Resonant Transition Radiation

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1. Introduction

The transition radiation occurs when a charged particle passes from one medium to another. Rearrangement of the particle field takes place. The formula obtained by Ginsburg and Frank is describing the radiation frequency-angular distribution[1]:

1. GinzburgV.L., Frank I.M. JETP 16, 15, 1946; J. Phys. USSR 9, 353, 1945 (in Russian)

Radiation spectral distribution for a particle moving in infinite layered medium is obtained by Faynberg and Khijnyak [2]:

2. FaynbergYa.B.,Khijnyak N.A. JETP, 35, 883, 1957 (in Russian)

The energy losses within X-ray frequency range for a relativistic particle passing through the boundary of two different semi-infinite mediums is obtained by Garibian [3]:

3. GaribianG.M.,Zh. Eksp. Teor. Fiz.,**37**, 1106, 1959 (in Russian)

Barsukov has shown that this energy losses are stimulated by X-ray transition radiation [4]:

4. BarsukovK.A.,Zh. Eksp. Teor. Fiz, **37**, 1106, 1959 (in Russian)

As a model for layered medium Ter-Mikaelyan were considering finite periodic medium with sinusoidal variation of the density.

$$\rho(z) = \rho_0(1 + \mu sin\frac{2\pi z}{l}) \quad (1),$$

where ρ_0 is the average electrons density in particle moving direction z, μ ($0<\mu<1$) is the parameter of electrons density variation, l is the spatial period of medium [5]:

5. Ter-MikhaelyanM.L.,Nucl. Phys., v. 24, No. 1, p. 43, 1961

The radiation intensity obtained by quasi-classical approach increases according to the number of density variation periods n. This relationship is relevant also in the case of medium with random holes [6].

6. GaribianG.M.,Gevorgian L.A., Yan Shi, Zh. Eks. Teor. Fiz. **66**, 552, 1974 (in Russian); NIM **125**, 133, 1975

The radiation intensity is transformed into series by Bessel's function as it is in the problem of undulator radiation. Due to the limitation of quasi-classical approach the argument of Bessel's function is smaller than one and only the first harmonic has a significant involvement in the radiation intensity.

The exact solution of this problem requires taking into account the fact that phase of radiation field has a summand that varies by sinusoidal law and its amplitude increases linearly along with *z*. As a result of quasi-classical approach this linearly increasing amplitude is substituted by constant value.

It was shown that exact solution of the problem leads to the essential modification of the emission characteristics. Particularly radiation harmonics have the same spectral distribution in the frequency range of the main harmonic [7].

7.GevorgianL.A., Proc. SPIE, 5974, 5974OU, 2005

On the other hand the harmonics in the soft frequency range are radiating coherently [8].

8. Lekdar Gevorgian, Proc. SPIE, 6634, 663413-1, 2007

The present work includes complete theory of RTR: frequency-angular distribution for separate harmonics as well as the integrated angular and frequency distributions.

2. The frequency-angular distribution of RTR

The exact solution of the problem for the *k*-th harmonic radiation gives per unit length of particle path the following frequency distribution:

(2)

Here *e* is the electron charge, *c* is the light speed in vacuum, β is the particle speed by *c* units, $\nu = \frac{\omega_p^2}{2\omega^2}$, ω_p is plasma frequency of the medium, $\Omega = 2\pi\beta c/l$ is the medium electrons density variation frequency, are the low and high limits of .

The two delta functions present in this formula prove that two photons are emitted simultaneously on different angles. It is true: also according to the energy and momentum conservation law the two photons should be emitted by different helicities. This is the condition when the radiation field phase is fixed, that is Huygence principle.

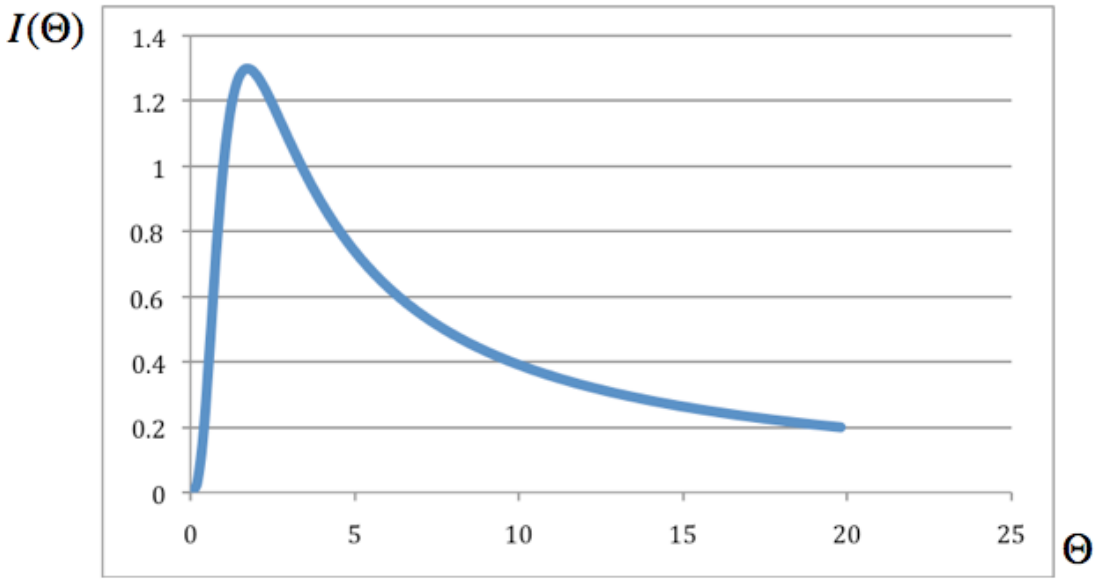
3. The angular distribution

Taking into account that with given angle two photons are emitted with different energies (complicated Doppler effect) from the expression (2) one will get the angular distribution of *k*-th harmonic radiation for *L=nl* particle path length

$$\frac{dW_k}{d\vartheta} = 2\pi\alpha n\mu\hbar\omega_p\gamma_p\gamma I(\vartheta) \tag{3}$$

$$I(\vartheta) = \frac{\sin^3\vartheta}{\gamma(1-\beta\cos\vartheta)^2} \tag{4}$$

All harmonics have the same angular distribution. Below is presented the angular distribution versus $I(\Theta)$, where $\Theta = \gamma\vartheta$.The angular distribution has a maximum on $\Theta_m = 2$.



4. RTR frequency distribution

For the spectral distribution one gets

(3)

The radiation occurs in the following frequency range

(4)

Where γ is the Lorentz factor of the particle, $r_k = \gamma_p\sqrt{1+\mu}/(\gamma k)$, $\gamma_p = \omega_p/\Omega$. The minimal value of the number of harmonics is determined from $r_k \leq 1$ condition

(5)

Within the cone with angular aperture ϑ_0 the maximal value of harmonic number is defined from $cos\vartheta_- \geq cos\vartheta_0$ condition

$$m = entier(\gamma_p[\frac{\omega}{\omega_p}(1-\beta cos\vartheta_0) + \frac{1-\mu}{2}\frac{\omega}{\omega_p}]) \quad (6)$$

The radiation frequency range now is the following

$$\omega_0 = \omega_1(m) \leq \omega \leq \omega_2(m) = \omega_c, \quad (7)$$

In this range are radiated harmonics with number $k \le m$. The shortest spectrum has the (1+m_0)-th harmonic and the longest spectrum has the m-th harmonic. The spectra of the harmonics are overlapping. The spectra of all harmonics are coinciding within (1+m_0)-th harmonic radiation range. Out of this range the number of harmonics decreases sequentially by one, thus the spectral distribution has a step-like form. If we transform into continues spectrum then one will have the following expression for the harmonics weight function depending on photons energy $E = \hbar\omega$

$$p(E) = (m - m_0)\left(1 - \frac{E_0}{E}\right)\left(1 - \frac{E}{E_c}\right) \quad (8)$$

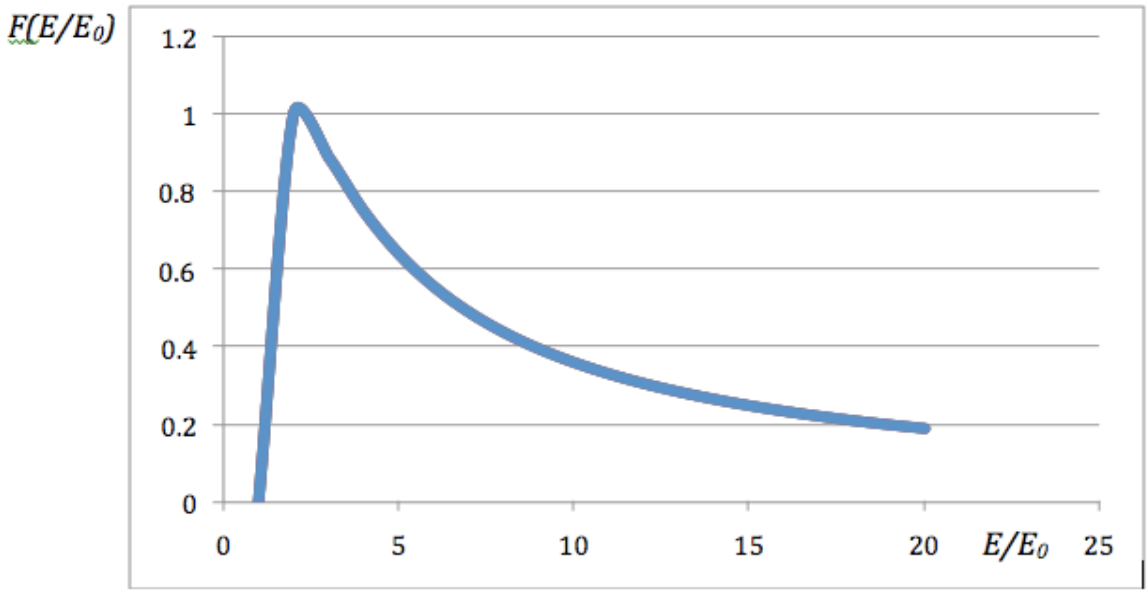
When $\vartheta_0 >> \sqrt{(1+\mu)/(1-\mu)}/\gamma$ then $m_0 << m = \gamma_p \vartheta_0 \sqrt{1-\mu}$ and for the spectral distribution of the total number of emitted photons we have

$$\frac{dN_{tot}}{dE} = \pi\alpha n\mu \frac{(\gamma_p \vartheta_0)^2}{2E_p} F(E), \qquad F(E/E_0) = 4\frac{E_0}{E}(1 - \frac{E_0}{E})(1 - \frac{E}{E_c}), \quad (9)$$

where

$$E_0 = E_p \frac{1+\mu}{2\vartheta_0\sqrt{1-\mu}}, \qquad E_c = 2\vartheta_0\gamma^2 E_p \sqrt{1-\mu}, \qquad E_p = \hbar\omega_p \quad (10)$$

Taking into account that $E_c >> E_p$ the spectral distribution of the emitted photons is the following



Note that in the case of plates stack ($\mu \to 1$) the spectral characteristics are significantly different from RTR.

Conclusion

The exact solution shows that a large number of harmonics have involvements in RTR formation. As consequence the radiation intensity increases by several orders. In the soft X-ray region one can obtain quasi–monochromatic directed powerful photon beam.

For example, if 1 GeV electron beam passes trough modulated gas with plasma wavelength, spatial period and then one electron emits 100 soft (0.5-1.5 KeV) photons within angular cone on *L=12.4 cm* path.