

# Phases of Gauge Theories

Francesco Sannino

CP<sup>3</sup> - Origins



Particle Physics & Origin of Mass

July 2010 @ Quark Matter - Roma

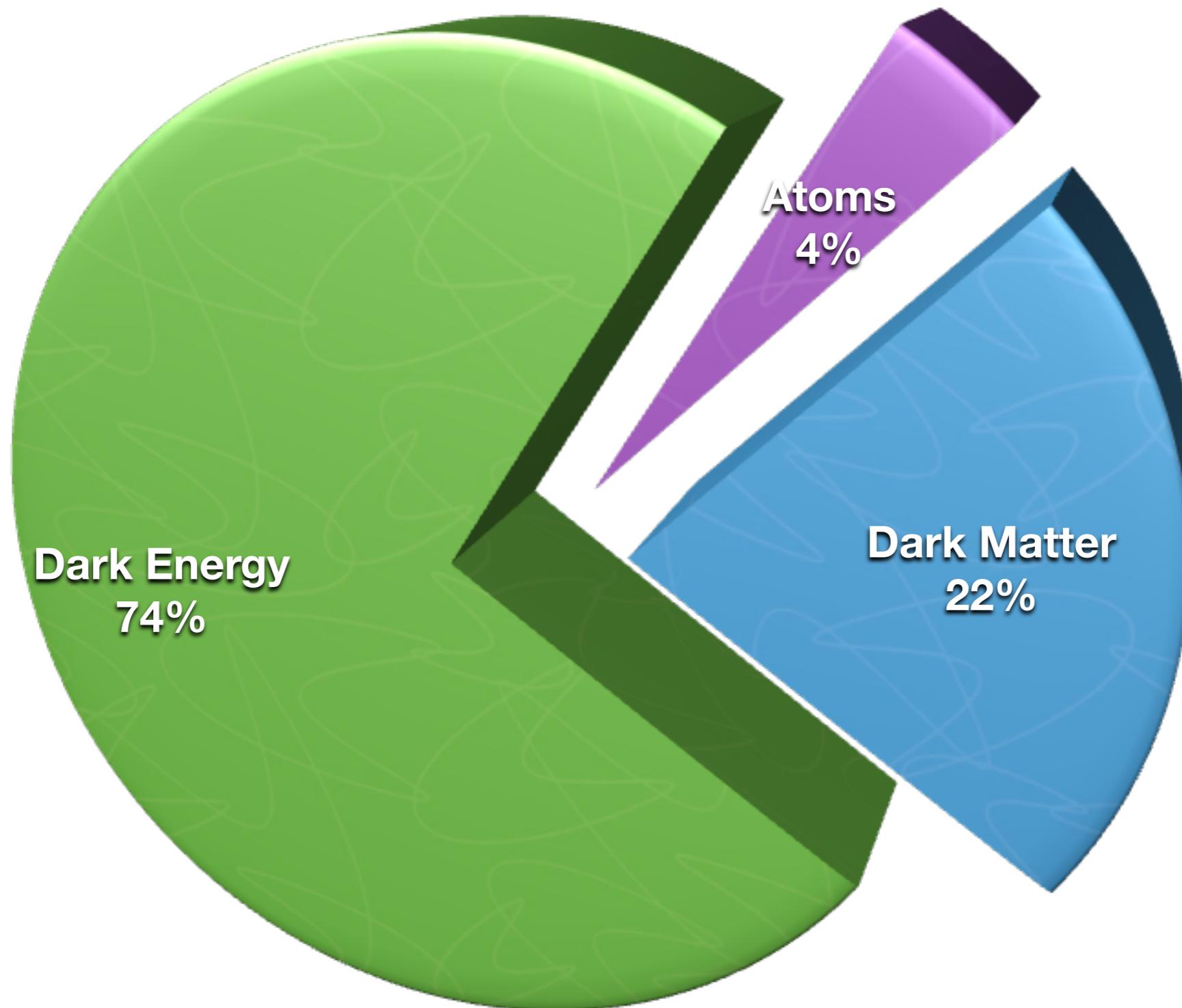






# Cosmology

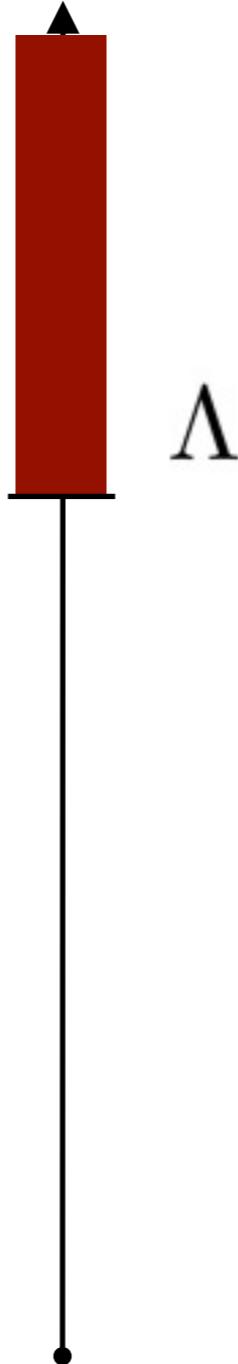
# Cosmology



# Standard Model

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*Energy*



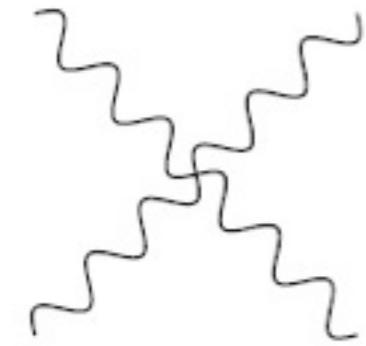
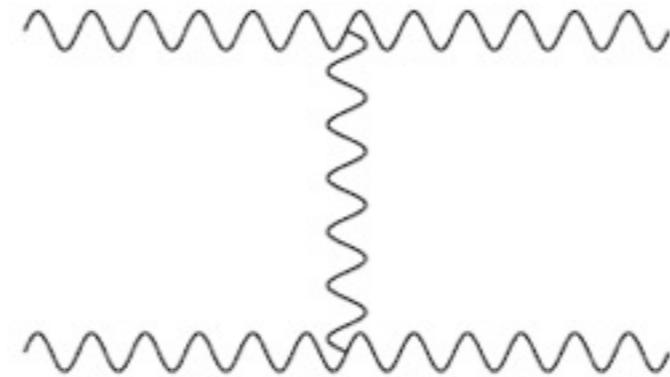
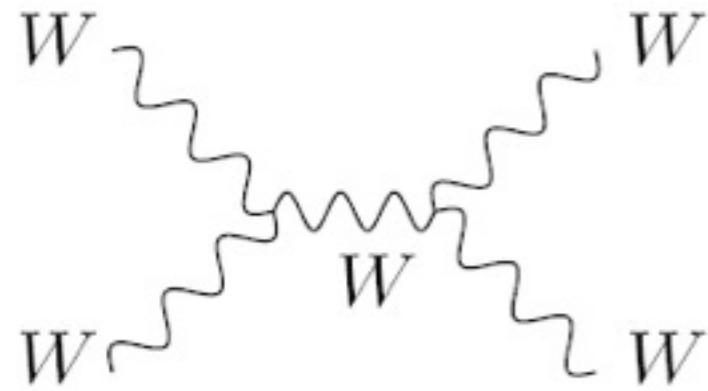
SM

The standard model					
Elementary particles					
Quarks	u up	c charm	t top	γ photon	
d	down	s strange	b bottom	Z Z boson	
Leptons	v <sub>e</sub> electron neutrino	v <sub>μ</sub> muon neutrino	v <sub>τ</sub> tau neutrino	W <sup>+</sup> W <sup>+</sup> boson	Force carriers
e	electron	μ	τ	W <sup>-</sup> W <sup>-</sup> boson	
				g gluon	
	Higgs* boson				

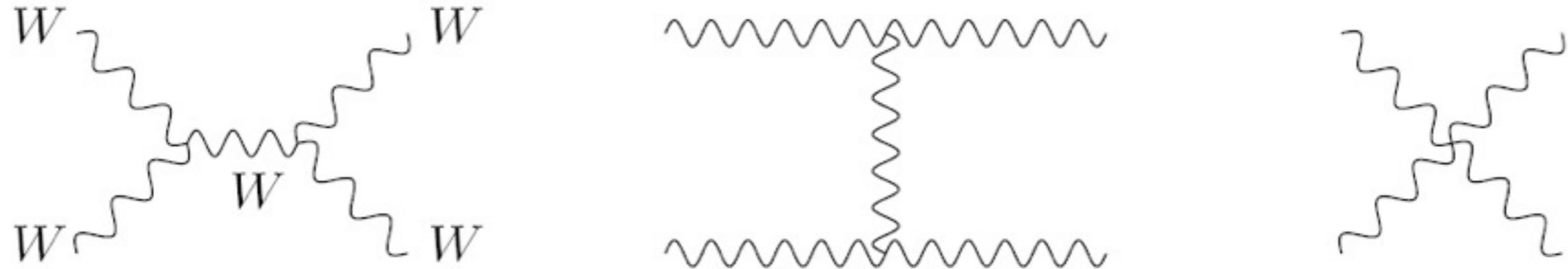
Source: AAAS      \*Yet to be confirmed

# WW Scattering

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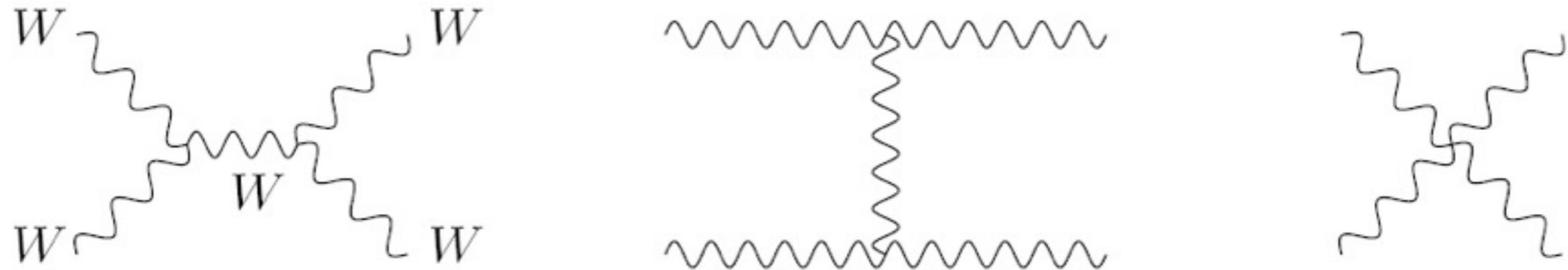
# WW Scattering



S-wave amplitude:

$$A_0 = \frac{G_F}{8\pi\sqrt{2}} s$$

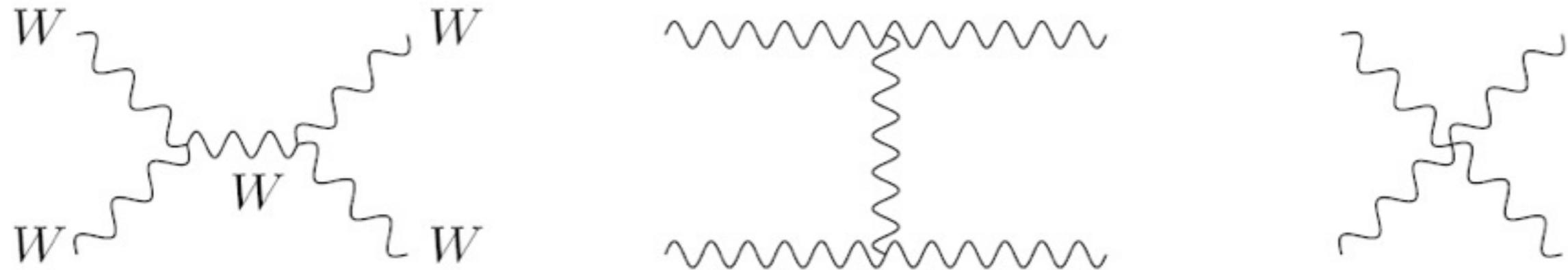
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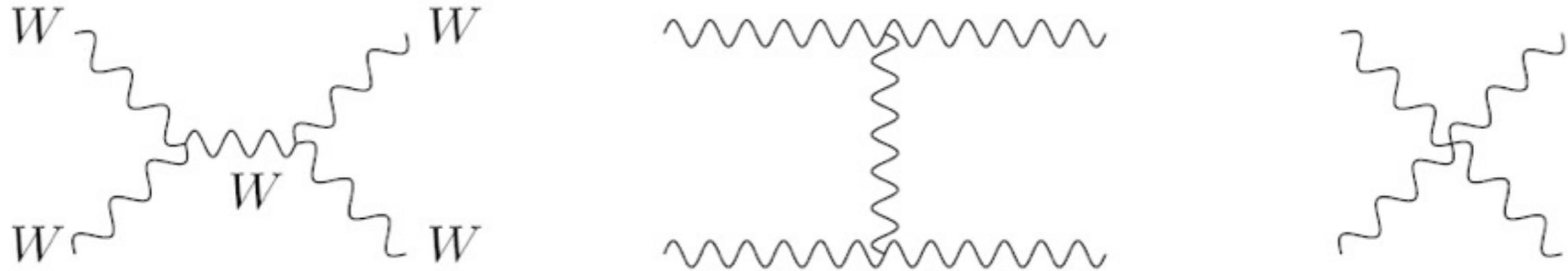
S-wave amplitude:

$$A_0 = \frac{G_F}{8\pi\sqrt{2}} s$$

$$G_F = \frac{g^2}{4\sqrt{2}M_W^2}$$

$$\simeq 1.14 \times 10^{-5} \text{GeV}^{-2}$$

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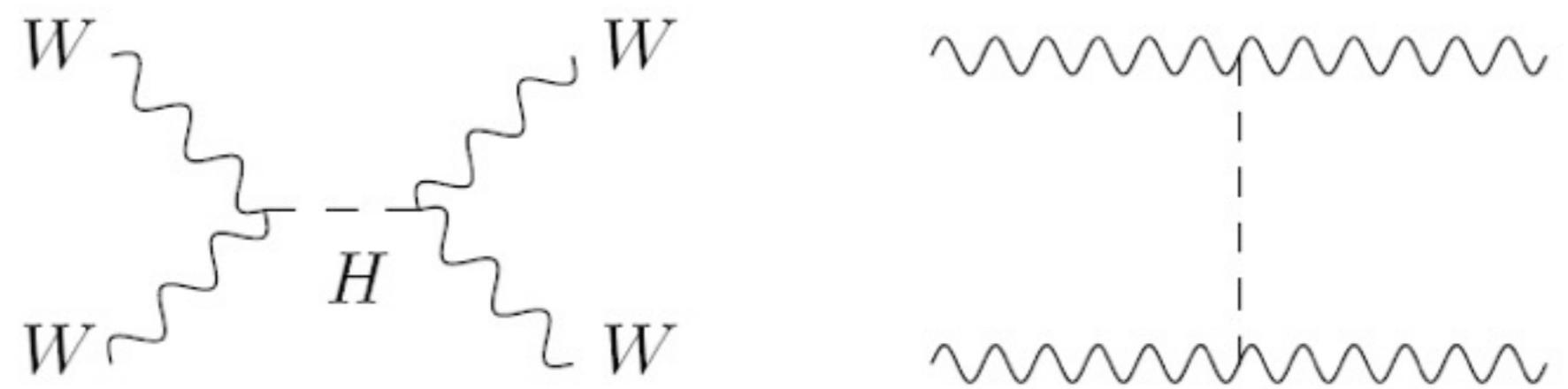
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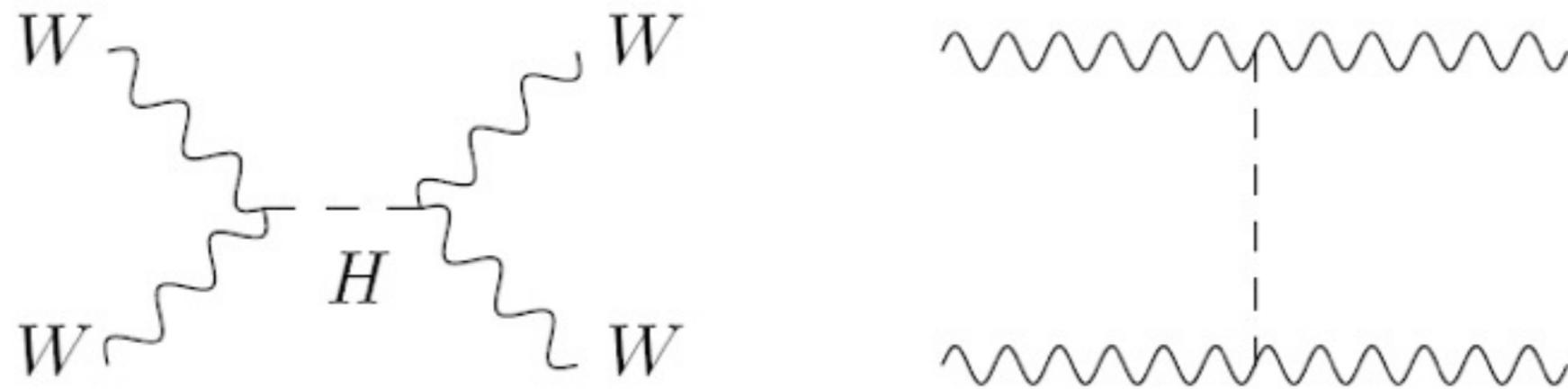
Unitarity:

$$\Re [A_0] \leq \frac{1}{2} \quad \longrightarrow \quad s \leq 4\pi\sqrt{2}/G_F \sim (1.2 \text{ TeV})^2$$





$$A'_0 = -\frac{G_F}{8\pi\sqrt{2}} \ s$$



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Theorem:

*Unitarity requires the existence of a weakly coupled Higgs particle or New Physics around the Terascale!*

# Origin of mass



# The Higgs

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_2 + i\pi_1 \\ \sigma - i\pi_3 \end{pmatrix}$$

# Custodial symmetries

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$$[i \tau_2 H^*, H] = \frac{1}{\sqrt{2}} (\sigma + i \vec{\tau} \cdot \vec{\pi}) \equiv M$$

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$$SU_L(2) \times SU_R(2)$$

$$g_{L/R} \in SU_{L/R}(2) \quad M \rightarrow g_L M g_R^\dagger$$

$$\mathcal{L}~=~\frac{1}{2}\text{Tr}\left[D_{\mu}M^{\dagger}D^{\mu}M\right]-\frac{m^2}{2}\text{Tr}\left[M^{\dagger}M\right]-\frac{\lambda}{4}\,\text{Tr}\left[M^{\dagger}M\right]^2$$

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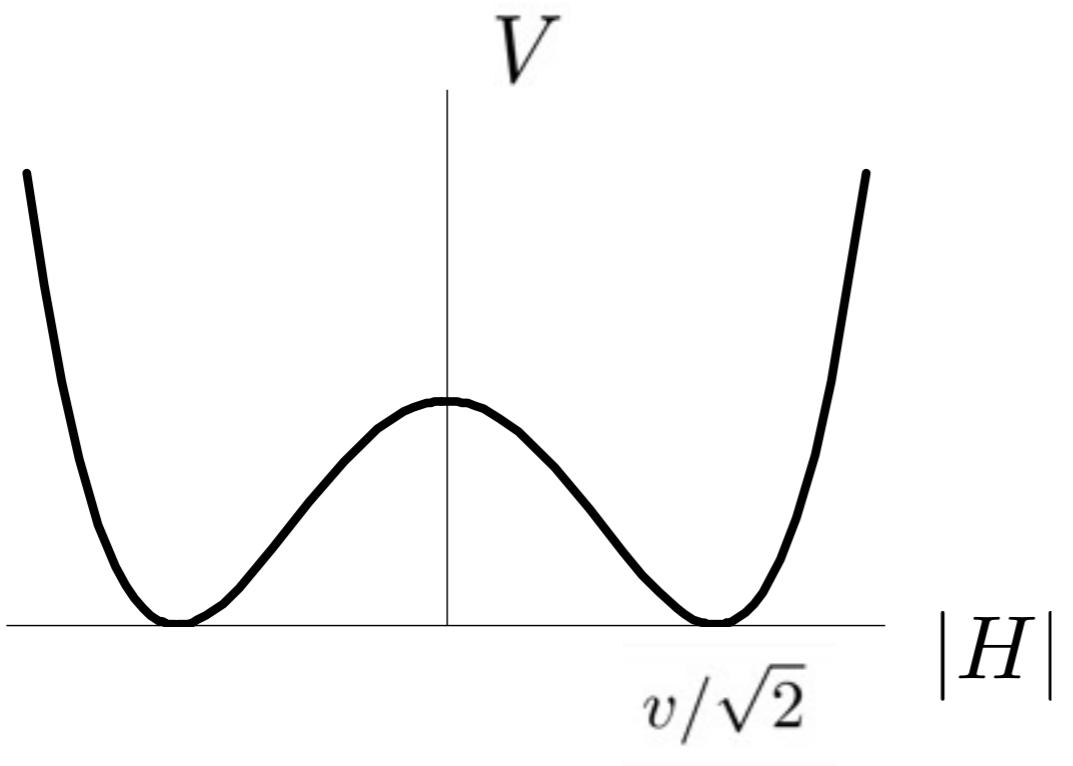


$$D_\mu M=\partial_\mu M-i\,g\,W_\mu M+i\,g'M\,B_\mu$$

$$W_\mu=W_\mu^a\frac{\tau^a}{2}\,,\quad B_\mu=B_\mu\frac{\tau^3}{2}$$

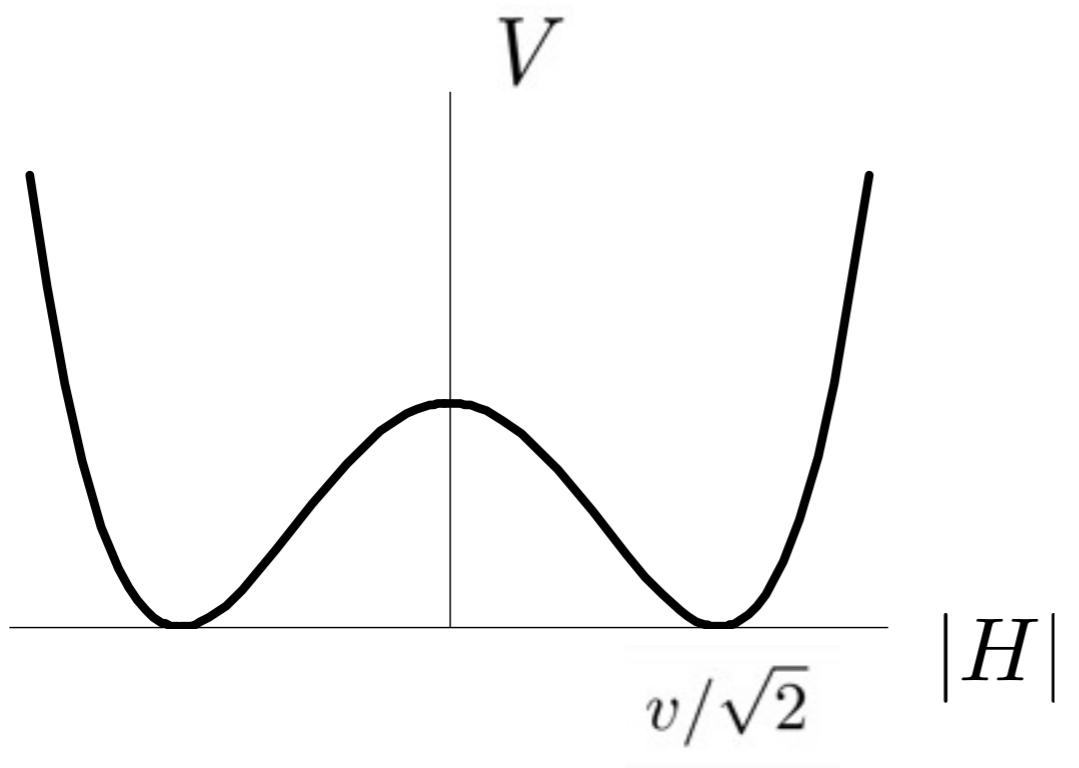
$$m^2<0$$

$$m^2 < 0$$



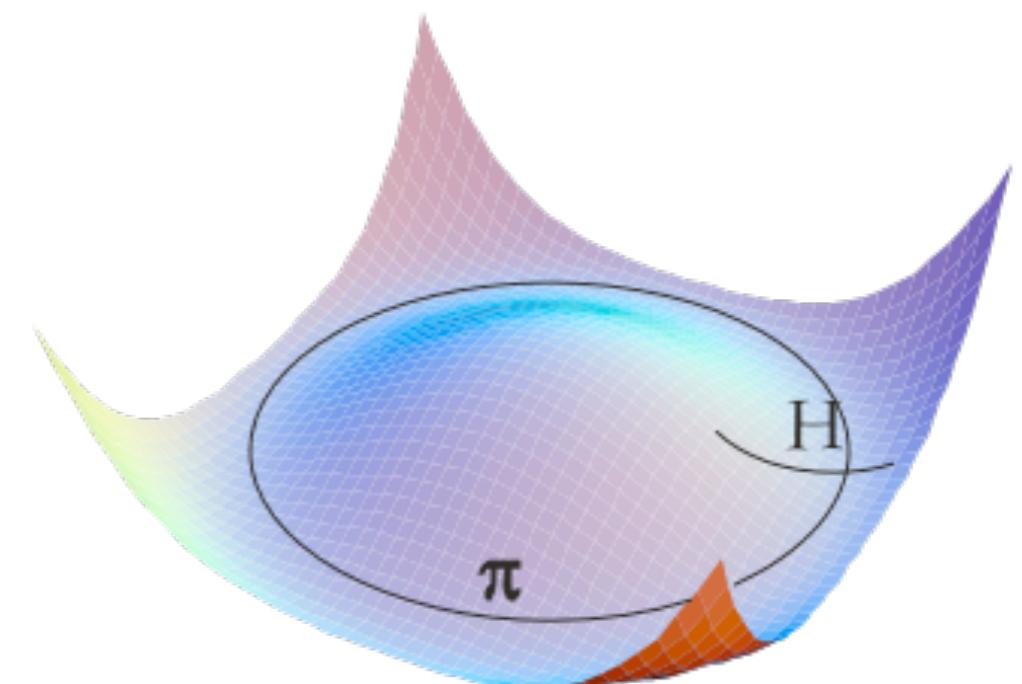
$$\langle \sigma^2 \rangle \equiv v^2 = \frac{|m^2|}{\lambda}$$

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$$\langle \sigma^2 \rangle \equiv v^2 = \frac{|m^2|}{\lambda}$$

$$\sigma = v + h$$



# Gauge Boson-Masses

$$\frac{1}{2}\mathrm{Tr}\left[D_{\mu}M^{\dagger}D^{\mu}M\right]$$

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$$\frac{1}{2} \text{Tr} \left[ D_\mu M^\dagger D^\mu M \right] \quad \longrightarrow \quad M_W = gv/2 = M_z \cos \theta_w$$

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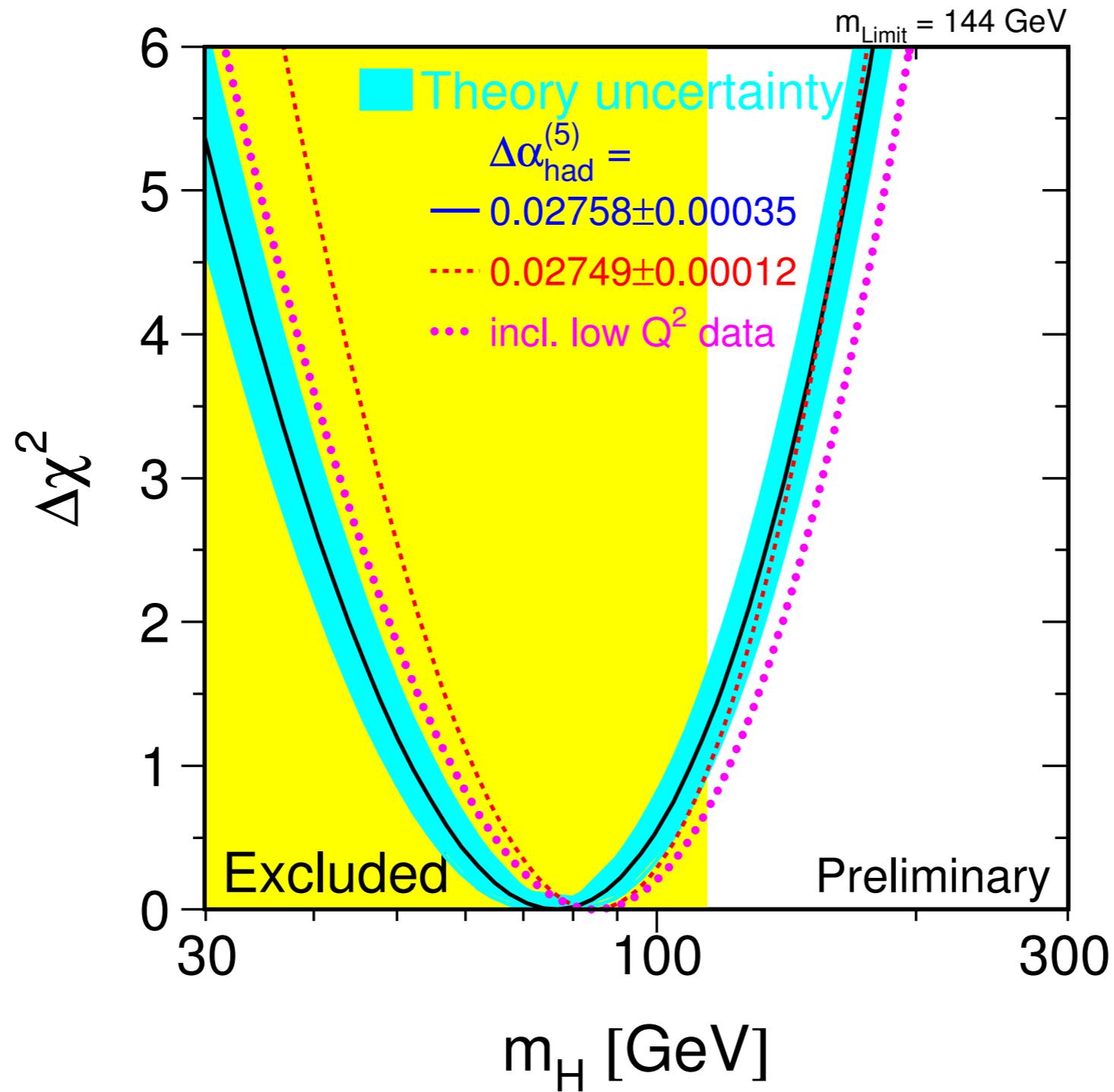
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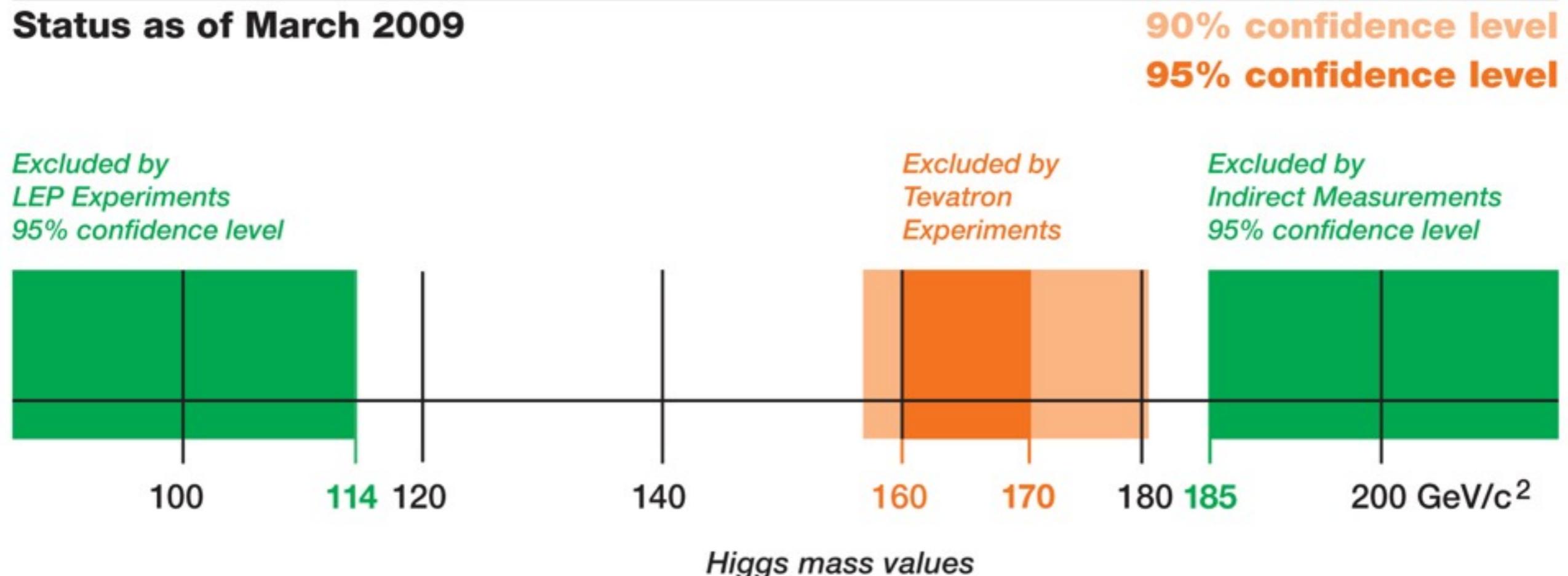
$$-\lambda_d \bar{Q}_L \cdot H d_R \rightarrow \boxed{m_d = \lambda_d v / \sqrt{2}}$$

# SM Higgs: Current Status:



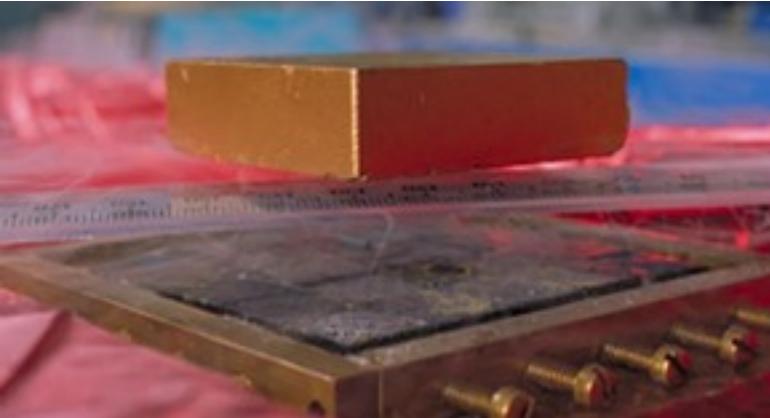
# Search for the Higgs Particle

Status as of March 2009



# Higgs mechanism in Nature

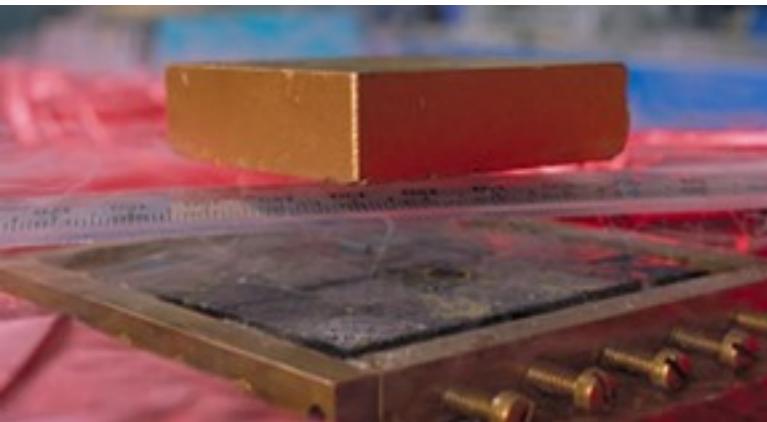
# Superconductivity



# Superconductivity

Macroscopic-Screening  
Non-Relativistic

SM-Screening  
Relativistic

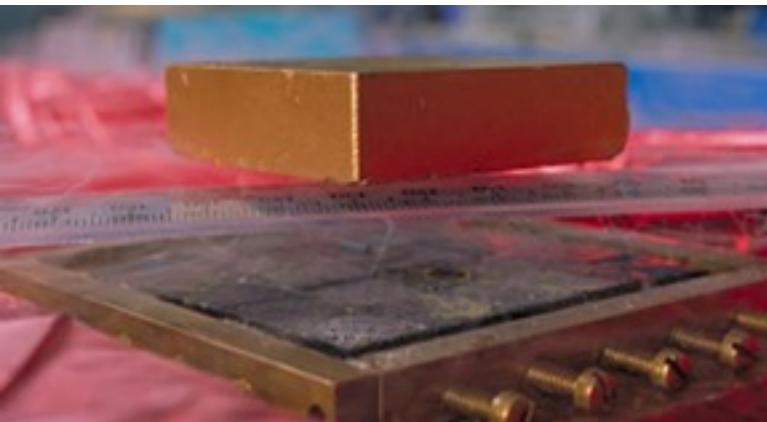


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$$T < T_c$$



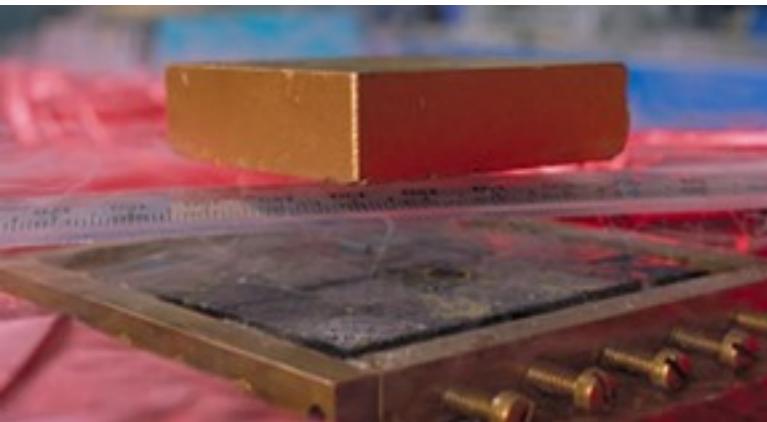
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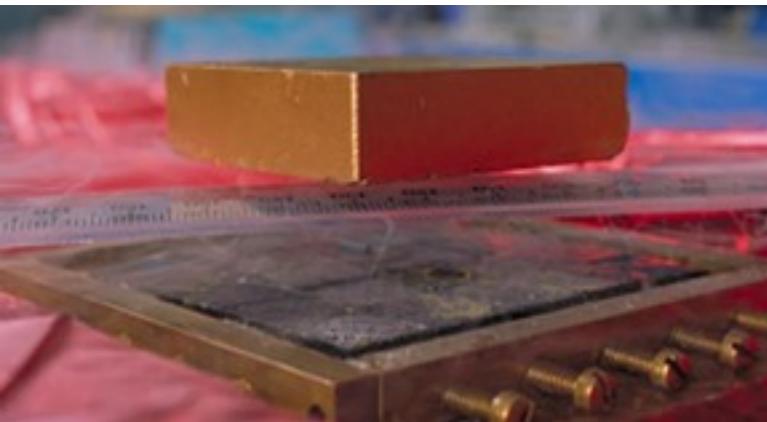
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Relativistic

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$$|\psi|^2 = n_C = \frac{n_s}{2}$$

$$|H|^2 = \frac{v^2}{2}$$



# Meissner-Mass Static Vector Potential

Weak-GB-Mass

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$$n_s \sim 4 \times 10^{28} m^{-3}$$

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$$M_W \sim 80~GeV$$

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Hidden structure

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????

# Instability of the Fermi Scale

$$v=1/\sqrt{\sqrt{2}G_F}\approx 246~\mathrm{GeV}$$

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$$M_H^2 = 2 \lambda \, v^2$$

# Quantum corrections

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No custodial symmetry protecting a scalar mass.

Hierarchy between the EW scale and the Planck Scale.

# Definition of Natural

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Small parameters stay small under radiative corrections.



# Electron mass

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If set to zero the  $U(1)_L \times U(1)_R$  forbids its regeneration

$$m_{eR} = R \times m_{eB}$$

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Naturalness begs an explanation of the origin of mass.

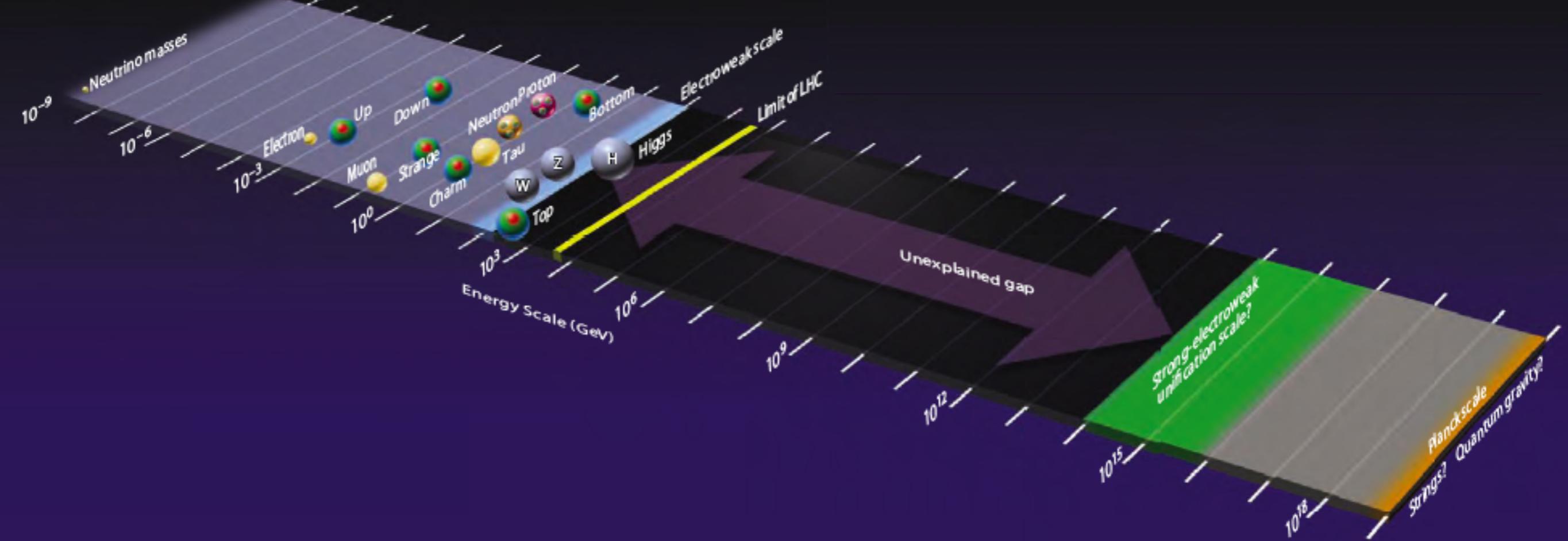
# Electron mass

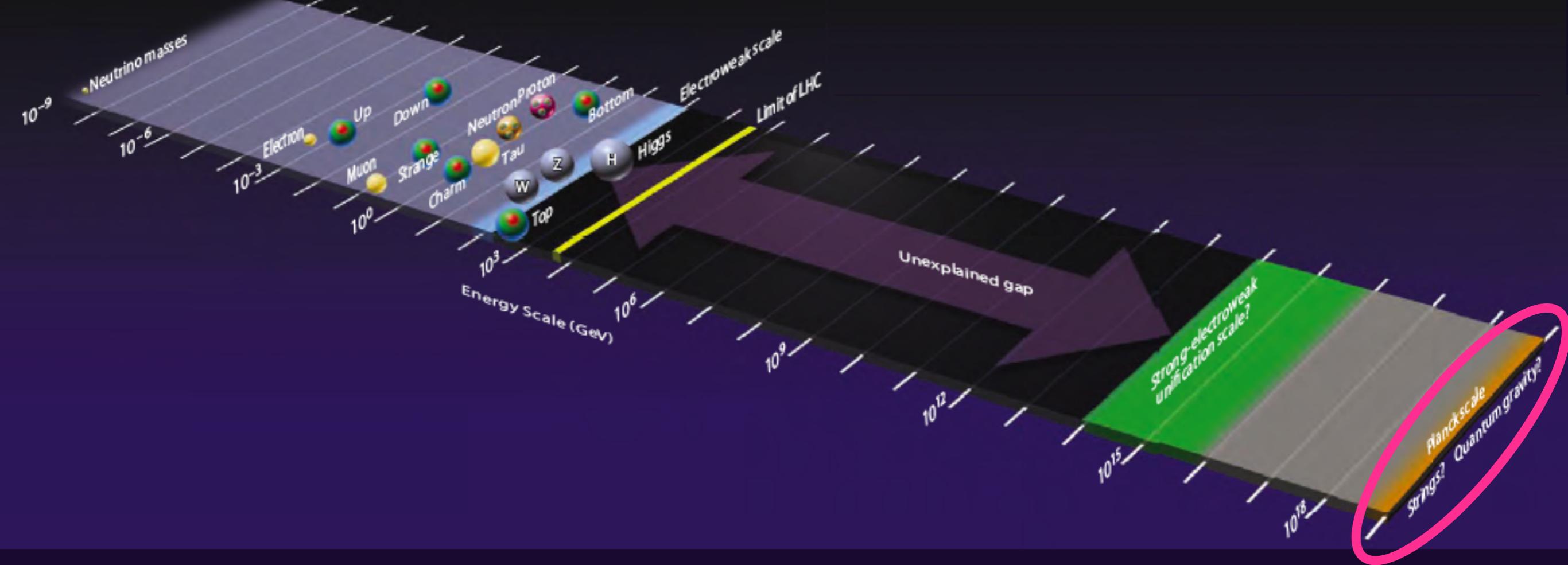
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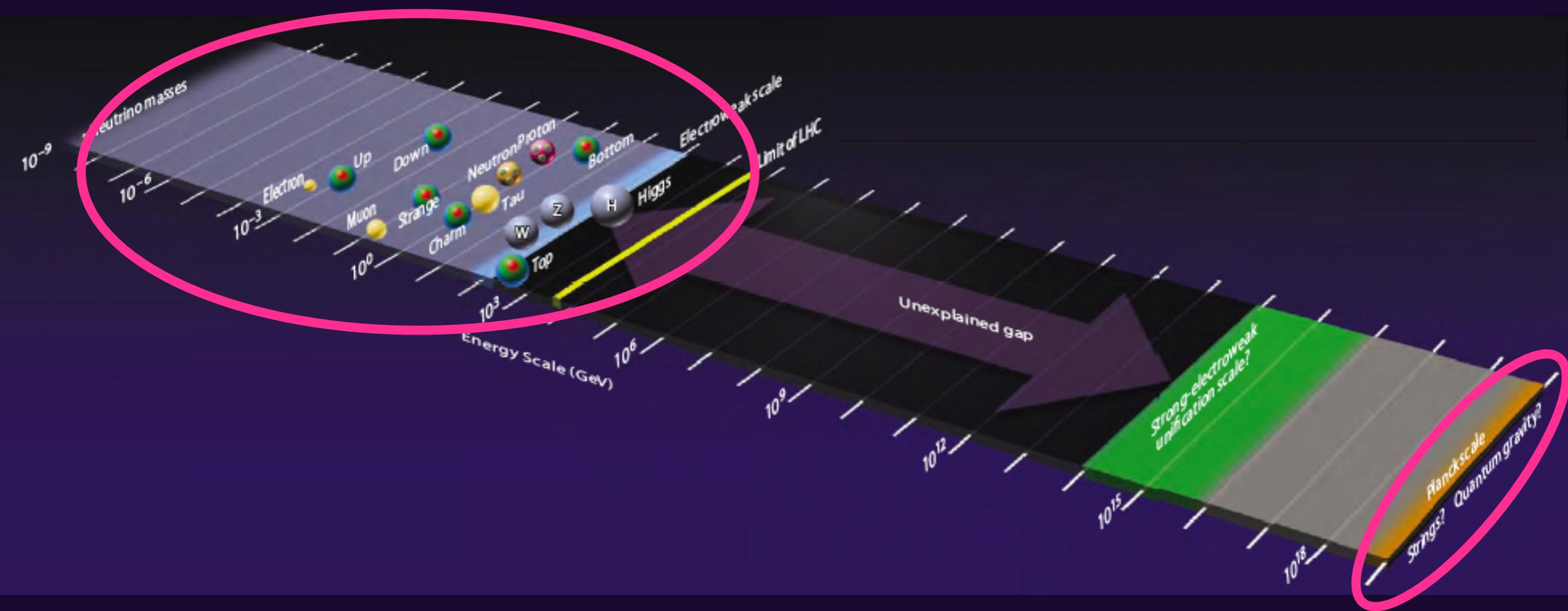
$$m_{eR} = R \times m_{eB}$$

Naturalness begs an explanation of the origin of mass.

No conflict with any small value of the electron mass







# Many Models

(?)MSSM

XLMS D

Technicolor

Branes

Unparticle

AdS/?

.....

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(?)MSSM

XLMS D

\$

Technicolor

Branes

.....

Unparticle

AdS/?

# Theory landscape

# Theory landscape

Gauge Group

# Theory landscape

Gauge Group

Matter

# Theory landscape

Gauge Group

Matter

SUSY

# Theory landscape

Gauge Group

Matter

SUSY

N=1

N=2

N=4

# Theory landscape

Gauge Group

Matter

SUSY

Non SUSY

N=1

N=2

N=4

# Theory landscape

Gauge Group

Matter

SUSY

Non SUSY

N=1

Fermions

N=2

Fermions + Bosons

N=4

Bosons

# Theory landscape

Gauge Group

Matter

SUSY

Non SUSY

N=1

Fermions

N=2

Fermions + Bosons

N=4

Bosons

Vector

# Theory landscape

Gauge Group

Matter

SUSY

Non SUSY

N=1

Fermions

N=2

Fermions + Bosons

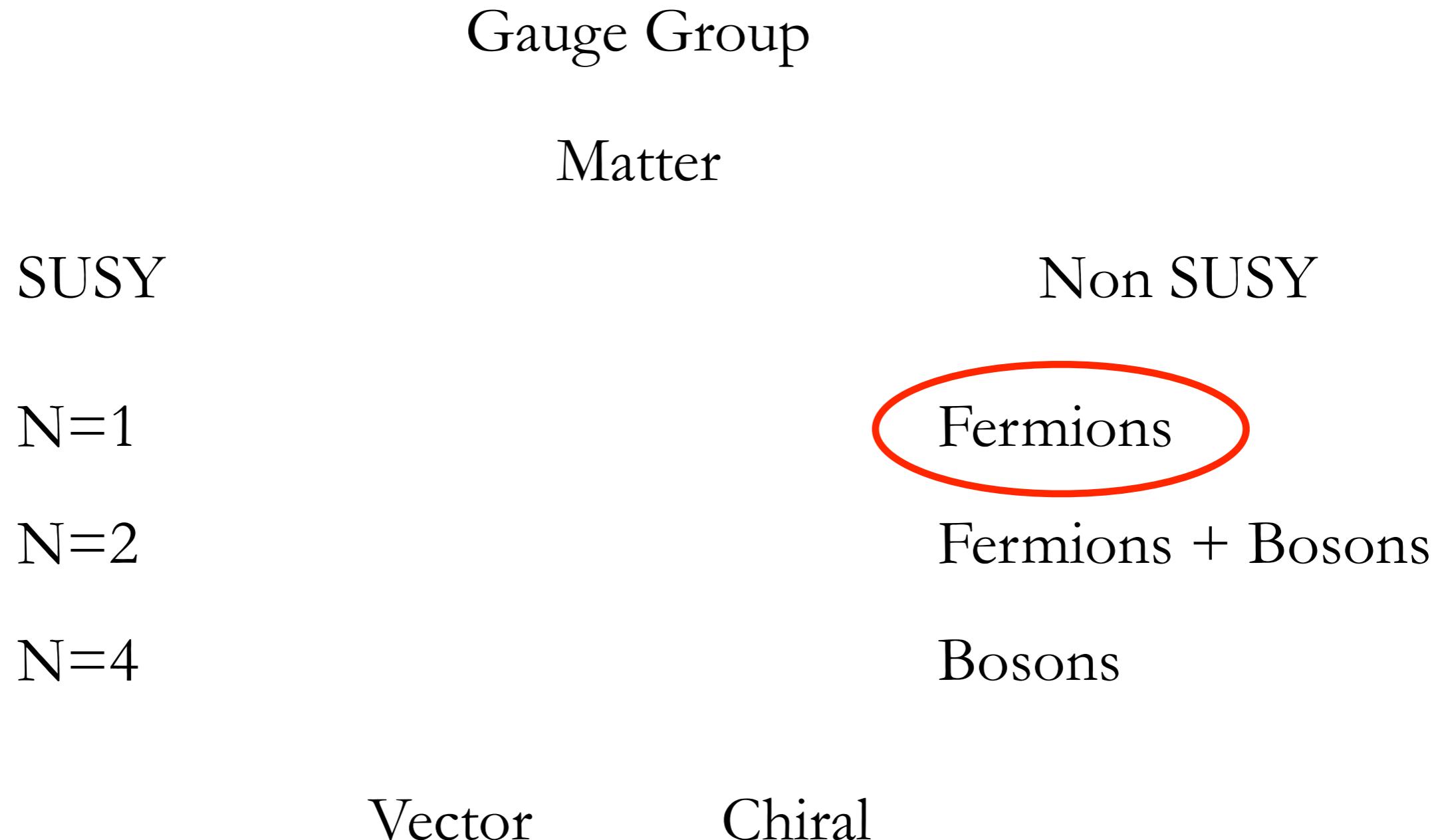
N=4

Bosons

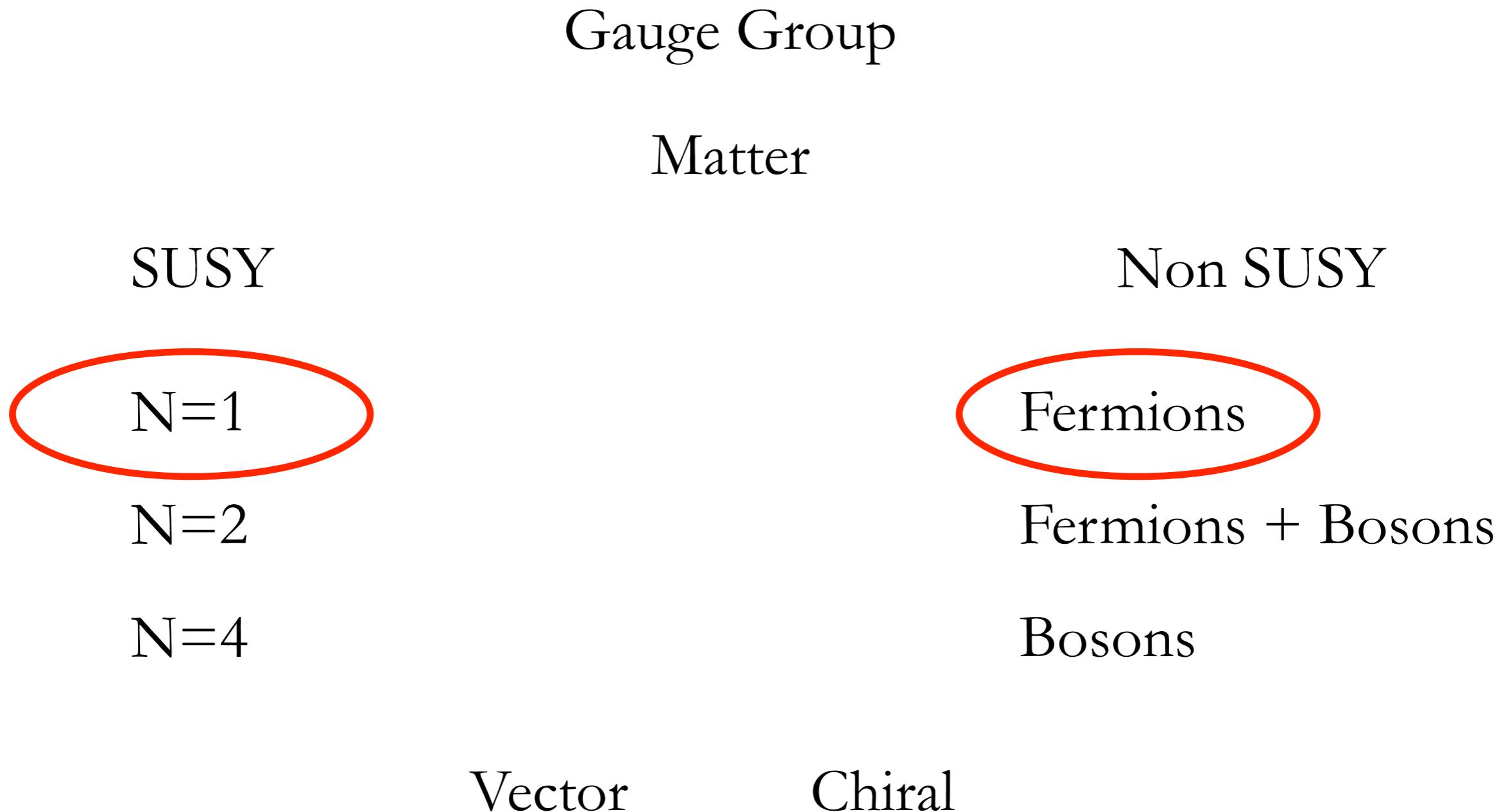
Vector

Chiral

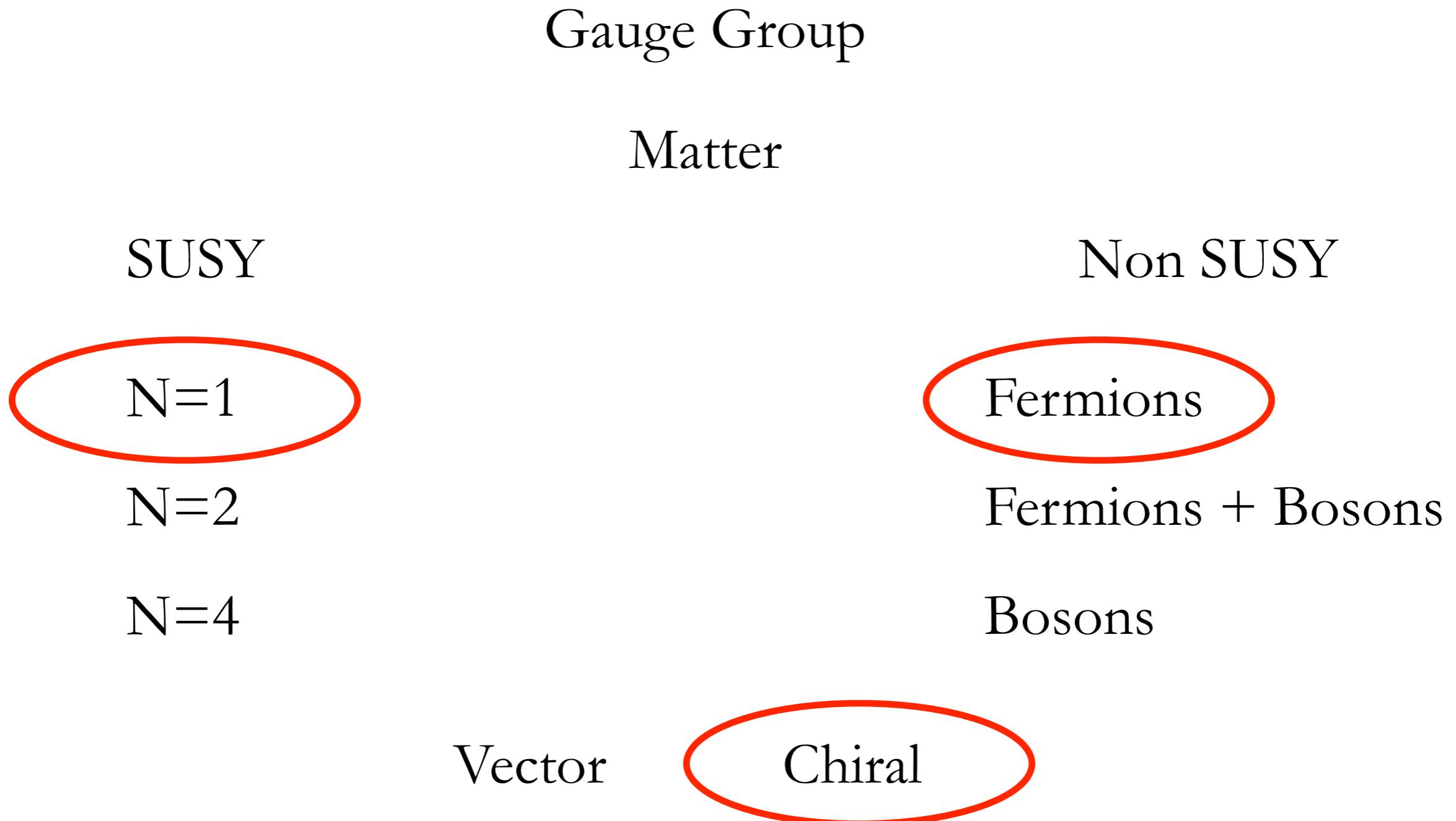
# Theory landscape



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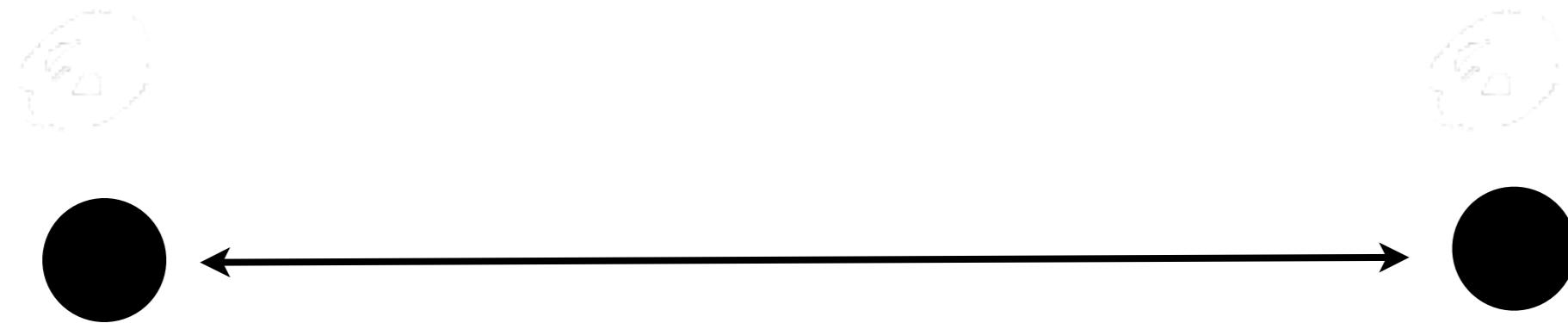
# Theory landscape





# Phases

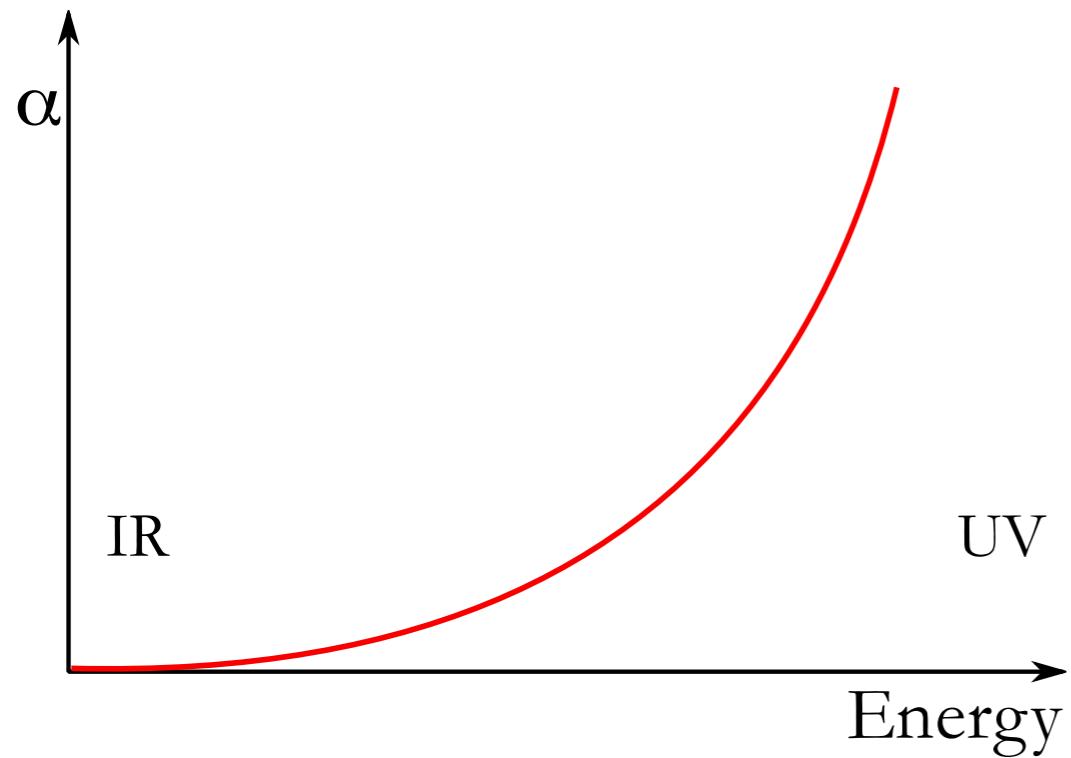
$$V(r)$$



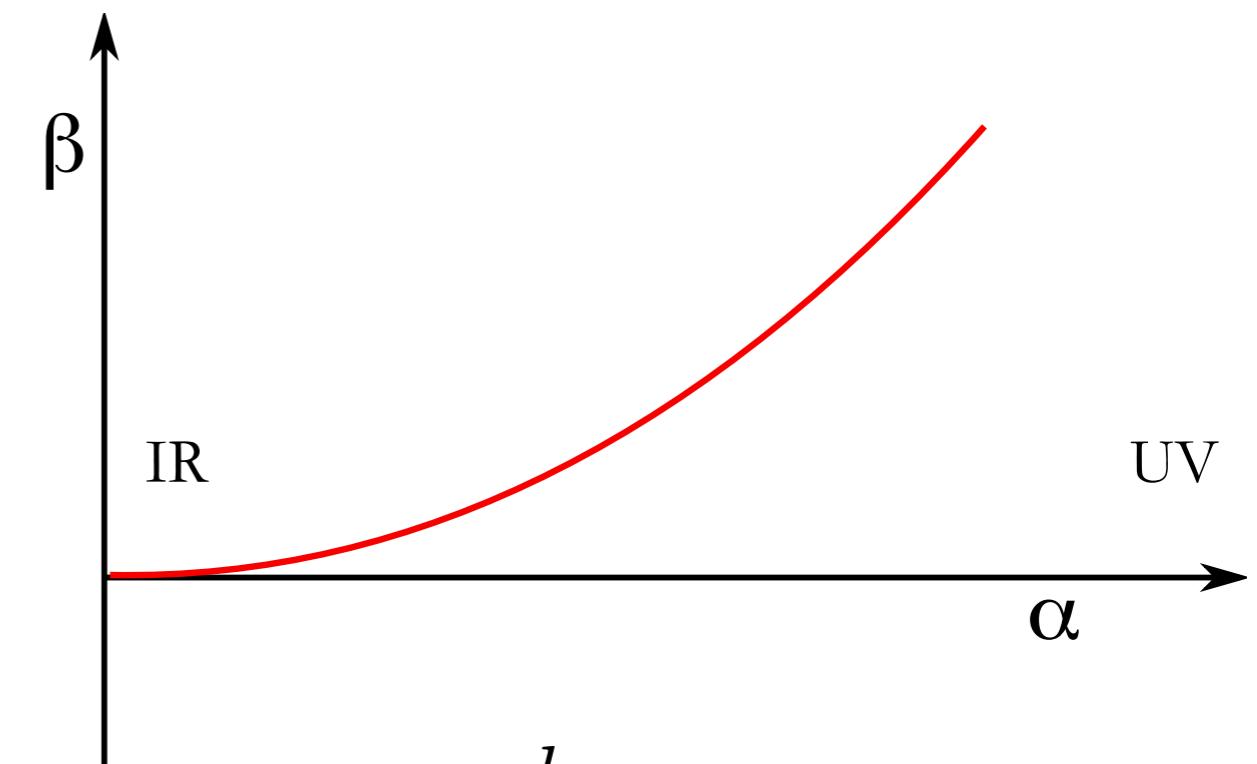
# Free Electric

$$V(r) \propto \frac{1}{r \log(r)}$$

$$\alpha(r) \rightarrow \frac{1}{\log(r)}$$



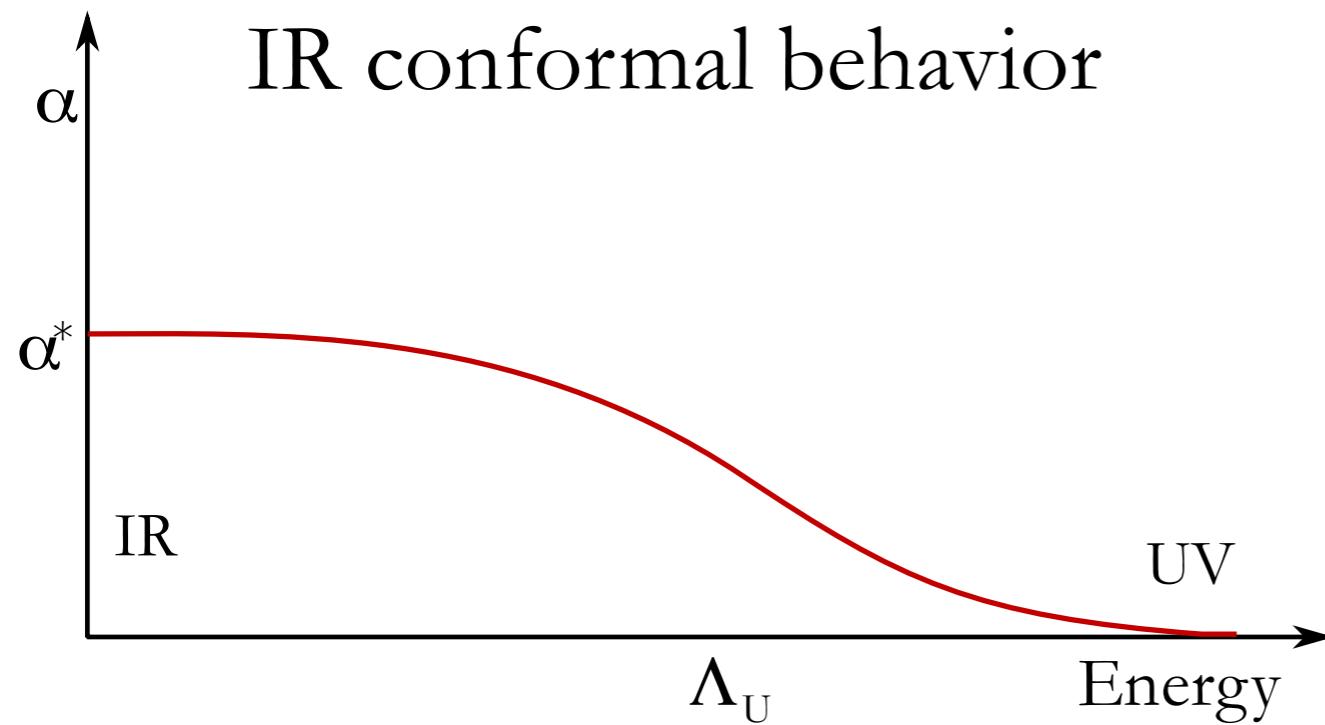
$$\alpha = \frac{g^2}{4\pi}$$



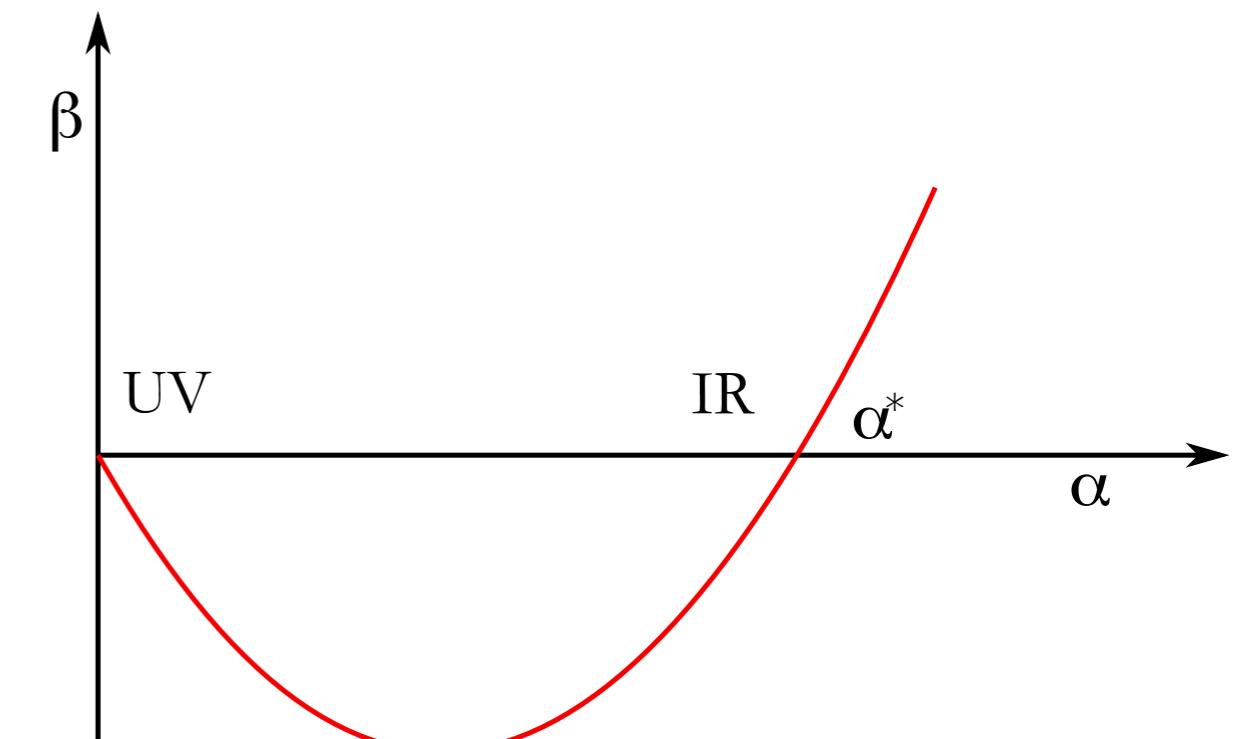
$$\beta = \frac{d g}{d \ln \mu}$$

# Coulomb

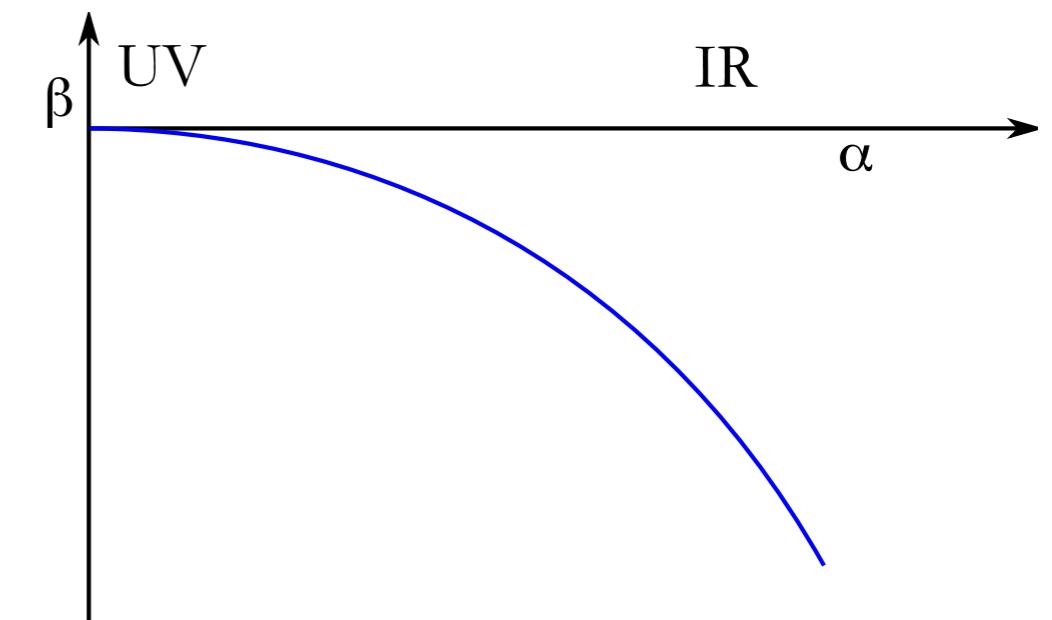
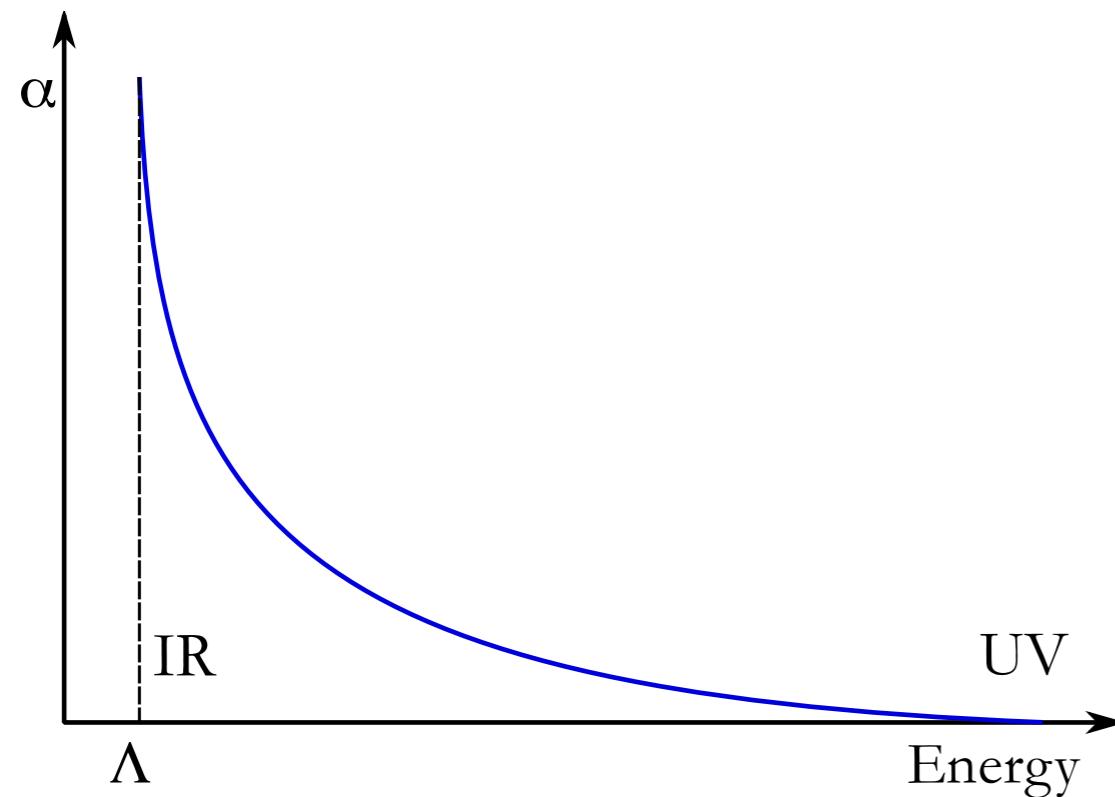
$$V(r) \propto \frac{1}{r}$$



IR Conformal Phase



# QCD - Like



$$V \propto \sigma r$$



# Knobs





# Knobs



Gauge Group, i.e. SU, SO, SP



# Knobs



Gauge Group, i.e. SU, SO, SP

Matter Representation



# Knobs



Gauge Group, i.e. SU, SO, SP

Matter Representation

# of Flavors per Representation

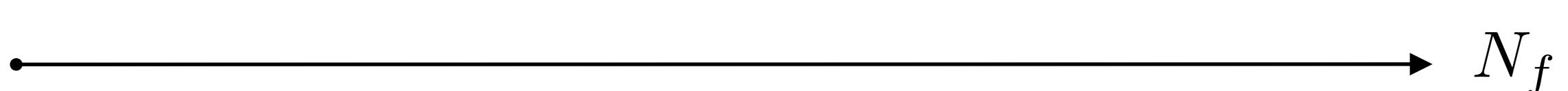
# Knobs



Gauge Group, i.e. SU, SO, SP

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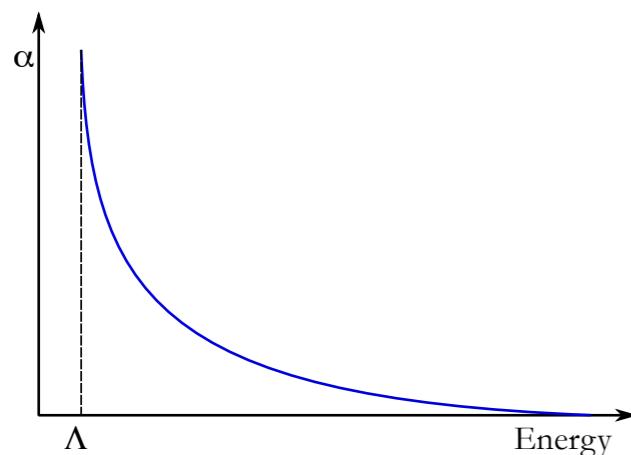
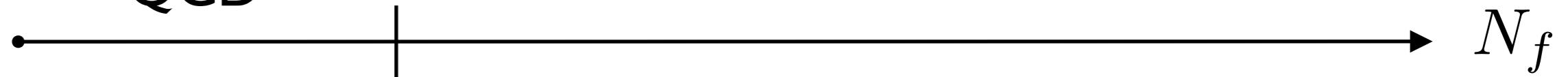


Gauge Group, i.e. SU, SO, SP

Matter Representation

# of Flavors per Representation

QCD



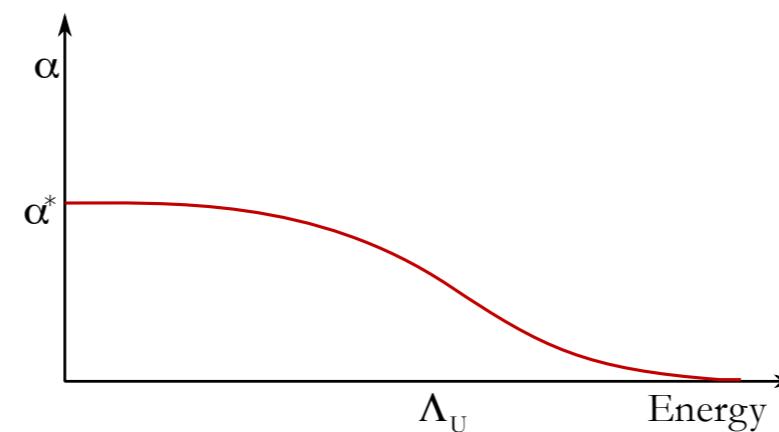
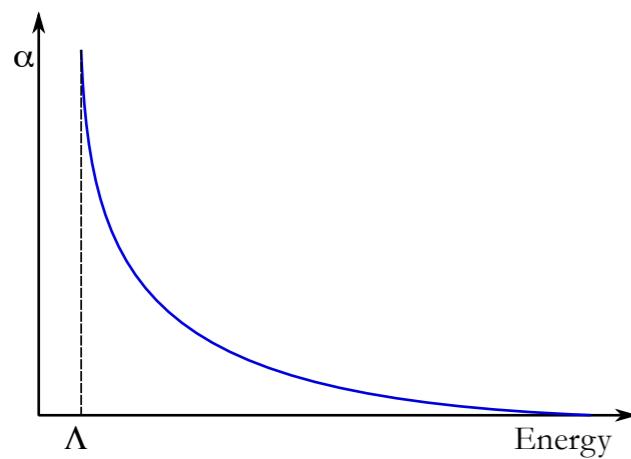


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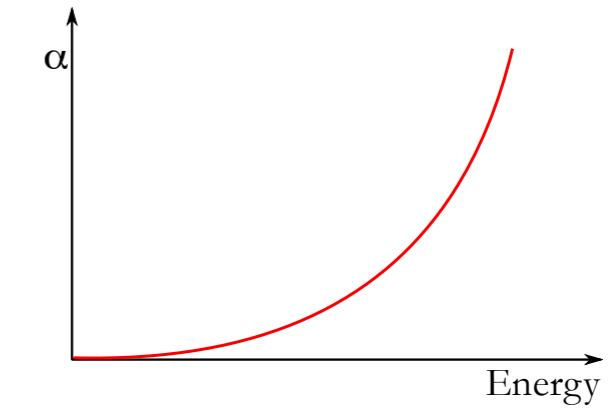
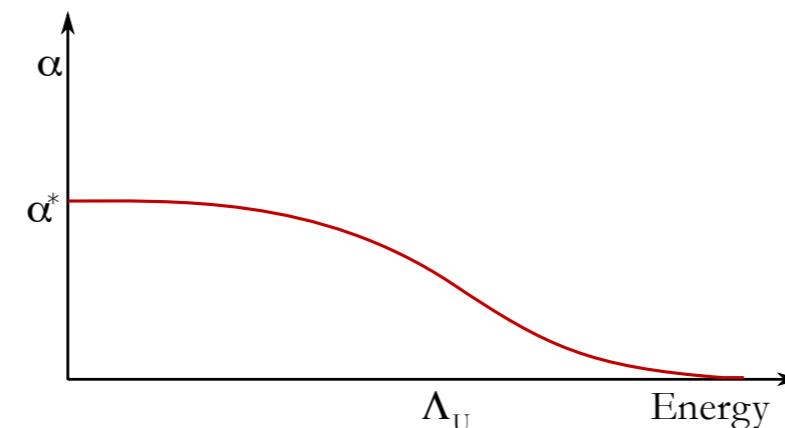
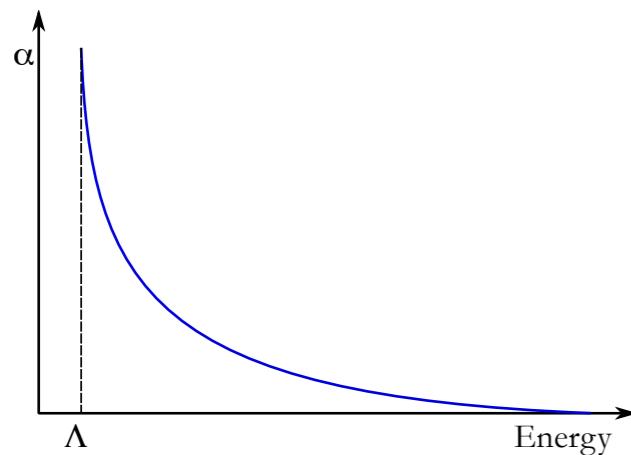
# Knobs



Gauge Group, i.e. SU, SO, SP

Matter Representation

# of Flavors per Representation



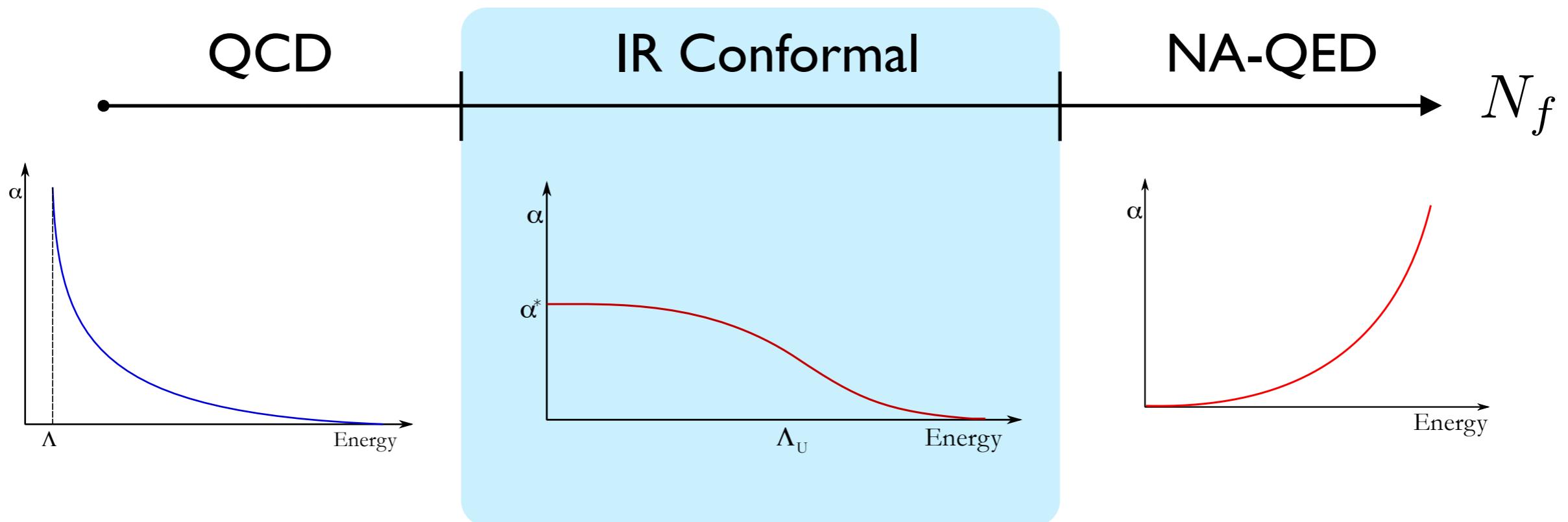
# Knobs



Gauge Group, i.e. SU, SO, SP

Matter Representation

# of Flavors per Representation



# Example



SU(N)

Adjoint Dirac Matter

Nf

Temperature = 0

Matter Density (i.e. Chemical Potential(s)) = 0

# Example



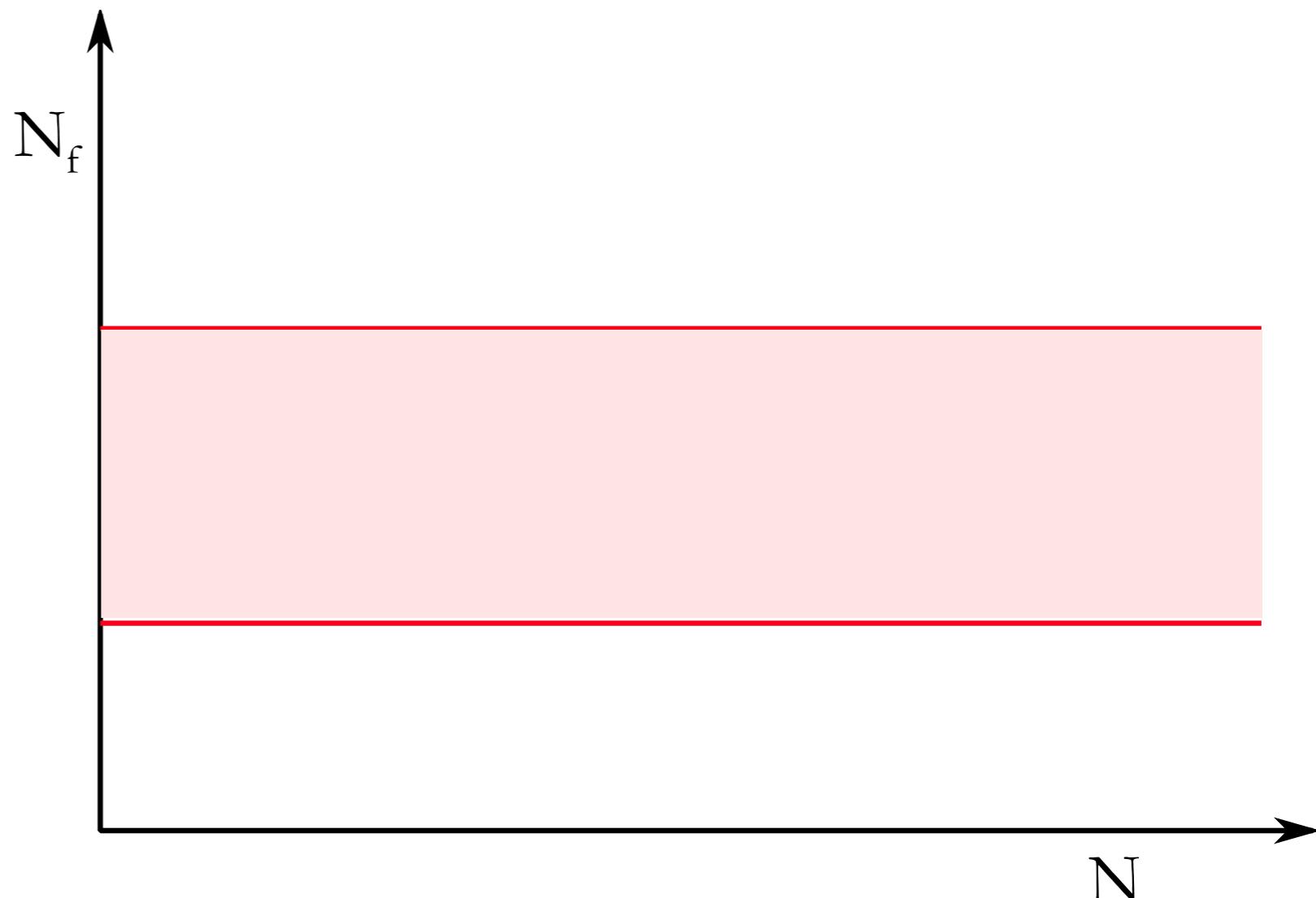
SU(N)

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# Example



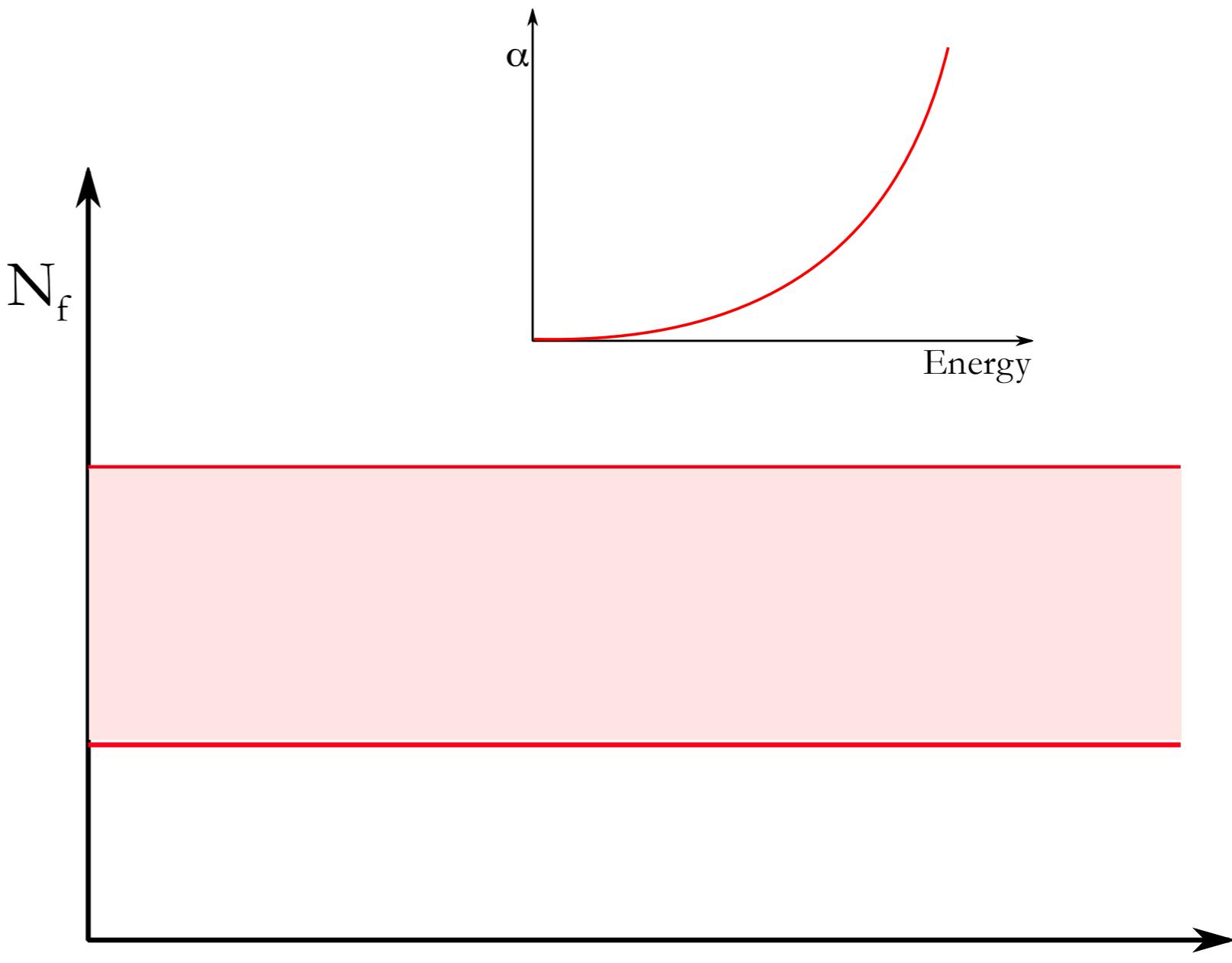
SU(N)

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# Example



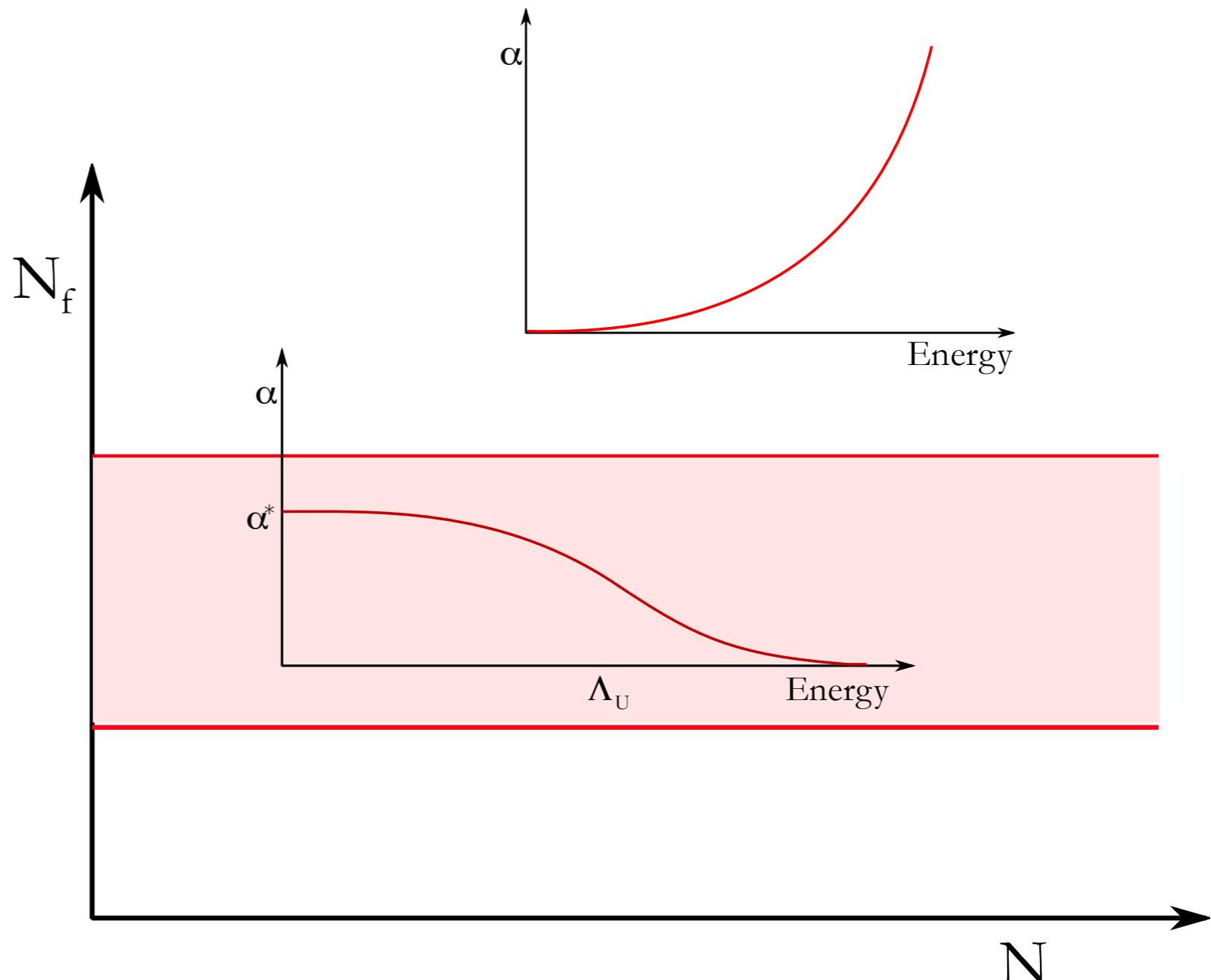
SU(N)

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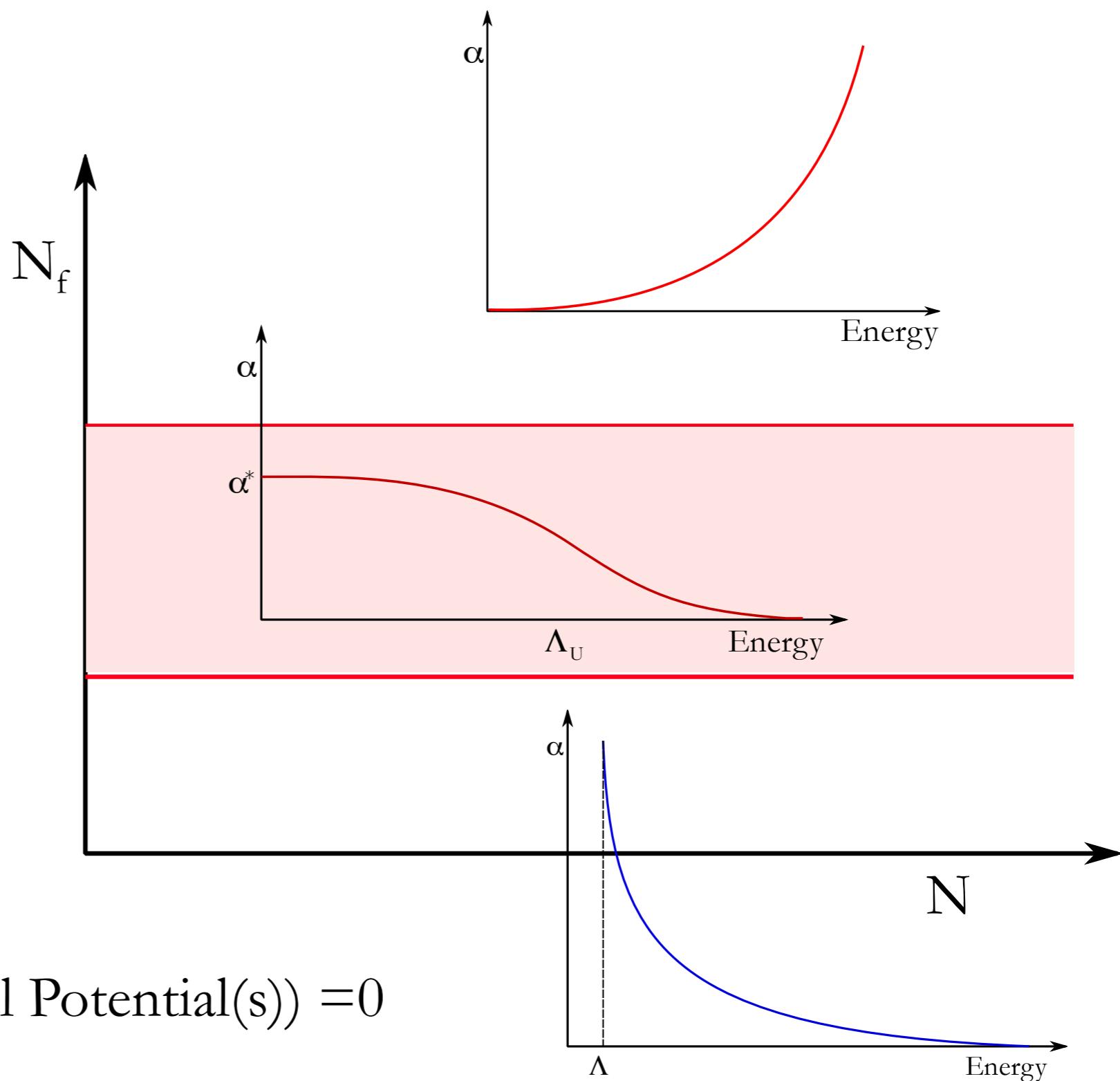
$SU(N)$

Adjoint Dirac Matter

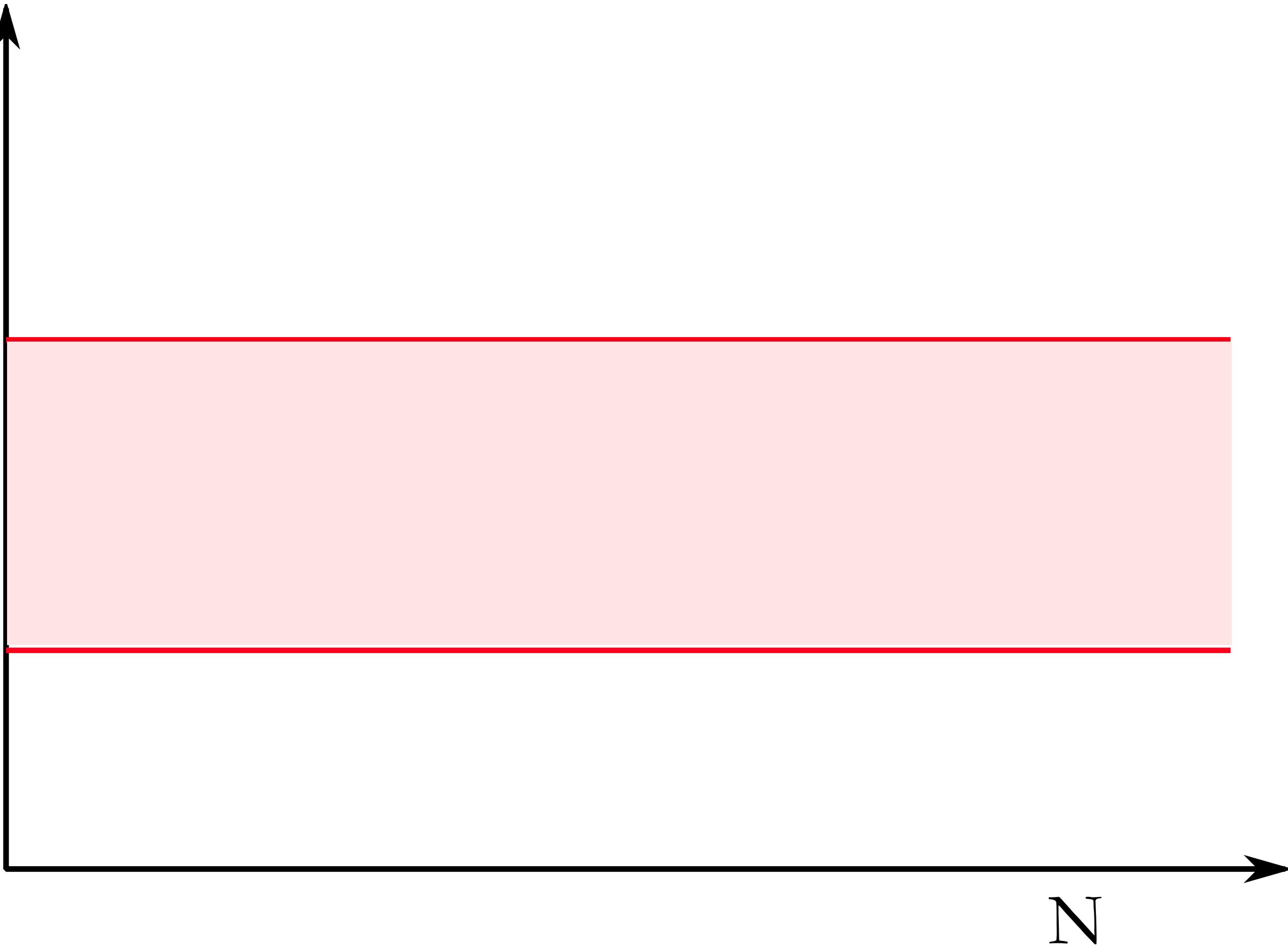
$N_f$

Temperature = 0

Matter Density (i.e. Chemical Potential(s)) = 0



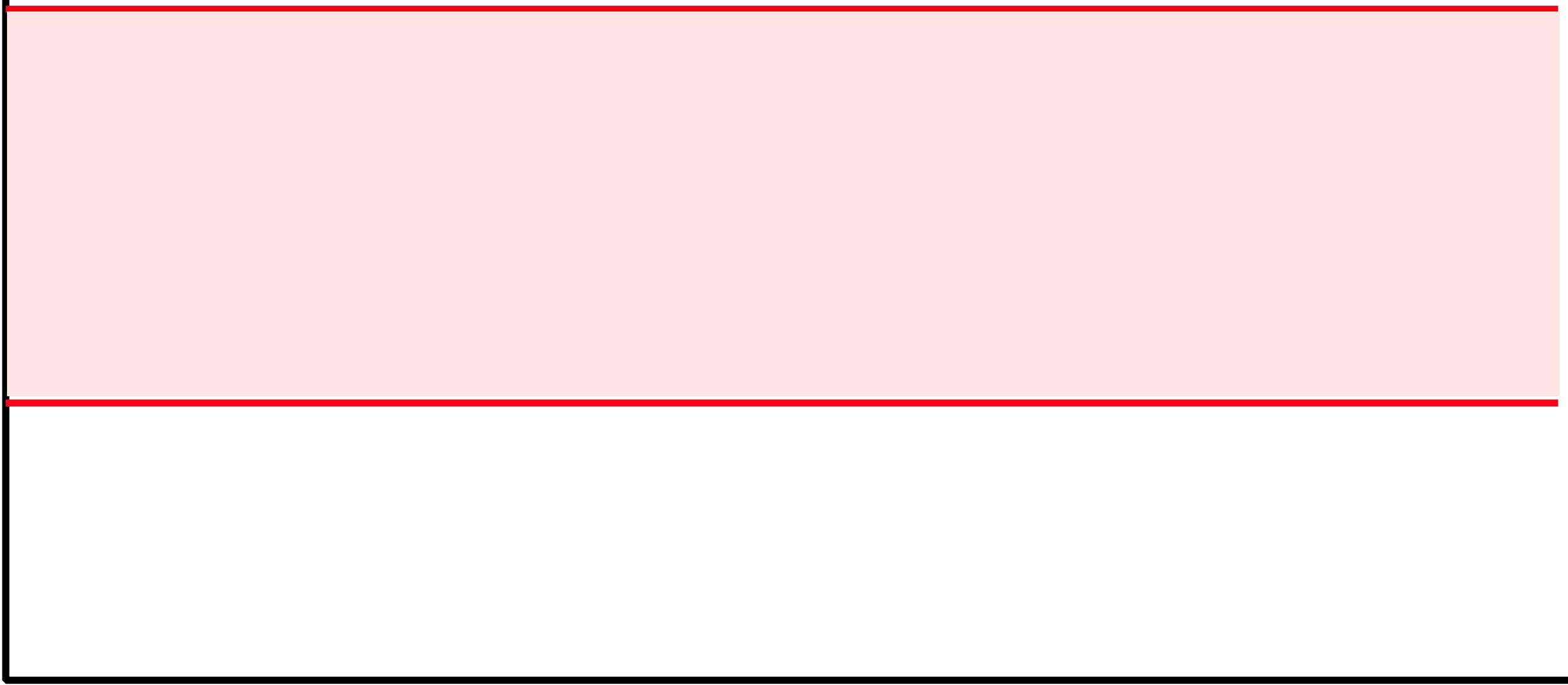
$N_f$



$N$

$N_f$

$$\beta_0 = 0$$



$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5 + \mathcal{O}(g^7)$$

$N$

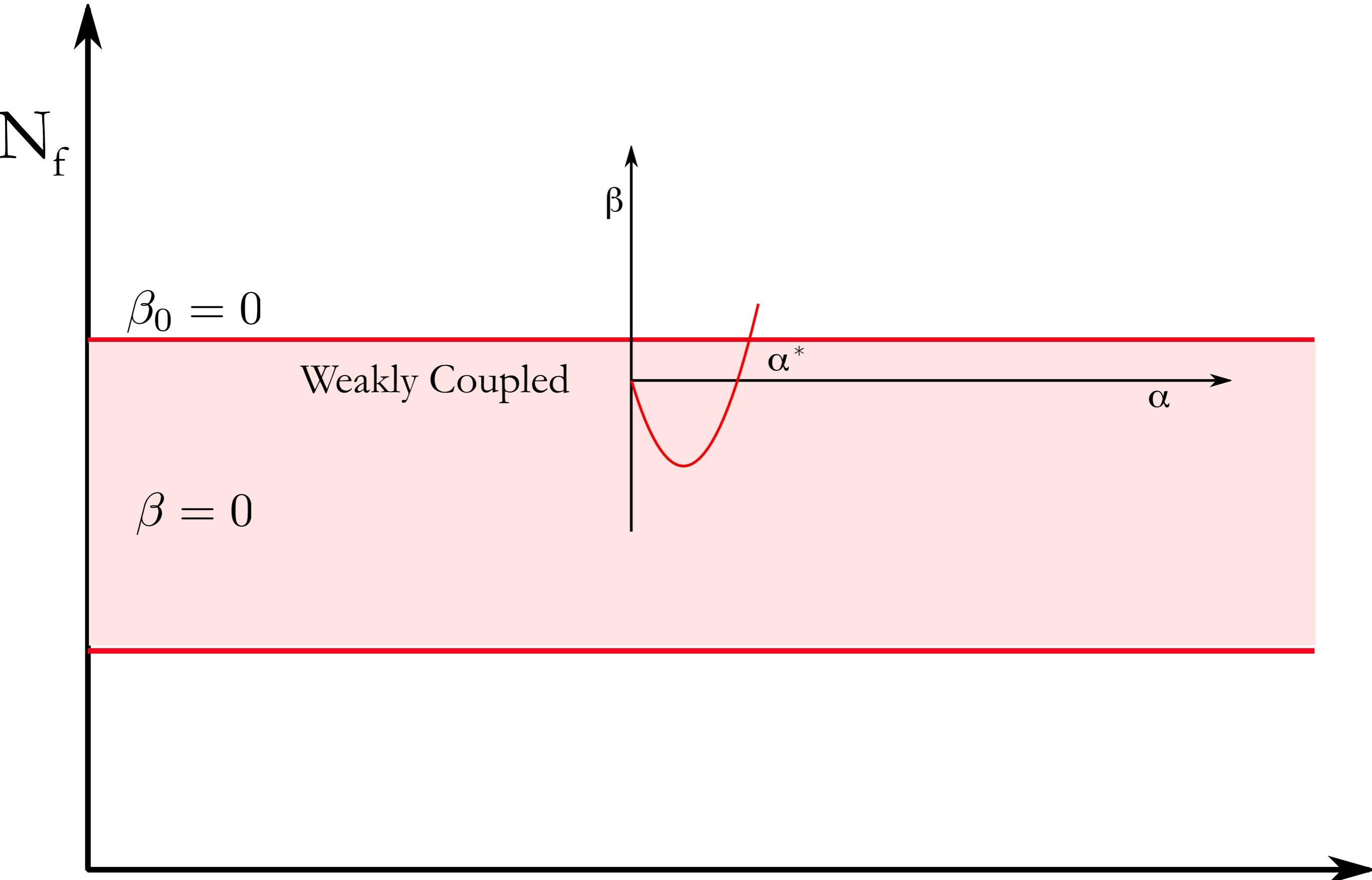
$N_f$

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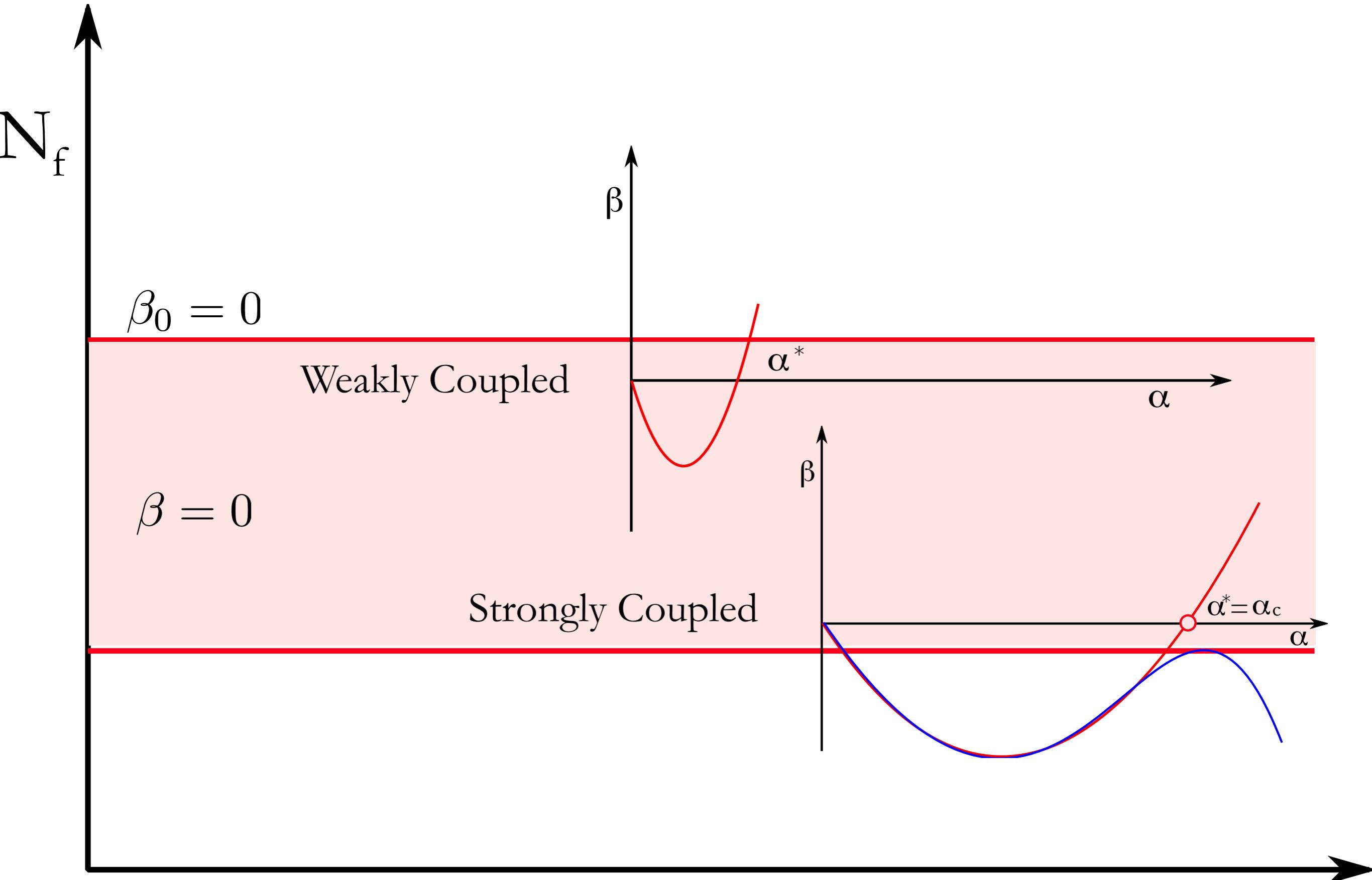
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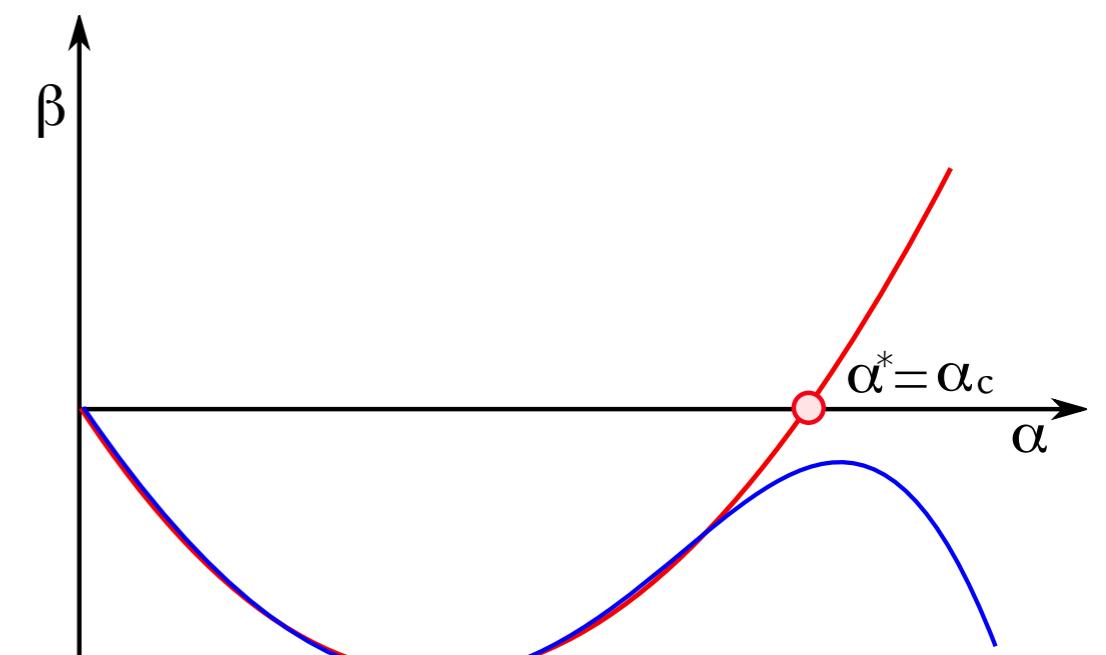
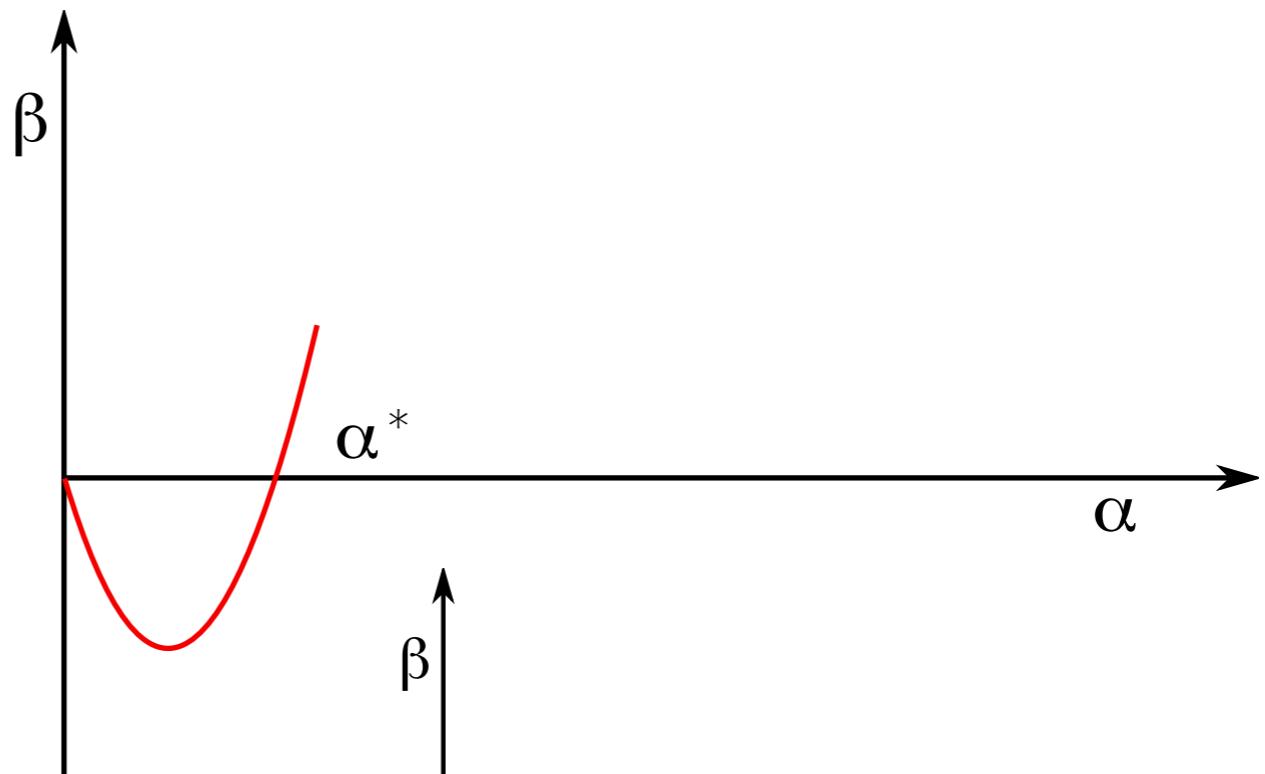
$N$

$$\beta_0 = 0$$

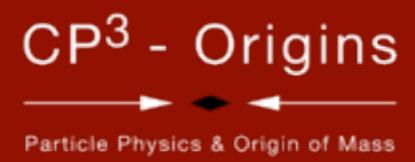
Weakly Coupled

$$\beta = 0$$

Strongly Coupled



$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5 + \mathcal{O}(g^7)$$



# Analytic Tools

# SUSY

The all orders beta function of NSVZ

Unitarity Bounds for Conformal Theories

Non-Renormalization of Superpotentials

‘t Hoofts Anomaly Matching Conditions

a-Maximization

Instanton Calculus

....

Seiberg, Intriligator, Novikov, Shifman, Vainshtein, Zakharov.

# Non SUSY

Different approaches

Schwinger - Dyson

Instanton Inspired Calculus

Thermal degrees of freedom count

Exact Renormalization Methods

Certain Topological excitations

Appelquist, Bowick, Chivukula, Cohen, Eichten, Gies, Hill, Holdom, Karabali,  
Jaeckel, Fisher, Lane, Litim, Mahanta, Miransky, Pawłowski, Percacci, Poppitz,  
Shrock, Simmons, Terning, Unsal, Wijewardhana, Yamawaki

# ..continued

Also:

The all orders beta function conjecture\* (RS)

Chiral Gauge Theories beta function (F.S.)

Unitarity of the Operators for Conformal Theories

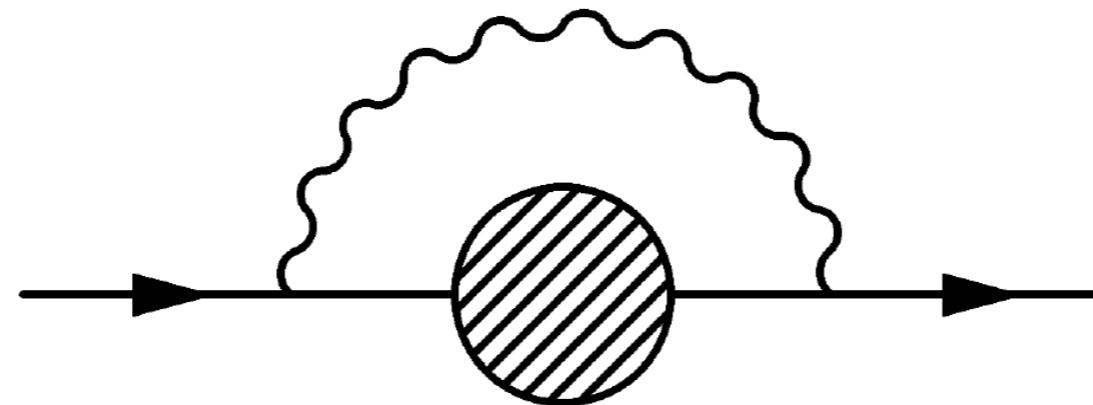
Gauge Dualities (F.S.)

First Principle Lattice Computations

\*Variations on the beta function (AT)

RS - Ryttov, F.S. 07  
F.S. 09  
AT - Antipin, Tuominen 09

# Schwinger - Dyson



The full nonperturbative fermion propagator reads:

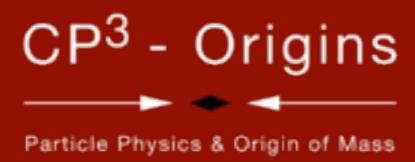
$$iS^{-1}(p) = Z(p) (\not{p} - \Sigma(p))$$

The Euclidianized gap equation in Landau gauge is:

$$\Sigma(p) = 3C_2(R) \int \frac{d^4 k}{(2\pi)^4} \frac{\alpha((k-p)^2)}{(k-p)^2} \frac{\Sigma(k^2)}{Z(k^2)k^2 + \cancel{\Sigma^2(k^2)}}$$

$$Z(k^2) = 1$$

$$\beta(\alpha) \simeq 0 \quad \alpha(\mu) \approx \alpha_c \quad \alpha_c = \frac{\pi}{3C_2(r)}$$



# In practice



$$\beta(g)=-\frac{\beta_0}{(4\pi)^2}g^3-\frac{\beta_1}{(4\pi)^4}g^5$$

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$$\beta(g)=-\frac{\beta_0}{(4\pi)^2}g^3-\frac{\beta_1}{(4\pi)^4}g^5$$

$$\beta=0 \hspace{1cm} \longrightarrow \hspace{1cm} \frac{\alpha^*}{4\pi}=-\frac{\beta_0}{\beta_1}$$

$$\alpha_c=\frac{\pi}{3C_2(r)}$$

$$\beta(g)=-\frac{\beta_0}{(4\pi)^2}g^3-\frac{\beta_1}{(4\pi)^4}g^5$$

$$\beta=0 \hspace{1.5cm} \longrightarrow \hspace{1.5cm} \frac{\alpha^*}{4\pi}=-\frac{\beta_0}{\beta_1}$$

$$\alpha_c=\frac{\pi}{3C_2(r)} \hspace{1cm} \longleftrightarrow$$

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$$\alpha^*\leq \alpha_c$$

$SU(N)$  Dirac Fermions in representation “r”

$$N_{f \text{ Ladder}}^c = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)} \frac{C_2(G)}{T(r)}$$

$$C_2(G) = C_2(Adj) = N$$

# Back to the example



SU(N)    Adjoint Dirac Matter

# Back to the example



SU(N)    Adjoint Dirac Matter

$$C_2(r) = C_2(G) = T(G) = N$$

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$$\beta_0 = \frac{11}{3}N - \frac{4}{3}NN_f = 0 , \quad \rightarrow \quad N_f^{\text{Asymp}} = \frac{11}{4} = 2.75$$

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$$N_f^c_{\text{Ladder}} = \frac{17 + 66}{10 + 30} = 2.075$$

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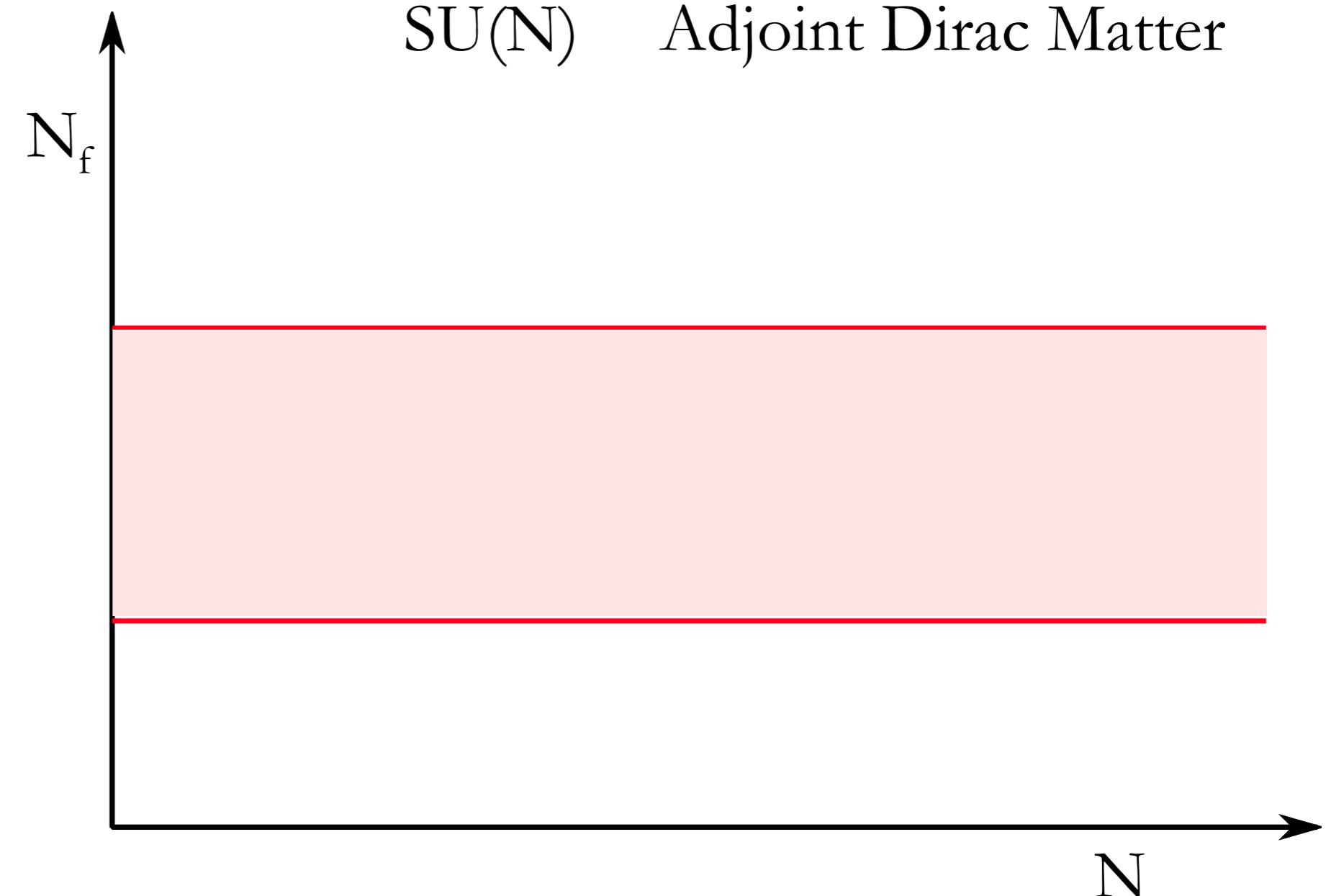
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# Back to the example



$$N_f^{\text{Asymp}} = 2.75$$

$$N_f^c_{\text{Ladder}} = 2.075$$



# Another Example

SU(N) 2-index Symmetric Rep

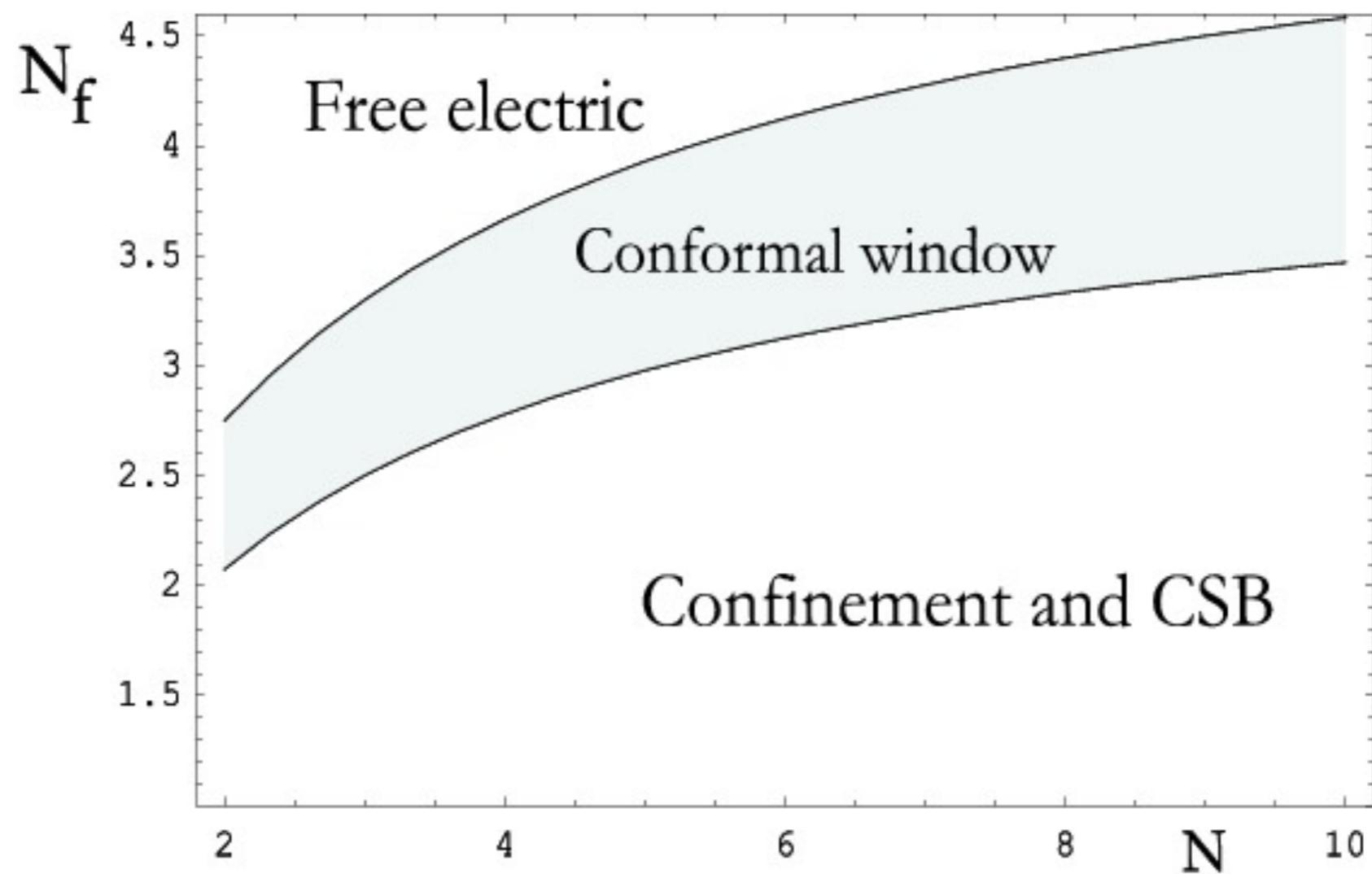
	$SU(N)$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	$U_A(1)$
$Q_{\{ij\}}$	$\boxed{\square}$	$\square$	1	1	1
$\tilde{Q}^{\{ij\}}$	$\overline{\boxed{\square}}$	1	$\overline{\square}$	-1	1
$G_\mu$	Adj	0	0	0	0



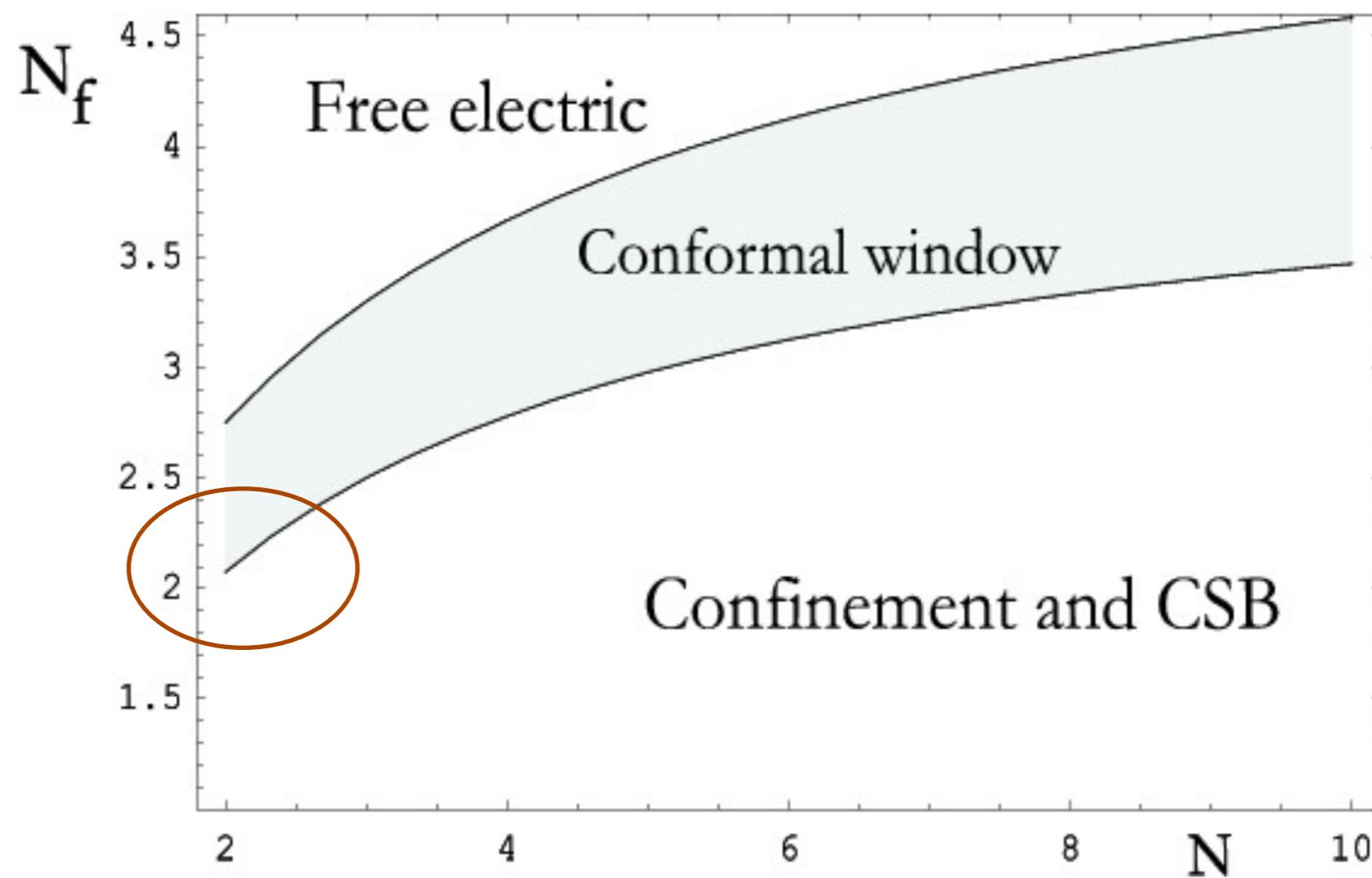
Here  $Q$  and  $\tilde{Q}$  are Weyl fermions.

The **A-type** is obtained by substituting  $\boxed{\square}$  with  $\boxed{\square}$ .

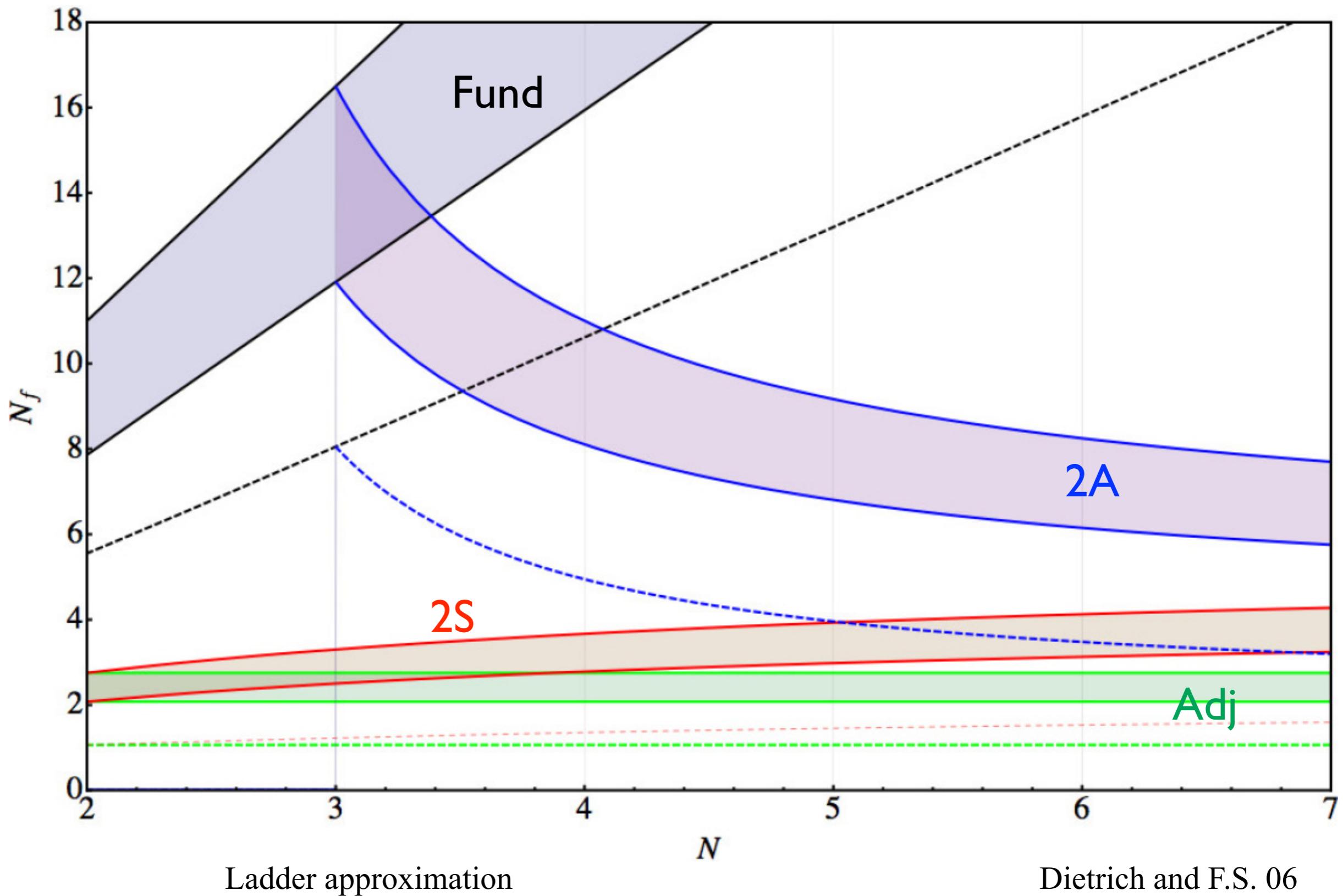
# Diagram for Symm. Rep.

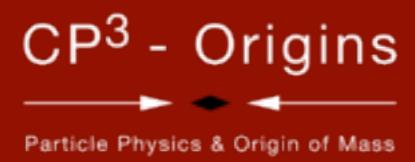


# Diagram for Symm. Rep.



# SD - Phase Diagram





# SUSY

# SUSY $\beta$ function

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$$\beta(g) = -\frac{g^3}{16\pi^2} \frac{\beta_0 + 2T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)}$$

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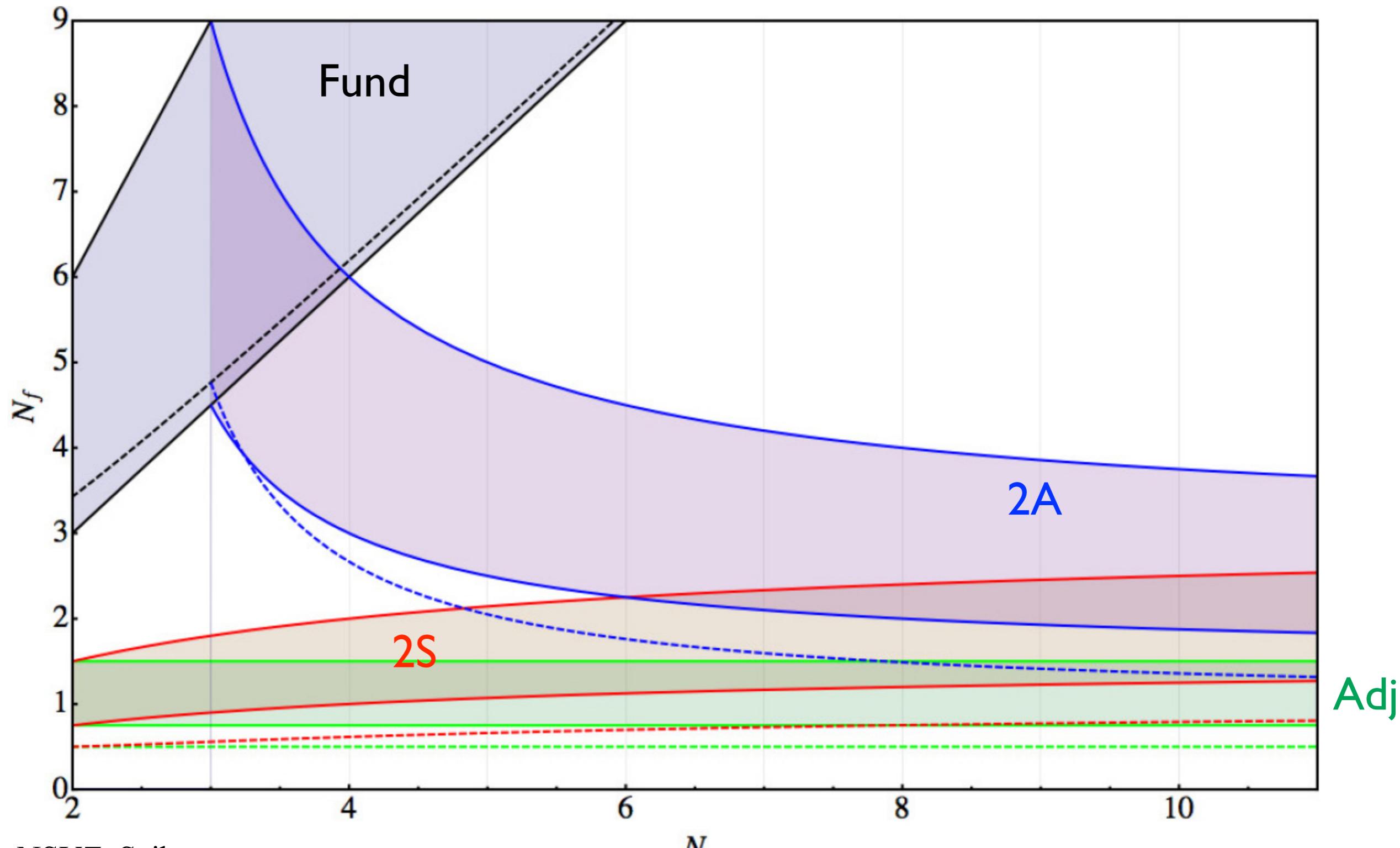
$$\gamma(g^2) = -\frac{g^2}{4\pi^2}C_2(r) + O(g^4)$$

$$\gamma(g^2) = -d \ln Z(\mu) / d \ln \mu$$

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$

NSVZ

# SUSY Phase Diagram



NSVZ, Seiberg  
 Intriligator-Seiberg

Ryttov and F.S. 07

# $\beta$ - function

Ryttov and F.S. 07

# $\beta$ function conjecture

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$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)(1 + \frac{2\beta'_0}{\beta_0})}$$

# $\beta$ function conjecture

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)(1 + \frac{2\beta'_0}{\beta_0})}$$

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$$\beta_0 = \frac{11}{3}C_2(G) - \frac{4}{3}T(r)N_f$$

$$\beta'_0 = C_2(G) - T(r)N_f$$

# Recovering SYM

$$SU(N) \quad N_f = \frac{1}{2} \quad \text{Adjoint Matter}$$

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$$\beta_{SYM}(g) = -\frac{g^3}{(4\pi)^2} \frac{3N}{1 - \frac{g^2}{8\pi^2} N}$$

# Recovering SYM

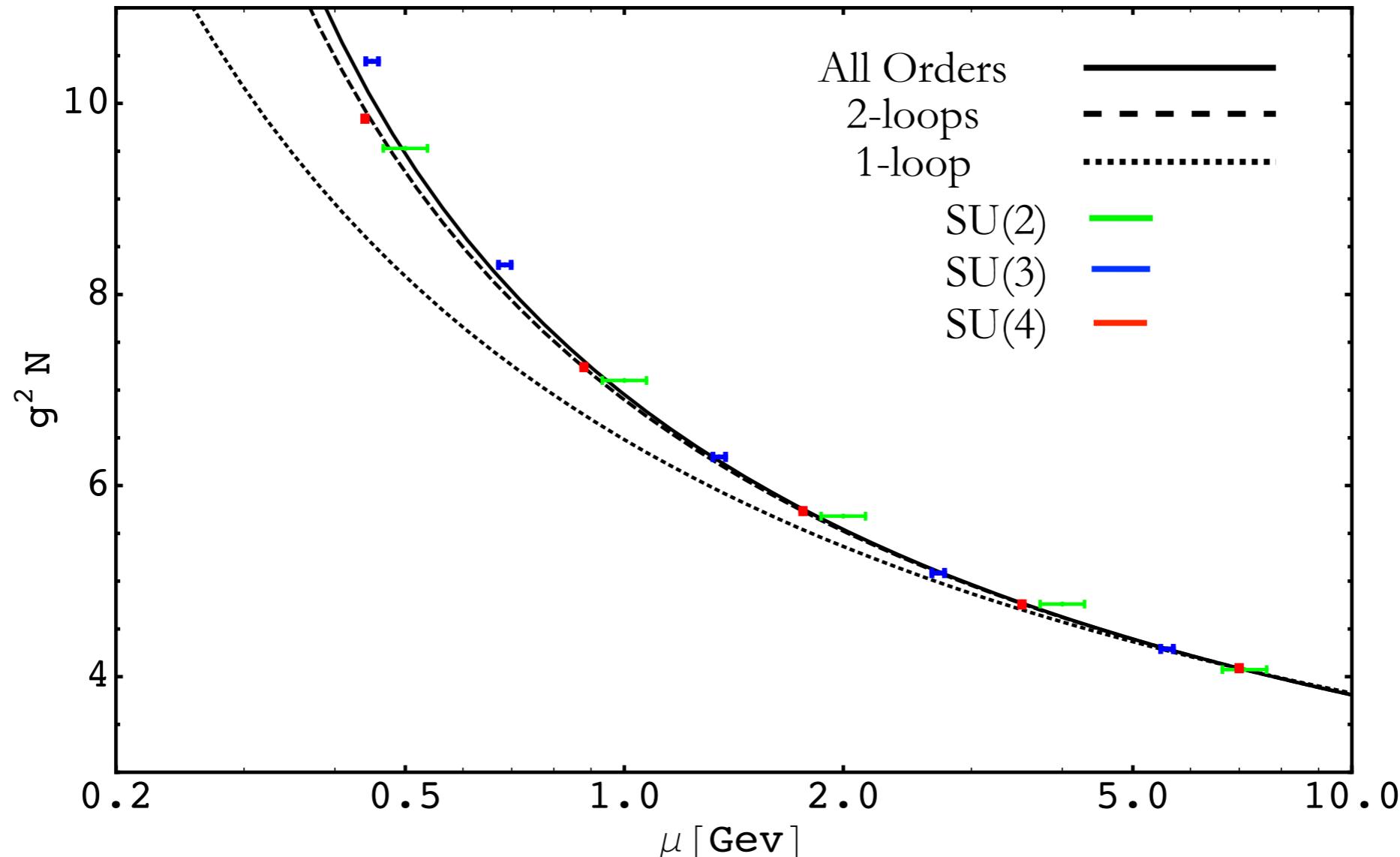
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$$\gamma_{\text{Adj}} = \frac{g^2}{8\pi^2} \frac{3N}{1 - \frac{g^2}{8\pi^2} N}$$

# Confronting with YM



- Luscher, Sommer, Wolff, Weisz, 92      SU(2)
- Luscher, Sommer, Weisz, Wolff 94      SU(3)
- Lucini and Moraitsis 07      SU(4)

# Bounding the window

$$\beta = 0 \quad \longrightarrow \quad \gamma = \frac{11C_2(G) - 4T(r)N_f}{2T(r)N_f}$$

Unitarity of the Conformal Operators demands:

$$\gamma \leq 2$$

# Confronting with YM

SU(N)    Adjoint Dirac Matter



$$\gamma \leq 2 , \quad \Rightarrow \quad N_f^c \geq \frac{11}{8}$$

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$$N_f^c_{\text{Ladder}} = 2.075$$

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SU(N)    Adjoint Dirac Matter



$$\gamma \leq 2 , \quad \Rightarrow \quad N_f^c \geq \frac{11}{8}$$

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Conformal in the IR

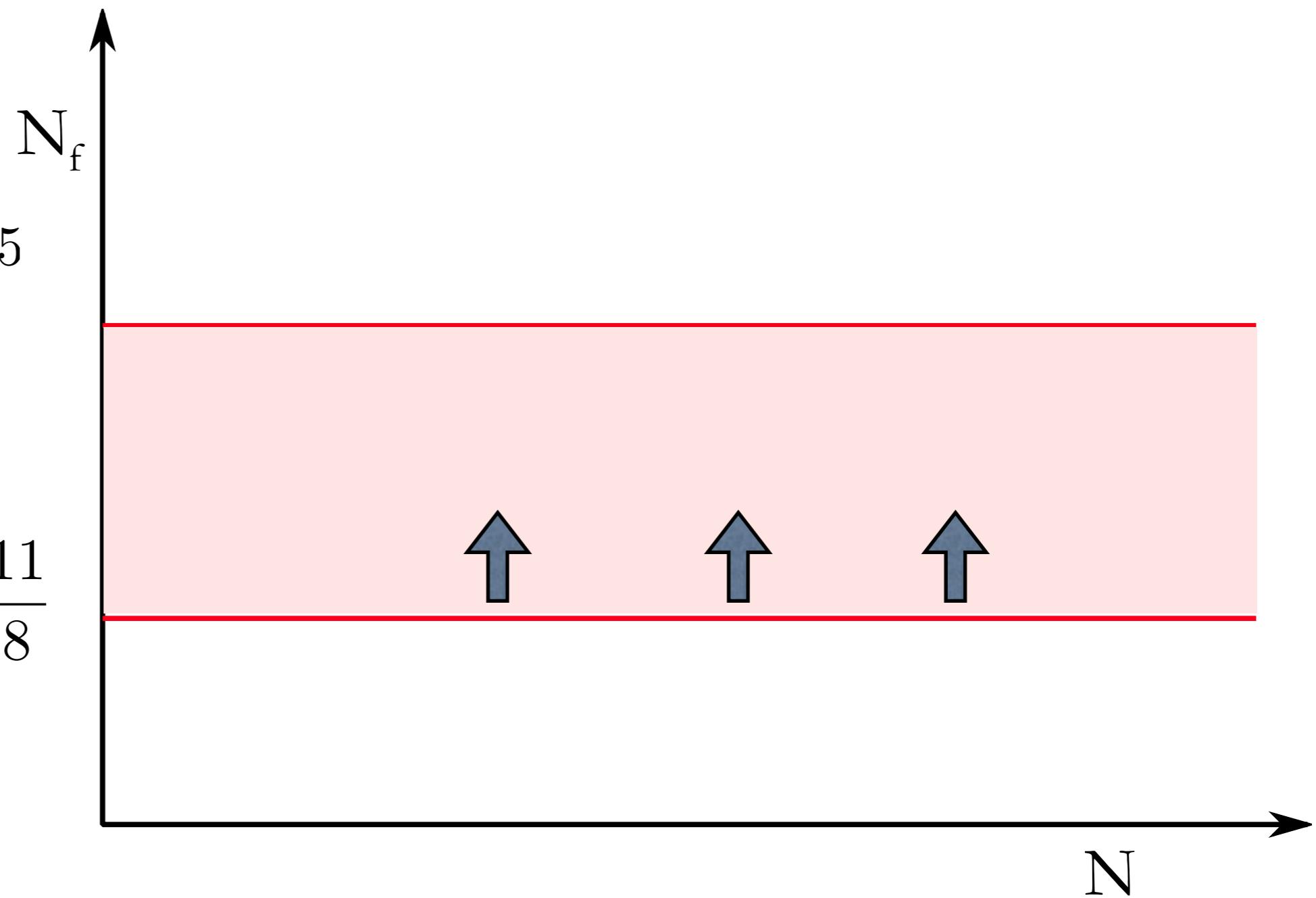
$N_f^c_{\text{Ladder}} = 2.075$

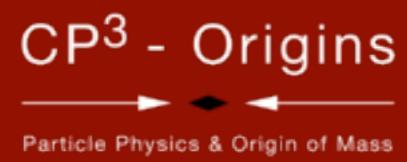
# Back to the example



SU(N)    Adjoint Dirac Matter

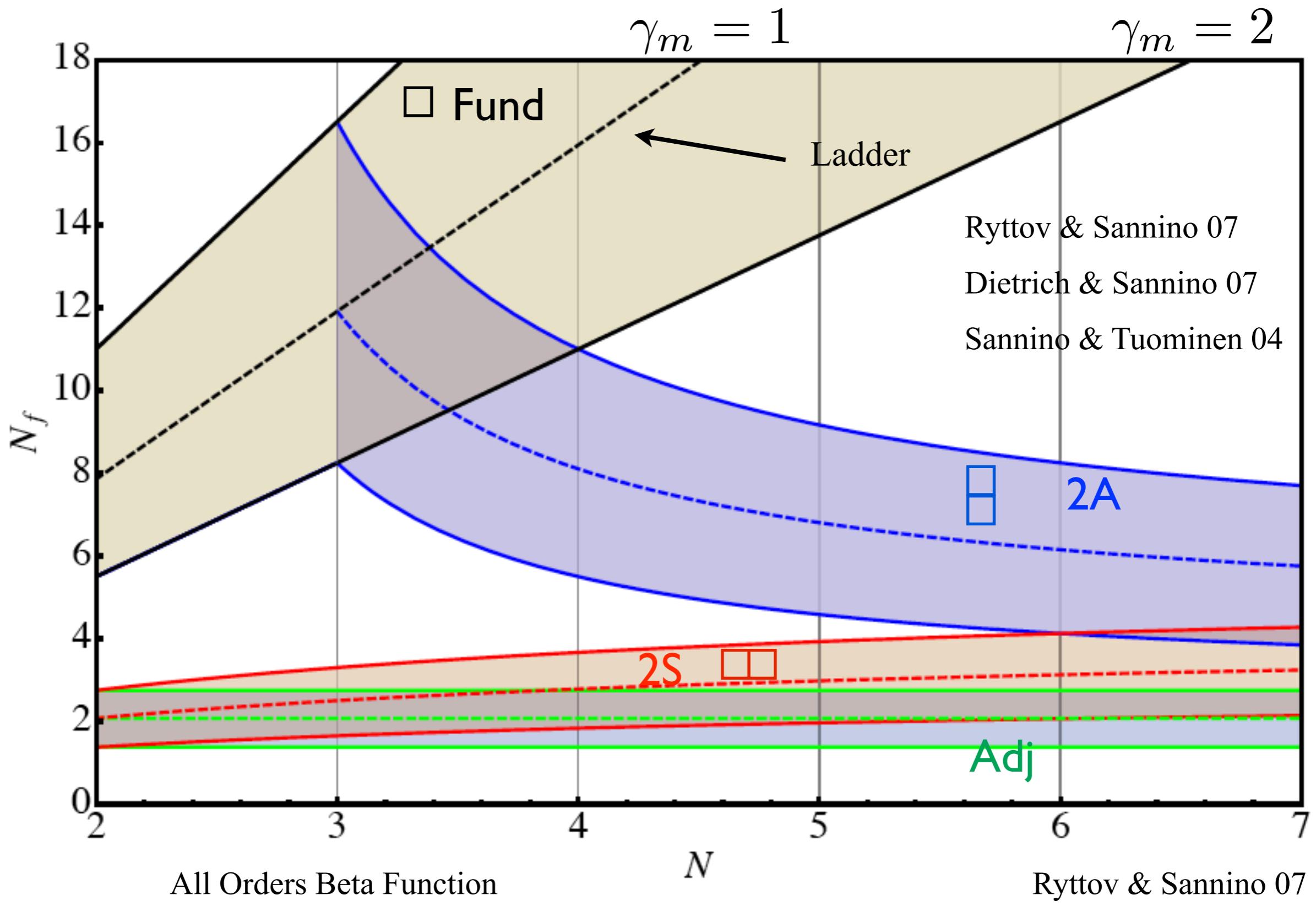
$$N_f^{\text{Asymp}} = 2.75$$
$$\gamma \leq 2 \rightarrow N_f^c \geq \frac{11}{8}$$



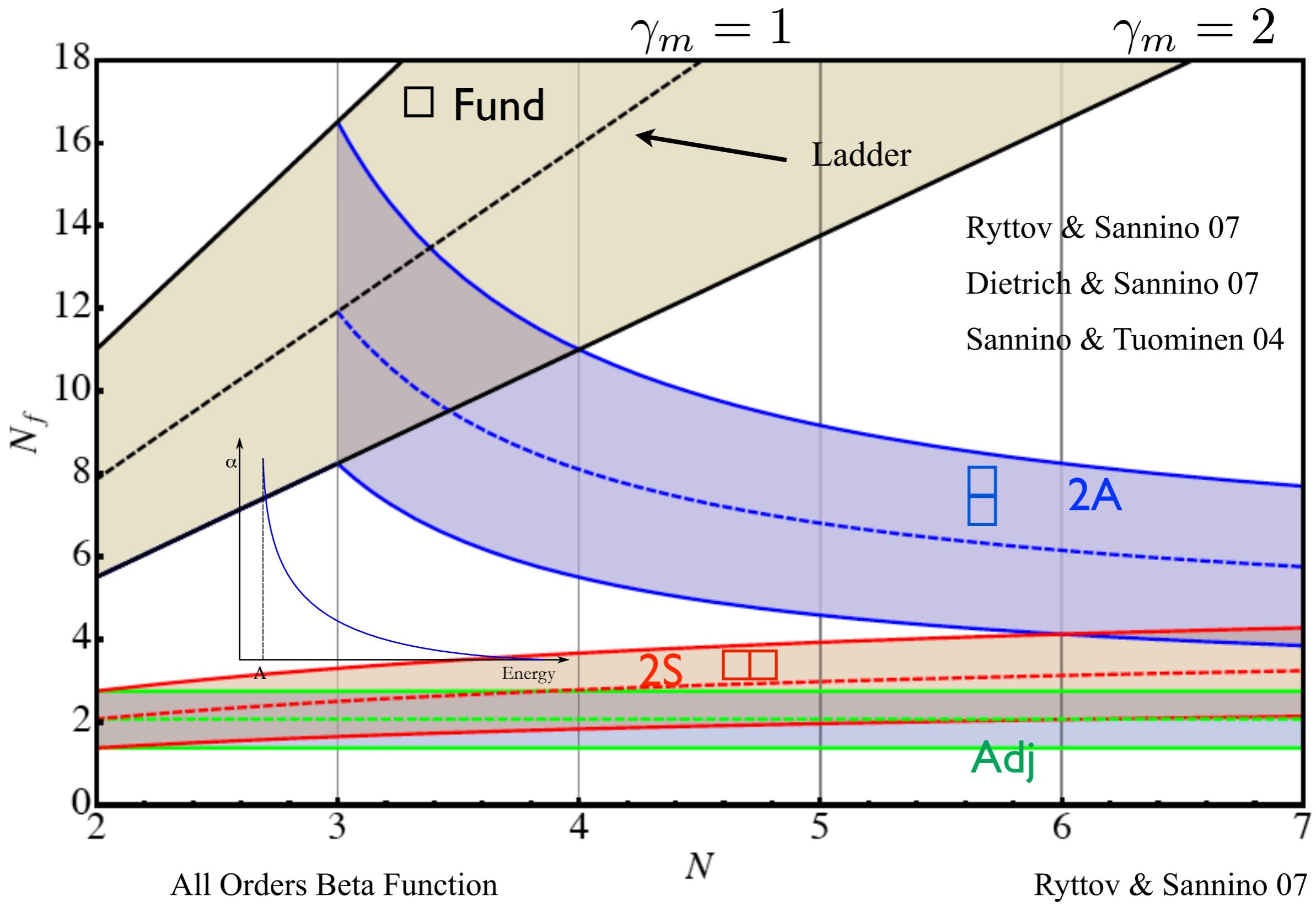


# Universal Picture

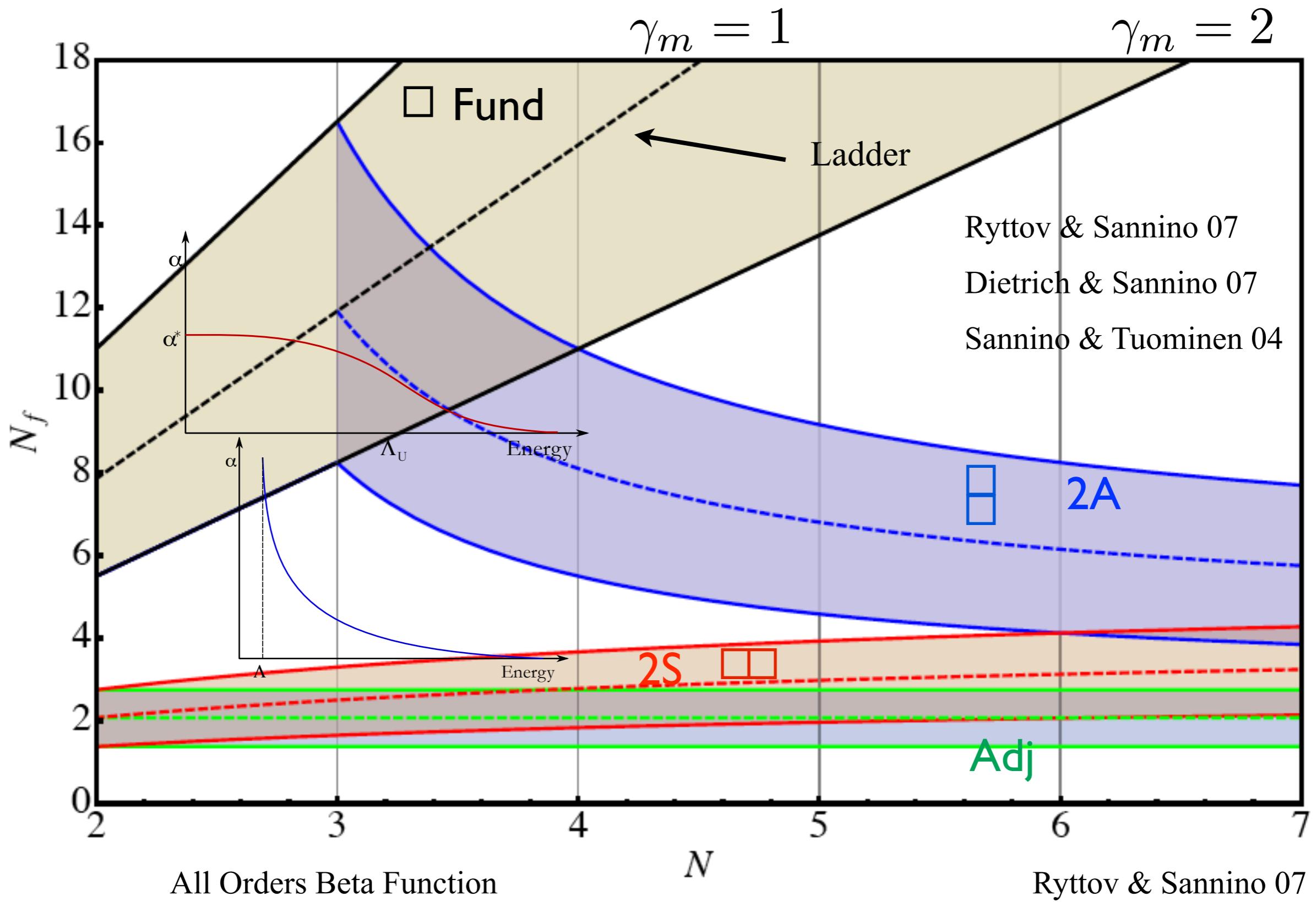
# SU(N) Phase Diagram



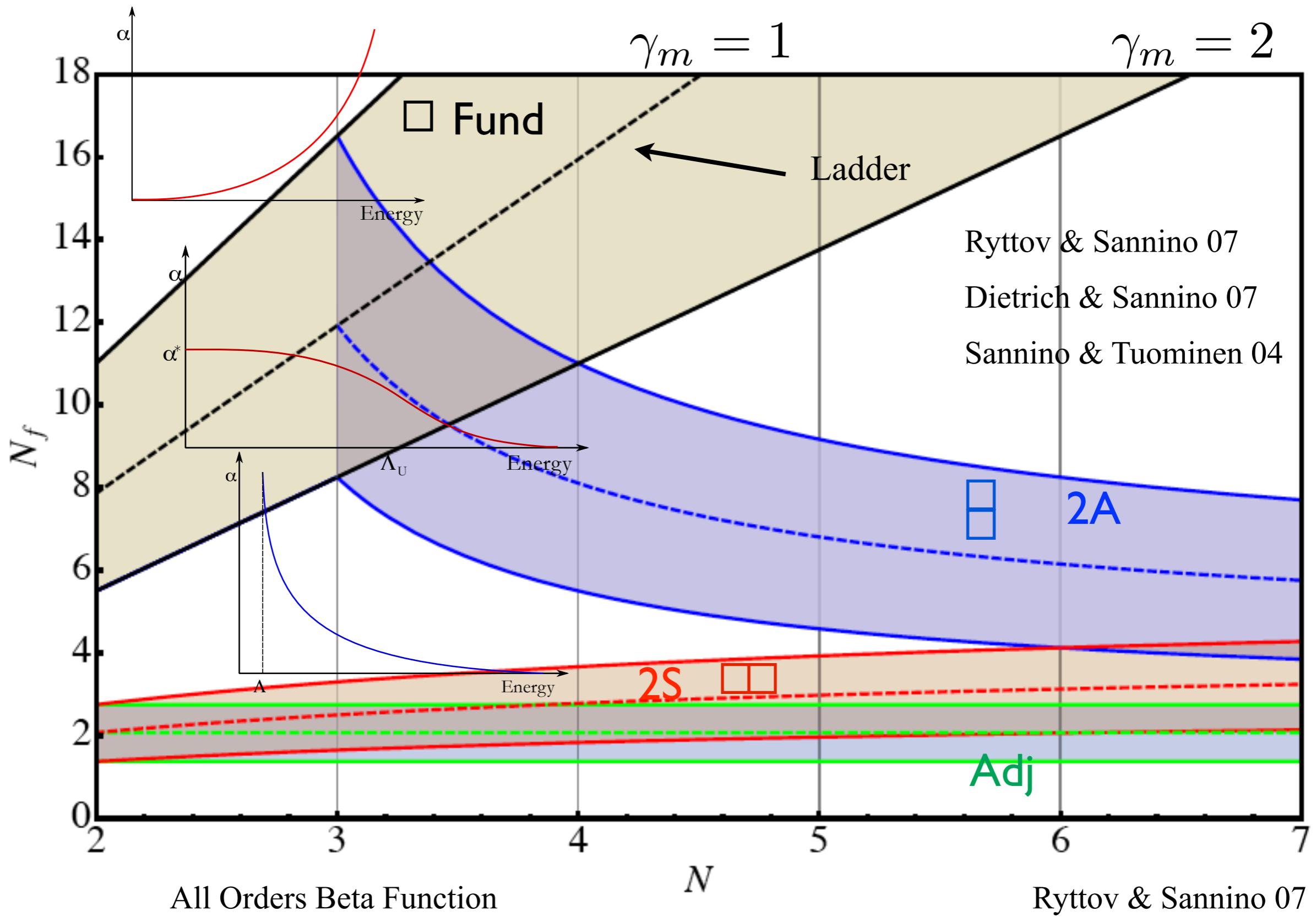
# SU(N) Phase Diagram



# SU(N) Phase Diagram



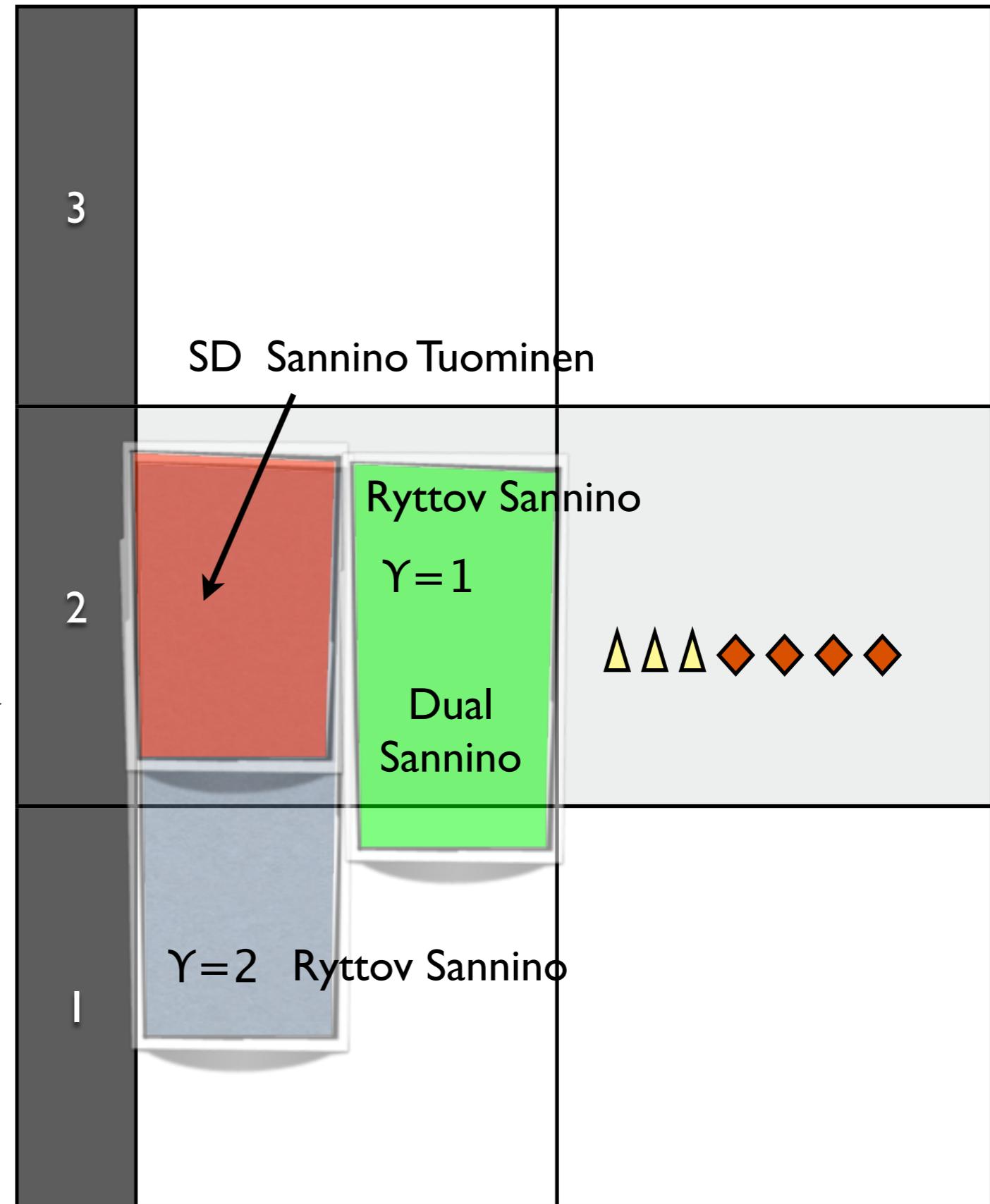
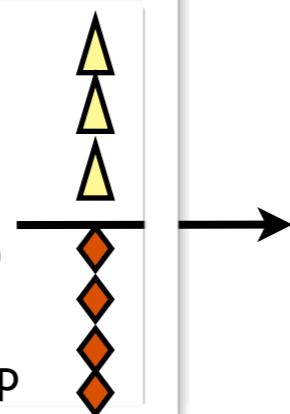
# SU(N) Phase Diagram



◆ Conformal    ★ Chiral Symmetry Breaks    △ ?

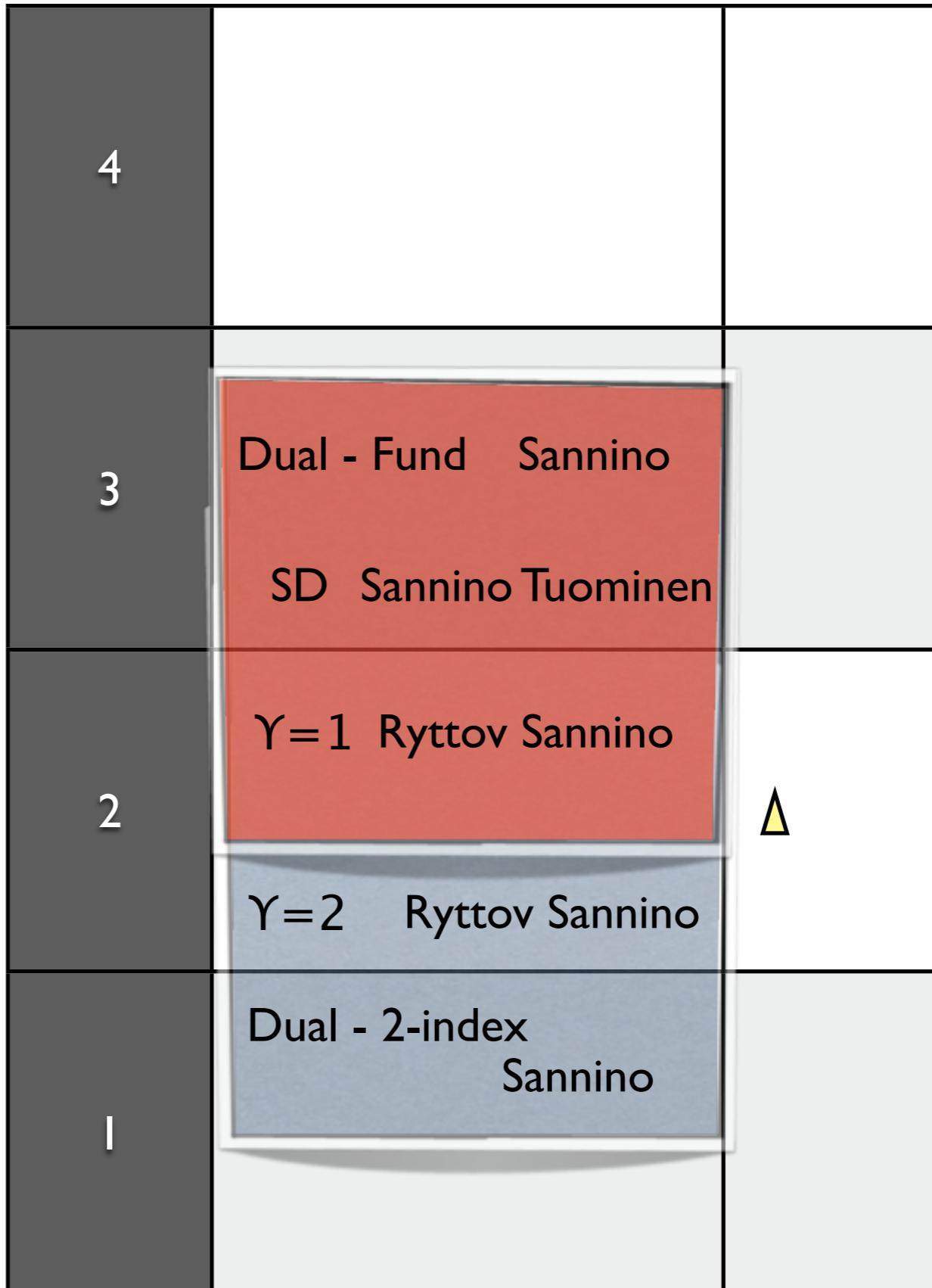
## Adjoint - SU(2) Minimal Walking Technicolor

Catterall, Sannino 07  
 Catterall, Gidet, Sannino, Schneible 08  
 Del Debbio, Patella, Pica 08  
 Del Debbio et al.. 09  
 Hietanen, Rummukainen, Tuominen 09  
 Catterall, Giedt, Sannino Schneible 09  
 Bursa, Del Debbio, Keegan, Pica, Pickup



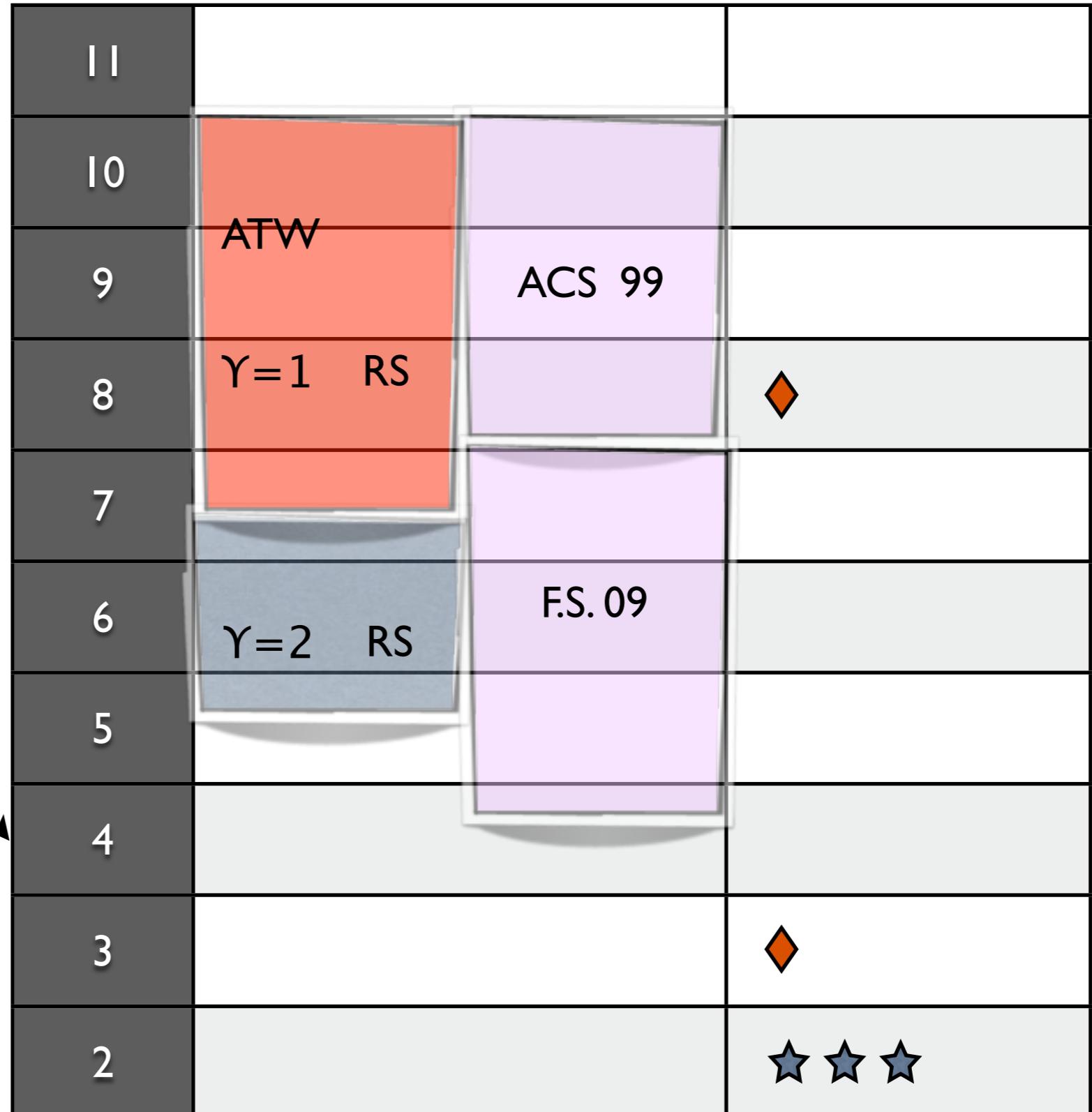
# Sym - SU(3)

◆ Conformal    ★ Chiral Symmetry Breaks    ▲ ?



# SU(2) Fundamental

◆ Conformal    ★ Chiral Symmetry Breaks    ▲ ?



RS = Ryttov-Sannino 2007

ATW = Appelquist-Terning-Wijewardhana 97

ACS = Appelquist-Cohen-Schmalz 99

# SU(3) Fundamental

◆ Conformal    ★ Chiral Symmetry Breaks    ▲ ?

Heller 98 ◆  
Hasenfratz 09 ◆  
Fodor et al. 09 ◆

Hasenfratz 09  
Fodor et al. 09  
Appelquist, Fleming, Neil 07  
Deuzeman, Lombardo, Pallante 09  
Jin, Mawhinney 09

Yamada et al. 09 ▲

Fodor et al. 09 ★

Appelquist, Fleming, Neil 07  
Deuzeman, Lombardo, Pallante 09  
Fodor et al. 09  
Jin, Mawhinney 09

Iwasaki et al. 04 ◆

Sui 01 ★  
Hasenfratz 09 ★  
Fodor et al. 09 ★

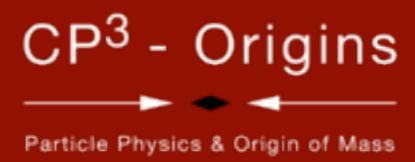
17			
16			◆ ◆ ◆
15	Υ=1	RS	
14	Dual	FS	
13	ATW	ACS	
12			★ ◆ ◆ ▲ ▲
11			
10	Υ=2	RS	▲
9	Dual	FS	★
8			★ ★ ★ ★
7			◆
6			
5			
4			★ ★ ★
3			

Dual S = Sannino 2009

RS = Ryttov-Sannino 2007

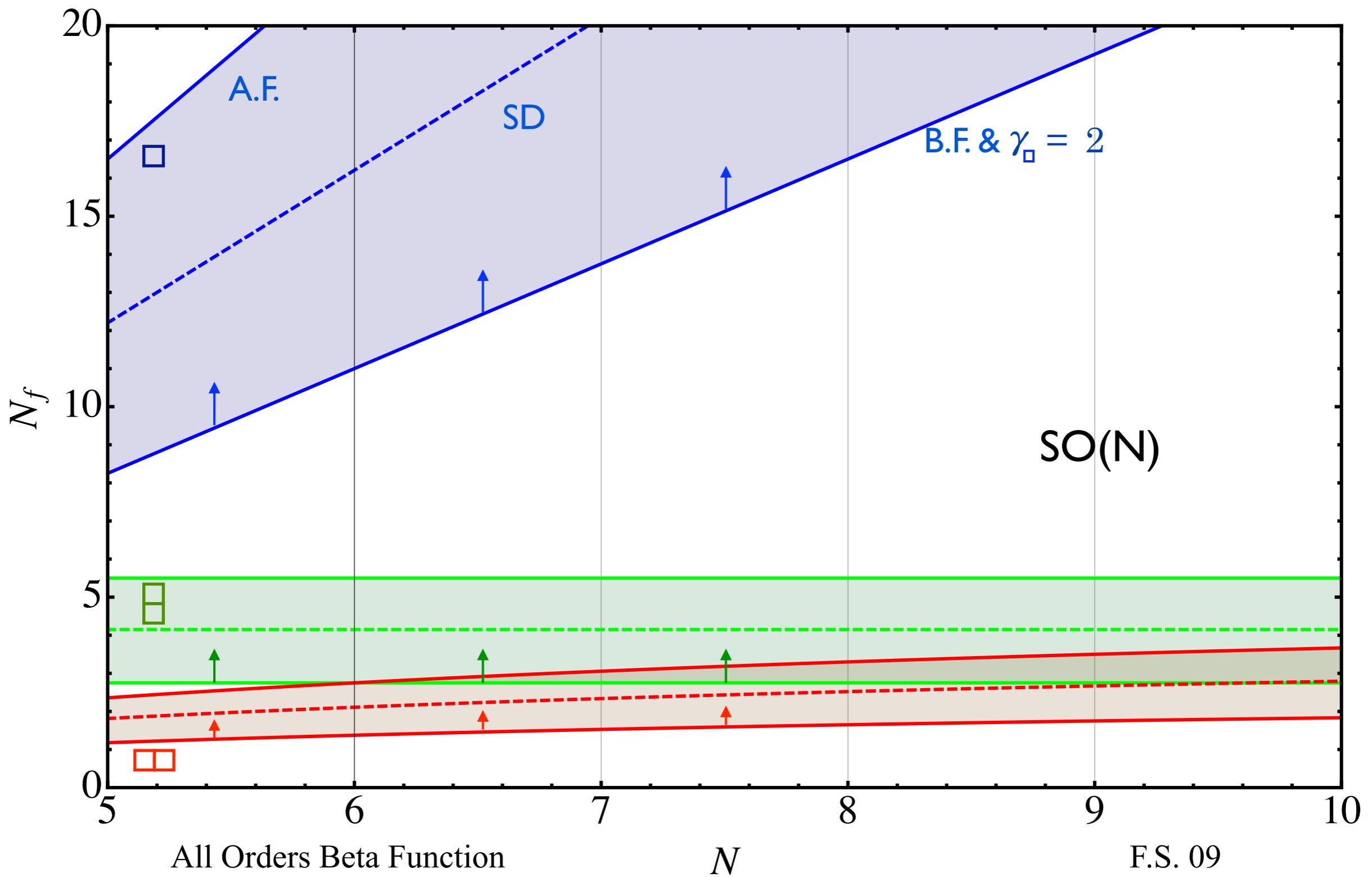
ATW = Appelquist-Terning-Wijewardhana 97

ACS = Appelquist-Cohen-Schmaltz 99

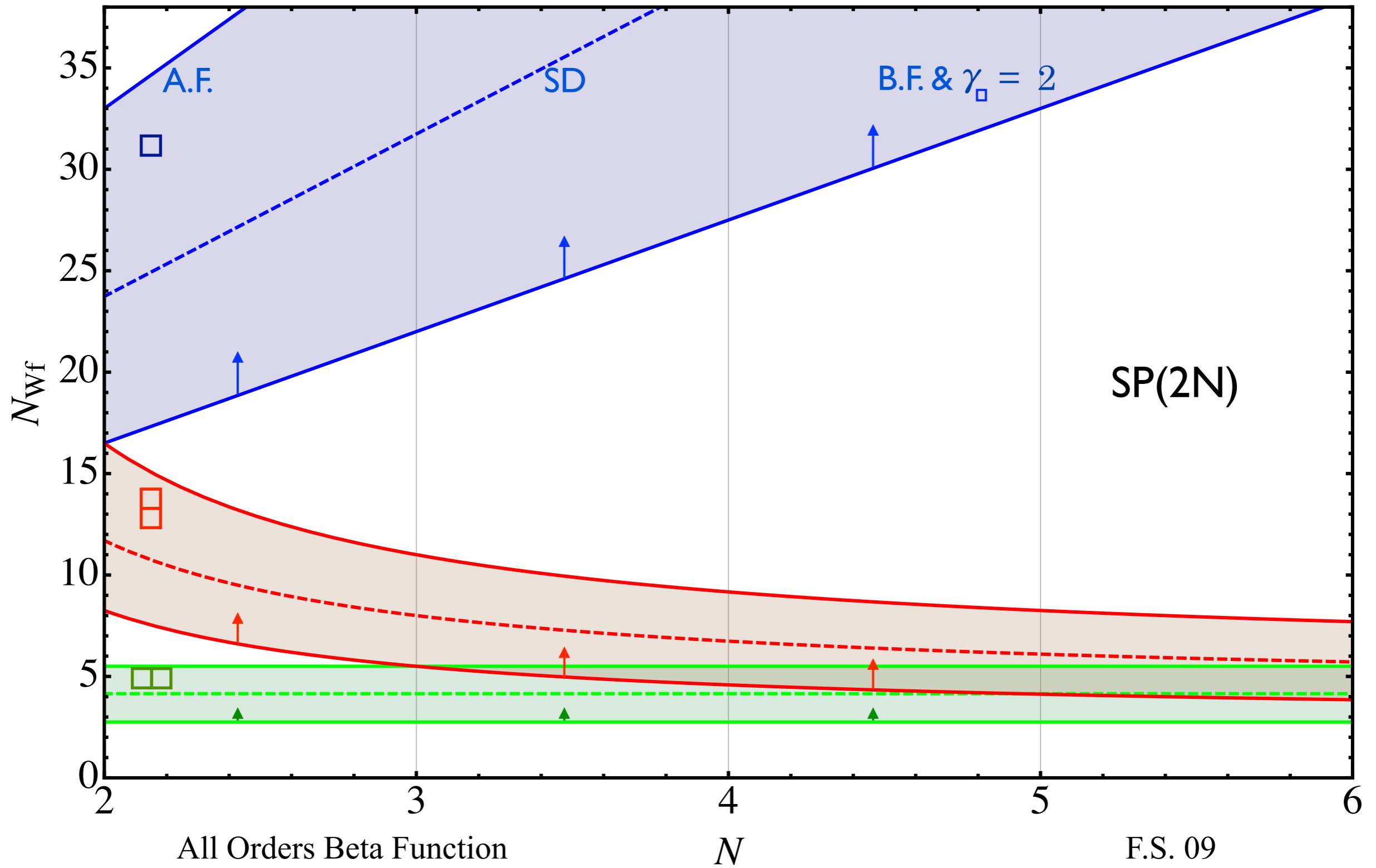


# Be imaginative!

# SO(N) Phase Diagram



# Sp(N) Phase Diagram



# Chiral Gauge Theories

# What does chiral mean?

Cannot give a mass term in the Lagrangian

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# Why study them?

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The Standard Model is chiral

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Grand Unifications

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Grand Unifications

Extended technicolor models

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Cannot give a mass term in the Lagrangian

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Grand Unifications

Extended technicolor models

First principle lattice computations are impossible

# Bars - Yankielowicz (BY)

Fields	$[SU(N)]$	$SU(N + 4 + p)$	$SU(p)$	$U_1(1)$	$U_2(1)$
$S$	□□	1	1	$N + 4$	$2p$
$\bar{F}$	□	□	1	$-(N + 2)$	$-p$
$F$	□	1	□	$N + 2$	$-(N - p)$

# Bars - Yankielowicz (BY)

Fields	$[SU(N)]$	$SU(N + 4 + p)$	$SU(p)$	$U_1(1)$	$U_2(1)$
$S$	$\square\square$	1	1	$N + 4$	$2p$
$\bar{F}$	$\bar{\square}$	$\bar{\square}$	1	$-(N + 2)$	$-p$
$F$	$\square$	1	$\square$	$N + 2$	$-(N - p)$

# Generalized Georgi-Glashow (GGG)

Fields	$[SU(N)]$	$SU(N - 4 + p)$	$SU(p)$	$U_1(1)$	$U_2(1)$
$A$	$\square\square$	1	1	$N - 4$	$2p$
$\bar{F}$	$\bar{\square}$	$\bar{\square}$	1	$-(N - 2)$	$-p$
$F$	$\square$	1	$\square$	$N - 2$	$-(N - p)$

# Chiral beta function conjecture

$$\beta_\chi(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} \sum_{i=1}^k T(r_i) p(r_i) \gamma_i(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta'_\chi}{\beta_0}\right)}$$

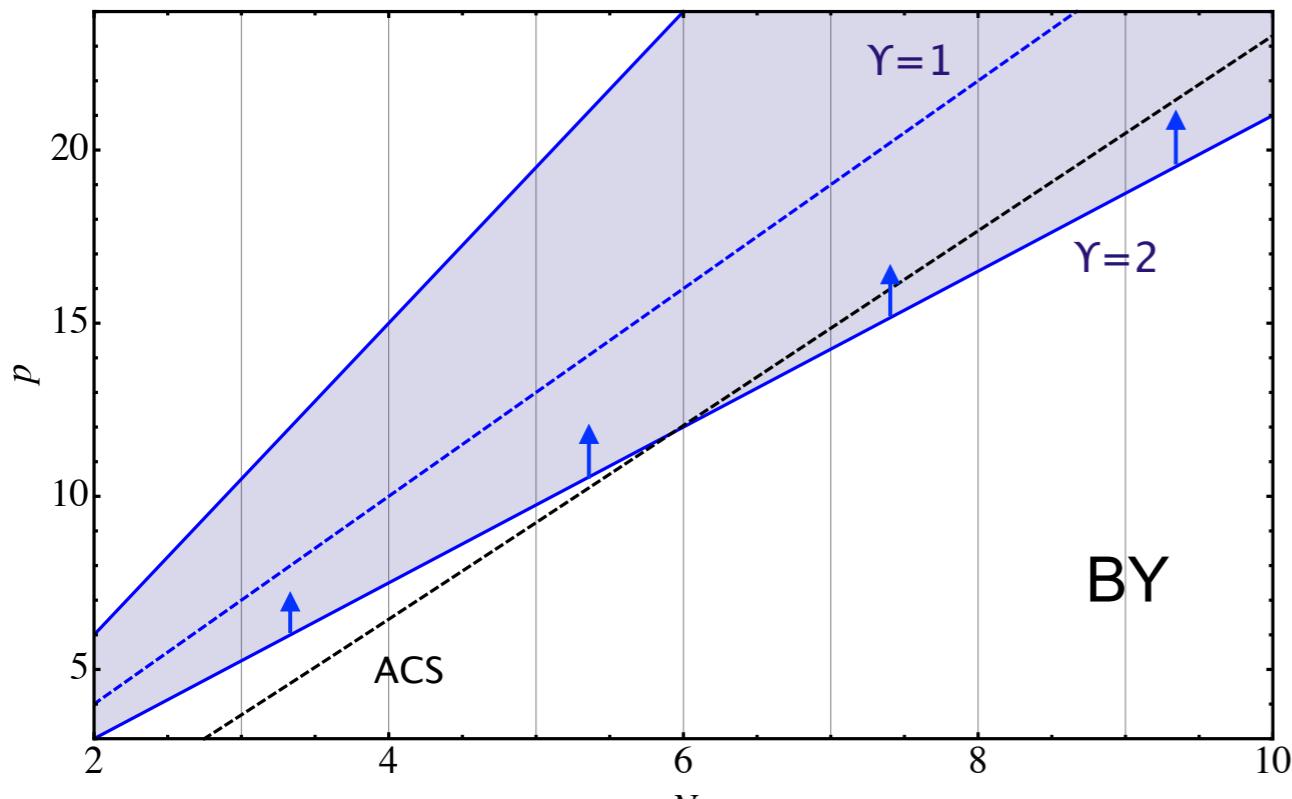
$p(r_i)$  vector-like pairs

$\gamma_i$  mass anomalous dimension

$\beta_0$  one-loop coefficient

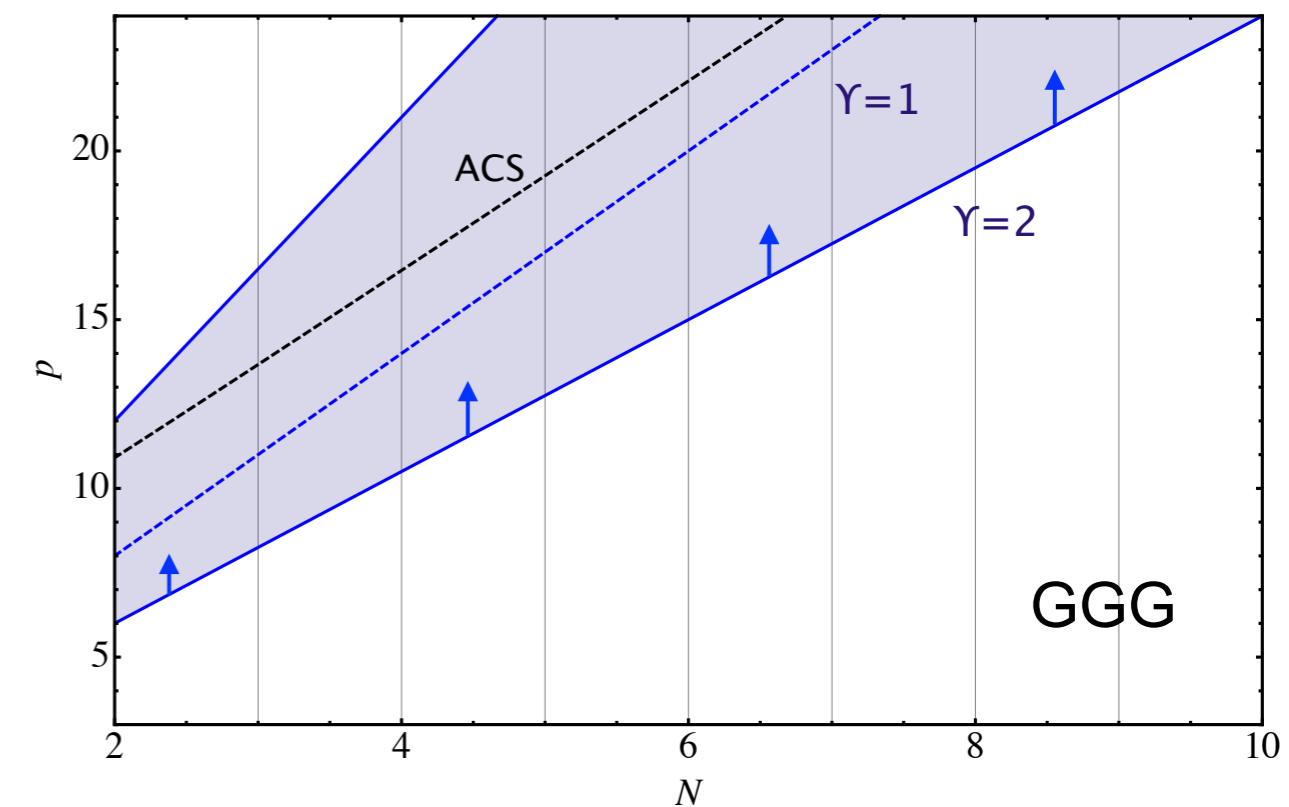
$\beta'_\chi$  Fixed to agree with the 2-loop coefficient

# Chiral Gauge Theories Phase Diagram



Chiral Beta Function

F.S. 09



Thermal: Appelquist, Cohen, Schmaltz, Shrock 99

Appelquist, Duan, F.S., 2000

Poppitz & Unsal have similar results with a different method

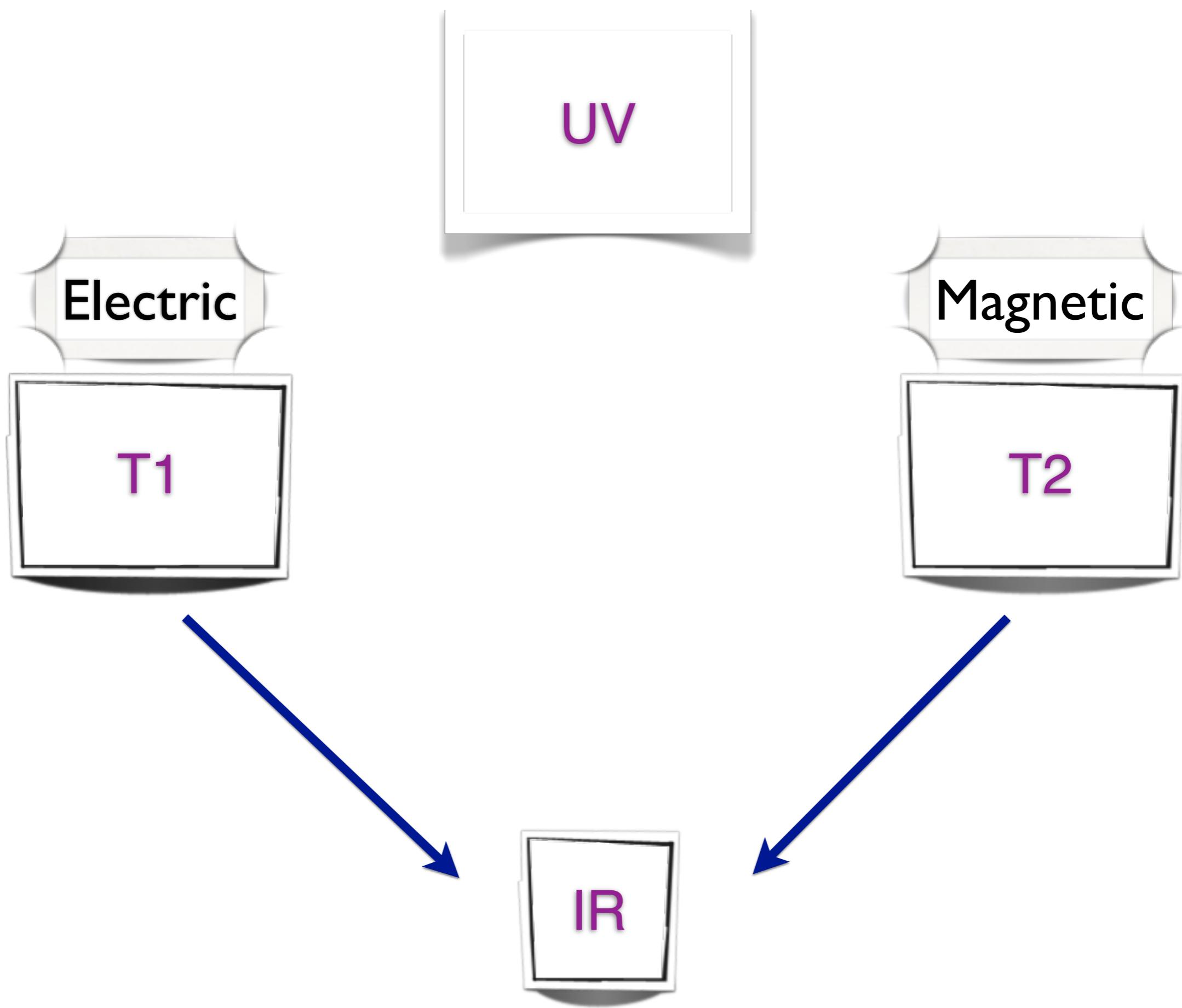
# Duals

**UV**

**T1**

**T2**





**Electric**

**Strong**

**Magnetic**

**Weak**



**Electric**

**Strong**

**Magnetic**

**Weak**



$$e g = 2 \pi n$$

# QCD Dual

F.S. 09

# QCD Dual

F.S. 09

**Within the Conformal Window**

# QCD Dual

F.S. 09

**Within the Conformal Window**

‘t Hooft Anomaly Conditions Respected

# QCD Dual

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‘t Hooft Anomaly Conditions Respected

Operator Matching

# QCD Dual

F.S. 09

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Operator Matching

Flavor Decoupling

# QCD Dual

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RS beta function

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Super QCD

Seiberg 96

# QCD Dual

F.S. 09

Within the Conformal Window

‘t Hooft Anomaly Conditions Respected

Operator Matching

Flavor Decoupling

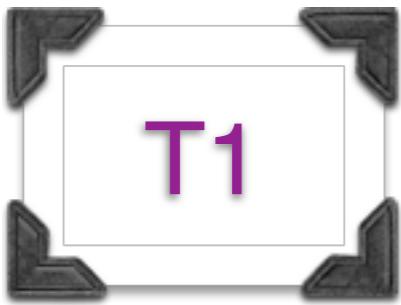
RS beta function

Super QCD

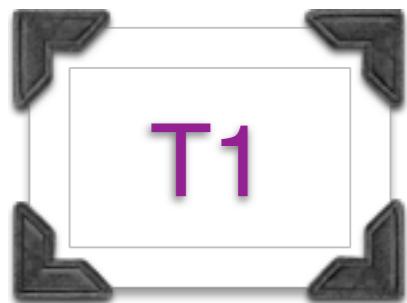
Seiberg 96

Previous attempt Terning 98

Fields	$[SU(3)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
$Q$	$\square$	$\square$	1	1
$\tilde{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1
$G_\mu$	Adj	1	1	1



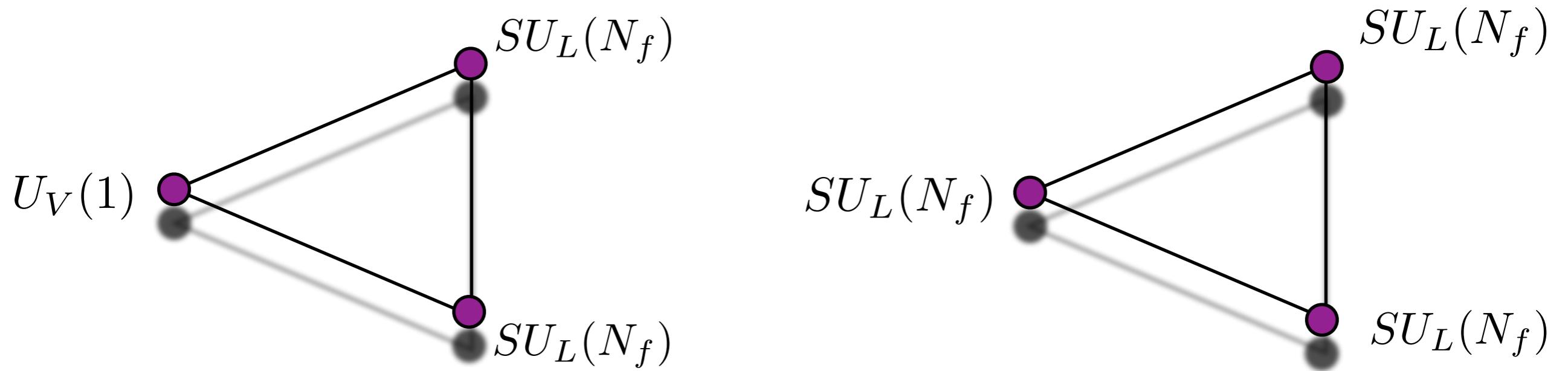
Fields	$[SU(3)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
$Q$	$\square$	$\square$	1	1
$\tilde{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1
$G_\mu$	Adj	1	1	1



T2

Fields	$[SU(X)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	# of copies
$q$	$\square$	$\square$	1	$y$	1
$\tilde{q}$	$\bar{\square}$	1	$\bar{\square}$	$-y$	1
$A$	1	$\begin{array}{ c } \hline \end{array}$	1	3	$\ell_A$
$S$	1	$\begin{array}{ c c c } \hline \end{array}$	1	3	$\ell_S$
$C$	1	$\begin{array}{ c c } \hline \end{array}$	1	3	$\ell_C$
$B_A$	1	$\begin{array}{ c } \hline \end{array}$	$\square$	3	$\ell_{B_A}$
$B_S$	1	$\begin{array}{ c c } \hline \end{array}$	$\square$	3	$\ell_{B_S}$
$D_A$	1	$\square$	$\begin{array}{ c } \hline \end{array}$	3	$\ell_{D_A}$
$D_S$	1	$\square$	$\begin{array}{ c c } \hline \end{array}$	3	$\ell_{D_S}$
$\tilde{A}$	1	1	$\begin{array}{ c } \hline \end{array}$	-3	$\ell_{\tilde{A}}$
$\tilde{S}$	1	1	$\begin{array}{ c c c } \hline \end{array}$	-3	$\ell_{\tilde{S}}$
$\tilde{C}$	1	1	$\begin{array}{ c c } \hline \end{array}$	-3	$\ell_{\tilde{C}}$

# ‘t Hooft Anomaly Matching



# Operator Matching

Magnetic

$A, S, \dots$

$M$

$\epsilon_{c_1 \dots c_X} q^{c_1} \dots q^{c_X}$

Electric

Baryons

$M \sim \bar{Q}Q$

Baryonic  
bound states

# A QCD Dual

F.S. 09

Fields	$[SU(2N_f - 5N)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	# of copies
$q$	$\square$	$\square$	1	$\frac{N(2N_f-5)}{2N_f-5N}$	1
$\tilde{q}$	$\overline{\square}$	1	$\overline{\square}$	$-\frac{N(2N_f-5)}{2N_f-5N}$	1
$A$	1	$\begin{array}{ c }\hline \square \\ \hline \end{array}$	1	3	2
$B_A$	1	$\begin{array}{ c }\hline \square \\ \hline \end{array}$	$\square$	3	-2
$D_A$	1	$\square$	$\begin{array}{ c }\hline \square \\ \hline \end{array}$	3	2
$\tilde{A}$	1	1	$\begin{array}{ c }\hline \overline{\square} \\ \hline \square \\ \hline \end{array}$	-3	2

# Dual Theory

$$L_{\text{Dual}} = L_{\text{Kin}} \left[ q, \tilde{q}, A, B_A, D_A, \tilde{A} \right] + L_M$$

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$$L_{\text{Dual}} = L_{\text{Kin}} \left[ q, \tilde{q}, A, B_A, D_A, \tilde{A} \right] + L_M$$

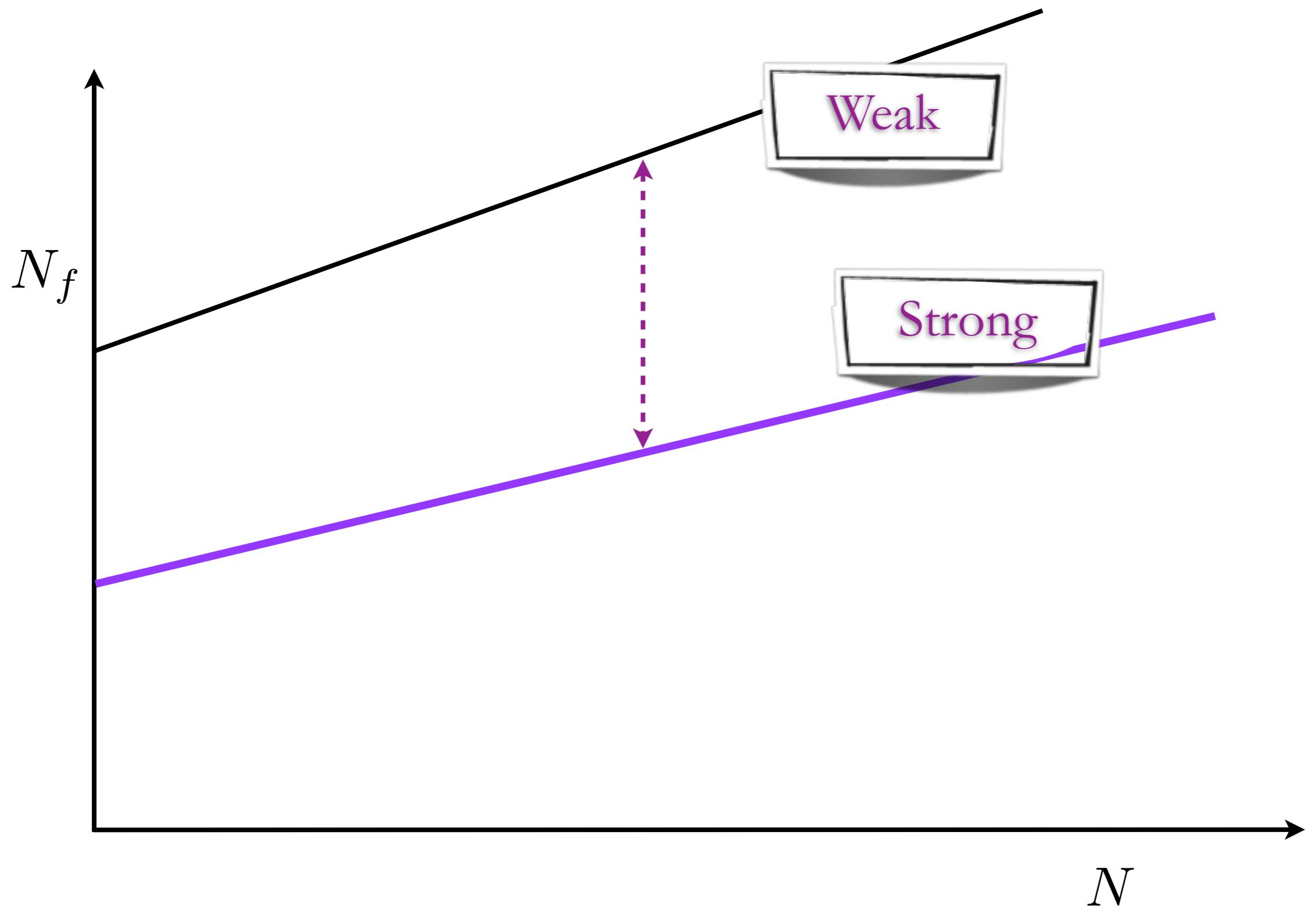
$$\begin{aligned} L_M &= Y_{q\tilde{q}} q M \tilde{q} + Y_{AB_A} A M \overline{B}_A + Y_{CB_A} C M \overline{B}_A + Y_{CB_S} C M \overline{B}_S + Y_{SB_S} S M \overline{B}_S + \\ &+ Y_{B_A D_A} B_A M \overline{D}_A + Y_{B_A D_S} B_A M \overline{D}_S + Y_{B_S D_A} B_S M \overline{D}_A + Y_{B_S D_S} B_S M \overline{D}_S + \text{h.c.} \end{aligned}$$

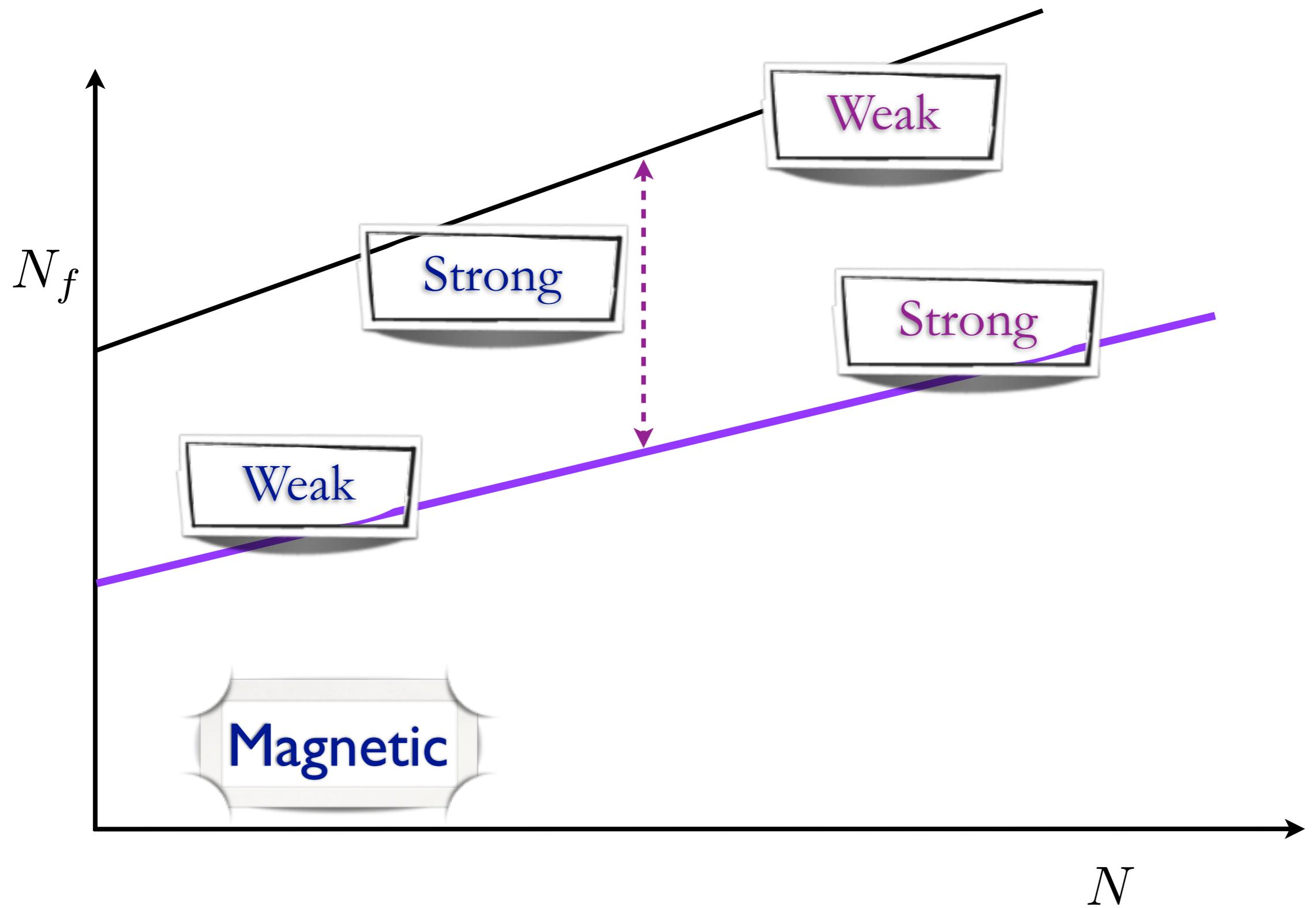
# Dual Theory

$$L_{\text{Dual}} = L_{\text{Kin}} \left[ q, \tilde{q}, A, B_A, D_A, \tilde{A} \right] + L_M$$

$$\begin{aligned} L_{\text{M}} &= Y_{q\tilde{q}} \, q \, M \, \tilde{q} + Y_{AB_A} \, A \, M \overline{B}_A + Y_{CB_A} \, C \, M \overline{B}_A + Y_{CB_S} \, C \, M \overline{B}_S + Y_{SB_S} \, S \, M \overline{B}_S + \\ &+ Y_{B_A D_A} \, B_A \, M \, \overline{D}_A + Y_{B_A D_S} \, B_A \, M \, \overline{D}_S + Y_{B_S D_A} \, B_S \, M \, \overline{D}_A + Y_{B_S D_S} \, B_S \, M \, \overline{D}_S + \text{h.c.} \end{aligned}$$

$$B[\epsilon_{c_1\dots c_X}q^{c_1}\cdots q^{c_X}] = 3~(2N_f-5) = \#\times B[\epsilon_{c_1\dots c_3}Q^{c_1}\cdots Q^{c_3}]$$





# Magnetic Asymp. Freedom

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$$\beta_0 = \frac{11}{3}(2N_f - 5N) - \frac{2}{3}N_f \geq 0$$

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$$N_f \geq \frac{11}{4}N$$

## Magnetic Asymp. Freedom

## Electric Chiral Sym. Restored

$$\beta_0 = \frac{11}{3}(2N_f - 5N) - \frac{2}{3}N_f \geq 0$$



$$N_f \geq \frac{11}{4}N$$



$$N_f^{BF}|_{\gamma=2} \geq \frac{11}{4}N$$

# Duality summary

- Duality can be tested on the Lattice
- Reduces the number of theories
- Use to compute nonperturbative quantities like the VV-AA correlator.

# Many Models

(?)MSSM

XLMS D

Technicolor

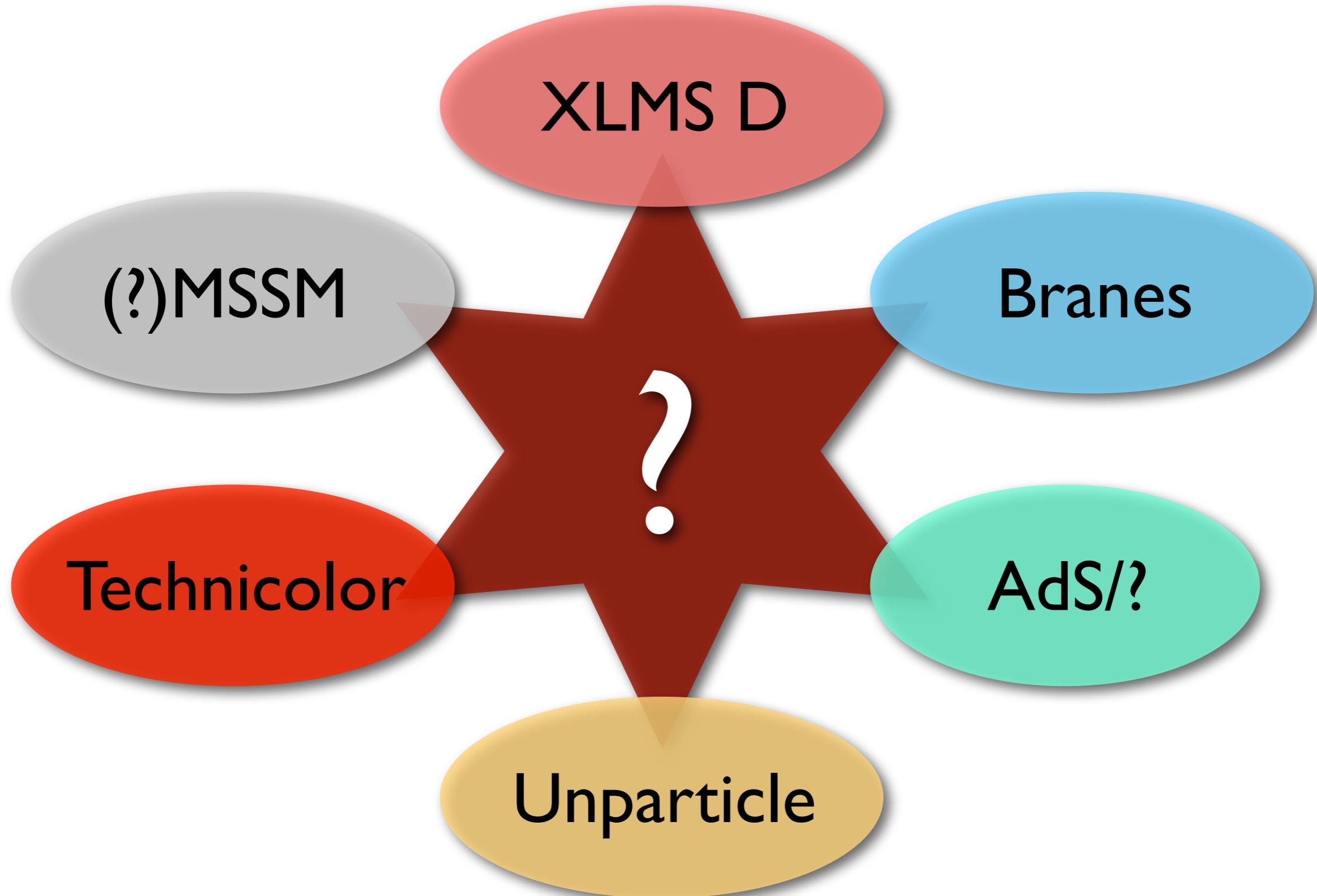
Branes

Unparticle

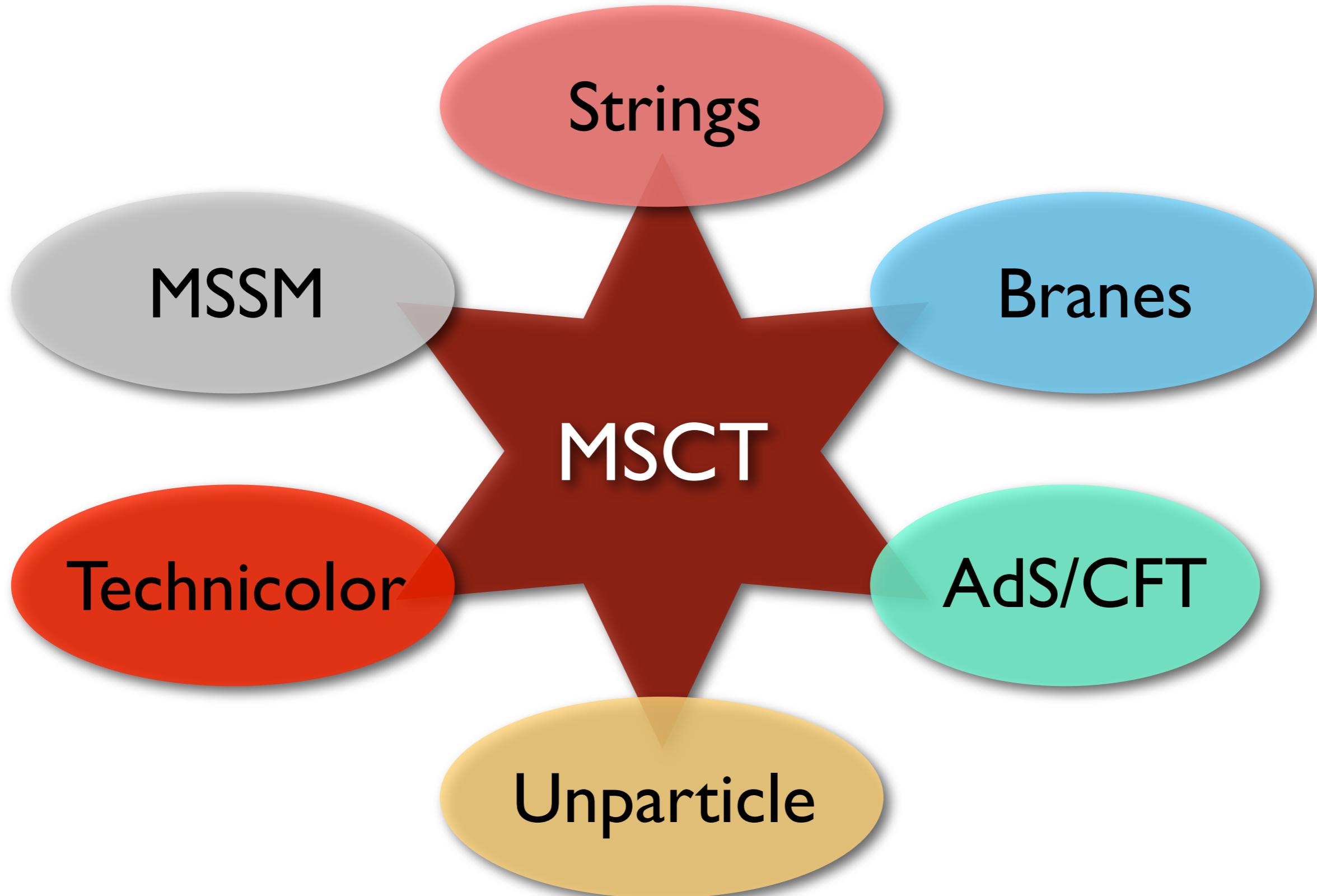
AdS/?

.....

# Unifying in Model Space



# Minimal Superconformal TC





# Conclusions

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- DEWSB can naturally occur at the LHC

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- Phase Diagram of strongly interacting theories

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# Conclusions

- DEWSB can naturally occur at the LHC
- Phase Diagram of strongly interacting theories
- Unification in theory space
- DEWSB cosmology is exciting.. another time