Channeling 2008, Erice, 26 oct. – 1 nov.

Acceleration (?) and radiation (?) in a helical wave guide

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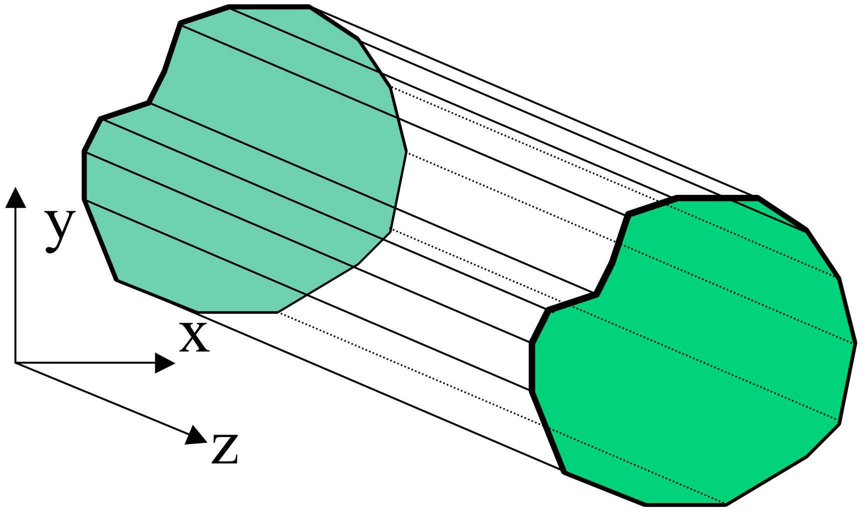
Recall: waves in a straight guide

Fields:

$\boldsymbol{F}^{\mu\nu}$ or, more simply,

$\boldsymbol{F} = \{\mathbf{E},\mathbf{B}\}$



Propagating mode:

$\boldsymbol{F}(t,x,y,z) = e^{-i\omega t}\, e^{iPz}\, \boldsymbol{f}(x,y)$

$v_{phase} = \omega/P > 1$ (in units, c=1)

There are 2 kinds of modes:

- *transverse electric* (TE)

  $E_z = 0,\ B_z \neq 0$

  cannot accelerate particles.

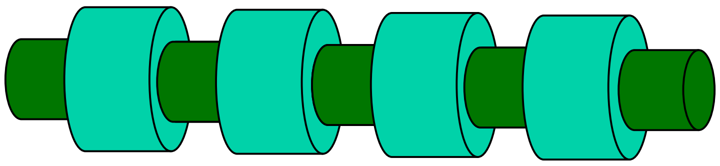
- *transverse magnetic* (TM)

  $B_z = 0,\ E_z \neq 0,$

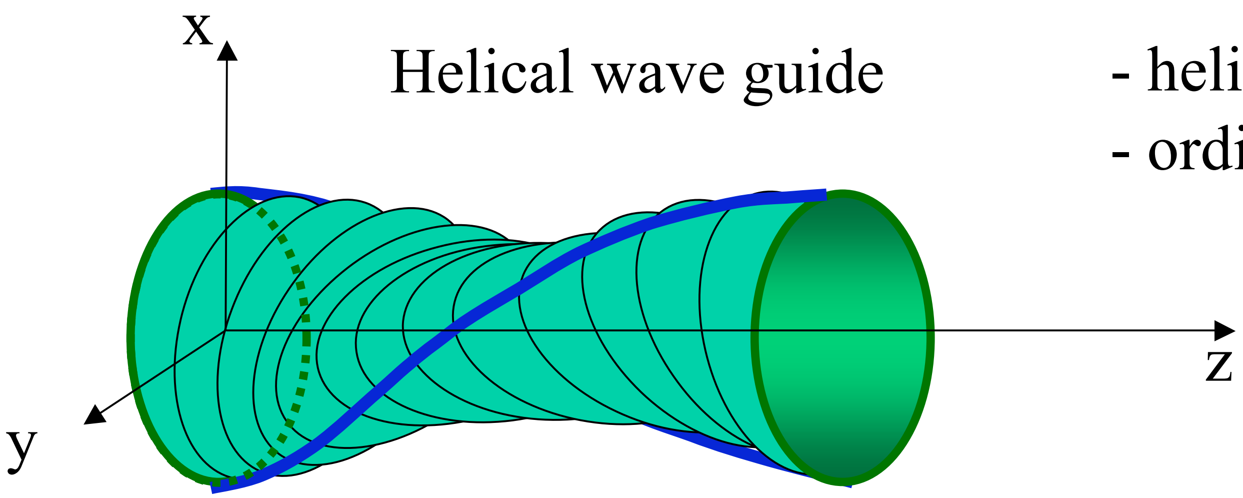
TM modes **can** accelerate particles, but not over long enough distances, since $v_{part} < 1 < v_{ph}$

Solutions to have v_{phase}<1

Cavities (usual solution)



Helical wave guide



- helix period: $2\pi/q$
- ordinary wave guide: q=0

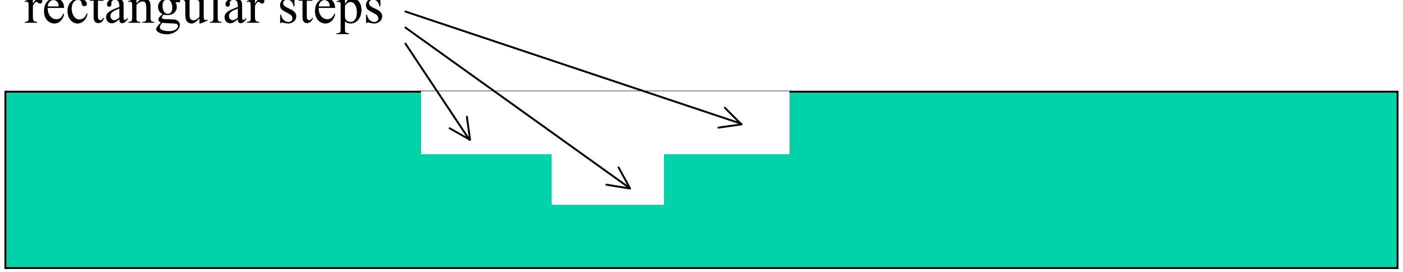
Previous works

The travelling wave tubes (TWT) is a particular kind of helical wave guide, but its wall is cylindrical; inside, a metallic rod is bent helically around the axis. It is difficult to cool this rod.

An helical waveguide can be realised by drawing helical grooves on the internal surface of an initially cylindrical tube. This is used for powerful microwave emitters. One advantage is a low dispersion.

T. Wallett, K. Vaden, J. Freeman and A. Haq Qureshi (NASA/TM-1998-207414) consider rectanglar grooves. They use a 3-dimensional code MAFIA to calculate the dispersion curve.

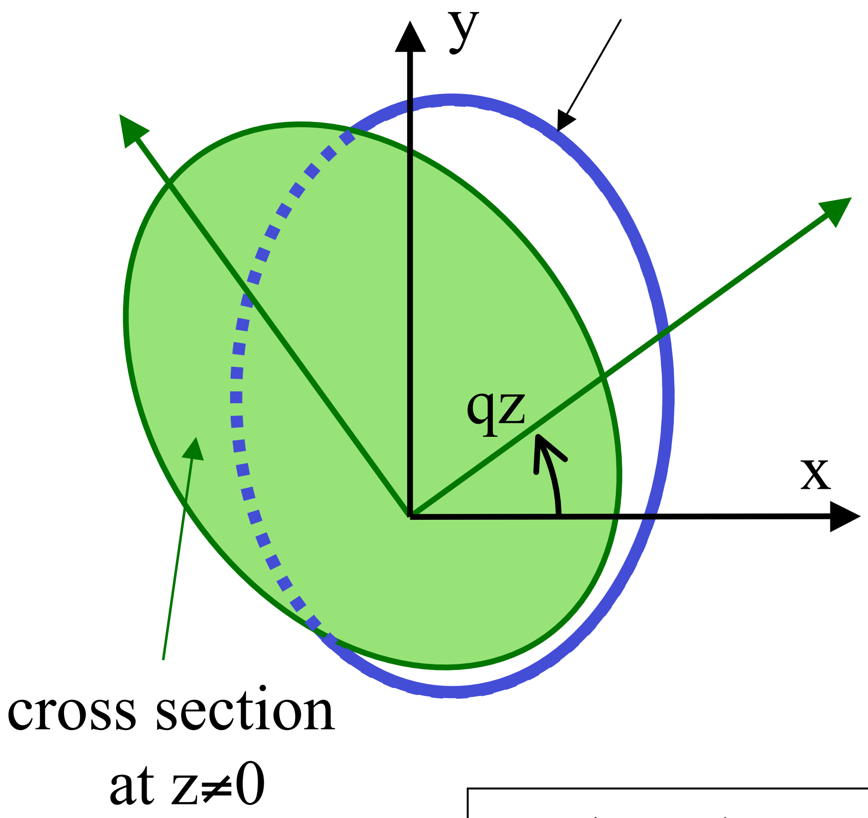
Y. Wei, W. Wang, G. Zhao, J. Sun, P. Zhou (*Int. Journal of Infrared and Millimeter Waves,* 1999) consider groove profiles rectangular or made of several rectangular steps



They calculate the modes by matching solutions valid in the different steps.

Invariance under translation × rotation

basic curve (section of the wall at z = 0)



$\mathbf{r}$ =(x,y) : transverse coordinates

$\mathbb{R}(qz)$: rotation of angle qz

$\mathbb{R}(-qz)\,\mathbf{r}$ = transverse coordinate in the co-rotating frame

$\mathbf{f}(\mathbf{r}) = \{\mathbf{B}(\mathbf{r}),\mathbf{E}(\mathbf{r})\}$: fields at z=0

$$\boxed{\boldsymbol{F}(t,\mathbf{r},z) = e^{-i\omega t}\, e^{iPz}\, \mathbb{R}(qz)\ \ \mathbf{f}\,[\mathbb{R}(-qz)\,\mathbf{r}]}$$

Maxwell → differential equations in 2 dimensions for $\mathbf{f}$(r)

Calculation of the modes

1) simpler case: scalar waves, Klein-Gordon equation

Partial wave decomposition:

$$\Psi(t,x,y,z) = e^{-i\omega t}\, e^{iPz} \sum_l a_l\, e^{il\phi}\, e^{-ilqz}\, J_l(k_l r)$$

action of $\mathbb{R}(-qz)$ on $\mathbf{r}$

Klein-Gordon relation: $k_l = [\omega^2 - (P - lq)^2]^{1/2}$

k_l : *radial momentum*, real for $P - \omega < lq < P + \omega$,
otherwise, pure imaginary

The *truncation-and-sutture* method

Scalar boundary condition : $\Psi(t,x,y,z) = 0$ on the wall.

It suffices to impose $f(\mathbf{r}) = \Psi(0,x,y,0) = 0$ on the basic curve.

Approximate method:

- restrict the partial wave expansion to N waves $\in [l_{min}, l_{max}]$
- impose $f(\mathbf{r}_n)=0$ on N "sutture points" $\{\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_N\}$ of the basic curve.

The coefficients a_l are then calculated by the finite linear system

$$f(\mathbf{r}_n) = \sum_l a_l \exp(il\phi_n)\, J_l(k_l r_n) \;=\; \sum_l C_{nl}\, a_l = 0$$

A non-zero solution exists only when

$$\det\{C_{nl}\} = 0$$

= the *dispersion relation* linking ω and P.

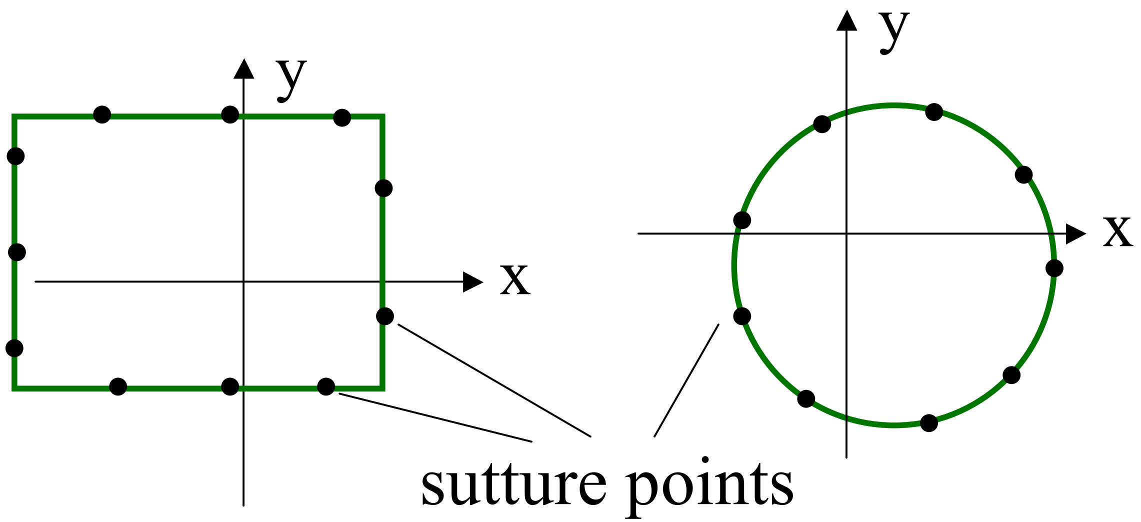
Check of the method

We applied it to the case of ordinary wave guides (q=0), of rectangular or cylindrical section. With only N ~10-15 waves and sutture points, the waves numbers $P_m(\omega)$ of the 2 or 3 lowest modes were close to the exact ones, to a few percent.

The stability of $P_m(\omega)$ was checked for

-a translation of $[l_{min}, l_{max}]$ by one or two units,

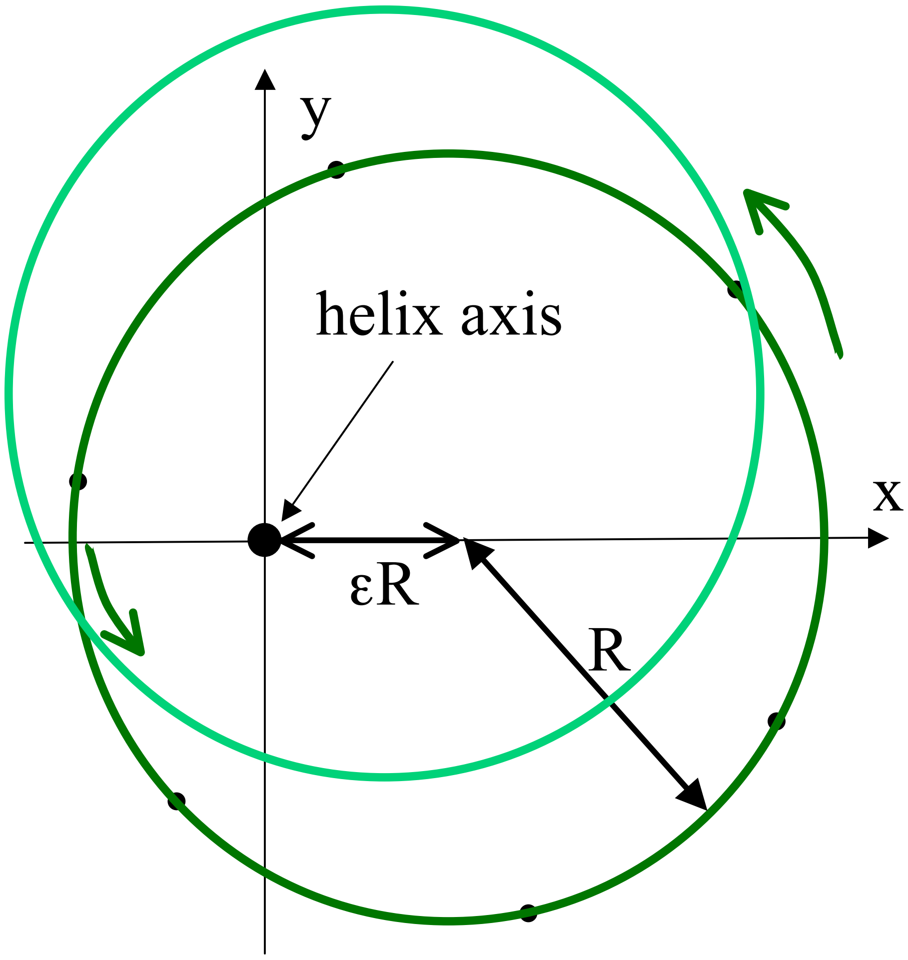
-a translation of the x,y origin from the centre of the rectangle one or circle.



Numerical application of the scalar wave case: $q \neq 0$, de-centered circular basis

Choosen guide parameters:

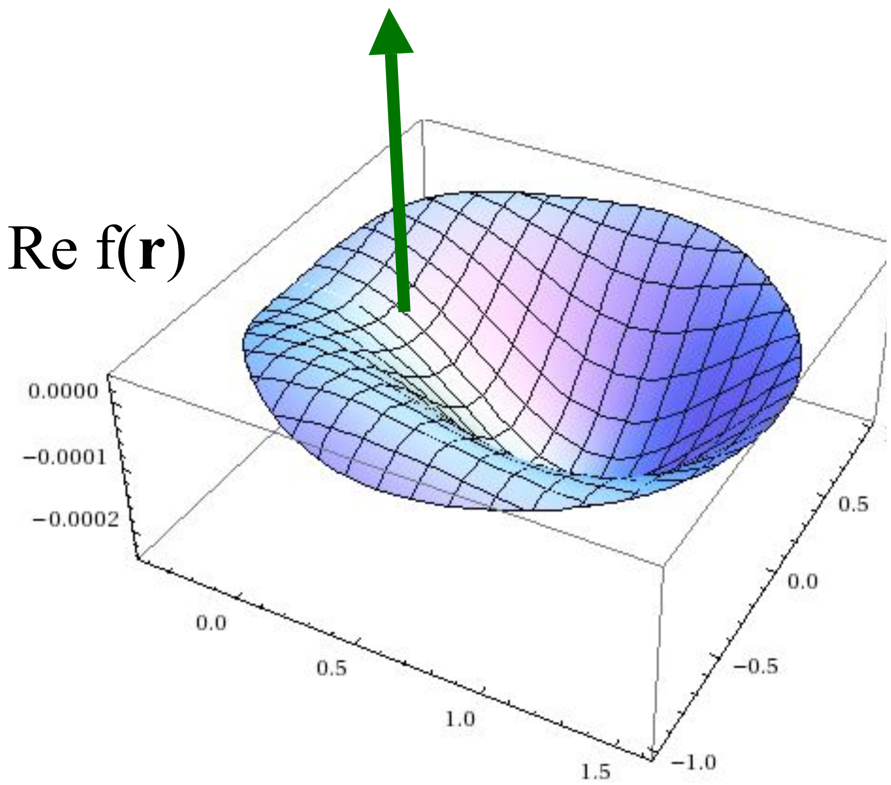
- Amplitude and wave number of the helical undulation:
  $\varepsilon R = 0.4\ R$; $q = 1.5\ R^{-1}$.
- Pitch angle of the largest $|\mathbf{r}|$ helix:
  $\tan(\alpha) = qR(1+\varepsilon) = 2.1$

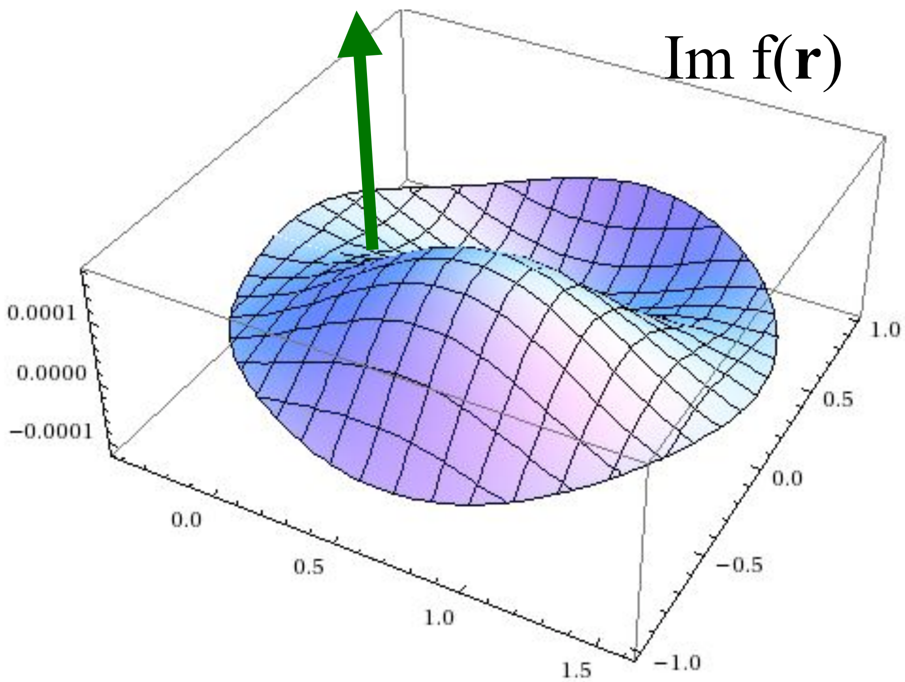


We looked for modes of phase velocity 1 (=c), i.e. we imposed $P = \omega$.

The lowest mode is found at
  $\omega = P = 3.3\ R^{-1}$,
with only 7 sutture points.

3-D plot of the (scalar) wave amplitude





- The centrifugeal effect is mainly visible on Re f (**r**)
- The asymmetry of Im f(**r**) in y corresponds to a motion with positive L_z, i.e., $\langle l \rangle > 0$

First conclusions, from the scalar case

- the truncation-and-sutture method is simple and robust.

- the helicity of the guide is able to slow down
  a fundamental mode, down to $v_{phase} < 1$,
  so that particles (not only tachyons !) can surf on it.

- the centrifugeal effect might be a problem : it may
  suppress the field near the axis, where the particle
  are supposed to be.

Calculation of the mode for electromagnetic waves

- Replace f(**r**) by the 6-component tensor **f**(**r**) ={**B**(**r**),**E**(**r**)}
- Introduce 2 sets coefficients : a_l for the TM modes,
  b_l for the TE modes
- Replace the radial wave function $J_l(k_l r)$ by the column 6-vector

$$\begin{Bmatrix} E_r \\ E_\phi \\ E_z \\ E_\phi \\ B_r \\ B_\phi \\ E_z \end{Bmatrix} = \begin{Bmatrix} i\, p_l\, k_l\, J'_l(k_l r) \\ -\, p_l\, l/r\, J_l(k_l r) \\ k^2_l \quad J_l(k_l r) \\ -p_l\, l/r\ \ J_l(k_l r) \\ \omega\, l/r \ \ J_l(k_l r) \\ i\,\omega\, k_l \ \ J'_l(k_l r) \\ 0 \end{Bmatrix}$$

for the TM partial wave

$p_l = P\text{-}lq, \quad k_l = [\omega^2 - p_l^2]^{1/2}$

- The TE partial wave is obtained by the duality E→B, B→ -E

Boundary conditions (electromagnetic case)

s = tangent vector to the basic curve at point **r**.
In polar coordinates,

$$E_r \,.\, s_r + E_\phi \,.\, s_\phi = 0,$$
$$qr\, E_\phi + E_z = 0,$$
$$B_r \,.\, s_\phi + (qr\, B_z - B_\phi)s_r = 0.$$

Only 2 of these 3 equations are sufficient. Thus, N sutture points give 2N boundary equations.
For N values of l, we have N coefficients a_l and N coefficients b_l.
We thus get a 2N×2N linear system.

Preliminary results for electromagnetic waves

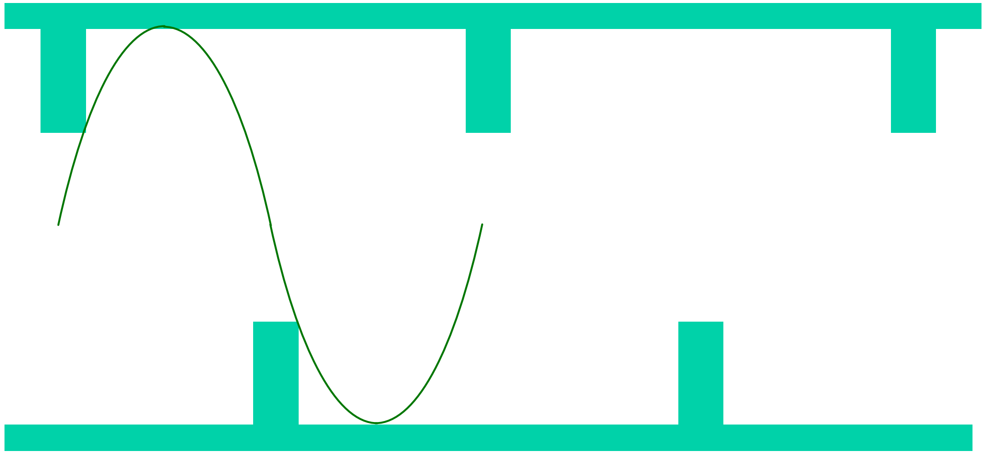
Due to lack of time, we could only calculate the lowest mode, without making the necessary tests (changes of l_{max} and l_{min}).

Our preliminary result (not shown here), if correct, shows a very small E_z on the axis. This may be related to the fact that, for an ordinary (q=0) circular wave guide, the lowest mode is TE.

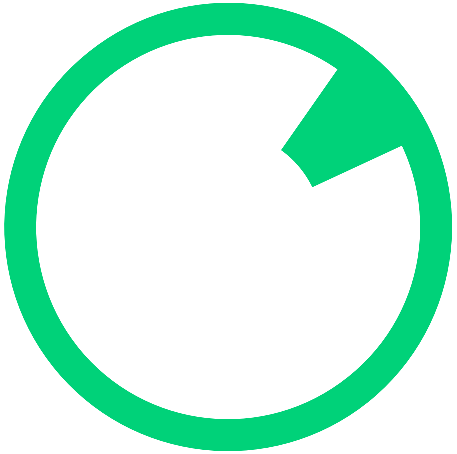
We hope, for acceleration purpose, that the second or third mode will be mainly of the TM type and have a large enough E_z.

Other cross section shapes.

1) asymmetrical basis.



Longitudinal cut view



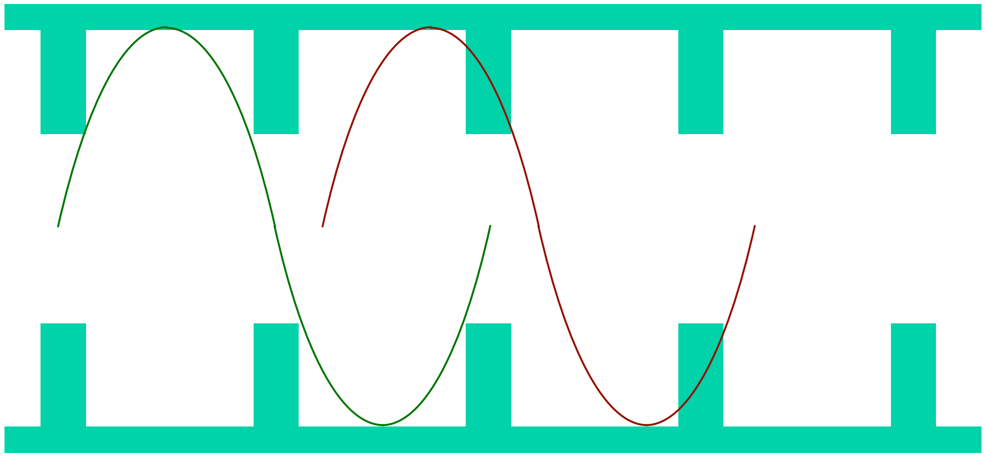
cross section

Due to the asymmetrical basis, there are non-vanishing transverse fields at $\mathbf{r} = 0$.

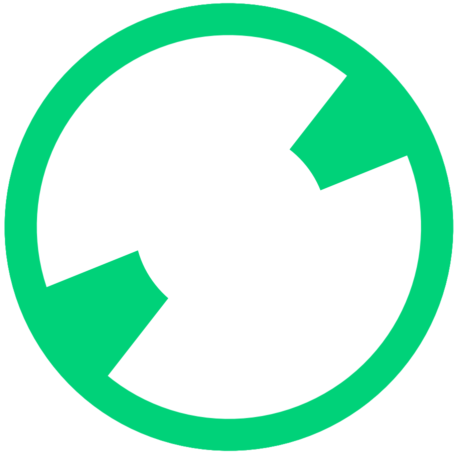
- image charge effect may be important
- for electrons, one may have parasitic synchrotron radiation.

Other cross section shapes

2) symmetrical basis



Longitudinal cut view



cross section

The fields are invariant under parity in the transverse plane.
There is no transverse field at **r** = 0.

Advantages:

- less space charge effect
- no parasitic synchrotron radiation in the case of electrons.

Résumé

1. **The helical waveguide is an interesting object from the theoretical point of view.**
2. **The truncation-and-sutture method seems well suited for this probem (as well as for many problems of electromagnetic cavities).**
3. **Low modes with phase velocity smaller than *c* are easily obtained.**
4. **Uncertainties remain about their efficiencies for particle accelation (or for the inverse process, stimulated emission of radiation). New numerical results are waited.**
5. **Concerning the spontaneous emission, the upper bound $dW/dz < C\, Z^2\alpha/b^2$ proposed in my previous talk should be applicable, if it really exists.**

Thanks you again for attention !