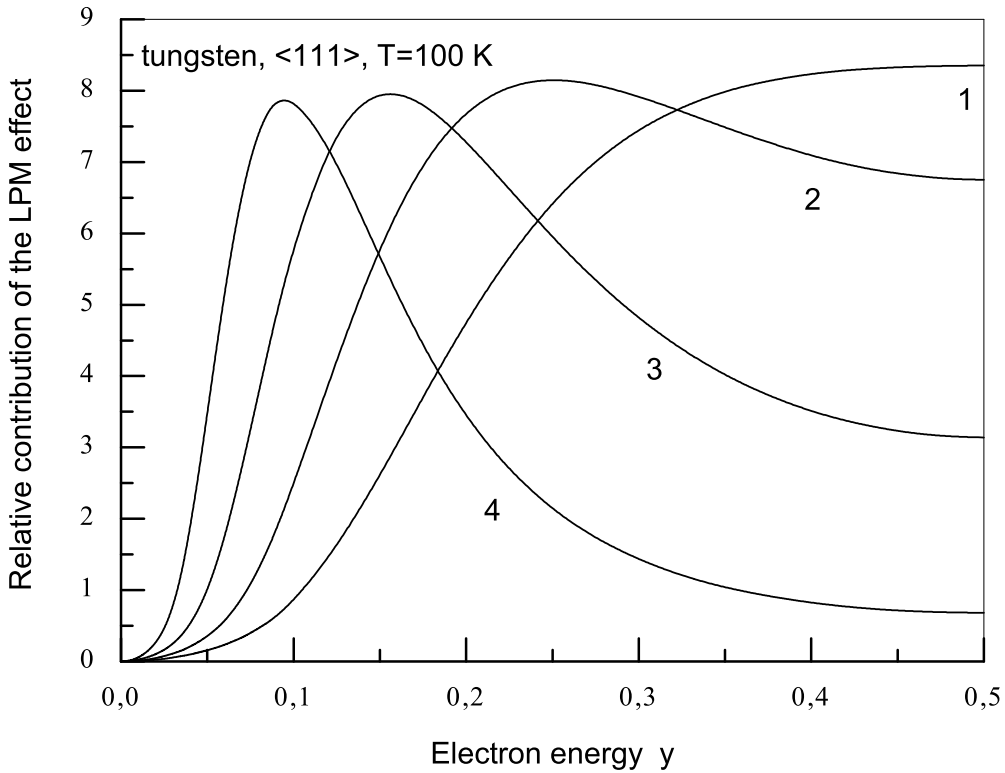

Figure 3: The relative contribution of the LPM effect in the spectral distribution of created electron (see Eq.(19)) $\Delta_p(\omega, y)$ (per cent). The curves 1, 2, 3, 4 are correspondingly for photon energies $\omega = 5,\ 7,\ 10,\ 15$ GeV, Δ_p^{max}=8.35 %.

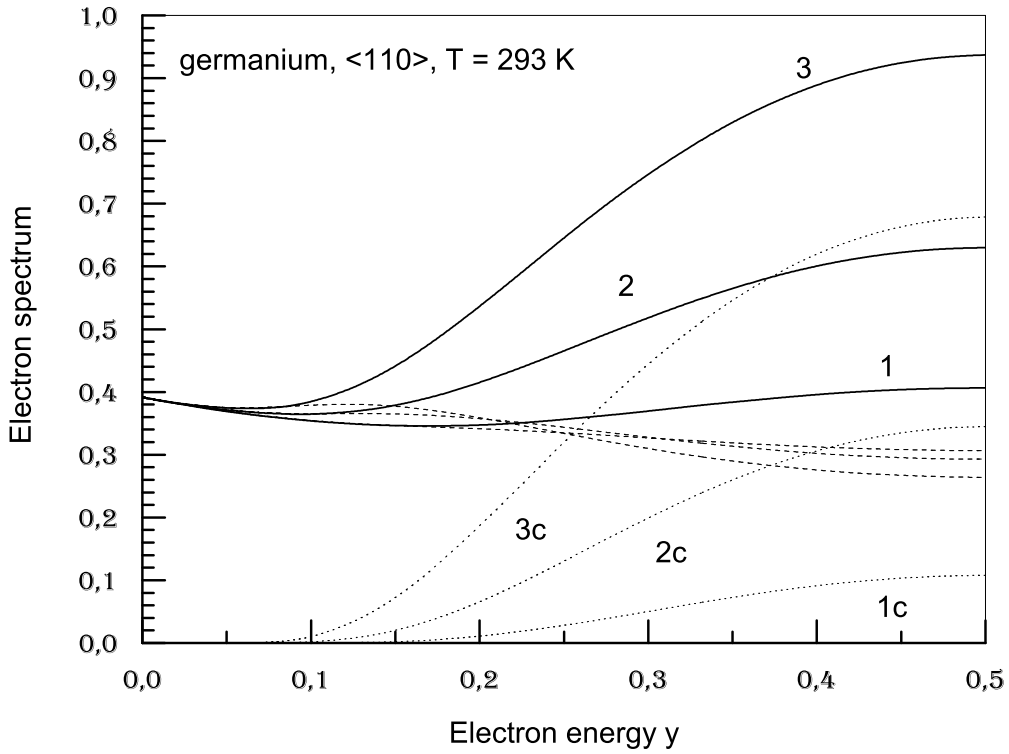

Figure 4: The spectral distribution of created by a photon pair (in units cm^{-1}) vs the electron energy $y = \varepsilon/\omega$ in germanium, axis $< 110 >$, temperature T=293 K. The curves 1, 2, 3, are the theory prediction $dW(\omega, y)/dy$ for photon energies $\omega = 55,\ 75,\ 95$ GeV respectively, The doted curves are the corresponding coherent contributions, the dashed curves present the incoherent contributions.

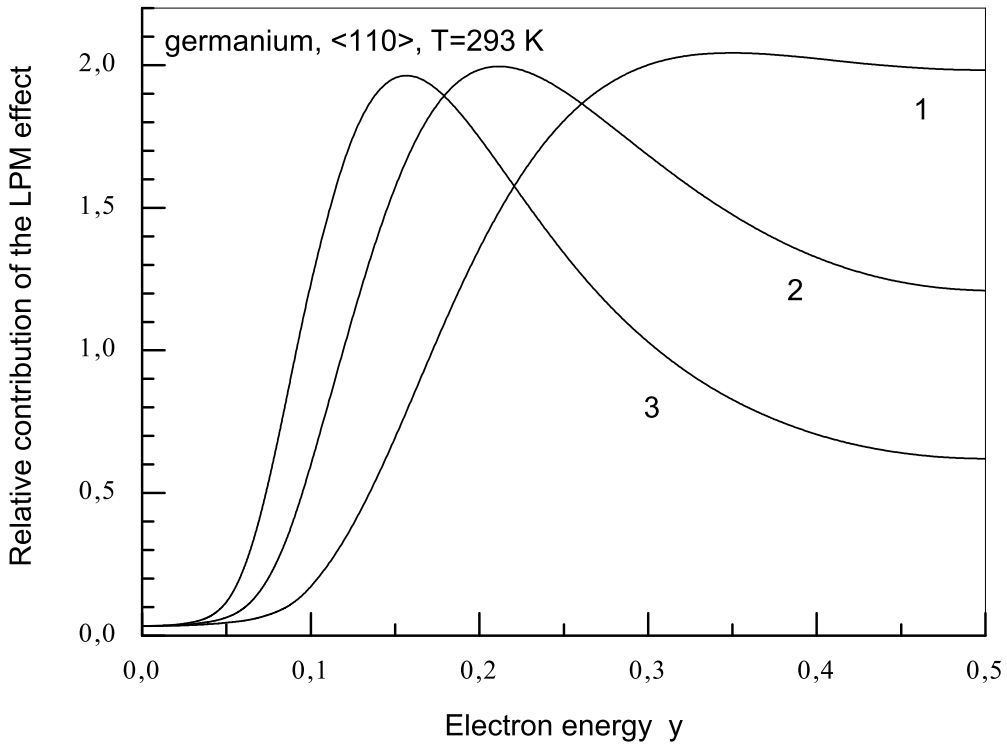

Figure 5: The relative contribution of the LPM effect in the spectral distribution of created electron $\Delta_p(\omega, y)$ (per cent). The curves 1, 2, 3 are correspondingly for photon energies $\omega = 55,\ 75,\ 95$ GeV.

Radiation

The expression for the spectral probability of radiation used in the above derivation can be found from the spectral distribution ($dW/dy = \omega dW/d\varepsilon$) using the standard QED substitution rules: $\varepsilon \to -\varepsilon,\ \omega \to -\omega,\ \varepsilon^2 d\varepsilon \to \omega^2 d\omega$ and exchange $\omega_c(x) \to 4\varepsilon_c(x)$. As a result one has for the spectral intensity $dI = \omega dW$

$$dI(\varepsilon, y_r) = \frac{\alpha m^2}{2\pi} \frac{y_r dy_r}{1-y_r} \int\limits_0^{x_0} \frac{dx}{x_0} G_r(x, y_r),$$

$$G_r(x, y_r) = \int\limits_0^{\infty} F_r(x, y_r, t) dt - r_3 \frac{\pi}{4},$$

$$F_r(x, y_r, t) = \mathrm{Im}\left\{ e^{\varphi_1(t)} \left[r_2 \nu_{0r}^2 (1 + i b_r) f_2(t) + r_3 f_3(t) \right] \right\}, \quad b_r = \frac{4\chi^2(x)}{u^2 \nu_{0r}^2},$$

$$y_r = \frac{\omega}{\varepsilon}, \quad u = \frac{y_r}{1-y_r}, \quad \varphi_1(t) = (i-1)t + b_r(1+i)(f_2(t) - t), \qquad (21)$$

where

$$r_2 = 1 + (1 - y_r)^2, \quad r_3 = 2(1 - y_r),$$
$$\nu_{0r}^2 = \frac{1 - y_r}{y_r} \frac{\varepsilon}{\varepsilon_c(x)}, \tag{22}$$

where the functions $f_2(t)$ and $f_3(t)$ are defined above. The local value of parameter $\chi(x)$ which determines the radiation probability in the field is

$$\chi(x) = -\frac{dU(\varrho)}{d\varrho} \frac{\varepsilon}{m^3} = 2\chi_s f(x), \quad \chi_s = \frac{V_0 \varepsilon}{m^3 a_s} \equiv \frac{\varepsilon}{\varepsilon_s}. \tag{23}$$

The functions and values are

$$\varepsilon_c(x) = \frac{\varepsilon_e(n_a)}{\xi(x) g_r(x)} = \frac{\varepsilon_0}{g_r(x)} e^{x/\eta_1},$$
$$g_r(x) = g_{r0} + \frac{1}{6L_0}\left[\ln\left(1 + \frac{\chi^2(x)}{u^2}\right) + \frac{6D_r\chi^2(x)}{12u^2 + \chi^2(x)}\right],$$
$$g_{r0} = 1 + \frac{1}{L_0}\left[\frac{1}{18} - h\left(\frac{u_1^2}{a^2}\right)\right], \tag{24}$$

where the function $g_r(x)$ determines the effective logarithm using the interpolation procedure: $L = L_0 g_r(x)$.

The expression for dI includes both the coherent and incoherent contributions as well as the influence of the multiple scattering (the LPM effect) on the photon emission process. The

intensity of the coherent radiation is the first term ($\nu_0^2 = 0$) of the decomposition over ν_{0r}^2

$$dI^{coh}(\varepsilon, y_r) = \frac{\alpha m^2}{\sqrt{3}\pi} \frac{y_r dy_r}{1 - y_r} \int_0^{x_0} \frac{dx}{x_0} \quad r_2 K_{2/3}(\lambda_r) - (1 - y_r) \int_{\lambda_r}^{\infty} K_{1/3}(z) dz \quad ,$$

$$\lambda_r = \lambda_r(x) = \frac{2u}{3\chi(x)}. \tag{25}$$

The intensity of the incoherent radiation is the second term ($\propto \nu_0^2$) of the mentioned decomposition

$$dI^{inc}(\varepsilon, y_r) = \frac{\alpha m^2}{60\pi} \frac{\varepsilon}{\varepsilon_0} dy_r \int_0^{\infty} \frac{dx}{x_0} e^{-x/\eta_1} f_r(x, y_r) g_r(x), \tag{26}$$

where

$$f_r(x, y_r) = \left[y_r^2 (f_1(z) + f_2(z)) + 2(1 - y_r) f_2(z) \right],$$

$$z = \left(\frac{u}{\chi(x)} \right)^{2/3}. \tag{27}$$

The next terms of decomposition of for the spectral intensity of radiation $I(\varepsilon, y_r)$ over ν_{0r}^2 describe the influence of multiple scattering on the photon emission process, the LPM effect. The third term ($\propto \nu_{0r}^4$) of the mentioned decomposition has the form

$$dI^{(3)}(\varepsilon, y_r) = -\frac{\alpha m^2 \sqrt{3}}{89600 \pi x_0} dy_r \left(\frac{\varepsilon}{\varepsilon_0} \right)^2 \int_0^{x_0} \frac{g_r^2(x)}{\chi(x)} \Phi(\lambda_r(x)) e^{-2x/\eta_1} dx, \tag{28}$$

where

$$\Phi(\lambda_r) = \lambda_r^2 (r_2 F_2(\lambda_r) + r_3 F_3(\lambda_r)). \tag{29}$$

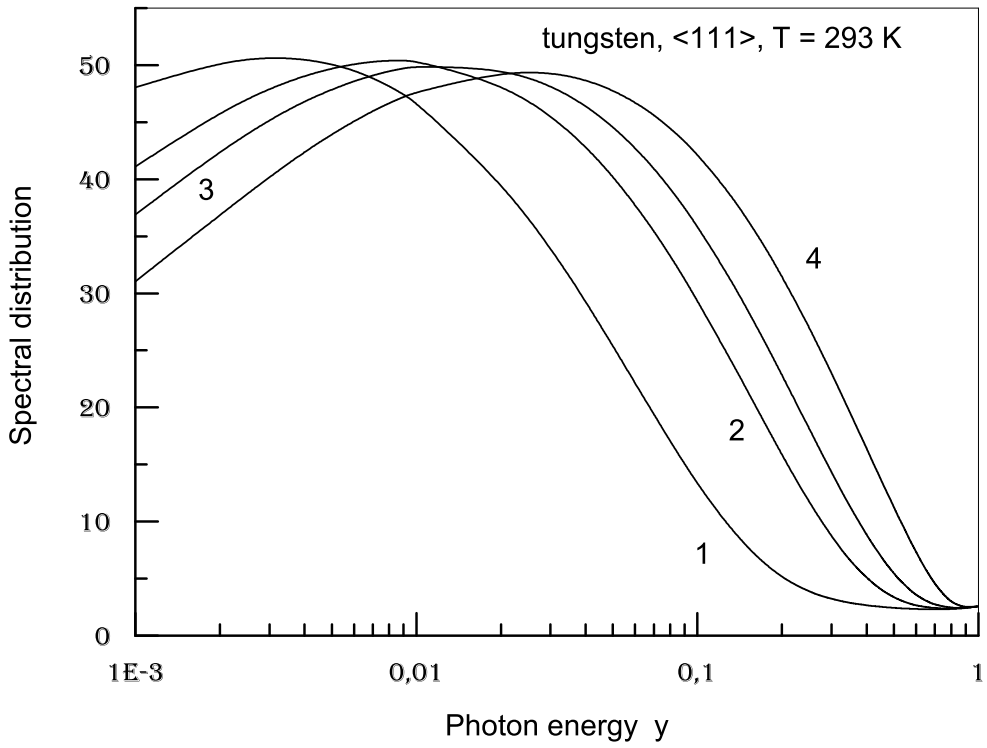

Figure 6: The radiation spectral intensity (in units cm^{-1}) vs the photon energy $y = \omega/\varepsilon$ in tungsten, axis $< 111 >$, temperature T=293 K. The intensity distribution $dI(\varepsilon, y_r)/d\omega$ The curves 1, 2, 3, 4 are the theory prediction for electron energies $\varepsilon = 1,\ 3,\ 5,\ 10$ GeV respectively.

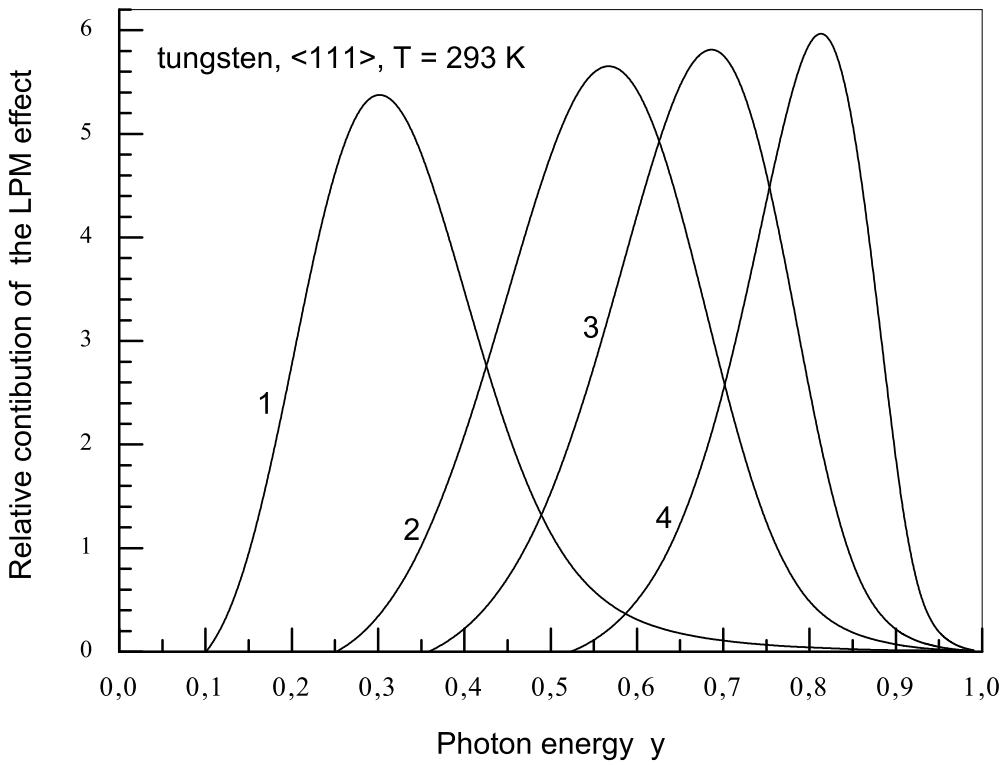

Figure 7: The relative contribution of the LPM effect in the spectral distribution of emitted photons Δ_r (per cent). The curves 1, 2, 3, 4 are correspondingly for the electron energies $\varepsilon = 1,\ 3,\ 5,\ 10$ GeV.

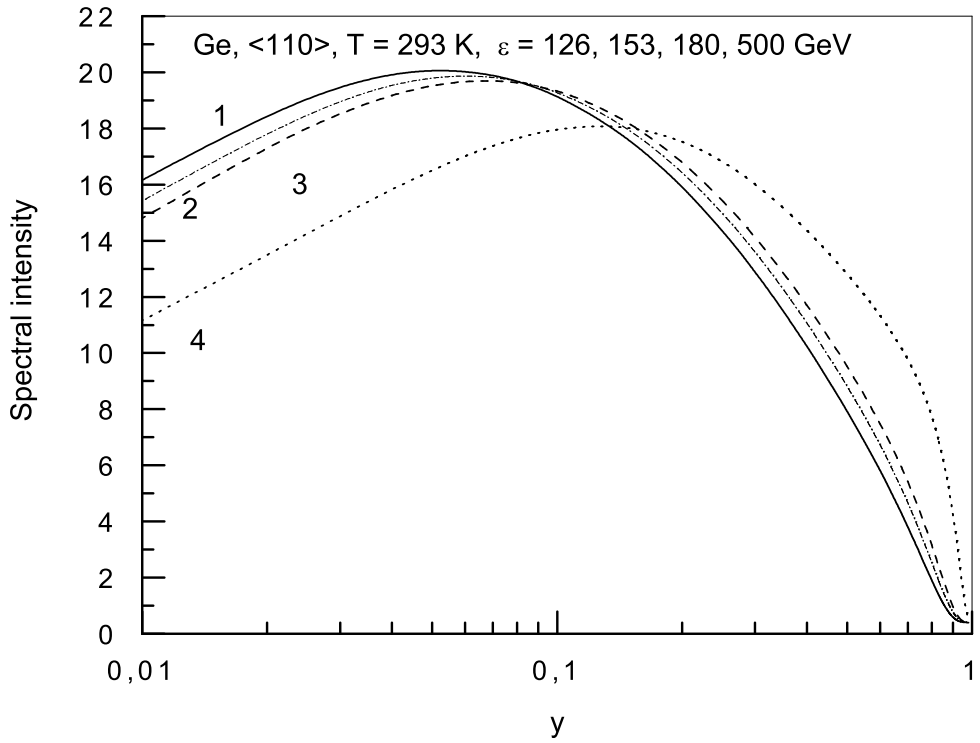

Figure 8: The radiation spectral intensity vs the photon energy $y = \omega/\varepsilon$ in germanium, axis $< 110 >$, temperature T=293 K. The intensity distribution $dI(\varepsilon, y_r)/d\omega$ (in units cm^{-1}) The curves are the theory prediction for the electron energy ε =126, 153, 180, 500 GeV.

Conclusion

At high energy $\omega \gg \omega_m$ ($\varepsilon \gg \varepsilon_m$) the influence of the multiple scattering on the process under consideration (the LPM effect) manifests itself for relatively low energy of one of the final charged particles ($\varepsilon_f \sim \varepsilon_m \ll \omega$ (ε)). In this region of spectrum $s_3(r_3) \simeq 0,\ s_2(r_2) \simeq 1$ one can present in the form

$$\frac{dW}{dy} = s_2(y)R_2(\omega y(1-y)) - s_3(y)R_3(\omega y(1-y)) \simeq R_2(\varepsilon) = R_2(\varepsilon_f),$$

$$\frac{dI}{d\omega} = r_2(y_r)R_2\left(\frac{\varepsilon}{u}\right) + r_3(y_r)R_3\left(\frac{\varepsilon}{u}\right) \simeq R_2(\varepsilon-\omega) = R_2(\varepsilon_f), \qquad (30)$$

So we have the scaling (dependence on the fixed combination of kinematic variables) not only for different energies of the initial particles in a given process, but also in the both crossing processes under consideration since this is the same combination $\omega y(1-y) = \varepsilon/u = \varepsilon(\varepsilon-\omega)/\omega$. For this reason at high energy of the initial particles the maximum value of the LPM effect for both

processes is defined by the maximum of the function $\Delta_{max} = \Delta(z_m)$, where

$$\Delta(z) = \frac{R_2^{coh}(z) + R_2^{inc}(z)}{R_2(z)} - 1, \quad z_m \simeq \frac{\varepsilon_m}{6}. \tag{31}$$

In the low energy region $\omega(\varepsilon) \leq \omega_m = \varepsilon_m$ this scaling remains only approximate one. Nevertheless the value of maximum and its position vary weakly. Just this energy region is suitable for the experimental study because the rather wide of spectrum $\Delta y \sim 1$ contributes. It should be emphasized that the LPM effect is large enough for heavy elements only (it is around 8 % in the maximum for tungsten at T=100 K).